Multiphase Microfluidics

Dr. Raghvendra Gupta
Department of Chemical Engineering
Indian Institute of Technology, Guwahati

Lecture - 10
Gas- Liquid Flow: Void Fraction and Pressure Drop

Hello in this lecture, we will be talking about the void fraction and pressure drop models for gas liquid flow in micro channels. The two quantities; the void fraction and the pressure drop or the pressure gradient; in the channel are of engineering importance, they are required to design any engineering device.

So, it is important to develop correlation and get an estimate to for the pressure drop or void frock fraction in the channel. In conventional channel or in large diameter channel, a number of correlations most of them empirical have been developed to measure pressure drop or calculate or credit or estimate the pressure drop as well as void fractions, in large channel diameters or large diameter channels.

Now, the first thing that one would like to do is look at the applicability of these correlations which have been developed for large diameter channels. And then one would ask one a question that edge those correlations, which are working fine for large diameter channels are they good enough for small diameter channels or any modification to them are is required. So, this is basically the theme of the lecture that we are going to have today.

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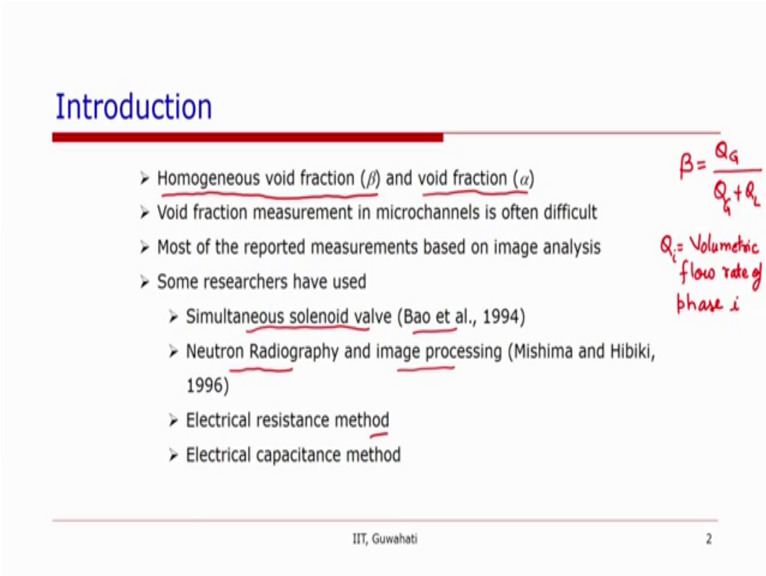

So, before going into this we will first talk about void fraction and then later in the lecture we will be talking about the pressure drop. So, a void fraction there are two different void fractions, that we are talking about here one is alpha and beta. As you see here that a beta is homogeneous void fraction and alpha is the void fraction beta, if we know the flow rates of the two phases.

Then we will know beta, because beta is defined as Q gas over Q gas plus Q liquid so; that means, a where Q is volumetric flow rate of phase i. Now beta is defined as Q G over Q G plus Q l. So, if we know the flow rates of the two phases, then we know what beta is beta is basically the volume fraction of the gas. If we assume that there is no slip between the two phases if both the phases are moving with the same velocity.

So, then the flow rates at which the two phases are introduced they will be move with the same velocity unfortunately in most of the cases it is not. So, for example, Taylor bubbles they move faster than the; average velocity. So, there is a certain amount of slip in Taylor bubbles, now void fraction measurements is not an easy task to do there are number of techniques that have been developed to measure void fraction in large channels and more recently the techniques are being employed or being modified to measure void fraction in small diameter channel.

So, some of these techniques are say for example, Bao et al., they have used simultaneous solenoid valve Mishima and Hibiki; they used neutron radiography apart from image processing. So, of course, image processing is one of the techniques that where one take 2 D image of the flow, and then try to correlate from yet to get the void fraction in the three dimensional channel ok. Then another method is electrical resistance method.

So, basically different phases will offer different resistance to the current flowing through it. So, one can measure the voltage and then estimate the resistance and based on that resistance overtime based on the resistance one can calculate or one can obtain the void fraction or at any cross section. In the channel similar concept where one measures the capacitance in place of resistances as electrical capacitors method ok.

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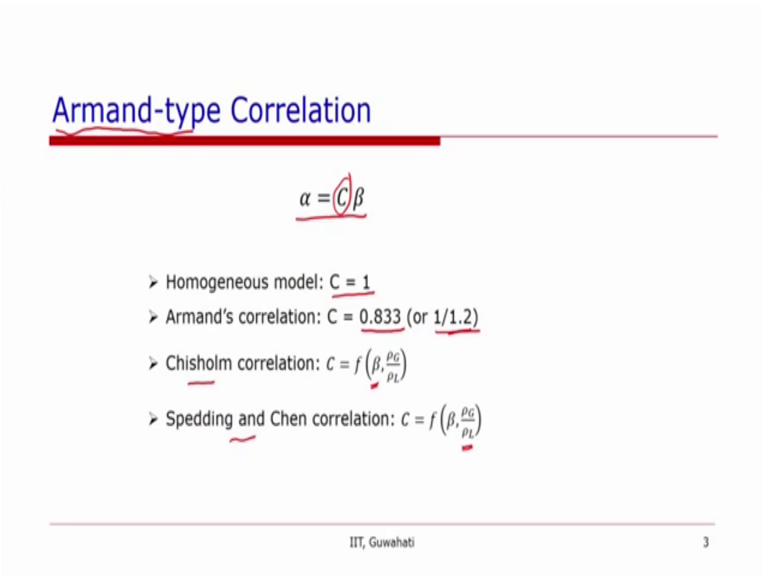

So now, let us look at the correlations a number of correlations have been developed, and it will be not possible to cover all the correlations. In this lecture, we will be talking about those correlations the correlation most of them have been developed for large diameter channels and then they have been modified for use in small diameter channels.

So, one of the correlations or one of the class of correlation are Armand type correlation. So, they are alpha is equal to sum constants C into beta. Now if it is a homogeneous model; that means, there is no slip between the two phases that will be probably true. When we have very small size of the bubbles and then they follow the flute flow faithfully then this C is equal to 1.

So, we have alpha is equal to beta. Now the next stage Armand's correlation so, in this C is 1 over 1.2 or 0.833, 1 over 1.2 is nothing, but 0.833. So, in some places you will see that C is equal 2.833. In other places beta over 1.2 ok, now these two correlations or Armand's correlation is quite widely used.

So, this is just saying say for example, for slack flow, that the Taylor bubble moves 1.2 times faster than the average velocity. Now, another two correlations that have been listed here by Chisholm and Spedding in Chen they also use. So, they have C, but C is not a constant, but C is a function of beta and the density ratio of the two phases. So, they take into account the density ratio and beta for both the phases.

So, these two correlations one can look at in any standard book on multiphase flow all in the literature in different papers.

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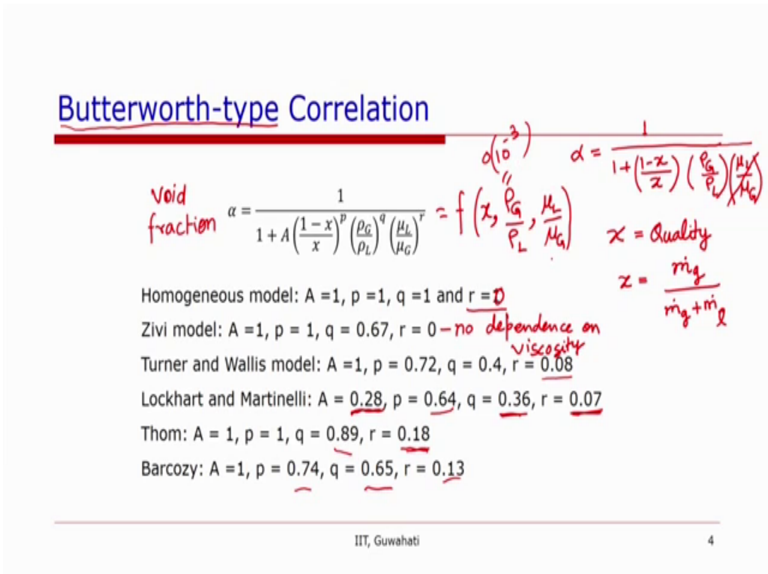

So, then another class of correlations is Butterworth correlation in which the void fraction alpha is defined as one over 1 plus A, where A is a constant into 1 minus x over x raise to the power P rho G over rho l raised to the power q and mu L over m G raised to the power r. I have tried to be as accurate as possible checking these correlations again and again; while typing while ah, but; however, there might be some errors which I could have overlooked. So, I suggest that when you use these correlations please verify these correlations with the literature you can find most of them with the most of them, in the two phase flow boiling and condensation in conventional and miniature channels by professor Musthaffa k C n.

Most of these most of the discussion, that we are doing here you can find that in the book. So, what the homogeneous model again if we define a P q r all of them equal to 1, then one can end up with having a Homogeneous model, which will basically be alpha is equal to 1 over 1 plus 1 minus x over x rho G over rho mu to mu L over mu G.

