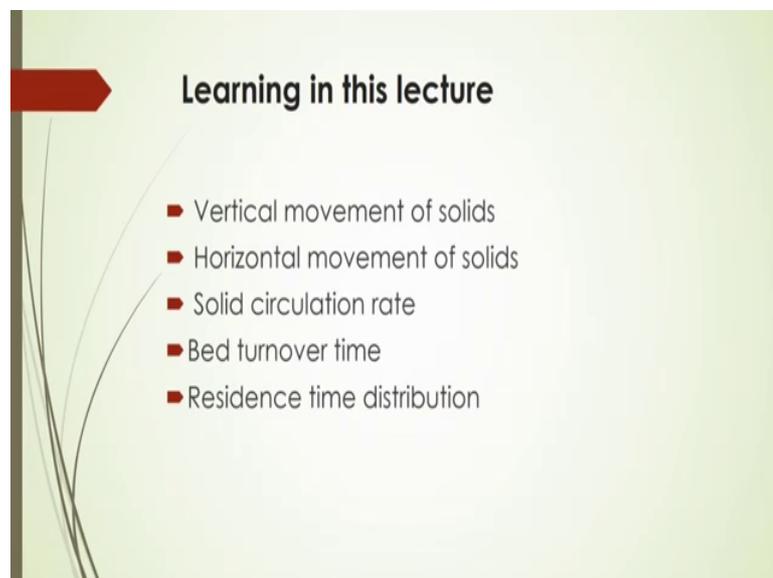


Fluidization Engineering
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Lecture – 24
Mixing: Gas-fluidized bed

Welcome to massive open online course on Fluidization Engineering. Today's lecture will be on mixing on gas fluidized bed system.

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In this lecture we will discuss about the vertical movement of solids, horizontal movement of solids, solid circulation rate, bed turn over time, and the residence time distribution.

Now, the previous lectures you have discussed about that entrainment elutriation and, because of who is the solid should be moving upward and then going downward, because of that terminal velocity of the solids and also type of the solids and also there will be some energy distribution by who is the bubbles go off and it will carry some solid particles and up to a certain height.

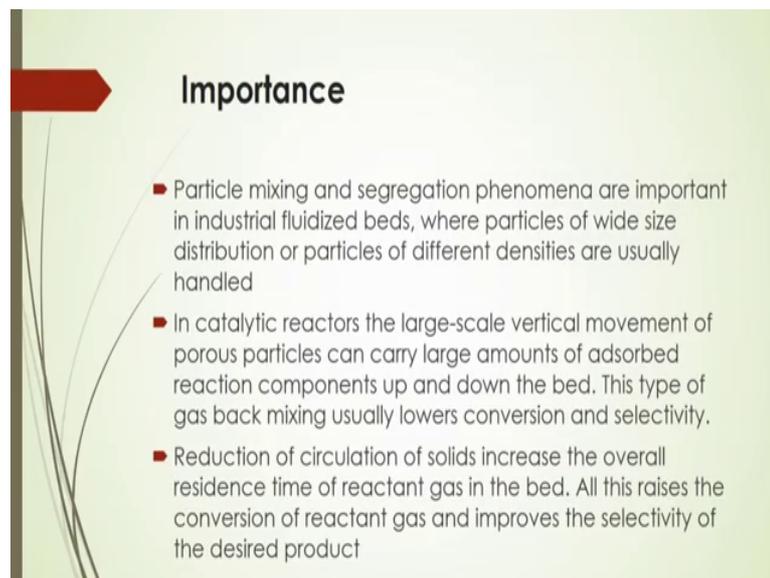
We will see that some solid particles will be break down, what because of some circulation of the solid pattern there; and you will see that that energy distribution also

inside the bed that will change the circulation rate inside the bed and, because of which that internal mixing of the phases inside the bed will be changing accordingly.

Now, how that vertical movement of the solids; that means, how vertically the solids are being mixed inside the bed that will be discussed and how to estimate that one and what should be the estimation parameter of the vertical or horizontal movement of the solids inside the bed that of course, you have to know and also, what will be the rate of that solid circulation flux; solid circulation flux and what are the different models to represent the solid circulation rate will be discussing here.

And then of course, a bed turnover time the solids how long it will be there inside the bed and during this operation and also what would be the residence time effective residence, time of the solids or in the phases inside the bed that of course, you have to know it will be very important for design of the particular efficient fluidized bed system.

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Importance

- Particle mixing and segregation phenomena are important in industrial fluidized beds, where particles of wide size distribution or particles of different densities are usually handled
- In catalytic reactors the large-scale vertical movement of porous particles can carry large amounts of adsorbed reaction components up and down the bed. This type of gas back mixing usually lowers conversion and selectivity.
- Reduction of circulation of solids increase the overall residence time of reactant gas in the bed. All this raises the conversion of reactant gas and improves the selectivity of the desired product

Now, vertical mixing and segregation phenomena are of course, the important in industrial fluidized beds, where particles of suppose wide size distribution or particles of different densities where these factors are very important in particular operations. So, that; in that case when this particle size distribution will be varying and also the density of the fluidizing particles are changing.

Then how this particle mixing will be inside the bed that you have to know this will be very important, because you know that some extent of this mixing will give the pattern of the solids inside the bed and also the selectivity and the conversion of the reactant gaseous reactant inside a bed that will be that will be actually affected by this solid mixing inside the bed.

As an example suppose the catalytic reactors in that case you will see large scale particle movement of porous particles are it is being used can carry large amount of activated or carbon or maybe some other catalyst particles for adsorption reaction, and then they are the large amounts of adsorbed reaction components they are of course, will be up and down movement of the bed because of the solids circulation and this type of gas back mixing usually lowers the conversion and a selectivity of the process.

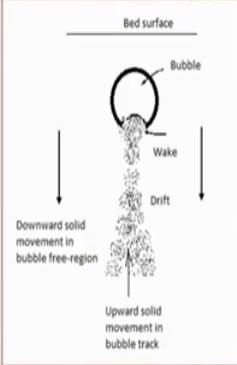
Now, if you reduce the circulation of the solids you can increase the overall residence time of the reactant gas which is sometimes in favor of the reaction, but too much back mixing also it will not be favorable in that case, but you have to get the optimum solid mixing or extent of mixing by which you can get the optimum mixing characteristics by which that optimum conversion and selectivity of the process can be obtained.

Now, all these raises the conversion and reactant gas and improve the selectivity of the desired product; if you know the extent of particle mixing and the segregation phenomena inside the fluidized bed the particle mixing in a gas fluidized bed generally induced by the passes of gas bubble you know that when bubbles will go up by the rising velocity.

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Solid mixing mechanism

- The particle mixing in a gas fluidized bed is entirely induced by the passage of gas bubbles.
- Particles are picked up by the bubbles in the bubble wakes at close to the gas distributor plate and are carried up to the top of the fluidized bed.
- Along the way up, the bubble wakes exchange their content with the particles in the rest of the bed in a phenomenon commonly known as **wake shedding**.



Solids mixing mechanism in fluidized beds —wake and drift

Of course, it will carry some solid particles from its wake and also there will be some drift effects by which the solid particles will be entrained to the bubble and then, it will be carrying over to the top of the bubble.

And in some extent you will see after certain height you will see some solid particles will be going downward also, then the movement of the solid will be there and some solid particles surrounding these bubbles may go again to the bottom part of the bubble in the wake region. And then it will be going up again from the top of the bubble region that is from head of the bubbles you will see some bubbles will be thrown away or swift away for which that solid particles will be moving some solid particles will be moving up and some solid particles will be going downward.

