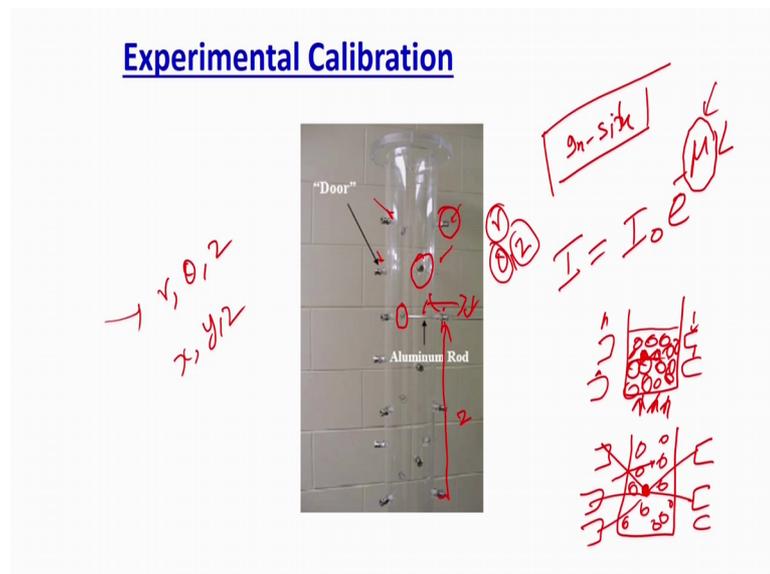


Measurement Technique in Multiphase Flows
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Lecture – 09
Radioactive Particle Tracking-II

Welcome back. What we were discussing yesterday is about the RPT technique radioactive particle tracking technique; and we have discussed the different steps. So, the first step was photon counting, second step was calibration. And calibration I said can be divided in two part one is the experimental way to do the calibration, another ways to software method or kind of numerical method to do the calibration, and then the last step was the reconstruction.

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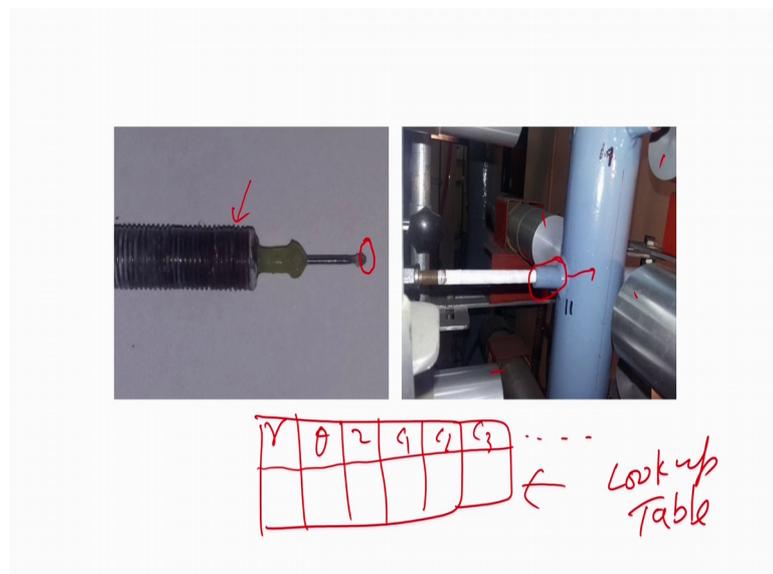
So, while discussion the experimental calibration method, what we have said that we have to actually put the particle the calibration is nothing we have to put the particle at several known location to create a lookup table in which we know that for a particular position how much count will be recorded on each detector. So, please again note it down that the calibration should be performed at in situ condition, so that the distribution of the phases would remain same.

So, in line of that, several people have done on whoever has done the RPT experiment have designed their own way to do the calibration which depends that on the size of the

system, type of the system, whether it is a gas liquid or liquid solid or it is a circulating fluidized bed, what is the size of the system, what is the temperature and pressure of the system. So, all those parameters they have taken into the count to design their own calibration device which will be able to precisely put the particles or place the particle at a particular location at in situ condition inside the director. And then we record the count on the each detector which are placed around the vessel of interest.

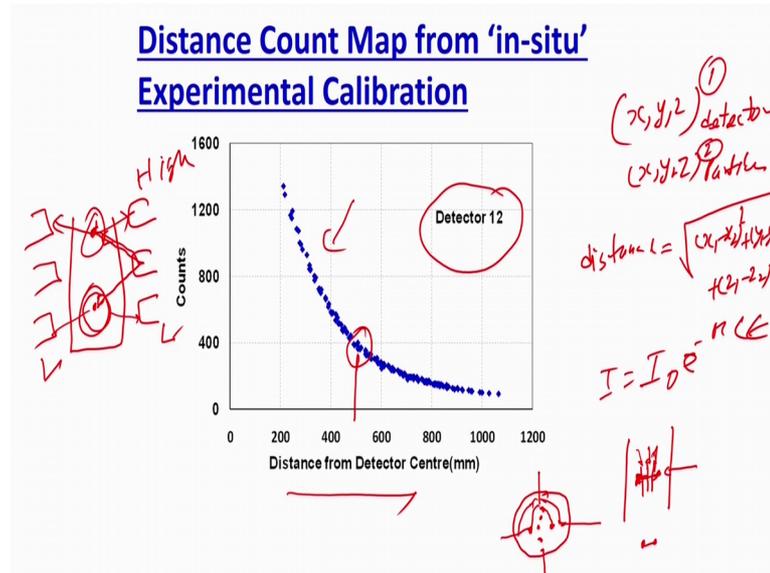
So, one of the method I said that we can use this way that we can have a port on the walls like this. And then you can insert a rod and put the particle on the top of this rod, and you can change the location of the particle. So, all though way once you fix the port, you fix your theta and z and then by changing moving the rods in and out what you can do you can change the r value by moving it in and out. So, you can very precisely the r, theta and z coordinate of the particle and you can record the count on the each detector.

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So, this is the same thing we have done for the opaque column, where we have put a particle tracer particle as it is shown here on the tip of a pin, and then we place this inside the column of interest, and we have noted down the counts recorded by these detectors. Whatever these detectors are they, which are placed in on the column of interest, you have recorded that count, so that is the way to do the experimental calibration.

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As I said that if you do that, you will get the position of the tracer particle to the count map for each detector. Now, what we can do we can also plot it in terms of the distance because I know the position of the particle, I know the coordinates of the detector, I have already fixed the detector. So, I know the coordinate of the detectors I can get the count versus distance map, so that is what we prepare for each detector. So, say I have given a number that this is for detected 12. So, if suppose in the system we are using 14 detector or 16 detectors, you have to do the same thing for all the detectors would have a distance called map. All you should have a lookup table available where you can see that for a particular position of the tracer particle how much count was recorded on the each detector.

Now, this as I said that this is experimental method is not feasible, it is very difficult to have the calibration position a calibration data point for all the possible location inside the reactor. Why, because there is some physical challenges which is kind of something which you cannot deal with because suppose if we are using these ports, this ports will take certain ideas I cannot put the port just next to it. I have to put ports somewhere here say this place or somewhere here at this place. So, what will happen I will not able to get any count within this range, because this particle is moving straight now in this case. So, it is almost impossible to put the particle at all the possible location inside the reactor.

So, what we are option we have, the option we have to do the calibration at certain point and do the interpolation in between these two points. So, suppose this is a column, I am placing the particle inside. So, I know that I have a point here, I have a point here, I can do the interpolation. And see that what will be the count in between say at this location or this location or this location, we can do that. But the problem is these multi phase flow reactors have a dynamic in nature. And the problem comes because they changes with the location and the dynamics also changes greatly sometimes with a small change in the location also.

So, once you do the interpolation, you assume that all the changes are linear; and it is just going to depend on the distance, and that is most of the time it is not true. So, the intermediate counts whatever you will say that with the interpolation it may or may not valid. It will be valid; it can be more accurate, if the distance between the two points between which you are doing interpolation is very, very low. Now, the problem remains same. Suppose, this is my cylindrical coordinate; I put a rod from this side say this is the four position I am putting the particle. If I know the particle here, if I know the particle count here, I know the particle count here, here and here, I have to do the interpolation for these points these points, these point or these points, between these points I have to do the interpolation.

So, this area is too big and they are quite far from each other, larger the column diameter CVL will be the problem. So, what is the possible thing that is why the interpolation cannot work and one has to develop some more numerical or scientific way to do the calculate the counting side, and that is the region that why we need some numerical approach to do the calibration.

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Photon Detection

- ❖ Tracer particle releases photons at solid angle of 4π .
- ❖ Only a small fraction of released photons are incident on the detector crystal.
- ❖ If the photon is capable of penetrating through the detector, it does not contribute to counts.
- ❖ Photons that are absorbed by the detector crystal are considered as a counts.

Source strength 10^5 sec^{-1}

50

5%

Tracer particle

Detector position is fixed

Not counted

3.7 x 10⁴

3.7 x 10³

1 mCi = 3.7 x 10¹⁰ disintegrations per sec

3.7 x 10¹⁰ disintegrations per sec

So, the numerical approach is actually based on the photon counting photon detection and counting system. From photon counting, we can say photon detection, transmission, and counting system. So, what it does photon counting or detection system actually we call it as a Monte-Carlo based program, but what this program do or what it does scientifically just to explain it briefly that suppose this is my tracer particle, this is my detector, this is detector and this is my tracer particle. So, for a fixed location of the detector and fixed location of the tracer particle, both are fixed. Now, I want to count that whether you find that how much count will be recorded on this particular detector for a fixed position of the tracer particle, detector position is already fixed. So, detector position is fixed.

So, what we can do, we can develop a scientific method or scientific way to calculate that how much counts will be recorded on the detector. So, we know that the tracer particle is going to emit the gamma rays on all the 4π solid angle. So, what will happen it will look like something onion, this is 2D. So, do not get confused it will be like an onion kind of a shape, which is moving out. So, this is kind of a 2D, I can show it like a circle. So, certain portion of the gamma rays because it is a solid particles, which emitting photons, it emitting the photon on a solid angle of 4π , so emitting all places like a sphere or like an onion.

Now, certain portion of that whatever has been emitted will actually incident on this detector. So, this is the portion which I am saying that which is going to be incidental on the detector, sorry this is the portion which is going to incident in or the detector. So, if I found the solid angle which tracer particle will form on the detector surface or detector crystal surface, I can find it out that only that much fraction of the overall solid angle will fall on the detector. So, if I know that suppose this fraction is 5 percent say, and I know that total particle the photon which has been emitted is say 1000, I know that how much photon is going to incident on this, it will be 1005 divided by 100. So, I will say that only out of 1000 only 50 photons will incident on top of this detector, they will only incident. So, we can find it out that how much photons will be incident. The only thing what we need is to find the solid angle.

Now, as I mentioned earlier also that all the photons which actually incident on the detector need not to be counted; only those photons which will be intervened in the detector and get adsorb within the gear detector crystal will be counted. If the photon is able suppose this is my detector, and this is my detector crystal, if a photon incident in this way say in this way let us assume a hypothetical case it is coming vertical and it just passed through it does not get adsorb. If it will not get adsorb, then it will not transfer the energy to this crystal. And in that case, the crystal will not go to the excited state, and while coming back it will not emit a photon or photoelectron.