So, basically where I am sorry r might be equal to 0. So, this term will go away. So, one can check and find out that alpha is equal to beta. Now x is a new term here ah. So, that it is not new for most of us x is quality and as beta is volumetric homogeneous void fraction or the homogeneous volume fraction and x is homogeneous mass fractions. So, as x can be defined as m dot gas over m dot gas plus m dot liquid where m dot is the mass flow rate of a phase ok.

So, then in the G v model they have assume or they have they have found that A is equal to 1 p is equal to 1 q is equal to 0.67 and r is equal to 0. So, they do not have any dependence on viscosity please check this ok. Now the Turner and Wallis model have a A is equal to 1 p is equal to 0.72 Q is equal to 0.4 and r is equal to 0.08. Lockhart and Martinelli correlation a is unlike other three correlations: A is 0.28 here and as you see other two correlation later also have value of A is equal to 1. So, out of all this only Lockhart and Martinelli correlation has A is equal to 0.28, p is equal to 0.64 and q is equal to 0.36 r is equal to 0.07.

So, if you look at these correlation all three of them all two of both of them have the value of p about 0.7 them Thom’s Correlation: p is equal to 1 and q is equal to 0.89, r is equal to 0.18. So, slightly stronger function of viscosity ratios and then Barcozy’s model p is equal 2.74 q is equal to 0.65, this q is equal to 0.89 and r is equal to 0.13.

So, there are basically different models and they have been grouped together as Butterworth type-correlation. So, we have to looked at Armand type correlation which is relatively simple A is equal to C beta form where beta is the volumetric flow rate. In this case volumetric homogeneous volume fraction, whereas; in this case we have alpha is equal to the function of x, which is quality the density ratio and viscosity ratio.

You might notice that the density ratio is will be of the order of 10 to the power minus 3 and viscosity ratio of the order of 10 to the power minus 2 or 10 to the power minus 1.

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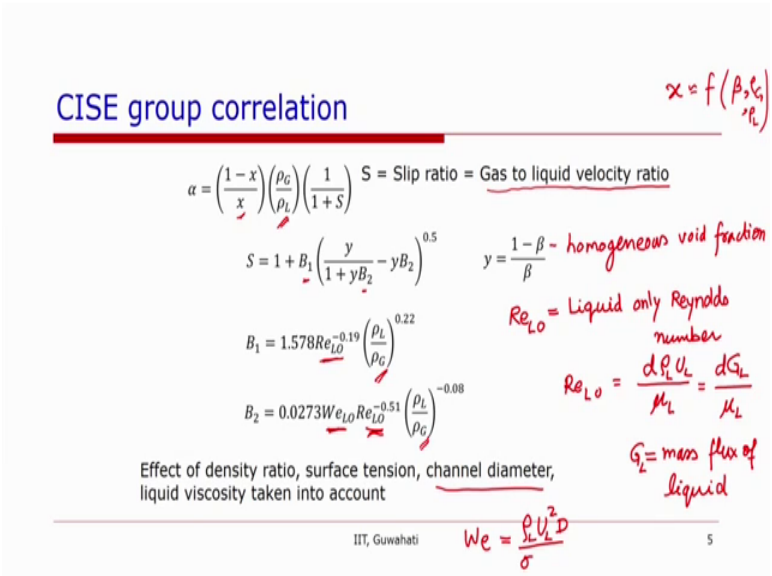

So, the third correlation is CISE group correlation. So, it has a again a similar to Butterworth type-correlation, that is; it uses quality 1 minus x over x is one group that they use then rho G over rho L is another group. So, same the density ratios and the ratio of fractions mass fractions of the two phases. Now, then they have 1 over 1 plus S; where S is slip ratio which is the ratio of the gas and liquid velocities.

So, most of the equations that we see here are two model this slip ratio between the two phases. So, S is equal to 1 plus B 1. We have two constants here B 1 and B 2 and y y is nothing, but it has been defined as 1 minus beta over beta, where beta is the homogeneous volume fraction or homogeneous void fraction with some simple considerations one can always find a relationship between the mass volume fraction or homogeneous mass fraction or not mass volume fraction mass fraction. So, one can find a relationship between x homogeneous mass fraction and beta which will also involve the density of the two phases.

So, I suggest that you should do this as an exercise, if you are already not if you have not done it earlier ok. So, s the slip ratio as a function of 1 plus B 1 into this term which has B 2 and y y we have already seen. So, now, what needs to found is B 1 and B 2. So, they have defined B 1 based on the liquid only Reynolds number and the density ratios now R e L o is liquid only Reynolds number and we will be using this quite often in this lecture. So, this is defined as R e L o is equal to channel diameter divided by mu L rho L into u L or liquid velocity or you can define as d is equal to d into G L over mu L where G is the mass flux. So, basically R e L o is the Reynolds number.

If only liquid phase would have been flowing in the channel and B 2 is equal to 0.0273 Waver number. So, if you do not remember Waver number let us just remind our self of that waver number is rho U square over sigma over d, because it is the ratio of inertial forces and a capillary forces. So, it can be defined in this manner, then we will have the properties of and the velocities of liquid in this case. So, that is Waver number.

So, if you look at this in this correlation care has been taken to take into account the density ratio. We can see the density ratio in a number of places it takes into account the viscosity of liquid, using Reynolds number it also takes into account surface tension using Waver number and it takes into account the flow rate ratio of the two phases or the mass flow rate ratio of the two phases using x.

So, this is a correlation which takes into account most of the physical effects that might occur, now surface tension is an important parameter which is there in the micro channels it also take into account the channel diameter by W e L o and R e L o. So, one would expect that this correlation might work for low Reynolds number or not low Reynolds number, low diameter small diameter channels and. So, let us look at some of the literature that people have found. So, in the small diameter channels for example, Bao et al., have compared their results with CILC correlation for channel size ranging between.

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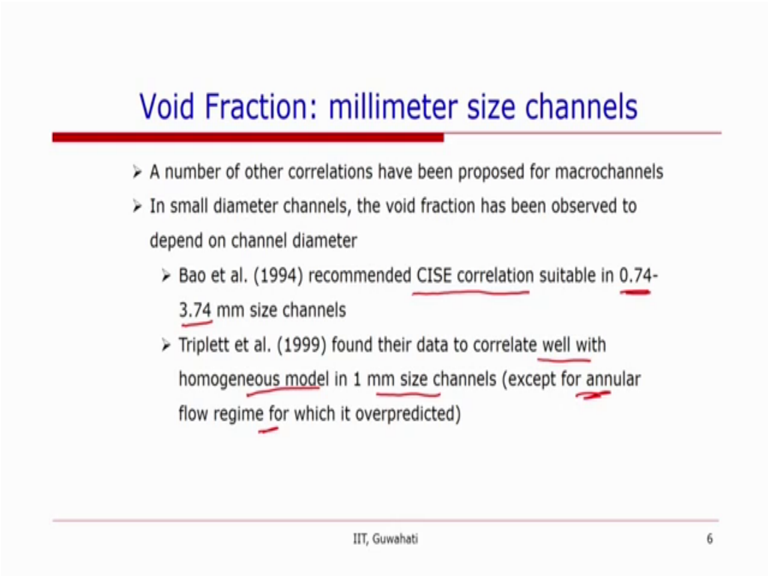

About 0.74 mm to 3.74 mm; so, from about 1 mm channel size of that order and then they found that CISE correlation is suitable for their purpose on the other hand Triplett et al., they found that their data correlate. As well with homogeneous model in 1 mm size channel except for annular flow regime for which it over predicted; now an exercise has not been taken up to look at the CISE correlation or at least I am may not aware where an extension comparison with the CISE correlation has been made with all the literature data for void fraction. In micro channels, it would be a really good to see if CISE correlation feds all the data values.

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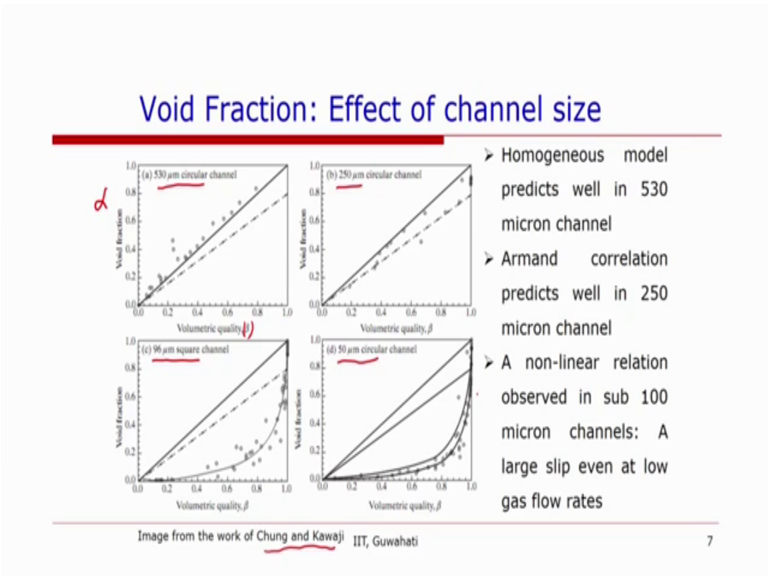

Now, Chung and Kawaji and other coworkers they have looked at the effect of channel diameter on the void fraction and you see an interesting trend in these graphs. So, there are four channels that they have looked at or that they have been plotted, they have actually looked at more channels in other articles.