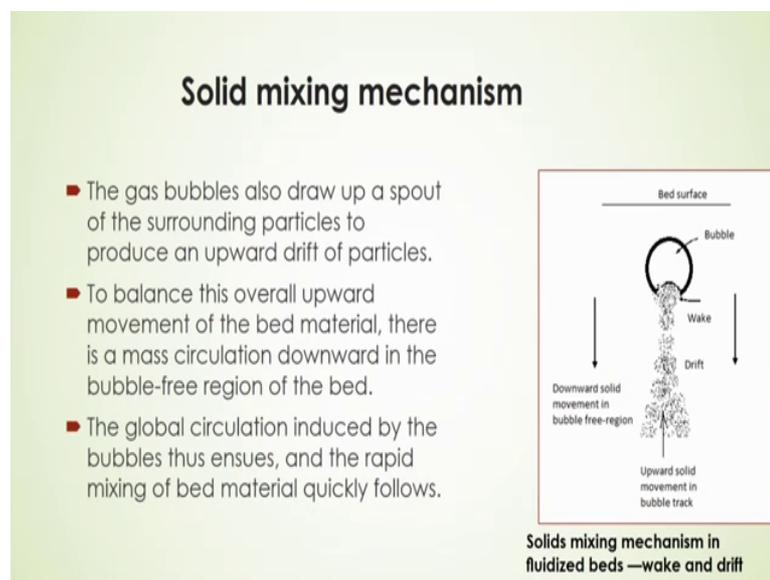
So, in this way there will be a movement of the solid particles in such a way that that along the axis of the bed there will be a circulation cell formation of the solid particles that depending on the other of course, factors like jet energy supplied there even the length of the bed and diameter of the bed; if you increase the diameter of the bed of course, there will be a circulation cell formation more circulation cell formation, and then more back mixing will be there inside the bed of that solid particles.

. So, during this mixing you will see that particles will be picked up by the bubbles in the bubble wakes at the close to the gas distributor plate and that will be carried off to the top of the fluidized bed. Later on and along the way of you will see that bubbles wakes

exchange their content during that bubble wake; that means, rising bubble and then in that case the bubble wakes will exchange their content with the particles in the rest of the bed and in this in the in such phenomena it is called that wake shedding there.

So, whenever bubble will rise by carrying that solid particles to the up from it is wake region it is seen that some solid particles will be actually exchanged from it is emulsion phase and from this wake to the emulsion and wake emulsion to this wake region they are in this way.

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So, this type of phenomena is generally called that wake shedding there so, because of this wake shedding also there will be some extent of solid mixing depends on and when the gas bubbles draw up a spout of surrounding particles to a produce an upward drift of particles.

There here in this picture it is given that in this case when gas bubbles drop a some particles and then the surrounding particles to produce and here upward drift of the particles here and to balance this you will see of course, some amount of solids will of course, move downward and because of that should be the balance of the mass there.

So, there is a mass circulation downward in the bubble free region of the bed to balance this happening and the global circulation that induced by the bubbles, thus ensures and the repeat mixing of the bed material quickly will be followed. So, in this case what is

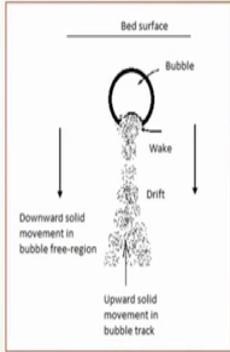
that solid mixing? Basically, the solid will be rising up by this wake up the bubbles when it will be rising up and then some solids due to the drift effect some other solids from the bottom it will be taken up and this type of phenomena is called drift.

And then when drift will be there will be a there will be a overall move upward movement of the bed material and for which there will be some imbalance, in that solid circulation solid movements and, because of which some masses circulation of the downward motion will be happened which is going to the bubble free region of the bed and in this way mass balance will be there overall mass balance will be there. So, this is the mechanism of solid in the fluidized bed that wake and drift mechanism.

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Solid mixing mechanism

- According to **Rowe (1971)**, the amount of solids induced upward by a single bubble is equal to about 0.6 times the bubble volume, of which about 60% is due to drift actions.
- **Fane and Nghiem (1983)**, who found that the volume of solids set in motion by a single bubble is about 0.5 to 0.8 times the bubble volume and approximately 75% was due to drift.

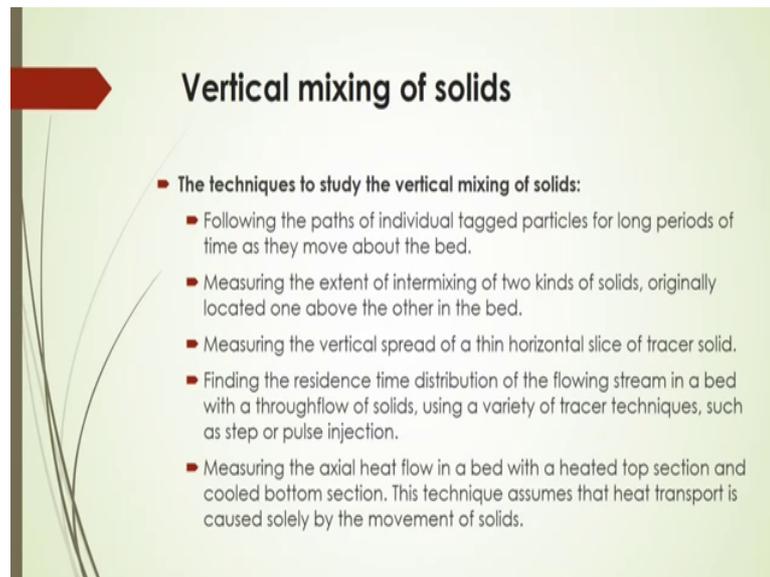


Solids mixing mechanism in fluidized beds —wake and drift

And now around 1970 1971 they have actually explained that the amount of solids that induced by upward induced upward by a single bubble which will be equal to 60 percent times the bubble volume that is 0.6 times the bubble volume of which we will see 60 percent is due to the drift action.

So, the whatever portion of the solids will be going up that will be 60 percent for action of the dream. And Fane and Nghiem 1983 found that the volume of solids that set in motion by a single bubble which will be about 0.5 to 0.8 times in the bubble volume and it will be approximately 75 percent due to the drift action.

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Vertical mixing of solids

- The techniques to study the vertical mixing of solids:
 - Following the paths of individual tagged particles for long periods of time as they move about the bed.
 - Measuring the extent of intermixing of two kinds of solids, originally located one above the other in the bed.
 - Measuring the vertical spread of a thin horizontal slice of tracer solid.
 - Finding the residence time distribution of the flowing stream in a bed with a throughflow of solids, using a variety of tracer techniques, such as step or pulse injection.
 - Measuring the axial heat flow in a bed with a heated top section and cooled bottom section. This technique assumes that heat transport is caused solely by the movement of solids.

And what should be the vertical mixing of the solids inside the bed to estimate how to estimate that vertical mixing of the solids there are several techniques to estimate that vertical mixing of the solids inside the bed.

The techniques of study the vertical mixing of solids are here given we are following there are sometimes you have to follow the paths of individual tagged particles or long periods of time as they move about the bed. So, you can estimate what will be the extent of vertical mixing of solids by following the paths of individual tagged particles individual tag some you have to insert some tag particles which is to be tagged in such a way that you have to observe that particles; how it will be moving what will be the track of that particles and how long it will take to cover the track. And in that way that you can find out for the period of time ah; when it will move about the bed and by, which you can say what could be the extent of mixing of that solids inside the bed.

You can also estimate the vertical mixing of the solids by measuring the extents of inter mixing of two kinds of solids. Originally that are located one above the other in the bed. Now in this case what you have to do that there will be a; solids which will be inter mixing some solid particles will be moving aside from each other; in such a way that that will change their location from one position to another position.

Now, there will be some inter mixing; like that one solids will go another position and from those position another solids will come in this position. So, there will be a exchange of particles from one portion one region of the bubble to that another region.

Now, what should the by exchange coefficient of that particles by which you can estimated the mixing of the solids inside the bed. Also you can measure the vertical mixing of the solids by measuring, this vertical spread of a thin horizontal slice of tracer solid. Suppose, if you add some tracer particles, but they are now how that tracer particles concentration will be changing, along the axis of the bed that you can obtain and from which you can calculate the extent of mixing of that solid particles inside the bed.

Again, there is another important method that by knowing the residence time of the particles also you will be able to find out what extent of mixing of the solid particles inside the bed happens; Now you can find first the residence time of the distribution; of the fallings of the flowing stream in a bed with a through flow up solids, and using a variety of research techniques such as step or pulse injection the you can obtain the extent of mixing of the solids there.

So, in this case you will see that whatever tracer particles will be used that should not be reacting with the adsorbed gas portion of the solids there and you have to use you have to use the tracer particle in such a way that that there will be knows actually concentration change with temperature also.

Now, if you add some tracer particles they are inside the bed; and if you if you take samples from different location of the bed and finding out the concentration of that the tracer particles they are with respect to time, then you will be able to calculate; what should be the mixing extent of mixing or intensity of the mixing inside the bed. Now, in that case you have to feed that concentration and time profiled; concentration time profile of the tracer particles there inside the bed with some distribution model.