So, in that case, there will be no signal generation. As I mentioned that how the detector works, because it goes to excited state, it emits a photo electron, it goes to the cathode it emits a electron the photo electrons converted to electron. Then it moves through a proper diode circuits to the N o 2 current a pulse of the current and that is what the pulse of the current we measure. Now, if the photon count is of photons or gamma ray have a sufficient energy to penetrate through this crystal, it is not going to adsorb. And if it is not going to absorb, it is not going to release any photon count oh photo electron, it means it will not be counted. So, it will be not counted. Only the case where it will be like this say this is my detector crystal, gamma ray which is coming in and is getting completely adsorb only those gamma rays will be counted.

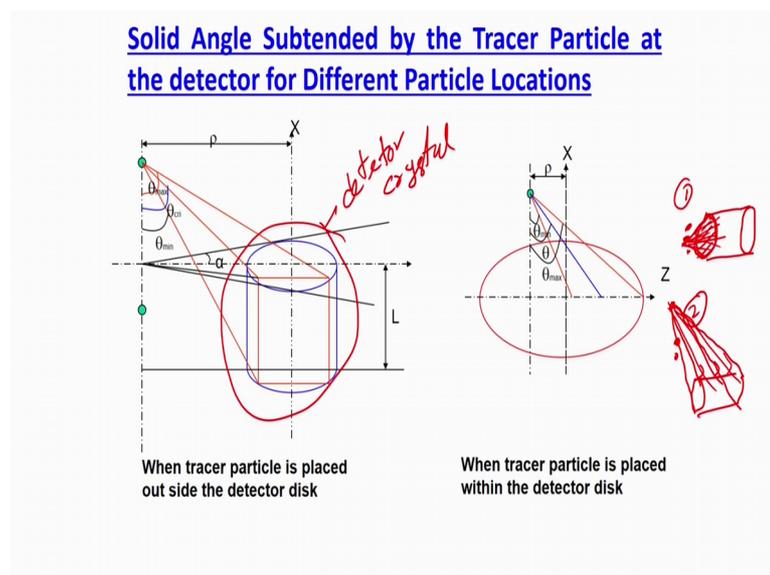
So, what I need to find, first I need to find the job is if I want to calculate I should know that what is my source strength, and why I need the source of strength. As I said that source strength says that how much photons or how much gamma ray it will emit per

disintegration. So, say if my source strength is 1 curie, 1 Ci, it is going to emit 3.7×10^{10} disintegrations per second. It means what if the 1 curie is the source strength, it is going to emit this much of the photon counts per second.

Now, if suppose I say that my source is of 100 milli curie then what will happen you have to multiply it by 10^{-3} into 100, so that way your disintegration will be modified. So, if I say that I have a 1 micro curie source, it means how much emission will take place, you have to multiply 10^{-6} to the power, it will be 3.7×10^4 disintegrations per second. So, it means you know that it should emit per second if suppose I say that I have a 1 micro curie source, it will emit 3.7×10^4 photons per second.

Now, out of that if I know this solid angle, and I say that only 5 percent is coming here or 10 percent is coming here, I can find it out how much of the photons is going to be incidental. It means suppose if I say that my solid angle is 10 percent, then 3.7×10^4 we will do into 10 divided by 100. So, what you are going to get that you are going to get 3.7×10^3 , this much of photons will actually incident on these detectors. Now, out of this 3700, how much of the photons will be counted, so that will we have to find that the trajectory of each photon. Now, the trajectory of each photon will depend that where the particle is.

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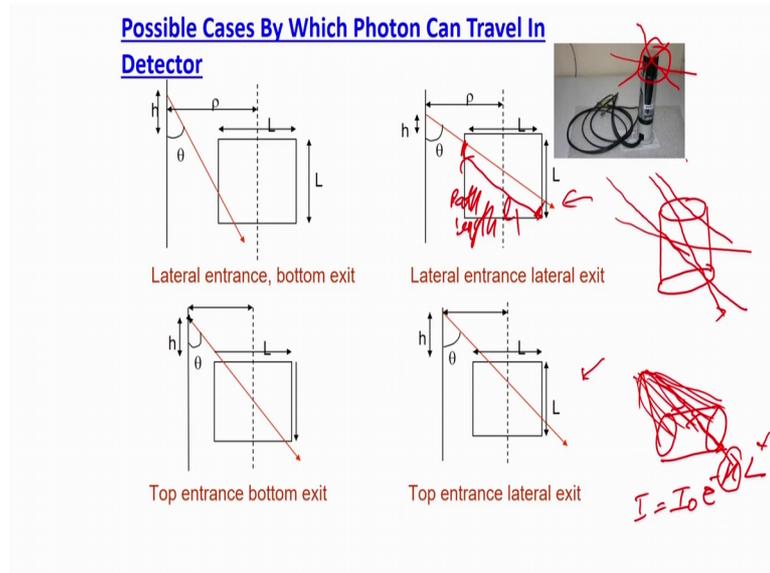


So, let us understand that where the particle can be in a global case. So, this is the detector crystal, because detectors are cylindrical. So, this is the detector crystal. And do not assume it to be kind of a vertical it is place, the detectors are place actually in this way horizontally and the particle is somewhere here, so that is what the way I tried to show here. So, what will happen if the detector if the particle is this, so what are the two cases the broad cases, the broad cases can be one that suppose this is my detector crystal. One is the particle is within this crystal the particle is within this crystal say a location here or here or here. And another one is that particle is outside of this crystal. So, suppose it is here, here or anyway. So, these are the two broad cases I can divide that how much of the photon incident it should actually get adsorb.

To calculate that I can divide my geometry in two major part, one is this part, this is the second part. Now, what it will happen say for any particular thing if this is my geometry it will form a solid angle, it will form a solid angle. So, this particle will form a solid angle on these detector crystal. I am not talking about the photomultiplier tube, I am just talking about the detector crystal - the initial part. So, it will make a solid angle and all the photons which will be incidented on this will be incidented within this solid angle only, this will be incidented within this solid angle.

So, similarly it will form for this location also it will form a solid angle this whole photon will form a solid angle, cone it will form on the detector surface. And all the photons which is being emitted will be fall within this range in this way in this direction. So, we can find it out the solid angle for a particular position of this particle and the detector. And we know that within that solid angle only all the photons will be incidented.

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So, what do we do we say that there are four possible condition through which it will be possible that condition will be it may come from the side. So, this is say for side wall. So, this is my detector crystal, again I have shown it here. So, this is my detector crystal let me zoom it out. So, say this is the way. So, what is the possibility and that its may enter from this side and may go from the bottom that is the one possibility. It may enter on the side goes from the side like second possibility. So, it can enter from this place, goes can in this place. It can enter from this place goes from this place bottom place. It can enter from the top place and goes from the sidewall; and it can enter from the top well. So, it can enter from this and it can go like this. This is the one possibility, which is the possibility number four.

And it is possible that it is entrance from this place, it is entering from this place and passing through this. So, these are the four possibility which you will find that how the photons incidented is going to penetrate or going to pass through the detector crystal. So, either it will become enter from the lateral entrance and go from the bottom exit. It can enter from the lateral inlet and goes throughout from the lateral exit. It can enter from the top and go out from the la land this bottom thing like this it is going. And it can enter from the top and go out from the lateral exit. So, this is the way these are the four condition. So, in all the four condition, I know the particular particle solid angle, so I know that we are actually it has an incidented.

So, suppose this is my detector crystal, if this the particle is here, I know that exactly how much if I know the solid angle for each tracer particle or each photon I can find it out that how much angle it is going to form and where it will be incident on the detector crystal. So, based on that, we can find it out the probability of all these four cases that whether it will enter from the lateral exit, lateral entrance, whether it will be a kind of bottom exit or lateral exit depending on the lateral in this kind of entrance. It can enter from the top it, and it can have a bottom exit and it can have a lateral exit. So, all these four cases it is fine.

So, for all these four, if I know that where the particle is incidenting, I can calculate that how much path length this length this I will say the path length, how much of the path length will be available for the photons to get adsorb. So, within that period only that much path length said this is the L_1 only the attenuation taken within the L_1 will decide whether the particle will be adsorb or not and it will be decided again the same which I equal to $I_0 e^{-\mu L}$. So, you know the attenuation coefficient you know the L . You have to see that whether this intensity is actually close to very very low or zero say, so that it means it would be adsorb inside if it is very very low.

So, we find the possibilities of that. We first find the solid angle, the fraction which will be incident in on the detector, then we found the trajectory of these paths and then we do the Monte-Carlo simulation. In Monte-Carlo simulation what we do you find the solid angle and so many number of photons within this solid angle, so many number of photons. And we see that what is the possibility of each photons which will be adsorb or it will not be adsorb. So, in that way, the photons which will be adsorb will be calculated as the count the photons which is not adsorb which will not be calculated as a count.

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Monte Carlo Simulation

Photon Count Rate:

$$C = \frac{TV\epsilon_{abs}\phi}{1 + \tau VA\epsilon_{abs}\phi}$$

Absolute Efficiency of Detector:

$$\epsilon_{abs} = \iint_{\Omega} \frac{\vec{r} \cdot \vec{n}}{r^3} \exp\left(-\sum_{j=1}^N \mu_j l_j\right) (1 - \exp(-\mu_D d)) d\Sigma$$

Handwritten notes:
 No. of gamma disintegration or strength
 Source Activity SC-46
 100-500 MCi
 889 1170
 Photopeak fraction
 PK



And to calculate that count we use this formula where T is nothing but your this you nu is nothing but the number of gamma disintegration of the source. So, this is the gamma disintegration number of gamma disintegration. So, like some of the sources has a property that per disintegration they emit two photons like scandium, say scandium 46 there is a typical example of this or disintegration its actually emit two number of photons. So, 46 actually gives the two energy one is 8891120. So, part disintegration gives two photons. So, in that case nu values will be equal to 2.

So, in that way it will be decided A is nothing but the source activity or strength please remember it is not the energy it is just activity. So, how what is the strength of the source or activity of the source like 200 micro curie and 300 micro curie which is a typical range of the radioactive source which you need or radioactive activity which you need for the RPT experiments. So, it is around 100 to say I will say maximum range will be 500 micro curie.

This phi is the photopeak fraction this is photopeak fraction, and why I am taking photopeak fraction. As I said that, we out of the complete spectrum which will receive say this is for the scandium, out of the complete structure we are going to adsorb or record the count which is fall within the photopeak rest of the things we are not at jobbing. So, you have to say that how much photopeak fraction is there. So, we know that how much fraction it will come. Then we calculate the efficiency of the detector and

which has been defined the absolute efficiency of the detector is has been defined that number of photon adsorb in the detector divided by the total number of photon incidented it on the detector.

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$$\epsilon_{abs} = \frac{\text{Number of photons adsorbed by the detector}}{\text{Number of photons incidented on the detector.}}$$

So, this epsilon absolute as I discussed has been defined number of photon absorbed by the detector divided by number of photon incidented on the detector, so that is the way epsilon absolute has been defined. We have already known that how many number of photons is going to incident depending on the activity and solid angle calculation. Then we inject that much photons on the randomly on this within that solid angle we see that what is the path line of each photons and then we try to find that whether the photon will get adsorb or not. If it is getting adsorb depending on the path length which is coming we say that the photon will be adsorb; if it is not coming we will say that photon will be not adsorb. If it is coming that photon is adsorb it means it will be counted as a count if it is coming that it is not adsorb that will not be considered as a count.