So, if you look at here 530 micron channel diameter 250 micron channel diameter. So, this is 0.5 mm 0.25 mm 96 about 100 micron channel diameter and 50 micron channel diameter. All of three channels are circular among these and one channel about 100 micron size channel is square channel and they found that for the millimeter size channel or.

So, you can look at that y axis is what is plotted alpha on the x axis is the homogeneous void fraction of beta has been plotted. So, they look at that homogeneous void fraction is good for 0.5 millimeter or 530 microns size channel as the channel size decreases, the ratio between alpha and beta it starts decreasing.

So, the slope of this line is decreasing and then, that is; why they say that alpha over beta is equal to 0.833 which is Armand type correlation or which is Armand correlation that predicted them at lower correlation they see a different train especially this is very clear at 50 micron channel diameter that that the ratio between the alpha and beta just small and then it increases sharply at a high quality or high gas fraction.

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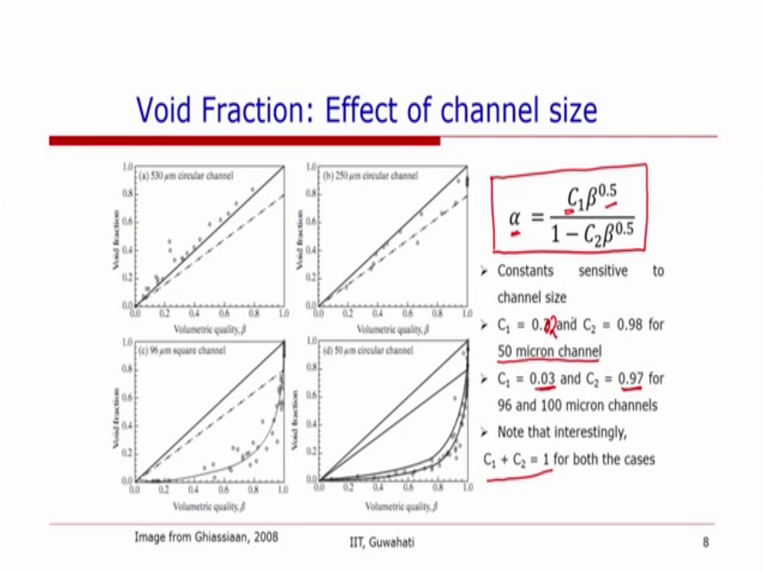

So, based on the results and these plots have been taken from Ghiassian book based on the work of Chung and Kawaji. So, they have developed a correlation which is that alpha is equal to a constant C 1 beta to the power 0.5 divided by 1 minus constant C 2 into beta to the power 0.5 and this costive is sensitive to channel size. So, they looked at for 50 micron channel C 1 is, sorry; this is 0.02, actually 0.02 and C is equal to 0.98 for 100 micron and 96 micron. Channel C 1 is equal to 0.03, C 2 is equal to 0.97 and in both the case its C 1 plus C 2 is C 1 plus C 2 is equal to 1 ok.

So, this is one correlation that has been developed for flow in micro channels and ah, but this does not take into account the size of the channel explicitly rather the; based on their observations from the experiments. They looked at that there are different values for different channel sizes; however, the difference between those is not very different as you can see from here.

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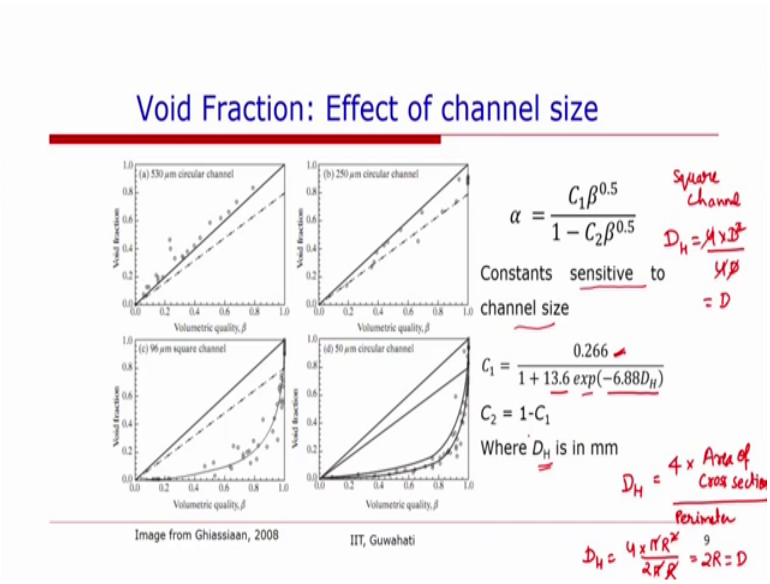

Now so, they further developed a that these constants they are sensitive to channel size and then they developed a correlation C 1 is equal to 0.266 divided by 1 plus 13.6 exponential minus 6.88 D H.

So, where D H is the hydraulic diameter of the channel, which is the channel diameter if it is DH is equal to 4, into area of cross section divided by perimeter. So, for a circles this D H is equal to 4 into pi R squared over 2 pi R RR cancel out pi pi will cancel out and one will have 2 R. So, a is same as the diameter of the channel for a square channel, if D is the channel dimension one will again get D H is equal to 4 into D square, where D is the dimension of the channel into divided by 4 D. So, 4 and 4 will cancel out and 1 will get D H is equal to D, where D is size of the channel for other channels one can similarly find out the hydraulic diameter.

So, the correlation the same correlation, but in this now C 1 is a function of the hydraulic diameter of the channel and C 2 is 1 minus C 1.

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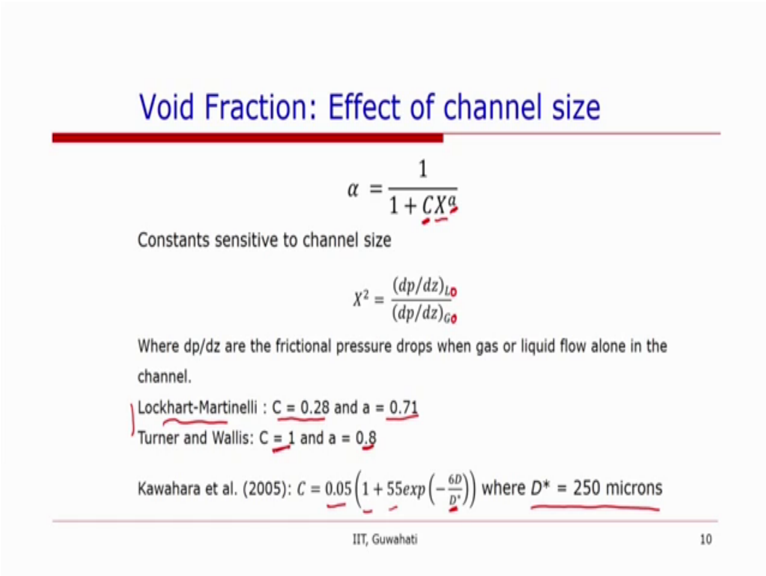

Now, another correlation that have been developed to look at the effect of channel size is where alpha is equal to 1 over 1 plus C into X to the power a. Now, we have a this X has not quality X is capital X here and now we have three unknown parameters here C xX and a. So, X is known as Martinelli or Lockhart Martinelli parameter which is defined as X square is equal to d p by d z liquid or you can say liquid only divided by d p by d z gas only. So, d p by d z liquid only or d p by d z gas only are the pressure drops in the channel if there is only single phase flow.

So, liquid only refers that if the there is only flow of liquid and gas only refers if there is flow only of gas. So, these are the frictional pressure drops which one can find using Poiseuilles law for laminar flow for fluid development laminar flow or Blazeou's law for turbulent flows, which we will be talking about later when we talk about the pressure drop. So, in this correlation which was for the large channels the Lockhart Martinelli correlation this it had C is equal to 0.28 and a is equal to 0.71 C constant 0.28 and constant a is equal to 0.71 Turner and Wallis they had C is equal to 1 and a is equal to 0.8.

Now, Kawahara et al., they have modified this correlation for to incorporate the effect of channel diameter in this. So, they had C is equal to 0.05 1 plus 55 exponential minus 6 D over D star and D star is reference diameter they took as 250 micron. So, there is this correlation has been modified again you can say that this was Lockhart Martinelli and Turner and Wallis correlation which has been modified by Kawahara for implementing in micro channels.