Now, generally axial dispersion model is the most used and sometimes you will see another important model that is called tank in (Refer Time: 15:44) models the; being used to calculate the intensity of the mixing inside the bed by this tracer techniques.

Another important method to find out the mixing of solids inside the bed by heat input method. In this case you have to measure the axial heat flow in a bed with a heated top section and cooled bottom section; and this technique assumes that that heat transport is caused solely by the movement of the solids.

In this case what will happen that; if you if you allow some heat from the top section either by solid particles or by fluid, then you have to measure how this temperature of this inlet it is changing at a certain distance of fluidized bed with respect to time and from which you will be able to calculate the temperature versus time profile and from which be able to calculate what will be the mixing characteristics from did from the what is that dispersion of the solids inside the bed.

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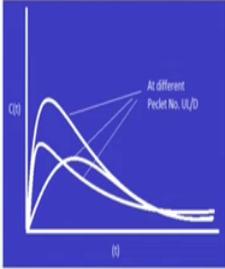
Dispersion Model

- The dispersion model equation for the dispersion process is

$$\frac{\partial C_p}{\partial t} = D_s \frac{\partial^2 C_p}{\partial z^2} - u_s \frac{\partial C_p}{\partial z} \quad (4)$$

C_p is the concentration of tagged particle

- The dispersion model does not always well represent the vertical mixing of solids
- The dispersion model well represented the mixing during turbulent fluidization where the bed looked close to homogeneous, but fit the data poorly when the bed is in the bubbling regime.
- The dispersion model to reasonably represent the vertical mixing in tall beds in which rather small-scale mixing is taking place.



Now let us discuss the dispersion model by which you can calculate what will be the dispersion? What will be the intensity of the mixing inside the bed? The dispersion model equation for the dispersion process is that; here how this concentration of the tracer particle with respect to time would change that will be is equal to $D_s \frac{\partial^2 C_p}{\partial z^2} - u_s \frac{\partial C_p}{\partial z}$.

So, in this case you can calculate the tracer concentration first you have to measure this tracer concentration with respect to time. And then feed this model; by boundary conditions that can be solved by numerically or by analytically also Levenspiel he has given the analytical solution of this equation and also you can solve this by a numerical

solution to find out this D_{sv} this D_{sv} is called this vertical dispersion coefficient of solid and C_{tp} is the concentration of tagged particle which is being used as a tracer particle inside the bed.

Now this dispersion model does not always well represent this vertical mixing of the solid sometimes some solid particles will be will be in a stagnant position inside the bed; in that stagnant position you will see some solid particles will be there. So, you will not be able to calculate that is the accurate measurement of the solid mixing inside the bed.

And the dispersion model well presented in the mixing during turbulent fluidization if you are fluidized bed in highly fluidization condition, then in that case that is fast fluidization or churn turbulent condition there may be less on of that stagnation of the solid particles in the bed; that means, dead zone of the solid particles or dead zone of the bed may be will be reduced at that first fluidization and churn turbulent condition.

And in that case you will see that maybe dispersion model will be well represented this phenomena of the solid mixing. And ah, but in this case the feed the data only, when the bed is in the bubbling regime sometimes, because of estimation of that concentration is quite difficult with respect to time in a particular at the highly turbulent condition.

And, the dispersion model to reasonably represent the vertical mixing in tall beds in which rather small scale mixing is taking place. So, this is important note to remember that this dispersion model will be beneficial only for that vertical mixing in tall beds.

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Vertical mixing of solids

- The vertical mixing rate in rather small beds is directly related to the gas velocity by
$$D_{sv} = 0.06 + 0.1u, \text{ [m}^2/\text{s]} \quad (1)$$
- The vertical mixing of solids is more rapid in large-diameter beds than in smaller beds, the relationship being given by
$$D_{sv} = 0.30d_t^{0.65} \text{ [m}^2/\text{s]} \quad (2)$$
- For the vertical mixing of solids in vigorously fluidized ($u > 10 \text{ cm/s}$) beds of fine Geldart A solids, **Miyachi et al. (1981)** found that the mixing data could reasonably be represented by the dispersion model with
$$D_{sv} = 12u^{0.5} d_t^{0.9}, \text{ [cm}^2/\text{s]} \quad (3)$$

Now, some observations that is given by different investigators of this vertical mixing of solids the vertical mixing rate in rather small beds is directly related to the gas velocity. So, Kuhnian Levenspiel they have given actually the relationship here this vertical this dispersion coefficient of the solid is related to the velocity of the; fluid or gas here.

So, this will be is equal to this is 0.06 plus 0.1 u; that means, here this vertical dispersion coefficient of solid is linearly related to this velocity of the fluid and the vertical mixing of solid is more rapid in large diameter beds; then in smaller beds, because in this case in the larger beds you will see there will be a more circulation cell of the solid or fluid stream inside the bed and, because of which that intensity of the mixing will be more; and whereas, in the smaller beds there will be no radial mixing for which only that vertical mixing will adsorb, but some adjacent to the wall there will be no actually that much mixing will be happen they are so, only in this case it is seen that for larger diameter it will be more dispersion coefficient and for smaller diameter tube it gives the smaller value of dispersion coefficient.

Now, Kuhnian Levenspiel they have given these correlations to calculate the; vertical dispersion coefficient of solids if how if you if you change that diameter of the column and from this equation you can calculate, what will be the value of dispersion coefficient of solids.

Now, if suppose a tube diameter is 1 meter then simple that D_{sv} is equal to 0.30 meter square per second and for the vertical mixing of solids in vigorously fluidized bed, where fluid velocity is greater than 10 centimeter per second. In that case, for fine Geldart solid particles Miyauchi et al 1981 developed one correlations with respect to the velocity of the gas and the tube diameter.

So, in that case they actually found that the mixing data reasonably actually well represented by the dispersion model with the dispersion coefficient of which is represented by this equation 3 as D_{sv} is equal to $12 u^{0.5} dt^{0.9}$; see here if you change both these gas velocity and tube diameter, then what should be the predicted value of the particle dispersion coefficient of solids you can calculate from this equation 3.

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Countercurrent solid circulation model

- This model views the solids moving in two streams, one rising and the other descending, with a crossflow or interchange between streams.
- Where the dispersion model does not fit well (gently bubbling and in not very deep beds), the countercurrent solid circulation model is often used.
- As per the model shown in Fig., some solids flowing up the bed and others flowing down the bed.
- This upflow and downflow with an interchange between streams is the basis for various counterflow models that have been proposed to account for the vertical mixing of solids.

Now, there are other models by which this solid circulation or solid mixing inside bed can be obtained here counter current solid circulation model; that is given by a Kuhnian Levenspiel in their textbook that, they are stated that this model be use the solids moving in a two streams one is the rising and another is the descending.

So, here in this case very interesting that whenever rising and down ward or descending movement is there; and parallely there will be a some exchange or interchange of the solid particles during the up flowing and down flowing solids there. So, this model

basically refers the refers the interchange of the solid particles during the movement of the solids inside the bed.

So, where the dispersion model does not fit well gently; you can say that for bubbling and in not very deep beds. So, in that case you can use this counter current solid circulation model for better results. Now, as per this model that is shown in figure; here you will see that some solids here the some solids going upward and some solids are flowing downward. So, in this case and other portions you will see there will be a cross flow that which will represent that interchange of the solid particles.

So, this up flow and down flow with an interchange between streams is the basis for the counter flow models that have been proposed to account for the vertical mixing the solids; here, if we if we segregate or you if we sectionalized the total bed into three regions; where this left side regions this is actually the portion or volume fraction of this total bed, that is represented by the gas which will be including bubble wake and emulsion and this portion this portion will be only the volume fraction through which the up flow solids will be moving upward and this portion will be representing the volume fraction through which the down solids will be moving downward.

. So, here if we if we denote that upward flowing fraction is f_u for the bed and downward fraction of the bed is f_d then $1 - f_u - f_d$ that will be represented by the gas volume fraction inside the bed along with the bubble wake and emulsion.