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Monte Carlo Simulation

Photon Count Rate:

$$C = \frac{TV\lambda\epsilon_{abs}\phi}{1 + \tau V\lambda\epsilon_{abs}\phi}$$

Absolute Efficiency of Detector:

$$\epsilon_{abs} = \int_{\Omega} \frac{\vec{r} \cdot \vec{n}}{r^3} \exp\left(-\sum_{j=1}^N \mu_j l_j\right) (1 - \exp(-\mu_p d)) d\Sigma$$

Handwritten notes:

- Time interval of Data Acquisition (pointing to T)
- No. of gamma disintegration per Scandium $\nu = 2$
- for ^{137}Cs , $\nu = 1$
- Dead time (pointing to τ)
- $T = 0.02 \text{ Sec}$
- 10 Hz
- $T = 0.1 \text{ Sec}$
- 150-200 (circled)

So, we find that absolute efficiency of the detector T here is the time interval of data acquisition, T is nothing but the time interval. So, suppose if I am acquiring the RPT data at a rate of 50 hertz, it means T will be 0.02 second; or if I am acquiring with the 10 hertz data, I am recording the data at 10 hertz, then T will be equal to 1 upon 10, it will be 0.1 second. So, we see that how the count is going to change on the detector it is going to be the function of what is the data acquisition time, what is the source activity, how much is the photopeak fraction in your this source particles was relevant energy for which we are performing the experiments. What is my absolute efficiency of the detector, it shows that how much kind of it will able to count out of incident it photons.

And this tau, this tau is nothing but dead time. So, any electronics we know have certain dead time; and because of that the electronics will not able to count the between the two subsequent events because the electronics have to be go to switch off mode or kind of come back to the normal state, and then again go back to your counting mode. So, because of that every electronics has certain dead time. And this tau is being represented with that dead time. So, this is the way we calculate the formula this nu is again same number of gamma disintegration per count integral integration of the photon or gamma and this source particle. So, it means what if suppose you have a scandium, this nu value will be 2. And if you have cesium nu value is equal to 1, because it just emit one energy photon. So, in that way we can calculate the count.

And the epsilon abs is being calculated based on this formula this is nothing but if you will see it is a Beer-Lambert's law which shows that how much the attenuation will take place. This is the overall length. Then suppose you are tracking say the particles for many times, every time you are calculating that what will be the attenuation coefficient what is the length of the travel if it is traversing and what is the kind of attenuation will play a role whether it will get attributed or not. So, these are the solid angle because of the solid angle so that you know that how much where the particle is falling. So, we can calculate the epsilon abs. So, with this method with this approach we can calculate that how the photon counts will change inside the column of interest.

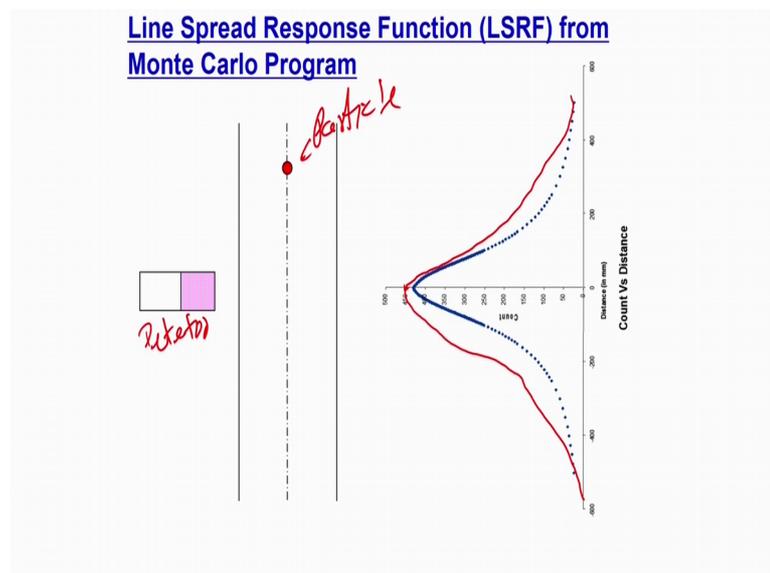
Now, there are several same problem you may feel that this is fantastic, now I do not need to do the experimental calibration, but that is not true. Because there are several parameters which you actually do not understand that how to do that and one of that carry this parameter is actually tau which changes with the change in the system also. So, we do not understand what will be the value of tau. Second thing what we have not done here and that is also critical is to find it out that; what is the fraction of the solids or of the gas or of the liquid which is being distributed inside so that because we know that the attenuation is going to be the function of that.

So, we do not know that that the fraction which will be distributed inside and I will say it in terms of the medium attenuation coefficient. So, I will say this, we do not know even this that what is the medium attenuation coefficient. So, we need to find that, we need to find that tau. So, these are some fitting parameters which we need to determine. Now, till you do not have the tau value, to do not have the mu value the mean this in situ condition attenuation coefficient, you cannot actually go ahead because you cannot calculate the counts.

So, for that we do experiments for certain locations say for 150 or 200 location depending upon the column geometry, and the locations you can do the experiments. And then we tune our Monte-Carlo program, we call it as a Monte-Carlo. Why Monte-Carlo, because we are randomly throwing the photon counts, so find that what is the solid angle and whether this will be kind of attributed just kind of attenuated or not within the bed and where it is coming in where it is going down going out. So, we do all those calculation and then we calculate the count, but there is those tuning parameter which I said need some of the experimental data.

So, it means what I can do in a system we can reduce the experimental data around for say 150 to 200 points, and then we can tune our Monte-Carlo program to for calculate the count at approximately all the location. And this is not based on the linear interpolation, this is based on that each position you are solving a equation. So, you get the data count you get the count which you can be more reliable and that is the way you get that. You do the calibration at a certain location, you do tune your Monte-Carlo program and they take each and every location you can calculate that; what is your in count which will be recorded on a detector for a fixed position of the tracer particle. So, you will get that count map and that can be done from the software thing.

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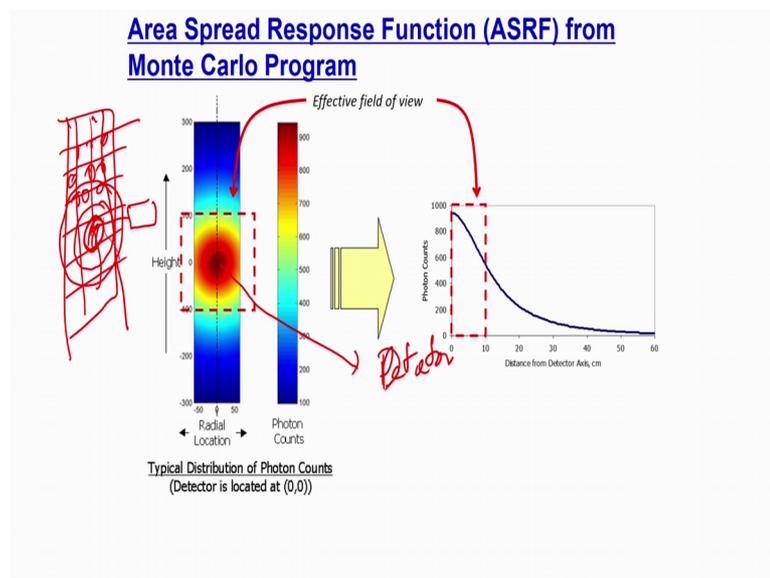


So, how it will look likes, if suppose are the same the same thing which I did in the experiments that move the particle in unidirectional, this is detector, this is particle. So, if it is moving in one direction, what will happen, it will keep on coming closer and the detector should record the maximum count, and then it will go down. So, it is once the particle is coming close to the detector, it is coming closer, it is going maximum at a location where it is right now and then it will be slowly start going down, so because now the counts are again going to be reduced.

So, I calculated this count map this is count map the way I have calculated for the experimental case I have calculated for this case that how the counts recorded on the detector will change. So, the job which you can do for the experiments by putting the

particle at a lot of location, you can put the particle at certain location, tune your program ready, and then you can generate the counts on within inside the detector at all the possible location. So, that job is done and I can say that yeah I will able to do the calibration. So, we can do the calibration in one-dimensional, we can do the calibration in all the three-dimensional, we can do the calibration in two dimensions and all the three-dimensional.

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So, in this case, what we did because we have now software we have luxury to create the counts wherever you want. We did the 2D kind of a column and there we put the particle at the center and then we start moving it kind of all the locations. So, I make a grid kind of a structure, I made this kind of a structure. And I placed the particle at all the location all the possible location, I record the count on the detector. And by using the Monte-Carlo program which has been tuned by some experimental point, and then we get this kind of a curve. Again if you see that this red colour which you are seeing inside the red colour this red colour dark red colour is actually the detector. So, this is my plane and detector is actually placed somewhere here. So, this is the say I assumed it to be center.

So, what will happen you will see the maximum count here, and you are going out, then you count away we keep on reducing in that way and that is exactly you have seen the blue colour shows the lowest count, red colour shows the maximum count. And you can see that this particle kind of how the counts on the within the column is changing with

the position, it is maximum here and keep on reducing as you go far from the detector. So, what we can do we can calculate the calibration curve or we can develop a calibration curve both through experimental as well as through numerical matters. So, now I did the experiment, I did my calibration, I have a lookup table available. The job is to compare these two to find the position because that is what we want to find.

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RPT Performance Parameters

Sensitivity: $S_r = \frac{1}{C} \frac{\partial C}{\partial r}$

- measure of ability of detector(s) to record a change in tracer particle location

Resolution: $\sigma_r = \sigma_c \frac{\partial r}{\partial C}$

- measure of uncertainty in locating tracer particle

Wish:

- highest sensitivity, i.e., high S
- highest resolution, i.e., low σ

Roy, Larachi, Al-Dahhan and Dudukovic (2002), *Appl. Radiat. & Isotopes*, **56**, 485-503.