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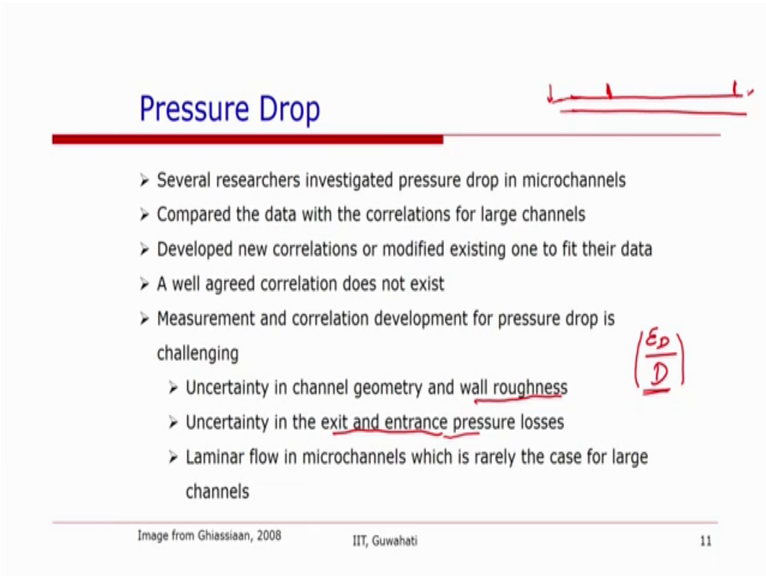

So, we have looked at that the void fraction measurement in micro channels depends on a number of parameters or it might be the homogeneous void fraction or homogeneous mass fraction. Then the density ratios viscosity ratios and the ratio of the two velocity is or the slip ratio and surface tension and the channel diameter.

So, there are some correlation which we have discussed; in there are some more correlations which we have not cited here, but people have developed those correlations and most of those correlations are dependent on the data that has been generated by the authors are some other data. And then they have compared their correlation with, but they does not seem to be an universal correlation which can predict the void fraction over all the channel diameters for all the fluids for all the flow conditions this is largely due to that if one goes at lower channel diameters. For example, 50 micron or lower one sees that the contact angle effects become important there which none of this take into account more over a range of flow rate ratios, there is a range of their they there are different flow regimes that we observe and in the each flow regime the arrangement of gas and liquid phases is different.

So, the void fraction or the slip between the two phases is very different in the flow regime. So, it is very challenging to develop a universal correlation which can predict all the data correctly. So, now, let us come back to pressure drop in micro channels and this has been investigated by several co researchers; in the past 20, 30 years and many of them have compared the data with the correlation for the large channels and most of them are had only limited success.

So, the data has been or the correlations have been modified to feed the data and really there is a well agreed correlation which can predict like void fraction. In the same is the case for pressure drops and there are number of uncertainties for pressure drop for example, when one measures pressure drop the flow will not be fully developed immediately. So, one might need to take into account the pressure drop in the developing regime.

So, one should be measuring the pressure drop one months the flow has reached at a fully developed state and the pressure drop between the those two say; for example, in the test section one can measure the pressure drop at these two regions, and it is not easy to have probs in a micro channel and such distances most of the times people measure the pressure drops and the inlet stream and then at the outlet or the outlet when it is open to the atmosphere.

So, then one need to take into account the; loses that happens at the mixers or at the walls. So, there exist uncertainty in the exit and entrance losses and this is more dominant in micro channels then the wall roughness if you look at the roughness. Epsilon D is a length scale for roughness and capital T is the channel diameter and looking at the accuracy of machining and another manufacturing the epsilon D over D will be very small for large channel diameter.

However, it will be not the case for micro channels. So, the roughness might have a stronger effect and one might need to take into account. So, that epsilon D over D can have an effect channel diameter channel geometry of course, will have any effect on the pressure drop. Then it is also not possible to immediately extrapolate the correlations that have been developed for large channels, because the flow in large channels is often turbulent where as the flow in a small diameter or micro channels is of laminar flow. So, it is not possible to immediately or to use the correlations for large diameter channels as it is for small diameter channels say one look at it study state the pressure drop will be a sum of the accelerational pressure drop the hydrostatic pressure drop.

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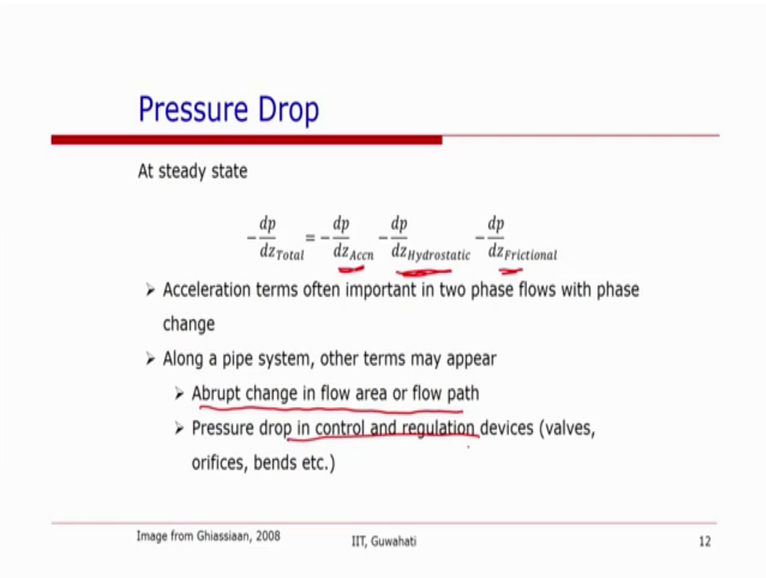

Because, of the gravity and the frictional pressure drop. So, the accelerational pressure drop is often important when one has phase change and the area of one phase is changing with respect to the other. So, it might be boiling or condensation.

So, because, we are talking about here adiabatic flows; so, we will not be talking about the accelerational pressure drop, hydrostatic pressure drop one can anyway take care by using rho G h using appropriate rho here. Now we will be talking about mostly the frictional pressure drop d p by d z some other pressured drops might come say; for example, when the there is an abrupt change in flow area or flow path or pressure drop in control and regulation devices as we discussed in the previous slide.

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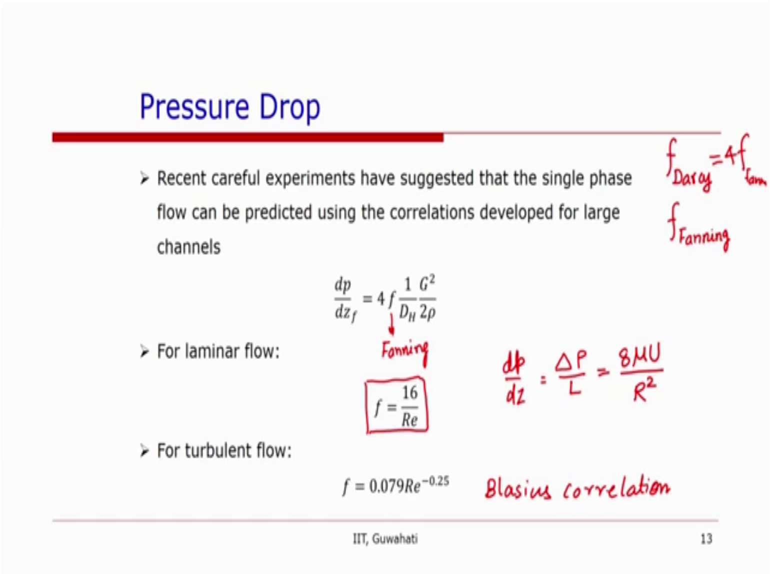

So, again for the pressure drop the pressure drop for two phase flow. In the correlations are generally based on the pressure drop for single phase flow and from that the correlations have been developed and the careful experiments. In last say past 10 to 20 years, it has shown that for 100 micron size channels or millimeter size channels of course, one can use the correlations that have been developed for conventional channels.

So, for the laminar single phase flow, in a channel those correlations are very well established for small diameter channels or. So, they can be easily used. So, let us look at it that for a laminar flow in a channel we have a pressure drop can be defined as delta P over L is equal to 8 mu U over R square. Now the pressure drop in the channel is often defined in terms of friction factor and there are two friction factors you might remember and it is confusing most of the time that there are two friction factors one is called Darcy friction factor and another is fanning friction factor.

So, it the difference between them is the definition that they have the one, we have here defined as f is equal to this is fanning friction factor and the Darcy friction factor is 4 times fanning friction factor. So, one need; to be careful that which friction factor one is using. So, the fanning friction factor for laminar flow is f is equal to 16 by Re, and this can be if you substitute the delta P by L or d p by d z, then one can find out f is equal to 19 by Re and for turbulent flows this is called Braziers correlation and it gives f is equal to 0.079 Reynolds number to the power minus 1 by 4.

So, these are the pressure drops relations for single phase flow.

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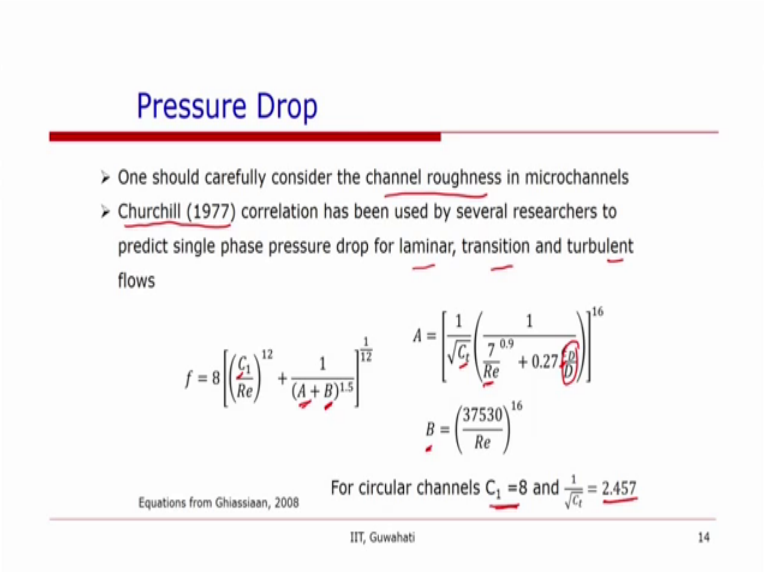

Now, because as we just talked about that the roughness might be important. So, a correlation, which has been used by several researchers to predict single phase pressure drop for laminar transition and turbulent. So, basically all the flow regimes for single phase flow taking into account the roughness of the channel is called Churchill's correlation.