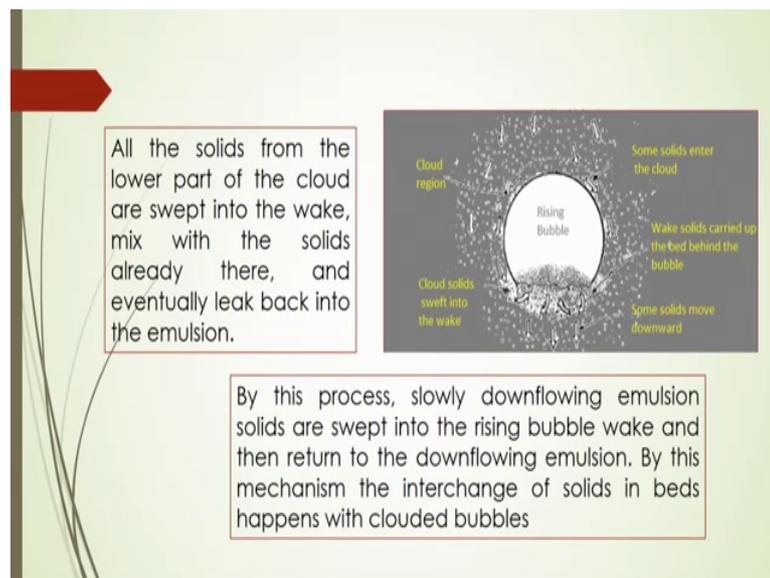
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Countercurrent solid circulation model

- The simplest version, introduced by **van Deemter (1967)**, divides the solids into two streams:
- One flowing up at a velocity u_{su}
- The other flowing down at u_{sd} with f_u and f_d ($= \text{m}^3 \text{ solids/m}^3 \text{ bed}$) being the bed fractions consisting of these streams
- Consider the movement of some labeled or tagged solids that constitute a concentrations of C_{su} and C_{sd} of the up- and downflowing streams

Now the simplest the simplest version that introduced by van deemter 1967 that divides the solids into two streams here one flowing up at a velocity.ah Let be U_{su} and the other that will be flowing down at U_{sd} , that is downward solid movements with u_u and u_d ; that means, meter cube of solid bar meter cube of bed. Being the bed fractions consisting of these streams and consider the movement of some labeled or tagged solids, that constitute a concentration of C_{su} and C_{sd} of the up and down flowing streams.

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If we consider that, that up flowing and down flowing stream with the u_u and u_d , then remaining that gas bubbles will be moving upward that $1 - u_u - u_d$; and the concentration for each of these texts each of this up flowing and solid volume fraction the concentration of the tags or it should be represented by that upward movement of solids for C_{su} and C_{sd} for that downward movement of the solids.

. So, all of these solids from the lower part of the cloud are that will be swept into the wake and mixed with the solids already there and eventually leak back into the emulsion. Now, by this process slowly down flowing emulsion solids are swept into the rising bubble wake and then returned to the down flowing emulsion by this mechanism the interchange of the solids in the beds happens; with the clouded bubbles as per shown in the figure.

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The differential equation describing the vertical movement of these tagged solids and their interchange can be expressed as

$$f_d \frac{\partial C_{sd}}{\partial t} + f_d u_{sd} \frac{\partial C_{sd}}{\partial z} + K_s (C_{sd} - C_{su}) = 0 \quad (5)$$

$$f_u \frac{\partial C_{su}}{\partial t} + f_u u_{su} \frac{\partial C_{su}}{\partial z} + K_s (-C_{sd} + C_{su}) = 0 \quad (6)$$

where the solids interchange coefficient K_s (m^3 tracer/ m^3 bed-s) represents the transfer of tagged solid from one stream to the other

Now, according to the actually that solid circulation concept the differential equation to describe the vertical movement of the tracer solids or you can say the tagged solids and their interchange can be expressed as by this equation 5 and 6. This 5 is for the differential equation for down flow solids and equation 6 for up flow solids.

So, for down flow solids here this you will see that f_d into this $\frac{\partial C_{sd}}{\partial t}$ this is nothing, but that the concentration of the solids are changing with respect to time, and this concentration change will be not only with respect to time it will be with respect to axial length also.

Now, whenever there will be a concentration change with axially and with respect to time and by varying the velocity of the gas the concentration of the solids downward solids and upward solids will be changing and this change of course, will be based on the internal circulation of the solids by interchanging or that interchanging or by just transferring of the solid particles from one location to the another location. For upward flowing of solids you will see that concentration also will be changing as per that; interchange of the solid particles or by exchange of the solid particles from one stream to another stream.

So, here this equation will represent this phenomena here this one important parameter in this model is called K_s this K_s will represented the solid interchange coefficient; and this will be actually basically the transfer of text solids from one stream to another stream.

So, solving this equation 5 and 6 to find out this K_s and from which you will be able to calculate what should be the actually intensity of the solids or degree of dispersion of the solids inside a bed; that can be obtained by this interchange coefficient solid interchange coefficient here.

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For a tall enough bed of fine particles and sufficiently large values of elapsed time, **van Deemter (1967)** showed that the changes in concentration of labeled solids could be represented by an effective dispersion coefficient given by

$$D_{sv} = \frac{f_d^2 u_{sd}^2}{K_s (f_d + f_w)} = \frac{f_d^2 u_{sd}^2}{K_s (1-\delta)(1-\epsilon_f)} \quad (7)$$

$$K_s = \frac{\text{volume of solids transferred from emulsion to the wake}}{\text{volume of bubble} \times \text{time}} \quad (8)$$

$$\approx \frac{3(1-\epsilon_{mf})u_{mf}}{(1-\delta)\epsilon_{mf}d_b}$$

Substituting (8) into (7)

$$K_s = \frac{3f_w u_{mf}}{2(1+f_w)\epsilon_{mf}d_b} \quad (9) \quad D_{sv} = \frac{f_w^2 \epsilon_{mf} \delta d_b^2 u_b^2}{3u_{mf}} \quad (10)$$

Now, for a tall enough bed of fine particles and sufficiently large the values of elapses time, you can say that van Deemter 1967 showed that the changes in concentration of the leveled solids or tagged solids and can be represented by an effective dispersion coefficient and how this effective dispersion coefficient will be depending on these interchange solid coefficient there inside the bed as per this solid circulation rate.

Now, this they he actually reported that this vertical dispersion coefficient of solids will be inversely proportional to this solid exchange coefficient if the solid exchange coefficient higher, then vertical dispersion coefficient will be lower.

So, here by this equation 7 you can calculate what should be the degree of this dispersion coefficient vertically for the solids. If you know that f_d , f_w means here the volume fraction of the bed through which the solids will be moving downward and if you know that this empty and u_{sd} and u_w also; so by this equation from this relationship you will be able to calculate, what should be the value for dispersion coefficient of solids.

Now this what; how what is the physical significance of this K_s ? The K_s nothing, but the ratio of volume of solids, that is transferred from emulsion to the wake to the volume of bubble into it is time. Now, it can be represented as $\frac{3}{1 - \epsilon_{mf}} \frac{u_{mf}}{u_{mf} \delta}$; what is this ϵ_{mf} ? ϵ_{mf} is nothing, but the minimum porosity of the bed and u_{mf} is the minimum velocity of the solids and δ is called what would be the volume fraction of the bubble, that is the of the total bed occupied by that bubble and the D_{sv} is the bubble diameter.

Now, this K_s you will see that since this k_{db} that is bubble diameter changing with respect to height that case also may change its height. So, accordingly vertical dispersion coefficient you can obtain axially also they are; how it will be changing. So, it may be you will see that if bubble diameter is actually increasing with height of the bed.

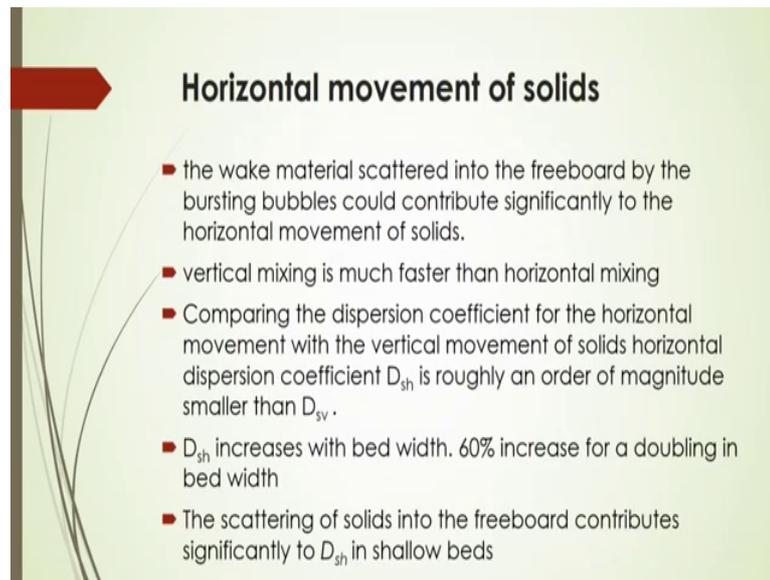
So, in that case the K_s value will change, now if K_s value will change; that means, K_s value if it is lower in that higher bed then; that means, from the away from the distributor then this axial dispersion coefficient of the solids will be increasing along with this height, because that K_s will be lower to the top.