For a combination of detectors:

$$S_r^2(x, y, z) = \sum_{i=1}^N S_i^2(x, y, z)$$

$$\frac{1}{\sigma_r^2(x, y, z)} = \sum_{i=1}^N \frac{1}{\sigma_i^2(x, y, z)}$$

Handwritten notes: 10, 11, 6.0, 25, 10, 1, 2, 1.2, 1.7, 2.4, 2.7, 2.8, 2.9, 3.0, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9, 4.0, 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 4.9, 5.0, 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, 5.9, 6.0, 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7, 6.8, 6.9, 7.0, 7.1, 7.2, 7.3, 7.4, 7.5, 7.6, 7.7, 7.8, 7.9, 8.0, 8.1, 8.2, 8.3, 8.4, 8.5, 8.6, 8.7, 8.8, 8.9, 9.0, 9.1, 9.2, 9.3, 9.4, 9.5, 9.6, 9.7, 9.8, 9.9, 10.0, 10.1, 10.2, 10.3, 10.4, 10.5, 10.6, 10.7, 10.8, 10.9, 11.0, 11.1, 11.2, 11.3, 11.4, 11.5, 11.6, 11.7, 11.8, 11.9, 12.0, 12.1, 12.2, 12.3, 12.4, 12.5, 12.6, 12.7, 12.8, 12.9, 13.0, 13.1, 13.2, 13.3, 13.4, 13.5, 13.6, 13.7, 13.8, 13.9, 14.0, 14.1, 14.2, 14.3, 14.4, 14.5, 14.6, 14.7, 14.8, 14.9, 15.0, 15.1, 15.2, 15.3, 15.4, 15.5, 15.6, 15.7, 15.8, 15.9, 16.0, 16.1, 16.2, 16.3, 16.4, 16.5, 16.6, 16.7, 16.8, 16.9, 17.0, 17.1, 17.2, 17.3, 17.4, 17.5, 17.6, 17.7, 17.8, 17.9, 18.0, 18.1, 18.2, 18.3, 18.4, 18.5, 18.6, 18.7, 18.8, 18.9, 19.0, 19.1, 19.2, 19.3, 19.4, 19.5, 19.6, 19.7, 19.8, 19.9, 20.0, 20.1, 20.2, 20.3, 20.4, 20.5, 20.6, 20.7, 20.8, 20.9, 21.0, 21.1, 21.2, 21.3, 21.4, 21.5, 21.6, 21.7, 21.8, 21.9, 22.0, 22.1, 22.2, 22.3, 22.4, 22.5, 22.6, 22.7, 22.8, 22.9, 23.0, 23.1, 23.2, 23.3, 23.4, 23.5, 23.6, 23.7, 23.8, 23.9, 24.0, 24.1, 24.2, 24.3, 24.4, 24.5, 24.6, 24.7, 24.8, 24.9, 25.0, 25.1, 25.2, 25.3, 25.4, 25.5, 25.6, 25.7, 25.8, 25.9, 26.0, 26.1, 26.2, 26.3, 26.4, 26.5, 26.6, 26.7, 26.8, 26.9, 27.0, 27.1, 27.2, 27.3, 27.4, 27.5, 27.6, 27.7, 27.8, 27.9, 28.0, 28.1, 28.2, 28.3, 28.4, 28.5, 28.6, 28.7, 28.8, 28.9, 29.0, 29.1, 29.2, 29.3, 29.4, 29.5, 29.6, 29.7, 29.8, 29.9, 30.0, 30.1, 30.2, 30.3, 30.4, 30.5, 30.6, 30.7, 30.8, 30.9, 31.0, 31.1, 31.2, 31.3, 31.4, 31.5, 31.6, 31.7, 31.8, 31.9, 32.0, 32.1, 32.2, 32.3, 32.4, 32.5, 32.6, 32.7, 32.8, 32.9, 33.0, 33.1, 33.2, 33.3, 33.4, 33.5, 33.6, 33.7, 33.8, 33.9, 34.0, 34.1, 34.2, 34.3, 34.4, 34.5, 34.6, 34.7, 34.8, 34.9, 35.0, 35.1, 35.2, 35.3, 35.4, 35.5, 35.6, 35.7, 35.8, 35.9, 36.0, 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52.7, 52.8, 52.9, 53.0, 53.1, 53.2, 53.3, 53.4, 53.5, 53.6, 53.7, 53.8, 53.9, 54.0, 54.1, 54.2, 54.3, 54.4, 54.5, 54.6, 54.7, 54.8, 54.9, 55.0, 55.1, 55.2, 55.3, 55.4, 55.5, 55.6, 55.7, 55.8, 55.9, 56.0, 56.1, 56.2, 56.3, 56.4, 56.5, 56.6, 56.7, 56.8, 56.9, 57.0, 57.1, 57.2, 57.3, 57.4, 57.5, 57.6, 57.7, 57.8, 57.9, 58.0, 58.1, 58.2, 58.3, 58.4, 58.5, 58.6, 58.7, 58.8, 58.9, 59.0, 59.1, 59.2, 59.3, 59.4, 59.5, 59.6, 59.7, 59.8, 59.9, 60.0, 60.1, 60.2, 60.3, 60.4, 60.5, 60.6, 60.7, 60.8, 60.9, 61.0, 61.1, 61.2, 61.3, 61.4, 61.5, 61.6, 61.7, 61.8, 61.9, 62.0, 62.1, 62.2, 62.3, 62.4, 62.5, 62.6, 62.7, 62.8, 62.9, 63.0, 63.1, 63.2, 63.3, 63.4, 63.5, 63.6, 63.7, 63.8, 63.9, 64.0, 64.1, 64.2, 64.3, 64.4, 64.5, 64.6, 64.7, 64.8, 64.9, 65.0, 65.1, 65.2, 65.3, 65.4, 65.5, 65.6, 65.7, 65.8, 65.9, 66.0, 66.1, 66.2, 66.3, 66.4, 66.5, 66.6, 66.7, 66.8, 66.9, 67.0, 67.1, 67.2, 67.3, 67.4, 67.5, 67.6, 67.7, 67.8, 67.9, 68.0, 68.1, 68.2, 68.3, 68.4, 68.5, 68.6, 68.7, 68.8, 68.9, 69.0, 69.1, 69.2, 69.3, 69.4, 69.5, 69.6, 69.7, 69.8, 69.9, 70.0, 70.1, 70.2, 70.3, 70.4, 70.5, 70.6, 70.7, 70.8, 70.9, 71.0, 71.1, 71.2, 71.3, 71.4, 71.5, 71.6, 71.7, 71.8, 71.9, 72.0, 72.1, 72.2, 72.3, 72.4, 72.5, 72.6, 72.7, 72.8, 72.9, 73.0, 73.1, 73.2, 73.3, 73.4, 73.5, 73.6, 73.7, 73.8, 73.9, 74.0, 74.1, 74.2, 74.3, 74.4, 74.5, 74.6, 74.7, 74.8, 74.9, 75.0, 75.1, 75.2, 75.3, 75.4, 75.5, 75.6, 75.7, 75.8, 75.9, 76.0, 76.1, 76.2, 76.3, 76.4, 76.5, 76.6, 76.7, 76.8, 76.9, 77.0, 77.1, 77.2, 77.3, 77.4, 77.5, 77.6, 77.7, 77.8, 77.9, 78.0, 78.1, 78.2, 78.3, 78.4, 78.5, 78.6, 78.7, 78.8, 78.9, 79.0, 79.1, 79.2, 79.3, 79.4, 79.5, 79.6, 79.7, 79.8, 79.9, 80.0, 80.1, 80.2, 80.3, 80.4, 80.5, 80.6, 80.7, 80.8, 80.9, 81.0, 81.1, 81.2, 81.3, 81.4, 81.5, 81.6, 81.7, 81.8, 81.9, 82.0, 82.1, 82.2, 82.3, 82.4, 82.5, 82.6, 82.7, 82.8, 82.9, 83.0, 83.1, 83.2, 83.3, 83.4, 83.5, 83.6, 83.7, 83.8, 83.9, 84.0, 84.1, 84.2, 84.3, 84.4, 84.5, 84.6, 84.7, 84.8, 84.9, 85.0, 85.1, 85.2, 85.3, 85.4, 85.5, 85.6, 85.7, 85.8, 85.9, 86.0, 86.1, 86.2, 86.3, 86.4, 86.5, 86.6, 86.7, 86.8, 86.9, 87.0, 87.1, 87.2, 87.3, 87.4, 87.5, 87.6, 87.7, 87.8, 87.9, 88.0, 88.1, 88.2, 88.3, 88.4, 88.5, 88.6, 88.7, 88.8, 88.9, 89.0, 89.1, 89.2, 89.3, 89.4, 89.5, 89.6, 89.7, 89.8, 89.9, 90.0, 90.1, 90.2, 90.3, 90.4, 90.5, 90.6, 90.7, 90.8, 90.9, 91.0, 91.1, 91.2, 91.3, 91.4, 91.5, 91.6, 91.7, 91.8, 91.9, 92.0, 92.1, 92.2, 92.3, 92.4, 92.5, 92.6, 92.7, 92.8, 92.9, 93.0, 93.1, 93.2, 93.3, 93.4, 93.5, 93.6, 93.7, 93.8, 93.9, 94.0, 94.1, 94.2, 94.3, 94.4, 94.5, 94.6, 94.7, 94.8, 94.9, 95.0, 95.1, 95.2, 95.3, 95.4, 95.5, 95.6, 95.7, 95.8, 95.9, 96.0, 96.1, 96.2, 96.3, 96.4, 96.5, 96.6, 96.7, 96.8, 96.9, 97.0, 97.1, 97.2, 97.3, 97.4, 97.5, 97.6, 97.7, 97.8, 97.9, 98.0, 98.1, 98.2, 98.3, 98.4, 98.5, 98.6, 98.7, 98.8, 98.9, 99.0, 99.1, 99.2, 99.3, 99.4, 99.5, 99.6, 99.7, 99.8, 99.9, 100.0

Now, before doing that we do certain more analysis which we call as a resolution and sensitivity analysis. Now, these are very critical because you are using a radioactive particle which is emitting the gamma rays. And we have our objective is to calculate or reconstruct the position of the tracer particle. Now, to do that we always design for any measurement, we design the resolution and sensitivity of the setup. So, here also we need to define that what is the resolution and sensitivity of the setup. And what you can reasonably achieve which you can what you can achieve what you cannot achieve. If you know that you can also make your system design or the calibration port design accordingly, so that you can accommodate more of the points.

So, how the sensitivity is defined, the sensitivity is defined that measure of ability of the detector to record in the tracer particle for the change in the tracer particle position. It means what if suppose if particle is here say suppose this is one location of the particle at time t equal to 0, at time t equal to 1, it moves from this place to this place, say this is now my another particle. So, sensitivity says that if the particle moves from this place to

this place, whether I will see any change in the count on the detector or not. Because what we are going to do we are going to reconstruct based on that if you have recorded any change in the detector count. So, if suppose here the detector count is 10, here the detector count is coming 11, obviously, it is very difficult to find that whether there is any movement or not because gamma radiation itself is a statistical quantity.

So, what you want, you want the difference like this that this is the particle one, this is the particle say it now moves here, this is particle two. Here it was 10; here it was 25. So, there is a significant difference in the count you have achieved by moving the particle from one location to another location and that is called the sensitivity of the measurement. So, it is defined as the sensitivity that the change in the count is recorded with the change in the position of the tracer particle. So, higher the sensitivity what will happen even the smaller movement of the tracer particle, you will see a significant change in the count on the detector, and you will be able to recognize it as a second position. So, higher the sensitivity is better for the RPT experiments.

Similarly, resolution is defined, and resolution says that its uncertainty between the locating the two subsequent position of the tracer particle. It is inherently opposite way a kind of we are defining, it exactly in the same way, but in the different method or different ways which is an instance. So, we will see that the sensitivity will be reciprocal of the resolution. So, what we are calling that if these two position of the tracer particle, if the particle moves in the moving continuously, so what is the uncertainty in locating the tracer particle. So, if suppose this is the particle one position, this is a particle say two second position, what will be uncertainty if the particle moves from this place to this place, whether I will be able to resolve it or not, what is the position in certainty.