And it has a f is equal to 8, this in this there are two constants as you can see A and B plus C 1. So, C 1 is 8 for circular channels and A is and again function of Reynolds number. The channel roughness and Ct and Ct is 1 over root C t is 2.457. So, this takes into account the roughness of channel and be constant B is 37530 over Re whole raise to the power 16. So, this correlation can be used to obtain friction factor and from there the pressure drop in rough channels for single phase flow.

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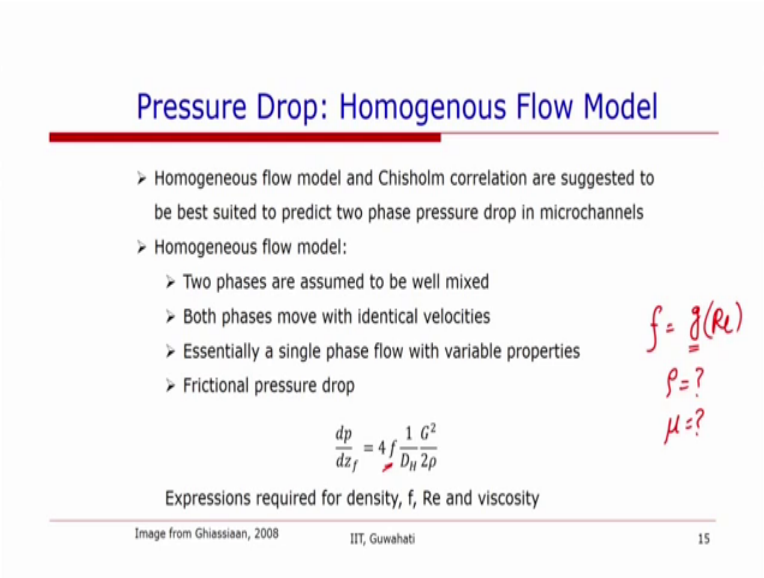

So like void fraction the simplest thing is looking at homogenous flow model. So, homogeneous flow model assume that the two phases move with the same velocity, there is no slip velocity between the two phases the two phases are both well mixed and. So, one need to take into account the only factor that, what is the f there f as we will see a f is as we have seen the f is equal to function of Re.

So, I have just written G here, because f is G Re. So, f is a function of reynolds number now one we will need to look at that what is rho and what is the mu, that one should be using here G of course, or the flow rate or the velocity one can use the mixture velocity.

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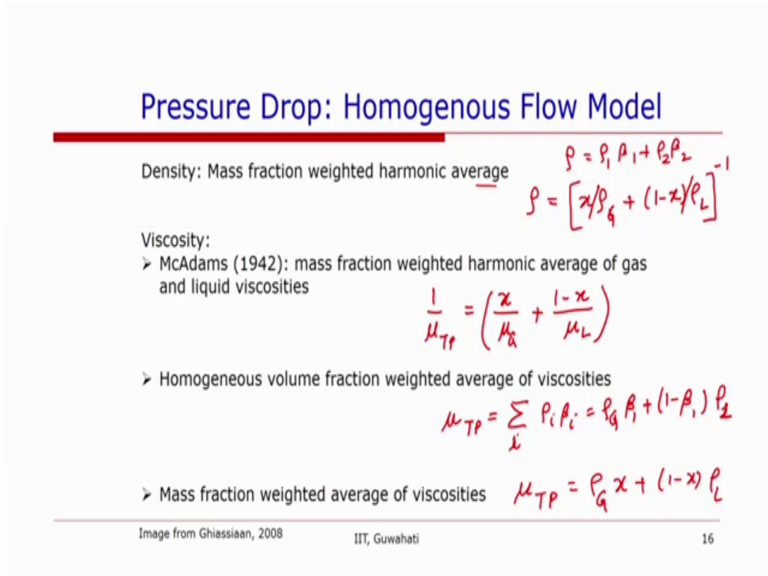

So, the density there are two definitions of densities one can use that rho is equal to rho 1 alpha 1 plus rho 2 alpha 2 or as it has been listed here that sometimes this definition of rho is used as rho is equal to x rho 1 or rho gas plus 1 minus x rho liquid (Refer Time: 42:16) which is harmonic average mass fraction weighted harmonic average of the densities ok.

Then there are different definitions for viscosities say one given by McAdams is mass fraction. So, 1 over mu for two phases is equal to x over mu G plus 1 minus x over mu L or volume fraction weighted average of the viscosities. So, one can have the viscosity to two phase viscosity is this basically will be hm, if we do not know alpha then we can use beta here. So, this will be sigma i rho i beta i which is basically rho 1 beta 1 plus 1 minus beta 1 rho 2 and this two is liquid and beta is for gas. So, we have rho gas.

And mass fraction weighted average of viscosities. So, where mu T P is equal to rho gas x plus 1 minus x rho liquid. So, different combinations of volume fraction or mass fraction weighted average of viscosities and densities have been used to predict the pressure drop using homogeneous flow model. In my opinion it is appropriate to use if the if the liquid phase is in contact only liquid phase in contact with the wall, then probably one can use the viscosity of water and try to see how well does it do part of correlation ok.

Now for pressure drop another very famous correlation specially in large diameter channels is known as Lockhart Martinelli correlation and then further down the constants in that given by (Refer Time: 44:43) and Lockhart Martinelli correlation and ah. So, it uses the concept of two phase multiplier. So, it gives that the pressure drop in two phase.

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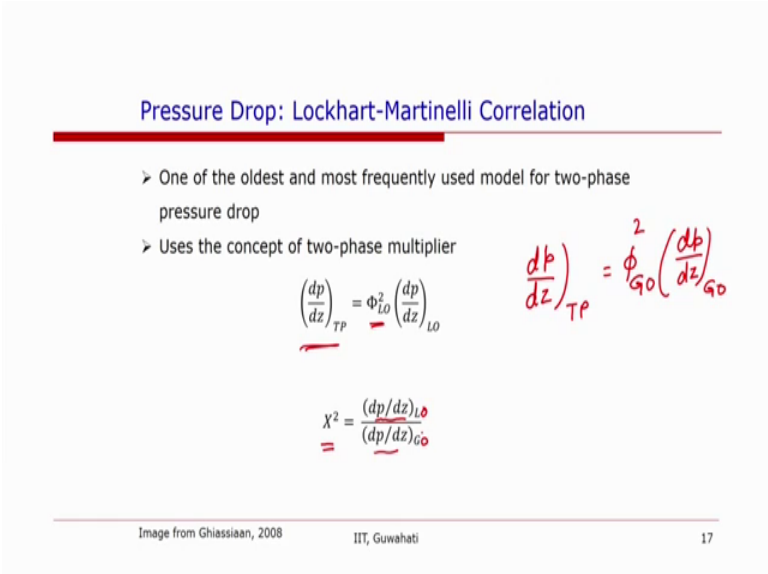

Is equal to a two phase multiplier, which has been given as phi L O square. So, a liquid only into d p by d z L O, one can also have d p over d z two phase is equal to 5 gas only squared and d p by d z for gas only. So, there are different correlations for this also.

Now, the Martinelli parameter there is that we have already discussed for void fraction is the ratio of the pressure gradient, if it is only liquid only flow and press the gradient if it is gas only flow. So, the x square is the ratio of d p by d z liquid only and d p by d z gas only.

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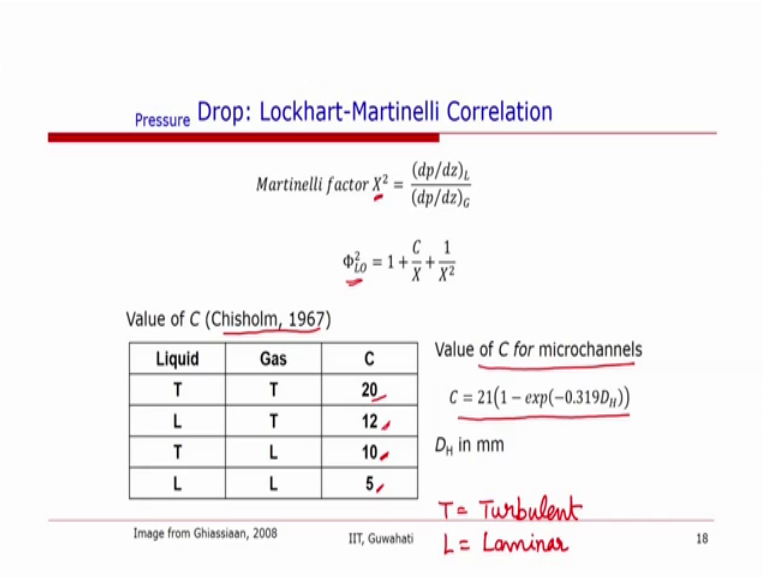

So once we have obtained Martinelli parameter then from Martinelli parameter one can calculate the two phase multiplier and remember this two phase multiplier is required to obtain the pressure drop in two phase. So, this is 1 plus C over X plus one over X square now X is Martinelli parameter. So, we know this from here and we already know how to calculate the d p by d z in liquid and gas cases one can calculate the Reynolds number for liquid phase for gas phase and obtain a and use the appropriate correlation either f is equal to 16 by Re or f is equal to 0.079 Reynolds to the power minus 1 by 4 the Braziers correlation.