And also it is seen that this K_s also will be changing with respect to that; what will be the wake section of the bubble in the bed. Now, if there is the wake fraction is suppose more than what will happen then the K_s may also will be more. Now, if suppose K_s is if volume fraction that the weight fraction of the bubble will come negligible; that means, for if you are having that that is bubble diameter is very small you will see that weight fraction will be negligible. So, according to that that K_s of course, will be small.

So, in that case that a vertical dispersion coefficient will be higher and ah; if you substitute this 7 and 8 into 7 then you can have this D_{sv} in terms of bubble diameter bubble rise velocity and the fraction of bubble volume inside the bed along with the wake fraction.

So, in this case what we observe that D_{sv} is proportional to the square of this wake fraction, and directly related to that minimum porosity and directly related to the volume fraction of the bubble directly related to the diameter of the bubble and directly related to the square of the bubble rise velocity, but inversely proportional to the minimum velocity of the bed gas velocity.

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Horizontal movement of solids

- the wake material scattered into the freeboard by the bursting bubbles could contribute significantly to the horizontal movement of solids.
- vertical mixing is much faster than horizontal mixing
- Comparing the dispersion coefficient for the horizontal movement with the vertical movement of solids horizontal dispersion coefficient D_{sh} is roughly an order of magnitude smaller than D_{sv} .
- D_{sh} increases with bed width. 60% increase for a doubling in bed width
- The scattering of solids into the freeboard contributes significantly to D_{sh} in shallow beds

Let us see what about that horizontal movement of the solids. Now, you will see that the wake is formed there; inside the bed behind the bubble; now the wake material that will be scattered into the freeboard by the bursting of bubbles that may contribute significantly to the horizontal movement of solids.

The vertical mixing is much faster than the horizontal mixing. Now, if you compare that dispersion coefficient for horizontal movement to the vertical movement of solids you will see that that horizontal dispersion coefficient is roughly an order of magnitude that will be smaller than vertical movement of solids.

Now horizontal dispersion coefficient increases with the bed width it is seen that 60 percent increase for a doubling in bed width the scattering of solids into the freeboard that will may contribute significantly to the horizontal dispersion coefficient in shallow beds. You will see that in shallow beds there will be a wider; that is distribution of the a gas portion and, because of who is that ejection of the jet and then also high kinetic energy of the jet gas jet; that means, more scattering of the solid particles near the distributor and because of which there will be a horizontal dispersion coefficient to be more larger than the vertical dispersion coefficient.

But along with that height of that bed you will see there will be a decrease of that horizontal dispersion compared to that vertical dispersion. So, this is basically this energy distribution as well as the solid circulation formation and also you will see that

how kinetic energy will be effective during the entrainment of the; solid particles inside the bed by the wake of the bubble.

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Mechanism for the horizontal movement of solids

- As a bubble rises, it pushed emulsion aside. However, the solids passing close to the bubble enter its cloud and are then drawn into the wake, whose diameter is roughly α times the bubble diameter.
- Solids mix uniformly in the wake and leave the wake from random positions, thereby giving rise to horizontal mixing. Solids further from the bubble move aside as the bubble passes, but return close to their original position

Kunii and Levenspiel (1991)

Now, mechanism for the horizontal movement of solids you will see as a bubble rise. So, it pushes emulsion aside and, then the solid passing close to the bubble which will be enters it is cloud and are then drawn into the wake and whose diameter is roughly α times the diameter as shown in figure. Here, now solids it will mix uniformly in the wake and leave the wake from the random position; and hence by giving rise to horizontal mixing and solids after that it will be move from the bubble aside as bubble passes and, but return close to their original position.

. So, this is the case here see you will see that this is the bubbles and the diameter of the bubble and surrounding these bubbles there will be cloud region and here this is the wake region. And in this case see how the mix mixed solids sometimes go up and some solid will be going downward also and that cloud that surrounding the bubbles which thickness will be ω into db and sometimes you will see that thickness also give you that, whether these bubbles will transferring the solid particles or not.

So, this you will see that wake region whether mixing; where mixing occurs there also you will see some solid particles will be coming to this mixing region and some particles will be going downward.

Now, if the cloud region will be more higher, what will happen; that is a thickness of the cloud region will be more higher, then you will see there will be a drag of solid particle that concentration of the solid particles surrounding, these bubbles will be more higher in that case.

The movement of the solid particles will be some extent hindering and some more solid particles will try to go downward to this wake region and after that and, because of the higher concentration and there will be a interaction of the solid particles some solid particles will go downward by the potential flow of the fluid there.

And you will see there will be a region where this actually lateral change of lateral exchange of the solid particles from this emulsion face to this wake region and wake region to the emulsion region will be changing and then this region; that is diameter wake region there the radius of that portion by which the solid particles will be moving horizontally that will be changed.

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Based on this mechanism the horizontal dispersion coefficient D_{sh} in terms of the Einstein random walk equation:

$$D_{sh} = \frac{(\text{fraction of solids that mix})(\text{mean square distance moved})}{4(\text{time interval considered})}$$

$$= \frac{1}{4} \left(\text{fraction of bed solids that enter bubble wakes} \right) \overline{\Delta r^2}$$

The mean square horizontal shift of a particle on passing through the bubble wake is given by

$$\overline{\Delta r^2} = \frac{(\alpha d_b)^2}{4} \quad (12)$$

where αd_b is the effective diameter of the wake

The diagram (11) shows a cross-section of a bubble with a cloud surrounding it. The cloud has a thickness αd_b . A trajectory of a particle is shown entering the bubble from the top. The wake region is shown below the bubble, where mixing occurs. The diagram also shows a plan view of the lateral movement of a particle, with the effective diameter of the wake αd_b and the time interval Δt indicated.

Kunii and Levenspiel (1991)

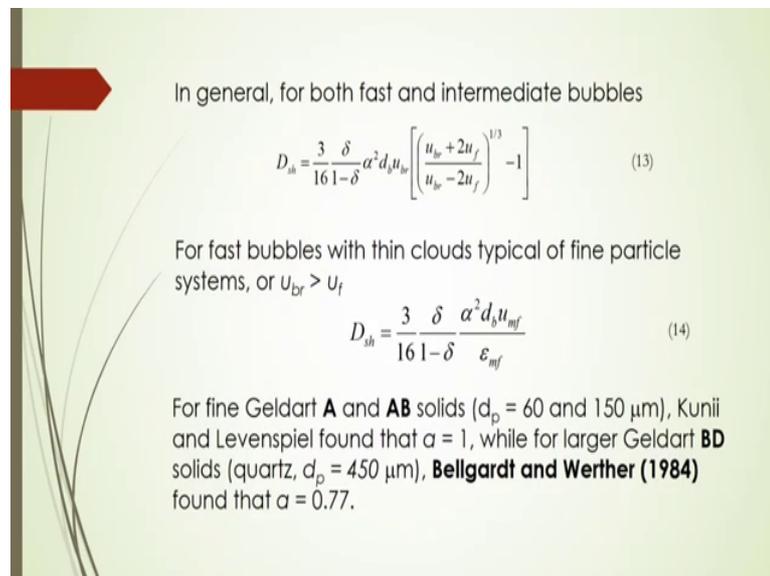
Now based on this mechanism the horizontal dispersion coefficient these D_{sh} ; in terms of that Einstein random walk equation you can see that D_{sh} that will be is equal to fraction of solids, that mix into mean square distance that will be moved by 4 into 4 times interval considered here.

So, what does it mean here you will see that that you can say 1 by 4 into fraction of bed solids that enter bubble wakes to mix their per unit time; that means, how many how much solids will be going to this wake and to mix there per unit time into this is delta r square this delta r square this is the distance this delta r.

So, this will be delta average of this delta r square this delta r square is nothing, but the mean square horizontal shift of a particle on passing through the bubble wake; that means, here from the emulsion phase to the wake or wake to the emulsion there will be a passage of the solid particles for movement.