So, if suppose position in certainty suppose the particle moves 1 mm in the two subsequent times t_1 it was here t_2 it was here and the difference is 1 mm say this was say 1 centimetre r equal to 1, and this is r equal to 2 say let us assume r equal to 2. So, the particle has actually move 1 mm. Now, if my position sensitivity or position uncertainty in the locating the tracer particle is 2 mm or 3 mm, obviously, I am not going to locate this particle, I will not be able to locate the particle because the minimum resolution I have is 2 mm. It means I will be able to sense the particle position change only if it moves more than 2 mm. If the uncertainty of locating the particle is 2 mm, I will really be able to say the part change in the particle position with a confidence only if it is

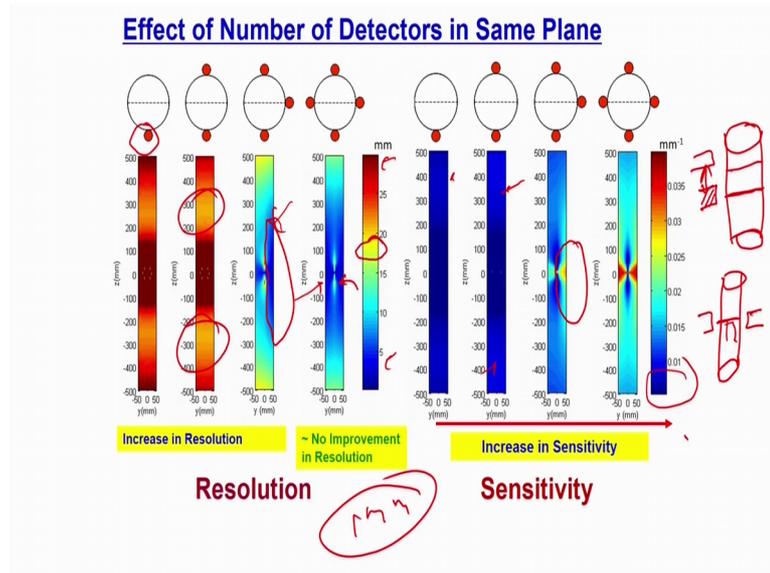
more than 2 mm. So, within 2 mm if it is moving somewhere, I do not know whether the particle is moving or not, so that is the way resolution is being defined.

Like you can other way, so you can that is the uncertainty of the particle locating the tracer particle. You can also define it as the minimum distance between the two subsequent position of the particle which can be recognized as a two different position. So, it means the particle is moving here and here. So, it is moving here if I say it is passing through all this position, but if suppose I record the data here, and I record the data here, my sensitivity of the resolution or measurement is not correct, it is pulling it back here.

So, what will happen you will see error in your results and that is why it is important that before going to the experiments, real case experiments, one should calculate the resolution and sensitivity and that you can do both experimentally as well as by using the Monte-Carlo method or kind of numerical method. Because what you need, you just need that how the Δr is changing with the Δc , where Δr is nothing but the position of the tracer particle, Δc is nothing but the count recorded on the detector. So, you can calculate this for individual detectors.

Then for multiple detectors sensitivity can be calculated you can just add the sensitivity of all the particles, all the detectors, you will get that what will be the sensitivity overall sensitivity at a particular location. Similarly, you can calculate the resolution and that will be one upon sigma square for all the detectors. So, each detector will have their own resolution. Now, you can do the similar exercise and then you can find it out that what will be the overall resolution.

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So, we can calculate it here in this way. And what we did here, we first put one detector at a plane. So, this is suppose my column. And at one plane, at any one plane, I just put only one detector, so that one red mark is showing that only one detector is there in this plane. So, I am talking about this plane and only one detector is being placed. How your resolution sensitivity will be changing. So, if you see that in this case the resolution is very very poor, because this value is very high; if this value is very high, it means the position uncertainty is very high 25 mm like in this case. So, it is very poor. We want as low as possible position in certainty.

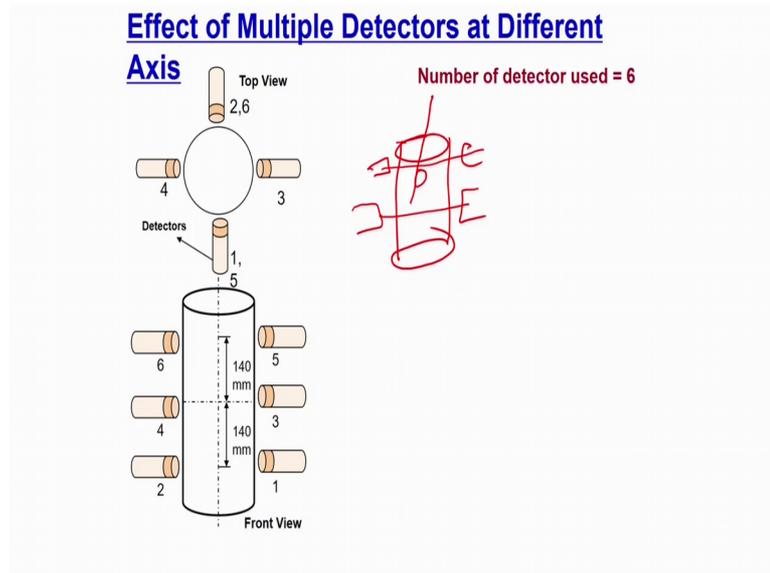
Now if you see the sensitivity again this whole colour is blue; it means the system in value is very very low, it means you will not change see much change in the position and count of the change in the particle position, so one detector. Now, what I did to increase that, because I want better resolution better sensitivity; I put the second detector on the same axis same axis. So, suppose this was the case. So, earlier I put just one detector now input the second detector. Now, what will happen, the second detector with the second detector we have seen that there is certain improvement in the sensitivity and you see that it is converting from dark red or red to no towards the yellowies, if you see this way yellowies colour. So, your sensitivity our resolution is actually improving. Now, it is from 25 mm or more than 25 mm was around 30 mm, it is coming somewhere in the range of 17 to 18 or 20 mm. So, it has improved.

Similarly, we have seen the sensitivity is also improved and this is the dark blue colour is actually now shifting towards the light blue. So, both with the second detector placed on the same axis both resolution and sensitivity has improved. Then we place the third detector. Now, we place the third detector here all these detectors are placed 90 degree apart. So, place the third detector. And we see that with placing the third detector, we have observed the drastical change in the resolution and the sensitivity values. So, if you see this resolution you can see that now the resolution at this location is at the range of less than 5 mm, say 2 mm or less than that.

So, now this has improved my resolution, it means I can the position uncertainty comes in the range of 1 to 2 mm. So, if the particle moves anything more than 1 or 2 mm, I will able to reconstruct it. So, we can do that similarly we see the sensitivity also that it has improved drastically from 0.001, it is now going the range of 0.035 or so. And once we put the four detector, we again observed that there is not much increase in the sensitivity or resolution only you get a symmetric profile. So, you are getting here half, you are getting here full that is it that is the only change you see. So, it means we can say that for this code geometry, the geometry is completely cylindrical, and there is no internals inside we are not assume any internal the geometry is very symmetric and cylindrical in nature you can say that three detectors of plane is a good option.

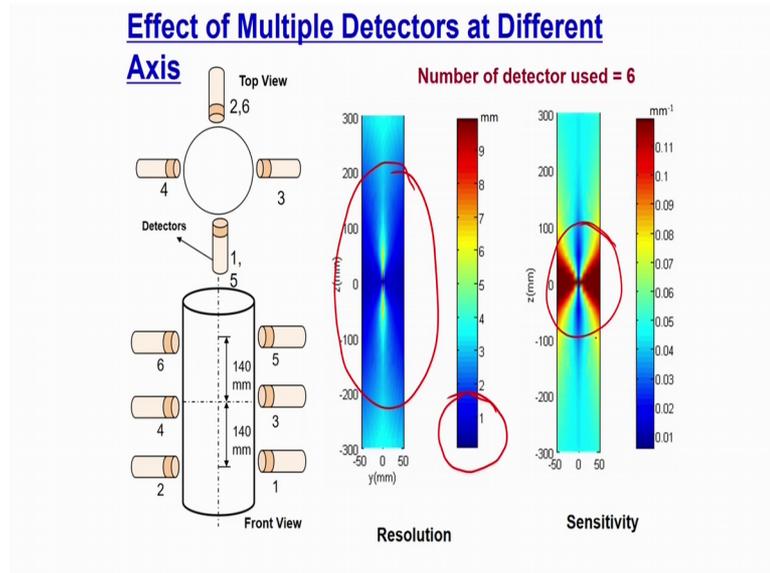
So, we can fix the location of the particles, we can also change. So, we can do the similar studies for the multiple planes. And I can see that how far I should put the second detector, so that the resolution and sensitivity between these two detectors would also be in the same range. It means suppose if one plane I am getting the minimum maximum resolution is 1 mm; I would like to maintain this one mm in throughout the column. So, it means now I have to fix that what should be the distance between the two axial detector plane.

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So, to do that we again did the experiments like this, we put the six detectors. And you close the two detectors on one plane and they staggered. Staggered means suppose this is the column. So, two detectors you display this place here in this angle. Two detectors are placed in this place it means this angle then again two detectors are placed in this place in this angle. So, they staggered. So, two on say 0 and 180, two on the other plane is 180 and 270, then again 0 and 180, then 90 and 270. So, in that way a staggering the detector, so that we can use it. And we are placing only two detectors because we came to know that the four detector actually just read the symmetry. If I place the dedication such a way that the very close then I do not need to put the third detector with the staggering I can do the job.

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So, that is why it is staggered and we found that the overall we get for the overall resolution of the detector for this column which we have done this thing is has been tremendously increased. And most of the places if you will see most of the places if you will see the solution is in the range of 1 mm, so that is the way we have improved and similarly the sensitivity has also improved drastically and other than the center plane most of the places the sensitivity is very high. Here you are seeing the resolution is very low at the central position somewhere because the column dimensions we have taken really big.

So, we have to take actually, but larger sources strength, so that I can get more counts. So, in that way clear to the RPT experiment you can design the RPT experiment by using this. You can say that how for your column geometry for your system, how many detectors you should place on a particular plane, what should be the distance between the two detector, whether it should be staggered or not and how to place it completely. And if you do that what will be your resolution. And whether that resolution is acceptable to you if not then you have to change the some of the parameter and that parameter can be source strength like the count depends on whatever the parameter the counts depend you have to change that.

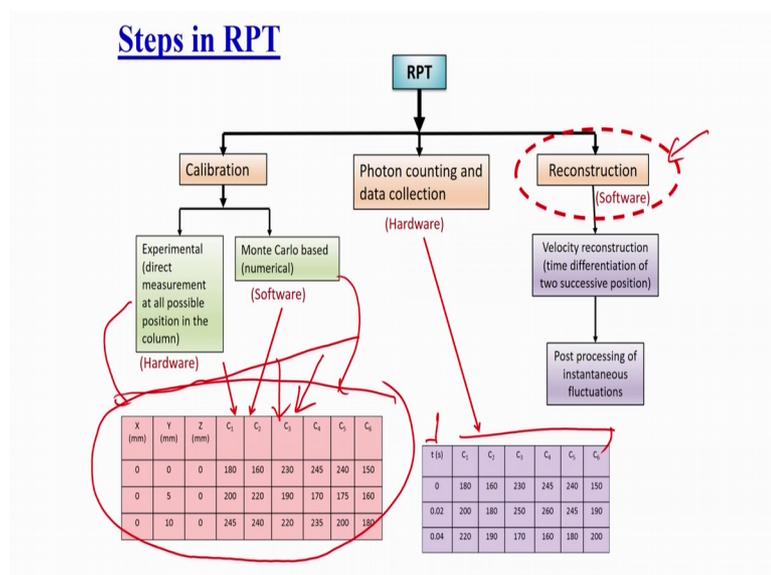
So, you have to change is that the data acquisition time, you have to change that the source is strength, you have to change either the source type of the source. So, instead of

one disintegration it may be two disintegration or for the peak fraction where it is higher in one peak you can take two peak source and all. All these things you need to change and then you have to find that; what is the desirable resolution you want to get a desirable accuracy in your system.