So, one would have obtain X from there now the value of C. So, Chisholm he gave a table that for turbulent. So, T here stands for turbulent and L is for laminar flow and for turbulent flow, the constant C is 20 for liquid laminar flow and gas turbulent flow it is C is equal to 12 and for turbulent liquid flow and laminar gas flow C is 10 and both laminar C is equal to 5.

Now the modification there has been a modification for micro channels and says that C is equal to 21 1 minus exponential minus 0.319 D H where D H is in millimeters. So, one has taken into account the effect of channel diameter into this. So, otherwise what we have is the from this the effect of the flow rates of the two phases the viscosity of the two phases and the density of the two phases in these correlations.

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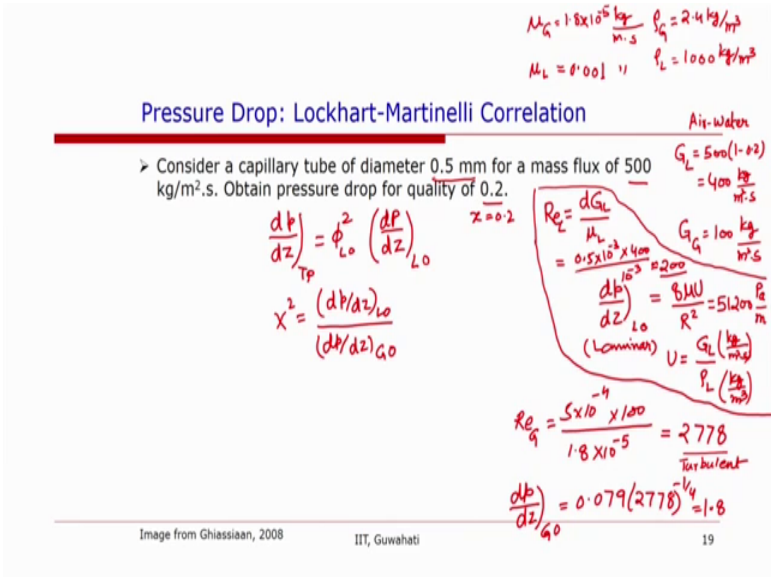

Now let us just look at that can we apply this Lockhart Martinelli in a correlation to obtain the pressure drop point quality of 0.2 in a capillary of diameter 0.5 mm for air water flow. And we can take the density of gas is about 2.4 kg per meter cube density of water. Let us take as 1,000 kg per meter cube viscosity of gas is about 1.8 into 10 to the power minus 5 kg per meter per second and viscosity of liquid is which is water.

So, again 0.001 kg per meter per second ok. So, if we have to calculate the pressure drop in such case we have given the mass flux and we have been given quality. So, quality is 0.2 and mass flux G is 500 kg per meter square per second. So, we have G of liquid is equal to 500 into 1 minus 0.2, which is 400 kg per meter square second and G of gas is equal to 100 kg per meter square second. So, now to obtain two phase pressure drop, remember the pressure drop that we will obtain using this is only frictional pressure drop. So, that will be equal to phi.

Let us take liquid only square. So, this is two phase multiplier in to d p by d z into liquid only pressure drop. So, first we need to find out the liquid only pressure drop. So, we can calculate d p by d z liquid only before that we need to calculate the Reynolds number for the liquid phase. So, Reynolds number is d channel diameter G liquid over mu liquid. So, if we substitute the values we will get 0.5; 10 to the power minus 3 into G L is 400 divided by 10 to the power minus 3 and this will give us a value of about Reynolds number of 200. So, basically the flow is laminar liquid flow.

So, we can say that for laminar flow d p by d z is equal to 8 mu U over R square and when we substitute the values what we will get is 51000 200 Pascal per meter G 1 can obtain or U, because U is not known. So, U 1 can obtained by G liquid by rho liquid you can say that this G L is k G per meter square per second and k G is k G per meter cube. So, units will be meter square second ok. So, we have obtained the pressure gradient for the liquid flow. Now, X square which is the martinelli parameter that is d p by d z liquid only divided by d p by d z gas only.

So, we need to find out the pressure drop in the gas phase as well. So, we will need to find that quality Reynolds number of liquid we need to find Reynolds of gas. So, that is again 5 into 10 to the power minus 3, which is channel its not minus 3 its 5 into 10 to the power minus 4 into 100 which is the gas flow rate divided by mu of gas which is 1.8 into 10 to the power minus 5.

So, we get the Reynolds number of gas phase is about 2,000 700 and 78. So, let us make this calculations from separated. So, that is the calculation for Reynolds number of liquid phase, this is Reynolds number of gas phase and from that, because this is more than 2,000, 100. So, we will use the Braziers correlation we will treat at as turbulent flow its not.

So, then we will have d p by d z gas only is equal to 0.0792778 raised to the power minus 1 by 4 and that will give us a value of about 1.8 its not d p by d z it is basically f.

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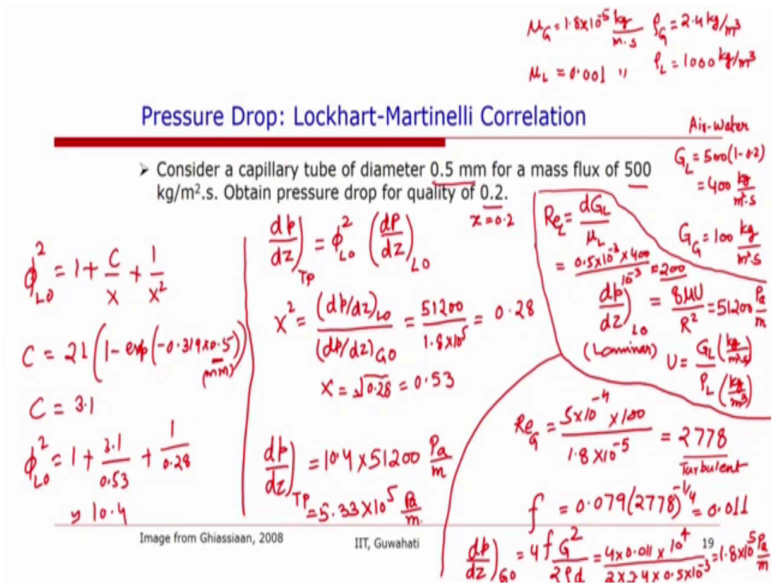

Frictional pressure drop so, once we have obtained this fanning friction which comes out to be about 0.011, then one can calculate d p by d z for gas only phase is equal to 4 f this is fanning friction factor into G squared over 2 rho and channel diameter. So, once we substitute here we get 4 into 0.011 into 10 raise to the power 4, which is 100 squared divided by 2 into rho is rho gas is 2.4. Channel diameter is 0.5 into 10 to the power minus 3 and when we substitute this we get value of 1.8 into 10 to the power 5 Pascal per meter.

So, we have got the pressure drop in the gas phase pressure drop in the liquid phase and from that we have got X square which is 51 200 Pascal per meter divided by 1.8 into 10 to the power 5 Pascal per meter and the ratio comes out to be 0.28 and we will get X is equal to square root of 0.28 which is about 0.53. Now, we need to find out phi L O square or two phase multiplier one plus C by X plus 1 over X squared. So, we know everything else the only thing we need to know its C and we have seen that C is equal to 21 into 1 minus exponential minus 31 minus D H

So, C is equal to 21 into 1 minus 0.319 oh sorry 1 minus exponential minus 0.319 into 0.5. So, one gets the value of C to be 3.1. Remember this is in mm. So, one gets the value C is equal to 0.31. So, phi L O square is equal to 1 plus 3.1 divided by 0.53 plus 1 over 0.28 and this comes out to be about 10.4. So, d p by d z for two phase is equal to phi L O square is 10.4 multiplied by liquid only dp by dz which is 51 1000, 200 Pascal per meter or this is about 5.33 into 10 to the power 5 Pascal per meter ok.

So, that is the pressure gradient for two phase flow and that we have used Lockhart Martinelli correlation here with the correction by (Refer Time: 59:56) changed for the value of C here. So, I would suggest that you should be a doing some of such exercises to obtain the feeling of numbers and to use these correlations to verify some of the literature data with this correlation.