So, what will be that width of that passage that will be represented by this delta r square? So, this average of this delta r square depends on the bubble diameter. So, it will be is equal to alpha into db whole square by 4. Now, this alpha into db is the effective diameter of the wake effective diame sometimes this bubble will be in spherical cap. So, there you will see that what will be the equivalent bubble diameter or effective diameter to be considered based on that spherical cap.

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In general, for both fast and intermediate bubbles

$$D_{sh} = \frac{3}{16} \frac{\delta}{1-\delta} \alpha^2 d_b u_{br} \left[\left(\frac{u_{br} + 2u_f}{u_{br} - 2u_f} \right)^{1/3} - 1 \right] \quad (13)$$

For fast bubbles with thin clouds typical of fine particle systems, or $u_{br} > u_f$

$$D_{sh} = \frac{3}{16} \frac{\delta}{1-\delta} \frac{\alpha^2 d_b u_{mf}}{\epsilon_{mf}} \quad (14)$$

For fine Geldart **A** and **AB** solids ($d_p = 60$ and $150 \mu\text{m}$), Kunii and Levenspiel found that $\alpha = 1$, while for larger Geldart **BD** solids (quartz, $d_p = 450 \mu\text{m}$), **Bellgardt and Werther (1984)** found that $\alpha = 0.77$.

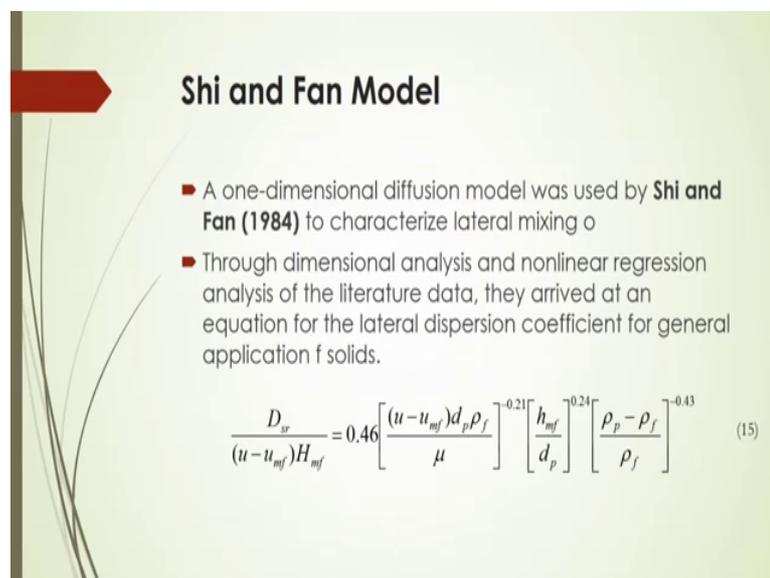
In general for both fast and intermediate bubbles use the Dsh is; that means, here dispersion coefficient of the solids horizontally that will be is equal to directly proportional to the bubble diameter and the bubble rise; velocity and you will see and actually it can be calculated by this equation 13; 13 by knowing that bubble rise velocity even fluid velocity and what will be the fraction of the bubble; that is occupied in the bed

and then you can you can you can calculate what will be the Dsh; that means, dispersion coefficient of solids horizontally.

Now for first bubbles with thin clouds typical value a fine particle system like; if u_{br} is greater than u_f bubble rise velocity, if it is greater than fluid velocity then horizontal dispersion coefficient of solids can be calculated by this here ; that means, here this u_{br} is the more dominant than fluid velocity. Then this fluid velocity to be will be negligible. So, simple from the equation 13 you can get this equation 14 to calculate the horizontal solid distribution coefficient for fast bubbles.

and for fine Geldart a type and mixture of this ab type solids it is diameter if suppose 60 and 150 micrometer Kunii and Levenspiel found that this alpha value generally is equal to 1 and while for larger Geldart bd solids and d_p is equal to 450 this alpha value it is coming 0.77. So, from this value you can calculate what will be the horizontal dispersion coefficient which can be obtained by this equation number 13 or 14.

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Shi and Fan Model

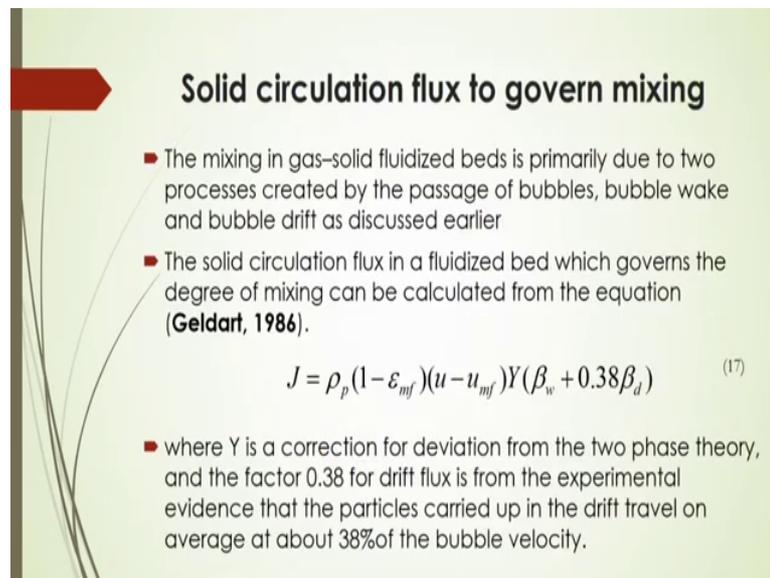
- A one-dimensional diffusion model was used by **Shi and Fan (1984)** to characterize lateral mixing o
- Through dimensional analysis and nonlinear regression analysis of the literature data, they arrived at an equation for the lateral dispersion coefficient for general application f solids.

$$\frac{D_{sr}}{(u - u_{mf})H_{mf}} = 0.46 \left[\frac{(u - u_{mf})d_p \rho_f}{\mu} \right]^{-0.21} \left[\frac{h_{mf}}{d_p} \right]^{0.24} \left[\frac{\rho_p - \rho_f}{\rho_f} \right]^{-0.43} \quad (15)$$

Now, there are other different models by different investigator. So, they have proposed to actually analyze the dispersion coefficient of solids inside the fluidized bed. Now Shi and Fan model they have given that one dimensional diffusion model they have given they have given one model based on that one dimensional diffusion model in 1984 to characterize the lateral mixing of the solids.

Now, through dimensional analysis and non-linear regression analysis of their experimental as well as literature data they arrived at an equation for the lateral dispersion coefficient for general application of solids that can be represented by this equation 15. So, this equation gives you the lateral dispersion coefficient; that means, the radial dispersion coefficient, this is a function of this is again the fluid velocity particle size fluid density viscosity and the minimum height of the fluidized bed. So, from this equation 15 you can calculate what should be the lateral dispersion coefficient.

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Solid circulation flux to govern mixing

- The mixing in gas-solid fluidized beds is primarily due to two processes created by the passage of bubbles, bubble wake and bubble drift as discussed earlier
- The solid circulation flux in a fluidized bed which governs the degree of mixing can be calculated from the equation (Geldart, 1986).

$$J = \rho_p (1 - \epsilon_{mf}) (u - u_{mf}) Y (\beta_w + 0.38 \beta_d) \quad (17)$$

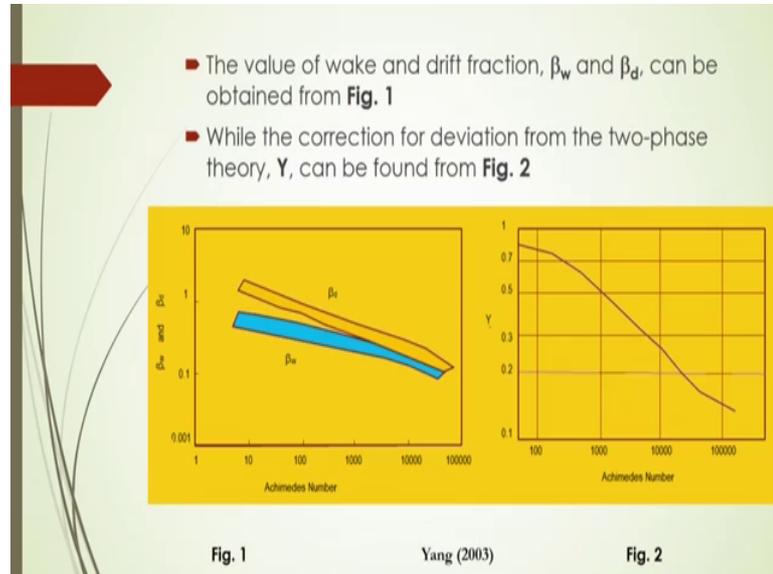
- where Y is a correction for deviation from the two phase theory, and the factor 0.38 for drift flux is from the experimental evidence that the particles carried up in the drift travel on average at about 38% of the bubble velocity.