So, this is called prior design of this. And we can also do that by using experimental data just what you need is the count at all the location. Again calculating this experimental data it is very tedious, very very to consuming time consuming. So, generally what we do we do at certain location, we tune our Monte-Carlo program and then we run the resolution and sensitivity and find it out whether it is coming within the acceptable limit.

So, this job is done my detector positioning is done, my experiment is done I have prepared my lookup table. So, first is to prepare these set the detectors at certain location we do the calibration at few points, tune our Monte-Carlo program, run the resolution in sensitivity and this plot we meet the plot. And we see that whether the detected position which we have done it is fine or not, whether the relation is coming our acceptable limit or not. If it is fine, then we go ahead we do the calibration even for certain more point in experiments we tune our Monte-Carlo program again, we do it the calibration numerical calibration.

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We generate a lookup table which says that how the position of the particle is changing with the time. So, we get this either through experiments or through this or through the

combined effect of it, both the combining together to this. We get a lookup table. in real experiments as I said the particle is free completely free to move wherever it wants to go you get the time versus count diagram so or table or diagram. Now, we have this two you know job is to calculate the position of the particle and then the velocity of the particle. So, we want to calculate the velocity. and for velocity we need actually the position. At once we have a position as I discuss that in the Cartesian coordinate, you can do it very simply calculate the velocity by Δx by Δt , you will get the velocity. The job is how to calculate the position reconstruction how to do that.

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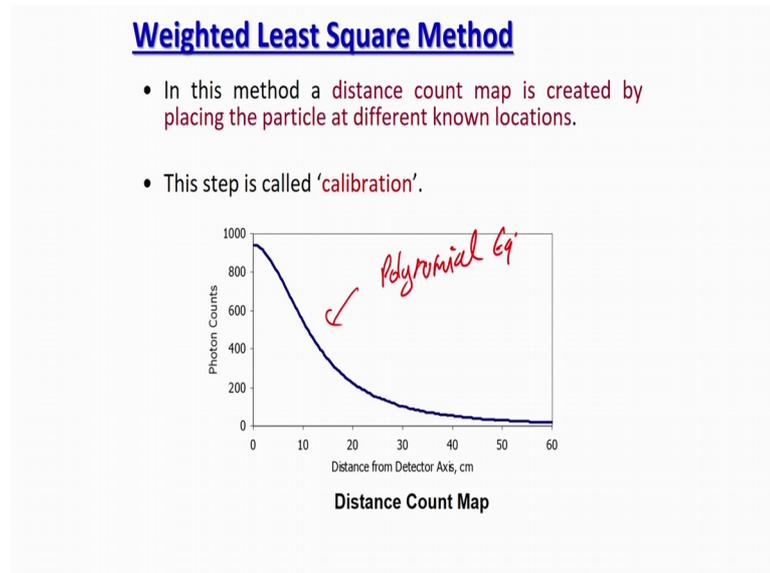
Reported Position Reconstruction Algorithms

- Weighted least squares algorithm
- Monte Carlo algorithm
- Neural network based reconstruction
- Cross correlation algorithm

Now, there several method is being used in the literature to do the position reconstruction. And some of these methods has been kind of listed here actually mostly these four methods are being used actually mostly these three methods are being used neural network is also used only once. So, these are the four method, weighted least square algorithm, the Monte-Carlo algorithm which we have already discussed that which will be used to generate the count the same algorithm can also be used to reconstruct the position. Neural network based algorithm you can use the neural network based program to calculate the particle position; you can use the cross correlation algorithm.

These two algorithms are very close to each other and we will discuss that what are these algorithm. So, we use this algorithm to calculate the position of the particle. So, now, we know that what I have I have lookup table, I have your photon count time series history.

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So, in the weighted least square method what it does we do the experiments to do the calibration. Now, we are not using the Monte-Carlo. So, I am not going to use the Monte-Carlo algorithm to generate the count. So, what we do we use to do the experiment this calibration experimentally only for lot of positions. And you will generate the photon count map on each detector. Again I am telling you this map will be for each detector. So, this is the distance and this is the photon counts. So, you can find it out from each detector distance count map you can generate.

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- Then the counts recorded during calibration is fitted in a polynomial equation and distance from each detector is calculated.

$$r_i = a_{0i} + a_{1i} \left(\frac{1}{I_i} \right) + a_{2i} \left(\frac{1}{I_i} \right)^2 + \dots$$



Where

r_i is the distance of tracer particle from the i^{th} detector
 I_i is the intensity recorded at the i^{th} detector.
and a_0, a_1, a_2 are constants.

- r_i and I_i is calculated from distance count map. Like, for detector 9 at 10 cm distance, 1200 counts is recorded on detector.

Then what we do in the weighted least square method we fit a polynomial which actually fit that the distance count map. So, to fit this, we fit it in a polynomial equation. So, we put a polynomial equation of this order second order, third order, fourth order wherever the fitting coefficient value is very high. So, r square value is very high, the confidence in the fitting is very high we use to fit a polynomial and where r_i is nothing but the distance of the tracer particle to detector. So, again I said that detector position is fixed in calibration you are putting the particle at several known location you know that what is the distance between the particle position and detectors.

So, this distance we already know like this and we know the intensity also which is nothing but the count recorded on the detector. So, we know this and we fit a polynomial equation. And it is going to be one of I , because we know that intensity will be high when the distance will be low. So, in that way we fix it. And we fit a polynomial for each detector. So, each detector we do this polynomial fitting and then what we do we start taking the kind of calculating that of the particle position.

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- Then actual RPT experiment is performed.
- Now the co-ordinate of tracer particle is calculated by estimating by least squares method:

$$|b| = (X^T W^{-1} X)^{-1} X^T W^{-1} Z$$

Where:

- b = co-ordinate of the tracer particle.
- Z = experimentally measured distance.
- X = hardware constant depend upon the co-ordinates of the centre of the detector.
- W = Weighting function

Z, X and W can be represented as:

$$W^{-1} = \begin{bmatrix} \frac{1}{\sigma_1^2} & & 0 \\ & \dots & \\ 0 & & \frac{1}{\sigma_N^2} \end{bmatrix}$$

Handwritten notes: $\mu_k = \sigma^2$, $\sigma = C$, Poisson distribution, and a matrix diagram with circled elements.

So, what we do we solve now the weighted least square method. So, this the equation we solve where b is nothing but the tracer particle coordinates. So, say I will say that say alpha, beta, gamma is the tracer particle coordinate or let it be let us confused I say that let me tell that x p, y p and z p is the tracer particle coordinate. We need to find that. If you need to find in the real experiment where the particle is moving with the time, my job is done. So, to do that we did the weighted least square method and that is nothing but you might be knowing the least square method. So, I have several equations now for each detector for all the distance, I have to solve all those things together, so that is why I did it with the weighted least square method because little bit optimization is needed they can be mini solution. So, each detector is going to give a distance.

So, there we have done this where x is nothing but it is a detector hardware parameter is you can say the detector parameter, and it is nothing but it shows that and we will discuss it that what it is. So, it is nothing but the detective coordinates. So, X what is the coordinate of the detectors, W is the weighting function which we are weighting with the counts recorded on the detectors. Then X I have already told; and Z is nothing but the distance from the particle to the detector and that is what we need to find that if you know the distance from the particle to detector and this you have calculated based on the lookup table, a calibration table.

We solve it weighted least square method. The W_i is being the kind upon one upon sigma square and all in a form of a matrix which is only tri diagonal matrix only the diagonal component is there; rest of the component is not there. Why because whatever the detector is placed only that detector you will get the count the other detectors will not get the count. So, suppose this is for detective 1, 2, 1, 2, 3, 4, 5. So, for first you will get the detector count; for second row you will get the detector second counts is this was you will get, then you will get second detector count all the other detector will not get the count because I am seeing the second detector. Now, similarly third detector four detector and this, so you will get a diagonal matrix.

And that diagonal matrix is nothing but the count recorded on the tracer particle for that position because as I this the count emitted or photon emission is actually a portion distribution process. you can think about the poison signed off read about the Poisson distribution process maybe in your mathematical courses or statistical courses this has been introduced. So, I am not going in detail of this, but what the Poisson process say that the mean is equal to sigma square. So, it means the sigma square is the standard deviation that is going to be equal to the mean count. And whatever the mean is here because its the photon counts which is the Poisson distribution you will see this kind of a distribution as I have discussed that day in the photopeak fraction the sigma square will be equal to c. So, mean count, which is being recorded on the detector. So, you would note this value.

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$$\sigma_i = r_i^m$$

$$|X| = \begin{bmatrix} 1 & -2x_1 & -2y_1 & -2z_1 \\ 1 & -2x_2 & -2y_2 & -2z_2 \\ \dots & \dots & \dots & \dots \\ 1 & -2x_N & -2y_N & -2z_N \end{bmatrix}$$

$$|Z| = \begin{bmatrix} (r_1^2 - x_1^2 - y_1^2 - z_1^2) \\ (r_2^2 - x_2^2 - y_2^2 - z_2^2) \\ \dots \\ (r_N^2 - x_N^2 - y_N^2 - z_N^2) \end{bmatrix}$$

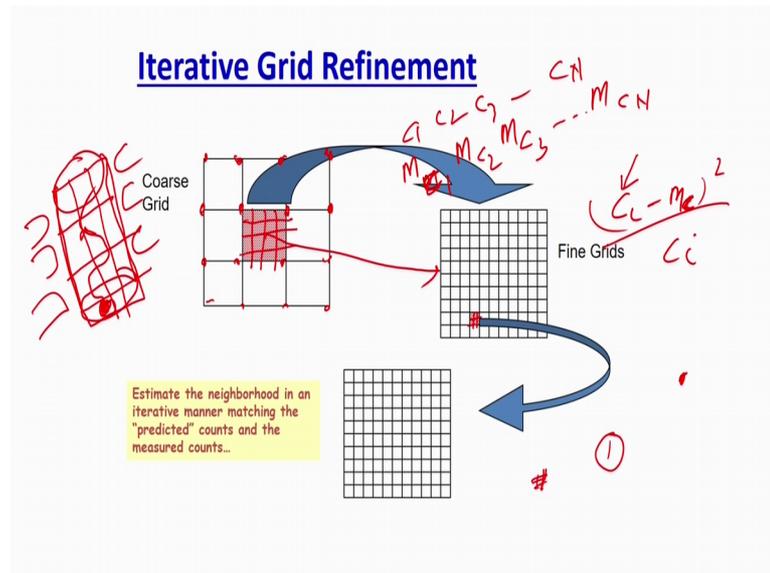
Where-
 x_i, y_i, z_i are the co-ordinates of the i^{th} detector.

You know the particle detector position, x_1 , y_1 and z_1 is the coordinate of the detector for a particular detector. So, you have say n number of detector, you will get a n number of these equations. You know the distance from all the detectors from all the particle positions. So, this will be r_1 , r_2 , r_3 and so on. So, this is the lookup table you have already know that. And then by if you have all this parameter you can solve this equation you know x now, you know w , you know z , you can get that what will be the position of the tracer particle x_p , y_p , z_p . So, you can reconstruct that how the tracer particle is moving clear. So, we can get this all these parameters known W we also known; Z we also know it is nothing but the distance. And you calculate the position of the tracer particle. So, this is called weighted least square algorithm.