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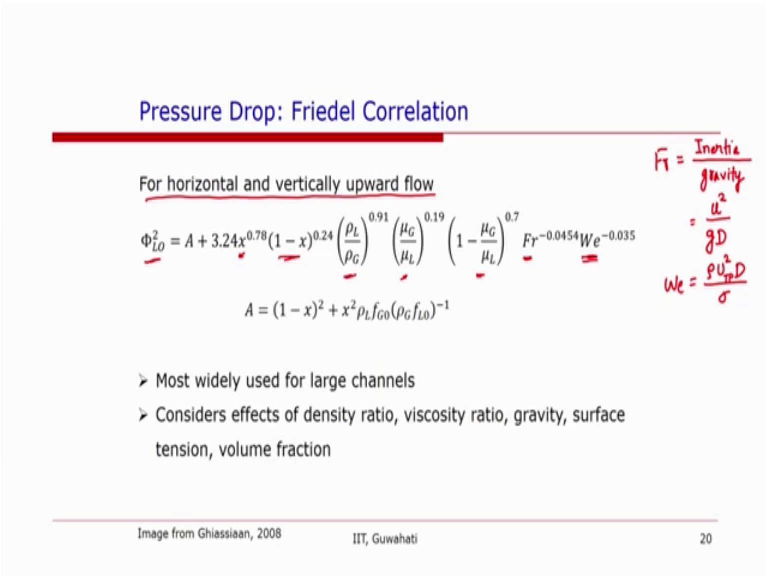

Another correlation which has been used in large channels is Friedel correlation and there are two different correlation one is for horizontal and vertical upward flow and the other one is for vertically downward flow I have not put it down here. So, this expression you see that the two phase multiplier. So, we are still using the same correlation like this where we have to have a two phase multiplier. So, this two phase multiplier is being defined by Friedel which is 1 plus.

Sorry, a plus 3.24 and it is a function of x and 1 minus x, where x is the quality, then the density ratios viscosity ratios and Proud number and Waver number. So, Proud number is ratio of inertia and gravity. So, that is basically u square over g D and one can use U T P here and Waver number we have already seen. So, that is rho U square D over sigma ok. And one can use U T P here ok.

So, in this correlation the while the Lockhart Martinelli correlation or the other correlation they do not have the effect of their surface tension whereas, the effect of surface tension has been taken into account in this case and. So, this might be a good idea to check the validity of Friedel correlation for micro channels. So, which has been done by some researchers ok.

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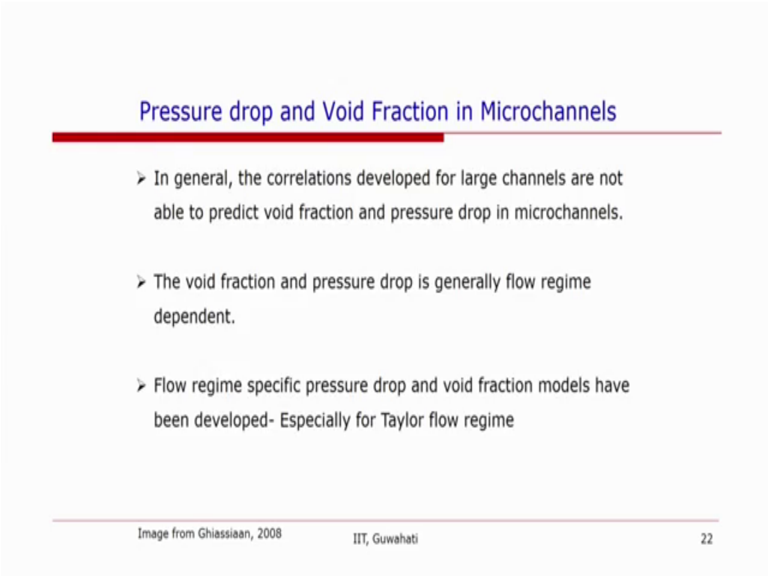

So, in general the correlations that has been developed for large channels they are not able to predict void fraction and pressure drop in micro channels. As it is some modifications has been suggested; one of the flow regimes that is; most important or that has been used for almost every application of micro channels is Taylor flow or slug flow region. So, it makes sense to get the void fraction and pressure drop correlation for this flow regime. So, we will just talk briefly about that correlation.

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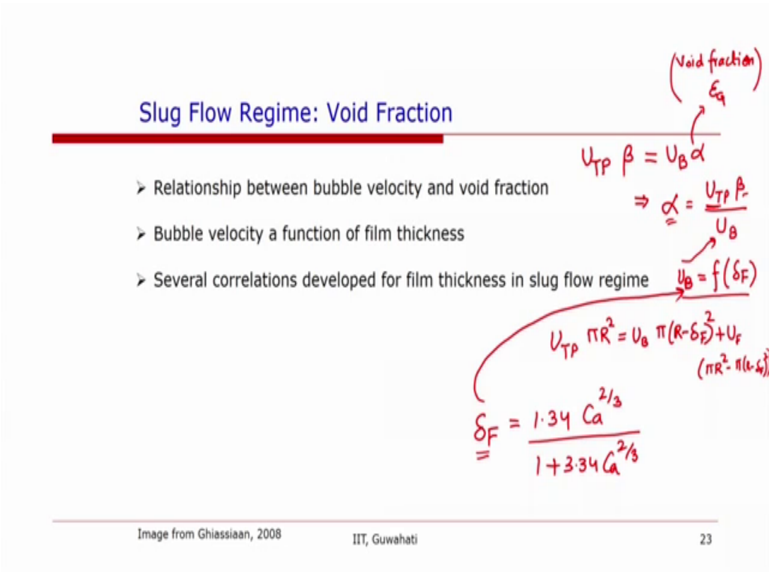

Now, for the slug flow regime, one can write a correlation between U T P we have already seen it for when we discussed slug flow or Taylor flow. So, one can write the relationship between the bubble velocity U B into alpha that is equal to U T P into beta.

So, if the bubble velocity has been measured or bubble velocity has been obtained, then one can obtain the void fraction the world void fraction or some places it is also called; we have termed it as epsilon G the void fraction. So, please do not confuse it is same as we have used in the Taylor flow discussion this void fraction epsilon G. So, this alpha is equal to U T P beta over U B.

Now we have also seen a relationship between U B as a function of film thickness that can be obtained as U T P into pi R square is equal T U B into pi R minus delta s squared plus U film, which is often neglected into pi R square minus pi delta s square plus U F which is (Refer Time: 65:11) pi r square minus pi R minus delta s square ok.

So, one can obtain U B as a function of film thickness. Now we have a number of correlations for film thickness the prominent or most popular of them being a Bretherton's correlations 1.34 C a to the power 2 by 3 and there is oscillation square modification which has been derived from class board. So, this is 1 plus 3 by 4, 1.34 C a power 2 by 3 divided by 1 plus 3.34 C a to the power 2 by 3.

So, one can use this correlation for film thickness and from this one can obtain bubble velocity and once one have optimal bubble velocity one can substitute here of course, one will know the flow rate of gas and liquid. So, one knows what is U T P and beta? And that is basically the super feasible velocity and gas. So, one obtain what is alpha? Now a small error in the film thickness you can make this you can test this for yourself that is small error in film thickness can have a large error in the velocity and consequent in the void fraction.

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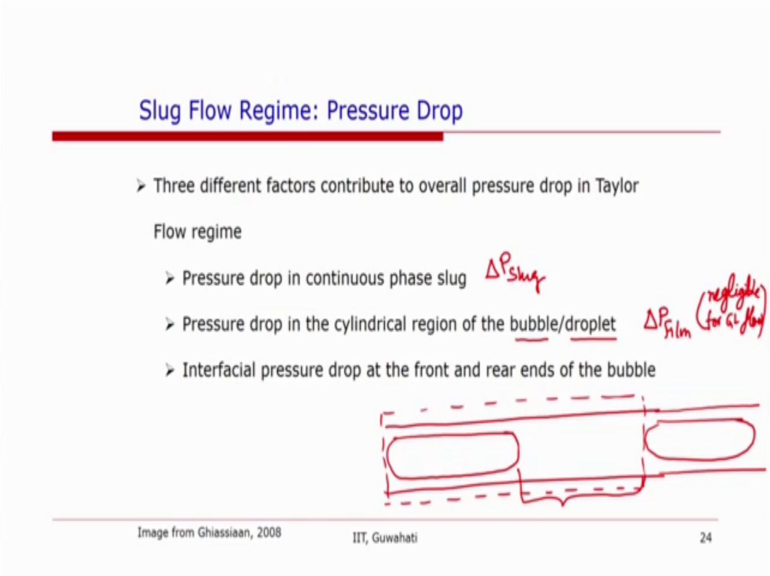

then for the pressure drop in the slug flow regime we can divide we take a unit cell which consists of a gas bubble. So, if we consider a unit cell consisting of one bubble and one liquid slug, then we have the there are different pressure drops that we need to consider the pressure drop in the continuous slug phase. So, delta P slug, then pressure drop in the cylindrical region or in the film region because this flow regime occurs in gas liquid and liquid flow.

So, one need to take into or one can make a generic or general correlation which can be valid for both the cases. So, one can say that that is the pressure drop in the film region which will be negligible for gas liquid flow then we have bubbles and then the interface pressure drop. So, the pressure drop and the front and back of the bubble.