Now, solid circulation flux to govern this mixing, now the mixing in gas solid fluidized bed is primarily that you know that due to the two processes created by the passage of bubbles bubble wake and bubble drift as discussed already. They are in the earlier section and the solid circulation flux in a fluidized bed, which governs the degree of mixing that can be calculated from the equation here as given by Geldart, 1986.

So, j is called solid circulation flux this can be is equal to rho p into one minus epsilon mf into u minus umf into y into beta w plus 0.38 beta d this Y this Y is a correction factor for the derivation from the two phase theory, they have derived it Geldart, 1986 he derived it, but the derivation is not given here ah, but according to them they have breached this final equation of the derivation and here; in this case capital Y is a correction factor for their equation and this will be is equal to 0.38 for drift flux and

which is from the experimental evidence that the particles carried up in the drift which is travelled on the average of about 38 percent of the bubble velocity.

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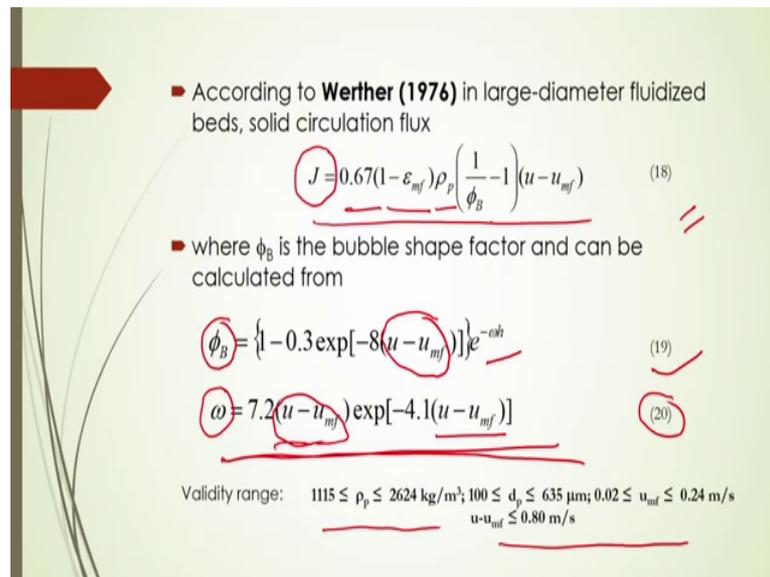


Now, this correction factor how it will be changing it is seen that that this correction factor Y it is the depending on the Archimedes number if Archimedes number is increasing, then this Y value is decreasing. So, you have to calculate this correction factor from this profile of given in equation given in figure tw2o here. So, from this figure for using for particular Archimedes number you have to calculate this Y factor.

And other factors are this parameter beta w and beta d this value of wake and drift fraction, this is beta w it is called wake fraction wake fraction and this beta d is called drift fraction. So, this can be calculated from this Archimedes number. Now there is a certain Archimedes number from which you can calculate this beta w or beta d from this profile given in the; is figure one here.

So, finally, you can calculate the solid circulation flux to govern the mixing by this equation 17 once you know this minimum porosity of the fluidized bed a minimum velocity of the bed and also what will be the velocity of the gas and getting the value of y beta w and beta d from the graph given here in figure 1 and figure 2 respectively.

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According to **Werther (1976)** in large-diameter fluidized beds, solid circulation flux

$$J = 0.67(1 - \epsilon_{mf}) \rho_p \left(\frac{1}{\phi_b} - 1 \right) (u - u_{mf}) \quad (18)$$

where ϕ_b is the bubble shape factor and can be calculated from

$$\phi_b = \left\{ 1 - 0.3 \exp[-8(u - u_{mf})] \right\} e^{-\omega h} \quad (19)$$
$$\omega = 7.2(u - u_{mf}) \exp[-4.1(u - u_{mf})] \quad (20)$$

Validity range: $1115 \leq \rho_p \leq 2624 \text{ kg/m}^3$; $100 \leq d_p \leq 635 \text{ }\mu\text{m}$; $0.02 \leq u_{mf} \leq 0.24 \text{ m/s}$
 $u - u_{mf} \leq 0.80 \text{ m/s}$

Now according to Werther 1976 in large diameter fluidized bed that solid this solid circulation flux can be expressed by another important factor that is called that is bubble shape factor. So, this is inversely proportional to this bubble shape factor here this bubble shape factor depends on that, what is that relative velocity of the fluid inside the bed and also it depends on the; what is that parameter; which is a function of relative velocity of the fluid; and here by equation 20 it can be calculated it is represented by omega.

So, here solid circulation flux is equal to 0.67 into 1 minus epsilon mf into particle density into 1 minus phi b minus 1 into u minus uv. So, this phi b is called bubble shape factor which will be calculated by this equation 19, once you know this omega omega can be calculated from this equation number 20. Now, this correlation actually is valid only within the range of this operating parameter given in the slides.

So, the solid circulation flux how to calculate it by Werther equation you can also calculate just if you are having that different bubble shape is there.

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Bed Turnover Time

- In actual practice where particles are continuously being fed and withdrawn, the bed turnover time should be as short as possible compared to the particles' residence time in the bed. Geldart (1986) suggested a ratio of residence time to turnover time of about 5 to 10.
- With the known solid circulation rate, the bed turnover time should be

$$t_T = \frac{H_{mf} A \rho_p (1 - \epsilon_{mf})}{M} \quad (21)$$

- In reality, particles spend different amounts of time in the bed, depending on
- the locations of feed and withdrawal nozzles,
- on flow and mixing patterns in the bed, and
- on many other factors.

- The particles' residence time is an even more important consideration in fluidized bed design.

Now, then what should be the bed turn over time what is that in actual you see that in practical field that where particles are continuously being fed and withdrawn. So, what will be the time spent for that the bubble the bed turnover time should be as short as possible compared to the particles. So, the residence time in the bed of course, it can be represented as those particles here

Now 1986 suggested a ratio of residence time to this turnover time of about 5 to 10 here this ratio; that means, here this turnover time to the; what is that ratio of ratios are residence time to the turnover time should be 5 to 10 here; now with the known solid circulation rate the bed turnover time should be this one. So, which can be calculated from the solid circulation flux. So, j is called solid circulation flux. So, this turnover time is inversely proportional to the solid circulation flux whereas, it will be directly proportional to the minimum height of the fluidizing condition.

So, here in reality particles that spend different amount of time in the bed that depends on the location of the bed and withdrawal nozzles on the flow and mixing patterns in the bed and also it depends on many other factors like; particle density viscosity of the fluids and the minimum porosity that had being actually controlled inside the bed, and the particles residence time is an even more important consideration in fluidized bed design, because anyway the what will be the residence time or retention time of the fluid

particles, that is contact with the fluid that you have to know that depends on that particle that is or you can see that reaction conversion as well as the selectivity of the process.

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Solids Residence Time Distribution in a Fluidized Bed

- Particle residence time distribution in a fluidized bed is more close to that of a stirred tank reactor (CSTR) or ideal backmix reactor than that of a plug flow reactor (Yagi and Kunii, 1961).
- In a perfect plug flow reactor, all particles have the same residence time, which is equivalent to the mean residence time of particles and can be calculated by

Handwritten notes and equations:

$$Pe = \frac{uL}{D} = \frac{Q_0}{-Q_0} \Rightarrow \bar{t}_R = \frac{W}{F_0}$$

W = weight of bed
F₀ = Solid feed rate

Now, solid residence time distribution in a fluidized bed. The particle residence time distribution in a fluidized bed is more close to that of a stirred tank reactor or ideal back mix reactor, then the that of a plug flow reactor very interesting that of course, any reactor whatever you are using for particular process that may not be that perfect mix reactor there may be not exactly the plug flow reactor there may be some deviation from the plug flow reactor.

So, that is why the no reactor can be as can be ideal, because there will be some back mixing of the solid fluid inside the bed at least for that solid gas particles there up flow there will be a internal fluid mixing. Internal solid circulation downward and upward movement of the solids may hindered the may deviate the ideal flow behavior of the reactor and it will be certain extent of non ideality will be there.