The only problem is here you need lot of calibration point because again I am saying that not using Monte-Carlo based method. So, you are generating this count map experimentally and least square method again because it is going to assume a linear assumptions, so that is why you have to actually do kind of do the experiments for several positions. So, it is very time consuming process.

Then what we do we can also do the calibration this for the standard method at that a standard method or kind of the way we have generated the photon count map numerically and that is called Monte-Carlo method or Monte-Carlo simulation method. And Monte-Carlo as I already explained what it does it takes the detector position for a fixed position of the particle, it find that how much fraction of the solids which were kind of photons which is emitted from the source is going to incident on this. It is going to calculate that walked off whatever has been incident it is going to be get adsorb inside. And based on that you can calculate that what will be the count recorded on the detector for a fixed position of the tracer particle and fixed position of the detector. By using this formula we have already discussed that. So, what we do we do the experiments; in the experiments we know that particle is moving completely.

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The Monte-Carlo program is ready which can calculate the count at any location inside the bed. So, what we do suppose this is my column, this is my column. And experiment it is moving freely, particle is moving freely. So, this is the particle tracer part the tracer is started here, it is moving freely. I have to reconstruct the position of the particle. I have already tuned by Monte-Carlo program which can give me the particle position of for a fixed location of the particle, it can give me the count on all the detectors. So, detectors were placed here.

So, what we are going to do we are going to divide the whole zone in a very coarse grid say I am dividing the whole flow zone in a very coarse grid. Please go and get confused in 2D, this all whatever I am talking about is the 3D, because it is the 2D and label just so the 2D. So, you divide your things in a coarse grid and then we run the Monte-Carlo program for all these nodes point. So, I know the position x, y, z position of this I know I do this. And then based on that wherever we do we kind of run it for all the discourse grid point, then we compare this data with whatever the data available during the experiments.

So, for the particular t time, we get a count on c_1, c_2, c_3 say c and on all the detectors. And all these node points, I will say it as a c calibrated say $C_c, C_{c_1}, C_{c_2}, C_{c_3}$ and so on say C_c , and I will get this points. What we do we calculate, we compare these two counts by using the psi square method and psi square method is what it is nothing but C_i

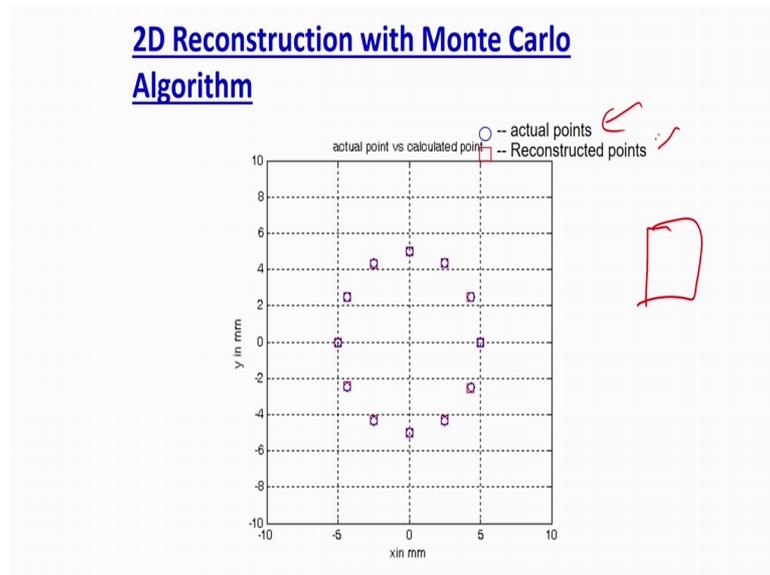
minus m_i the square upon C_i , we minimize this value. We see that where the difference is going to be the low; lower the difference ideally if the size square value is zero it means the calculated or this disturb this see say lets write it at the end, so to review the confusion. So, this is the major count major count m , and this is also m .

And I will say the c calculated in this and c major if I calculate this, this is the experimental this is the major if it is zero then it means you are exactly locating the tracer particle position. Now, we are doing it a very coarse grid I will get some value I will not get the value 0. So, for each node point I will get a ψ square value. Now, for the node point or for the cell where the ψ square values are coming very low definitely it will be very low in this region it is this cell this whole cell it will be very very low or it will be low. Then what we do we further discretize this cell in a very small grid again.

So, now we look this portion again I am discretizing it means I am putting small grid sheer again now only this portion. So, I am computationally saving the time. If I prepare the complete lookup table for all the possible position, it will take a lot of time. So, I am saving the time, and first divided in the small closed grid. Then now we are doing it in this way that we have divided it into the small parts, it will be like this. Then again I will do the ψ square test everywhere and I will find it out the location where the tracer particle where the ψ square values are less, so this position.

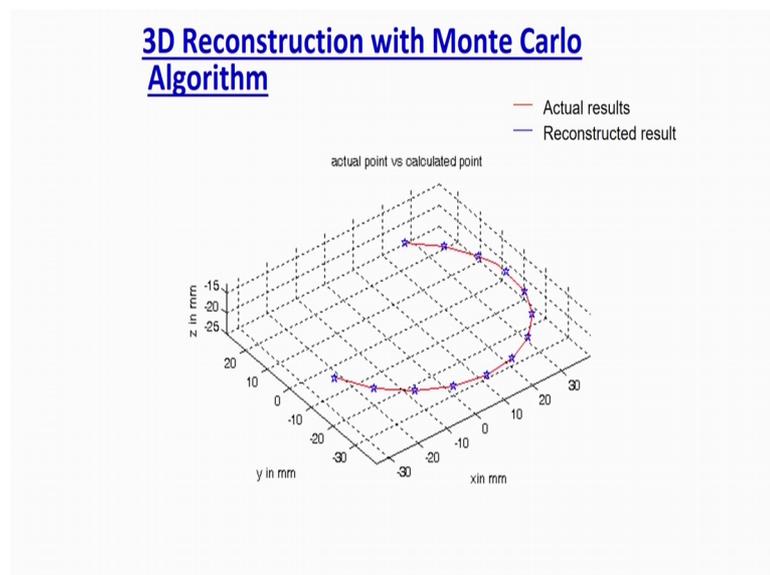
Again I will do the same operation, I will keep on reducing it further I will again discretize it. And I will keep on doing it till the point I am not achieving my minimum resolution that this grid size should not be lower than the resolution of the particle location because we know that that image is the particle uncertainty. So, suppose if my uncertainty is 1 mm, there is no point of making the grid which is less than the 1 mm because you are not able to get that if the two particle position is less than 1 mm, you will not able to position the particle. So, I will keep on reducing it and calculating the size square value. And with this iterative method, I will get the position of the tracer particle, same I will repeat for the other position, other position, and I will get that how the position of the tracer particle is changing inside. So, I will keep on doing it.

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So, we did it and we compared the result what we did we actually put the particle at several known location say we make a simple set up and we put the particle at several known location in a cylinder in a circle. And we have reconstructed the value and we found that the actual position was blue colour, reconstructed value was red colour and you see that exactly able to locate that where is the particle.

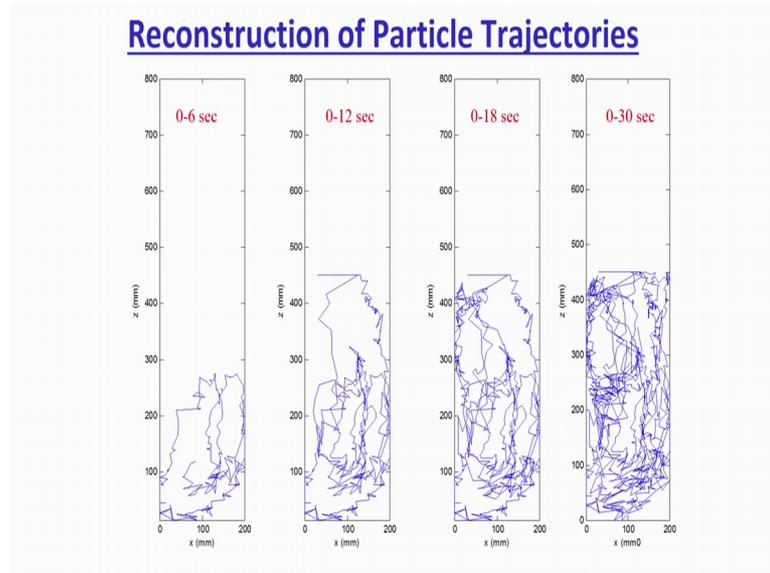
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Similarly, we move the particle in arbitrarily path in the three-dimensional that was a stationary. Now, this is the particle is moving in the three-dimensional path. And we

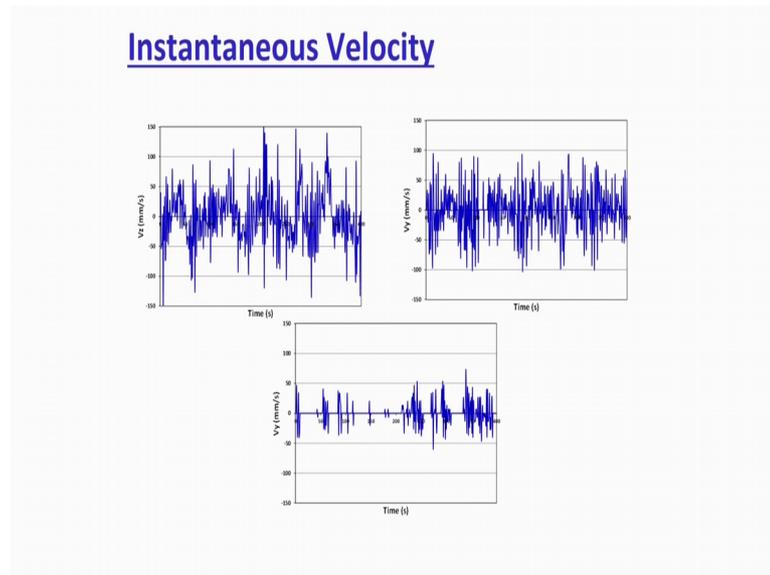
reconstructed the position by using the Monte-Carlo algorithm. And you can find that this is the result which we have reconstructed. So, this is the we actually find that the particle location you are able to exactly reconstruct the path of the particle. So, the Monte-Carlo reconstruction without them it means is working fine.

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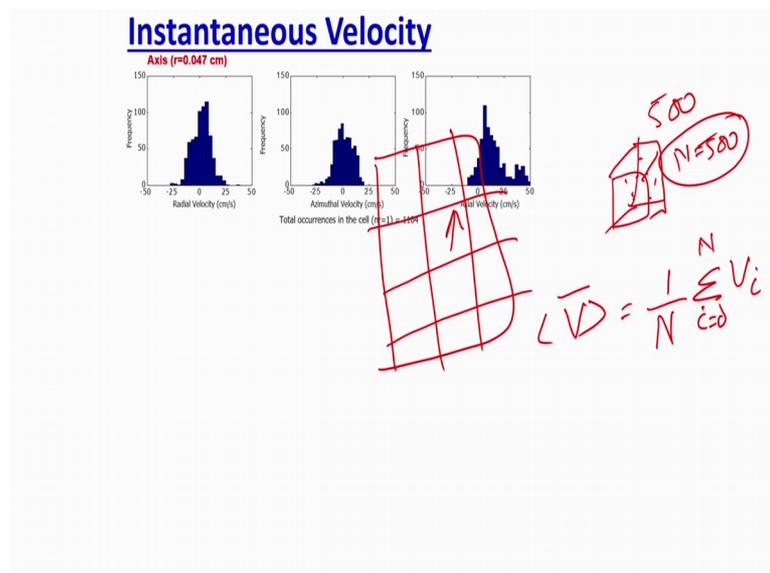
So, what we get we get position of the particle. So, we see that how the position of the particle is changing with the time. So, this is say for particular instance how the particle position is started with this and it is moving it is moving in this line. So, how this particle position is changing with the time, and once the particle position we know that how it is changing in the time.