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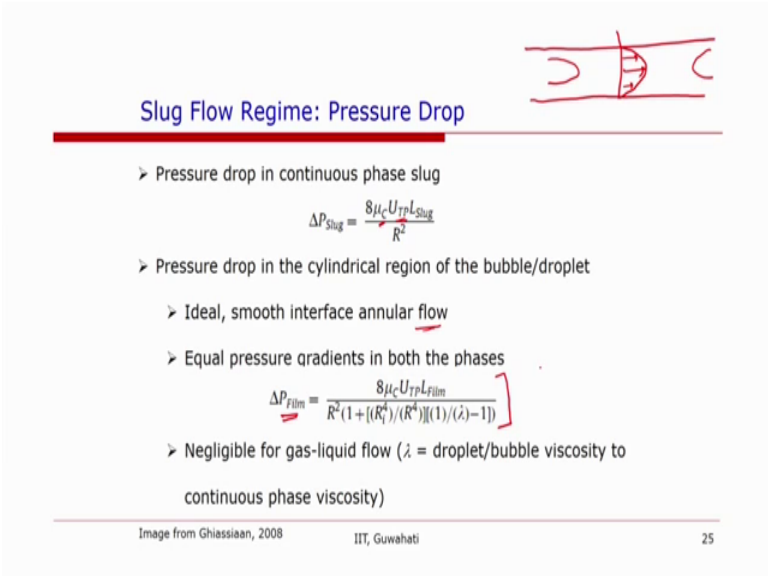

So it has been observed that if the slugs are long enough then one can use the single phase flow.

So, let us remind our self that the slug have a velocity profile which is close to parabolic velocity profile here, and if we accept near the ends of the bubble. So, one can use for long slugs. So, one can use the (Refer Time: 69:03) correlation from where one can obtained that the pressure drop is eight mu U L over R square where C is the continues phase U T P is the two phase velocity some of the costume the two phase is L is the length of the slug.

So, the pressure drop or the in the slug phase is given by this. Now pressure drop in the cylindrical region one can assume an ideal smooth interface and over flow which we will be discussing in a separate lecture. So, from that one can calculate the drift pressure drop in the film region and that comes out to be this for high value of lambda where lambda is the droplet or bubble viscosity to continuous phase viscosity.

So, this can be negligible for gas liquid phases.

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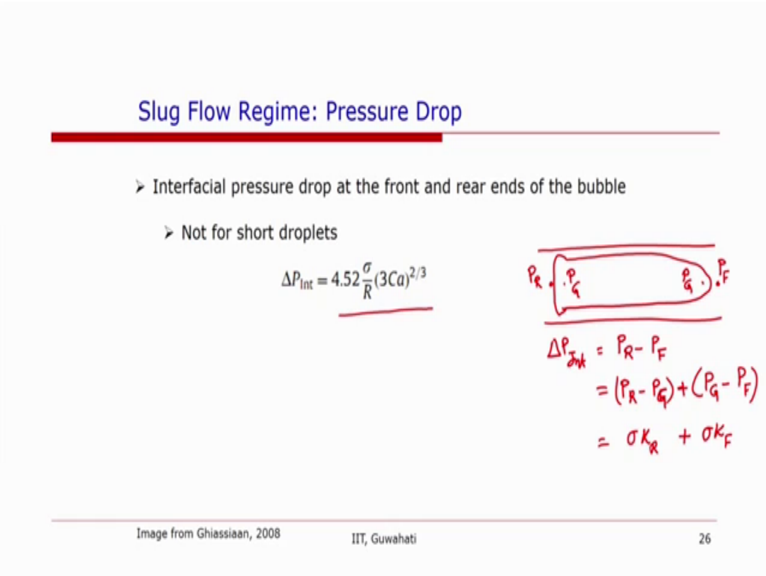

And then the third is in the pressure drop at the front and rear and rub the bubble side it will depend on the curvature and the two phase. So, if you consider the point here and point here the pressure drop at the front the; because the pressure in the gas is almost uniform. So, the pressure you can say the pressure in gas here and pressure in gas here is same. Now, you can say the pressure at front and pressure at rear. So, delta P because of the interface is delta P and the interface is basically P R minus PF and that one can write that PR minus PG minus P R minus PG minus P G minus P F or P R minus P G plus P G minus P F.

So, that we can give that the pressure drop at the front is two sigma or one can have this as sigma kappa and the front and this is sigma kappa at the rear. So, based on this one can calculate the pressure drop at the front and back and using Brethertons model following Bretherton this is the pressure drop that has been calculated because of the bubble.

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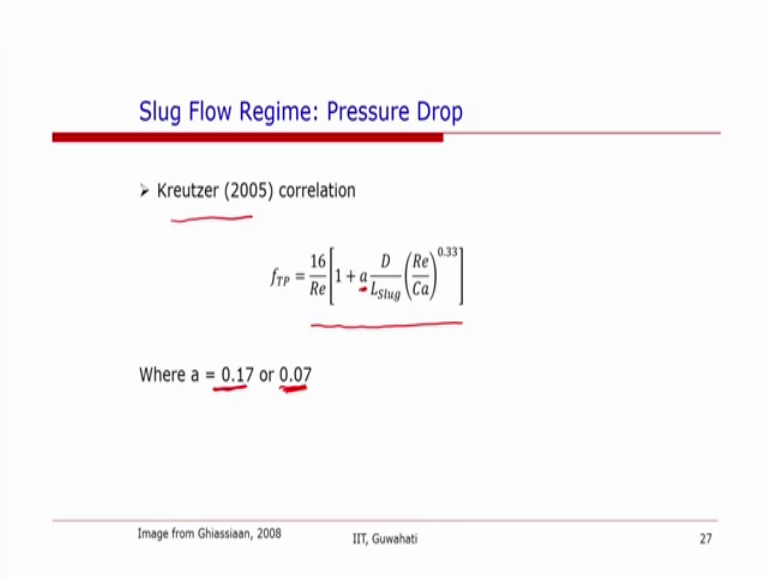

So, if we sum these two and this has been done by Kreutzer and he has obtained a frictional pressure drop or the friction factor for two phase flow in the slug flow regime which is 16 by Re 1 plus a D over where D is the channel diameter L is the length of slug and Re over C a raise to the power 0.33 and this value of a, what he obtained from his correlation was 0.17, and then he compared this with the numerical simulation and he obtain found that 0.07 fit well his numerical simulation whereas, from experimental data it was 0.17.

So, this is one correlation which is used or which can be used for pressure drop in the slug flow regime.

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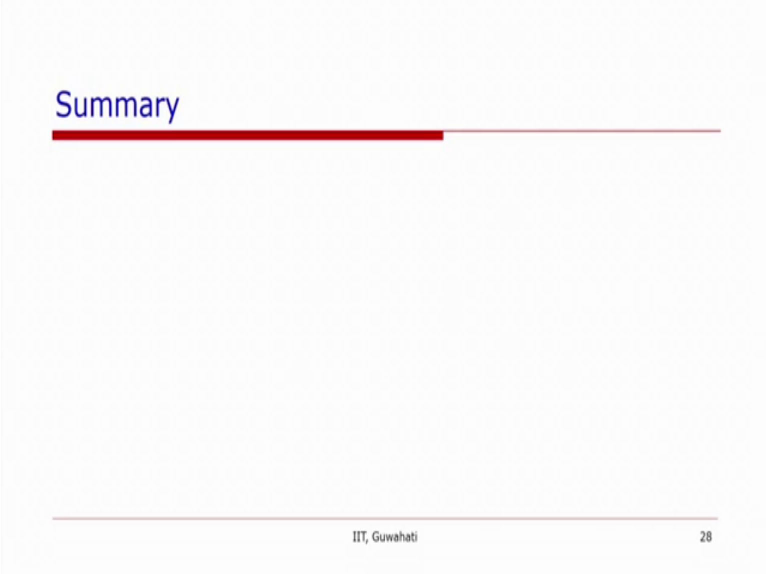

So in summary what we have looked at today is the void fraction and pressure drop correlations for large channels, and how they have been modified for small diameter channels. Please note that we have not discussed all the correlations that are available, there in the literature for large channels..

Neither have we discussed the correlations all the correlations that have been developed for micro channels or that have been modified; all the modifications are there for micro channels. We are not put together all the message is that one can modify these correlations and look at the; what are physical factors which are being taken into account into the small diameter channels and try to see a universal correlation can be there which seems to be almost impossible to me.

I would suggest some of you can take up the task of verifying the correlations. Especially; the once which include all the effect of all the factors such as surface tension and the channel diameters for example, once that come to my mind that CISE equal to relation for the volume fraction and Friedel correlation for pressure drop and look at their validity for a large bunch of data; if that can be those can be good enough for the small diameter channels ok.

And then we have looked at briefly the pressure drop and void fraction, how it can be measured? How the Taylor flow regime there has been some descent modifications in the in the recent paper by class board for pressure drop in the Taylor flow regime. So, I suggest that you look into it and that that generally mostly talk about the pressure drop the interfacial pressure drop that is there so.

Thank you.