So, what will be that non ideality factor; that non ideality of the reactor can be estimated by that mixing characteristics, and then mixing characteristics. Actually one important parameter is called that retention time or back mixing time is there as well as the what is that dispersion coefficient and dispersion number also.

What is that dispersion number? Dispersion number is nothing, but one day pecklet number, what is that pecklet number? Pecklet number is defined that; what is that the ratio of spread by dispersion solid dispersion by and by conviction and by what is that dispersion by that molecular diffusion.

So, in that case this pecklet number is nothing, but is defined as UL by D . Now 1 by pecklet number is equal to D by UL this D by UL is called dispersion number or 1 by pecklet number. Now, if you increase if there is a dispersion number increase, then you can say that more mixing will be there if 1 by pecklet number in infinite; that means, they are the the reactor is going to it is ideal or tends to idea or plug flow reactor and this if 1 by pecklet number is tends to 0 means here you can say that dispersion will be less; that means, here are back mixing back mixing more back mixing will be there or there will be no mixing at all.

So, in a perfect plug flow reactor all particles have the same residence time, which is not at all possible in the fluidized bed and which is equivalent to the mean residence time of the particles and can be calculated by this \bar{t}_R that will be is equal to W by F_0 , what is W ? W is the weight of the bed and F_0 is the solids speed rate.

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For a completely mixed stirred tank reactor, the residence time distribution can be expressed by following ideal backmix reactor model

$$R(t) = \frac{1}{\bar{t}_R} e^{-t/\bar{t}_R}$$

\bar{t}_R = mean residence time of particles

$R(t)dt$ is the fraction of solids staying in the bed for the time period between t and $t + dt$.

Yagi and Kunii (1961) found experimentally that the equation represented the particle residence time in a fluidized bed quite well.

For a completely mixed stirred tank reactor the residence time distribution can be expressed by the following ideal bat mix reactor model here this is R t that will be is equal to 1 by $t R$ t to the power minus t by $t R$ bar here $t R$ bar is nothing, but W by F_0 .

Now this $R t dt$ this $R t dt$ is the fraction of solids that stays in the bed for a time period between t and the small period of t plus Δt here. Now Yagi and Kunii 1961 found that the equation represented the particle residence time in a fluidized bed will be quite well by this equation.

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The fraction of solids spending less than time t can then be calculated from

$$f = 1 - e^{-t/\bar{t}_R}$$

From the equation it can be said that about 18.2%, spends less than 20% of the average particle residence time in the bed.

\bar{t}_R = mean residence time of particles

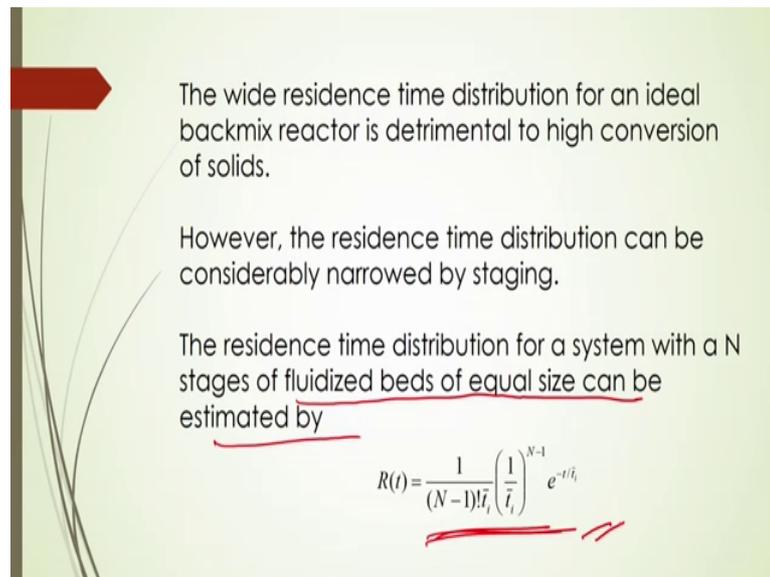
$$\bar{t}_R = \frac{W}{F_0}$$

W = weight of bed
F₀ = Solid feed rate

And the fraction of solids spending less than time t can then be calculated from this equation f is equal to 1 minus e to the power minus t by \bar{t}_R .

Now, from this equation it can be said that about 18.2 percent spends less than 20 percent of the average particle residence time in the bed. Where \bar{t}_R will be is equal to mean residence time of the particles which will be W by F_0 .

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The wide residence time distribution for an ideal backmix reactor is detrimental to high conversion of solids.

However, the residence time distribution can be considerably narrowed by staging.

The residence time distribution for a system with a N stages of fluidized beds of equal size can be estimated by

$$R(t) = \frac{1}{(N-1)! \bar{t}_i} \left(\frac{t}{\bar{t}_i} \right)^{N-1} e^{-t/\bar{t}_i}$$

So what we can say that wide residence time distribution sometimes will be favorable for this reaction kinetics and ah, but they are if there is more mixing then more retention time it will be there, but then also more mixing also sometimes it will be hindered the reaction, conversion and selectivity. So, you have to obtain the optimum condition of that mean residence time inside the bed so, that you can get high conversion of the solids or a reaction there.

Now, important is that you have to then reduce the back mixing inside the bed. So, nowadays there are several options are, actually by different investigators they have actually finding out that if the micro reactors are being developed for this fluidization also; then it may be useful for getting the plug flow phenomena of the fluidized bed ah.

Now, the research direction may also change to that developing the plug flow reactor. So, that you can get the high conversion of the solids inside the bed and ah; however, the residence time distribution can be considerably narrowed by the staging it also sometimes advantages to get the lower residence time there the residence time distribution of the system with a N stage of fluidized bed of equal size that can be estimated by this equation. If you are staging N stages there then residence time also will be there and based on this you can get the more reaction conversion of the solids.

So, I think in this lecture we have learned lot of things about that how what would be the mechanism of the solid mixing inside the bed; and how vertically and how horizontally

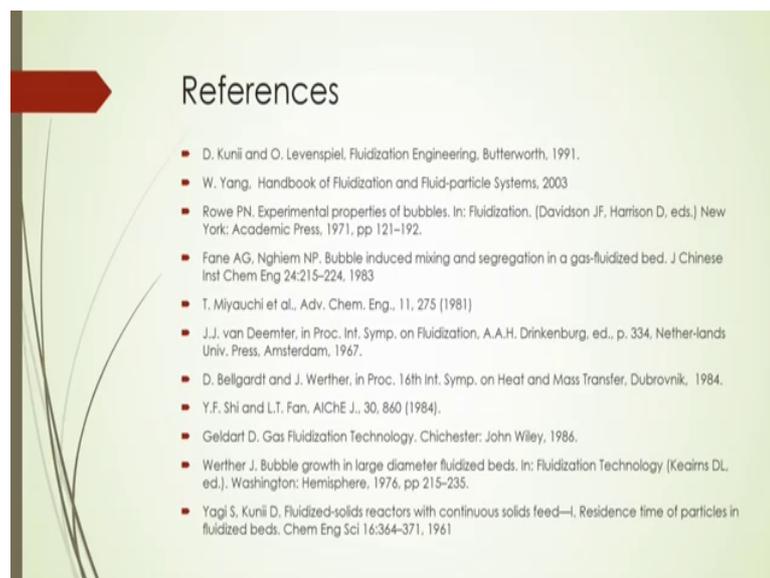
the solid mixings are being happened inside the bed; and how bubbles actually take part in the mixing of the solids. Then how solids are moving downward and upward, and if we do the model of that exchange rate of the solids, then you can obtain what will be the actually degree of mixing of solids inside the bed.

And, this is it is very important what would the extent of conversion or reaction inside the bed is taking place important also and sometimes the heat distribution also more residence time is required, from who is also you have to know that; what should be the residence time of the particle and also how this residence time will take part in the intensity of the heat transfer of the fluidized bed.

For taking part in different reaction as well as physical operation also and how this mixing characteristics can be modeled and how the size of the bubbles and the exchange rate of the solids, actually change the vertical and horizontal distribution horizontal dispersion coefficient of the solids inside the bed; that we can calculate here by the equation given by different investigators as shown in the slides here in this lecture.

So, thank you for this lecture and we will be discussing more about the particle segregation inside the bed here.

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In the next lecture thank you some references that you can follow for further reading.

Thank you.