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We can get the velocity by v_z by v_r , we can get the v_z velocity, we can get the v_x velocity we can get the v_y velocity, all the instantaneous velocity we can get that is called Lagrangian tracer velocity.

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Then from that instantaneous velocity the good thing as I said earlier also why this technique is so powerful because we are doing we are tracking one particle for very long time. So, each cell the particle will come several times. So, now, once we have the instantaneous velocity, we divide the whole column again in small grids. Now, do not get

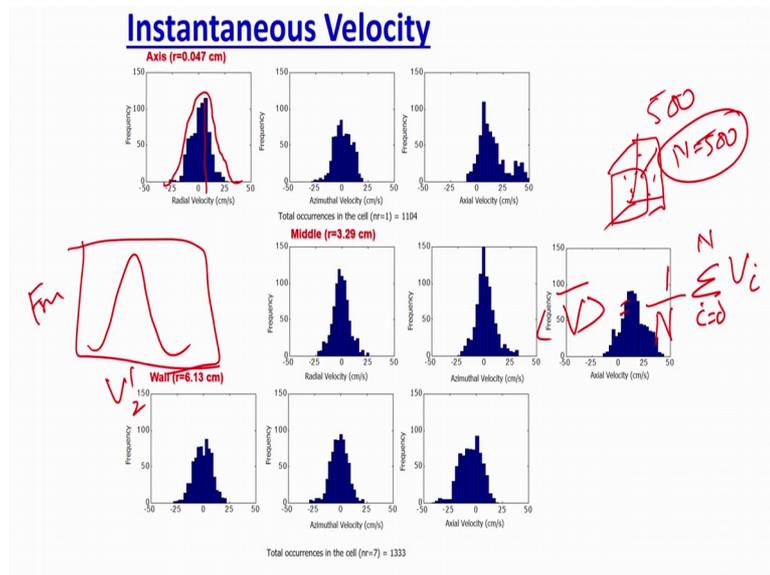
confused if grids are not very big they are very small grids. So, we divide the columns in a small grid in three-dimensional grid. And we see that say if I did the experiment for 8 hours, how many times the particle comes inside. Whatever time the particle comes inside we do the ensemble average and ensemble average is what is nothing but the ensemble average mean velocity is nothing but it is $\frac{1}{n} \sum V_i$ where i equal to zero to n .

So, if suppose in a 8 hours experiment in this particular cell particle comes 500 times and because 500 for all the 500 times you know the velocity you just add those velocities divided by 500, you will get the mean velocity of that location. So, this cell I will know that this is the mean velocity. Powerful point of the technique is because we are tracking one particle for sufficiently long time, each location particle is multiple time particle is visiting you can calculate not only the mean velocity, you can also calculate the PDF. It means you can calculate the probability distribution function of the instantaneous velocity that what will be the velocity distribution you can achieve for your system because multi phase flow again is very dynamic in nature is there is no steady state concept, there is no concept of the fully developed flow. So, the particle velocity will be different.

So, mean velocity which we say is just a statistical quantity ideally you never see sometimes mean velocity you see somewhere plus or somewhere minus. So, how much plus, how much minus that information is also needed, if you want to do a proper scale up, if you want to really understand the system. Like in this case if you see that the mean velocity will come in the range of say around 1 or 2 centimetres or 5 centimetre per second while the instantaneous velocity is changing from 20 centimetre plus minus, so wide distribution.

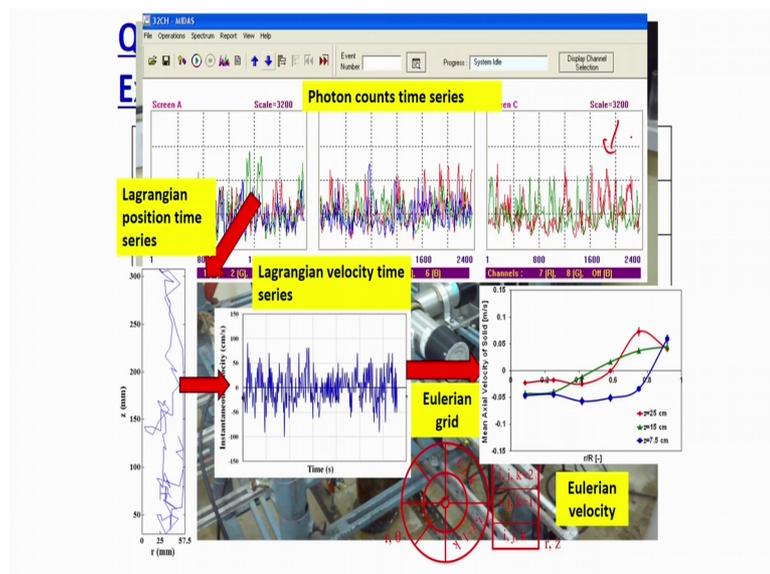
So, this is the powerful technique it not just give you the mean velocity, it gives that what kind of distribution you can achieve there. You can see that if there is any reaction what is the maximum possibility of maximum extent of reaction is there, what is the minimum extent of reaction is there, you can get everything, you can get the mixing. So, that is the most powerful that is why this technique is because you did not literally mean you get the PDF of instantaneous. Now, I have a mean for each cell I have instant a p d f of instantaneous for each cell I can calculate the PDF of fluctuation.

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So, similarly I can plot the frequency versus V_z prime, I can calculate that how much V_z prime is changing. And that is not only for V_z prime, you can do it for V_r prime, V_θ prime like the PDF of instantaneous velocity they can do it for the radial velocity a velocity axial velocity all the velocities you can do. So, similarly you can calculate for the fluctuation also, so that is why the technique is very very powerful.

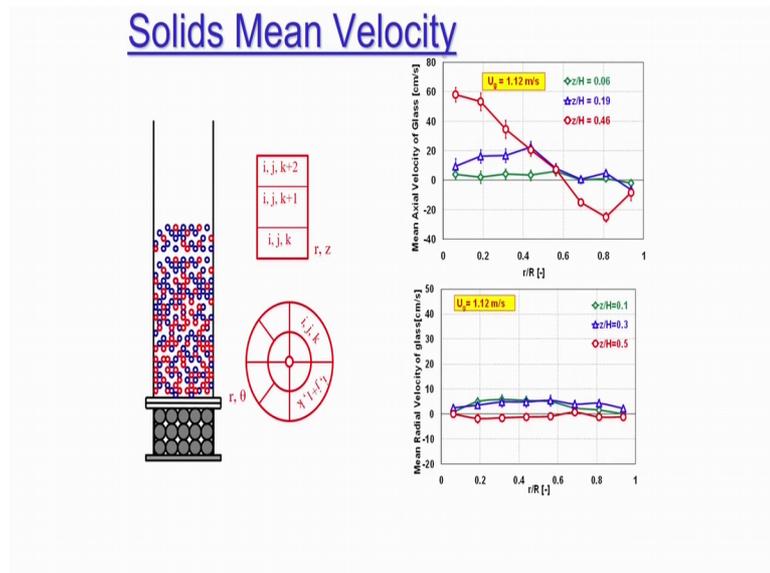
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So, at the end as I said the overall this remains like this only that you have the photon count time series from that you calculate the instantaneous velocity that sorry

instantaneous position how the particle position is changing with the time. From there you calculate the instantaneous velocity or Lagrangian velocity of the particle. And then from there you would divide the column in small cells and you get the mean velocity. So, from this kind of a data which looks very vague, very rough, very chaotic you get a very systematic profile.

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So, this is the solid mean velocity profile which we are discussing for the system for a gas solid system. And you can see that from this data which is very (Refer Time: 61:49) looks very chaotic the count profile. You get a proper profile of the mean velocity which says that for this condition particle is moving up at the center, going down near the wall. So, you get this kind of an information.

You get lot of data, lot of information from this time series analysis can be done, I am not covering in part of this, but you can go and see that. We can do the time series analysis, we can calculate the diffusion value that how the particles are fluid are diffusing from one location to another location. How the solids are diffusing, you can calculate the autocorrelation, you can calculate the cross correlation, you can calculate the Kolmogorov entropy, you can calculate the quality of mixedness, all this quantity you can calculate. So, you can do a very regress analysis. So, that is the way the technique is powerful.

Now, whatever we have discussed till now is only the advantage of the technique. They are certain disadvantage. The first disadvantage, the major disadvantages it uses radioactive particle. The moment radioactive particle comes into the picture there is a fear and we always forget afraid that what will happen to my safety and that is the big question and big concern that is why the person who is handling this technique should be a trained person. He should have a radiation training radiation safety training one is the biggest requirement, one has to do that. And one should know how to safely deal with the setup though they are pretty much within the safe limit that the activity we use your source of strength we use is very very low compared to whatever we use in the medical application. It is very very low we use generally 100 to 500 micro curie range, but still one has to be careful with the safety precaution.

So, one has to do a dedicated worker is needed to do this, a dedicated lab certified lab is needed, so that you can have kind of facility to handle the radiation. And in case of any emergency you are trained enough to handle it. So, dedicated lab facilities needed, dedicated approval is needed to use this kind of thing. Then this is not an (Refer Time: 63:51) technique. So, what you need to do you have to do everything assembling yourself. So, one need proper understanding, you do not it is not an on and off device that you would go with a camera take a photograph job done.

So, it cannot be like that you need to be involved, this is very very involved experiments. You need to plant the experiment properly, so that you get the desired accuracy like where the detector should be placed, how far this would be placed from each other, how many detectors would be placed for each axis. And the result which I have shown is just a kind of representative results, so it just the typical results. It will change by changing the dimensions of the system, changing the type of the systems, if you put internals inside, this will change if you feel keep on changing. So, you have to always do this exercise calibration is a very very tough job, it is very time consuming. And the job where all your accuracy of the RPT experiment is actually depend on the accuracy of the calibration.

So, need to be because we are tuning even if you are using Monte-Carlo, you are tuning your Monte-Carlo based on the points which you have calibrated experimentally. So, calibration is a very typical step it is very time consuming. And many times what is possible is happen that doing the calibration at industrial scale system is sometimes is

very very difficult, because how you will do that calibration, how will put the hole inside your reactor to do it. And sometimes the operating conditions reactors are being operated say at a temperature of around 500, 600 degree centigrade at a pressure of say 100 bar or so or 50 bar or so, it is very difficult to do the calibration or that step. So, these are the disadvantage of the technique.

Again the resolution of this technique, a special resolution of the technique is not very high it is not in the range of what MRI can give you or what the PIV can give you it is little bit more, it is in the range of 1 mm or so. So, resolution wise also this technique is not very great, but the versatility wise this technique is really great you can practically or theoretically you can implement it anyplace wherever you want.

So, with this, this portion is over and next time we will discuss another technique.