# Fluidization Engineering

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## Lecture – 10
## Analysis of Frictional Pressure Drop in Fluidized Bed By Different Models

Welcome to the massive open online course on Fluidization Engineering. Today's lectures on analysis of friction of pressure drop in fluidized bed by different models. So, in the previous lecture we have discussed how to estimate the fictional pressure drop in three phase fluidized bed. And also we have given example that what will be the different contribution of different portion of frictional pressure drop and also total pressure drop that has already been discussed and also given example.

Now, in this lecture how to analyze this frictional pressure drop by the different models. This models maybe correlation models and from the basic or there may be some mechanistic models. So, let us see what will be the difference models by which we can analyze the frictional pressure drop in the fluidized bed.

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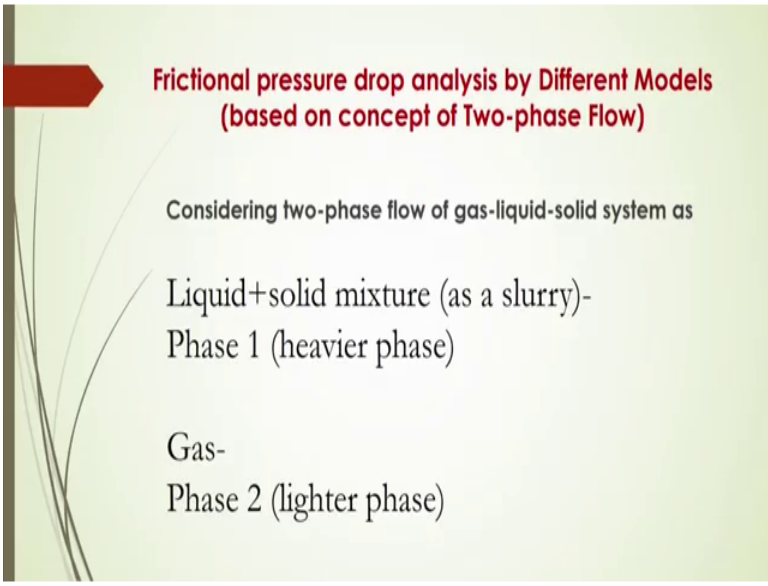


So, for this analysis different models of course, the models will be based on the concept of two phase flow. Considering two phase flow of the gas liquid solid system as; in this case the liquid plus solid mixture will be considered as a slurry phase and it will be considered as a phase 1 and which is heavier phase and to gas that will be considered as a phase 2 that is lighter phase.

So, three phase will be converted to two phase just by considering liquid and solid mixture as one phase and gas is another phase. And there are several models to analyze the frictional pressure drop in two phase flow Lockhart- Martinelli model is the basic model that is developed 1949 ah..

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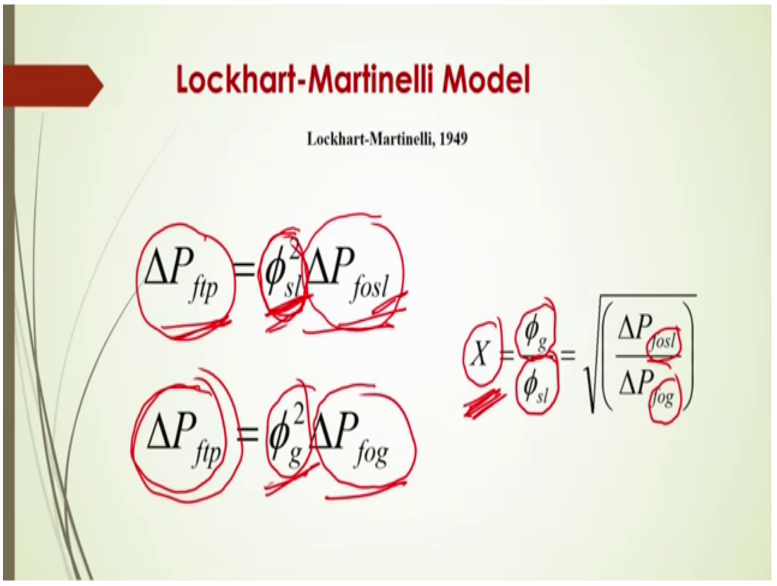


And based on this concept of Lockhart-Martinelli model now what should be the frictional pressure drop or what should be the multi; what should what should be the frictional pressure drop if we know the if we know the single phase pressure drop inside the bed?

So, if we do this frictional pressure drop of this two phase pressure; in this case two phase means here what? Slurry is the one phase, gas is the another phase. So, frictional pressure drop of these two phase; that means, in the three phase of this fluidized bed, then you can write this frictional pressure drop of these two phase will be is equal to phi s l square into delta P of fosl; what does it mean? This delta P fosl is nothing, but the frictional pressure drop of the single phase slurry. And phi sl is nothing, but the multiplication factor or Lockhart-Martinelli multiplier for this slurry phase..

So, if we multiply these multiplier of this phi s l square to the single phase pressure drop then you can have this three phase pressure drop. Even if you consider the single phase pressure drop of gas, then what should be the multiplication factor by which you can get the total frictional pressure drop in the bed?

Now, this you have to remember this Lockhart-Martinelli model is basically for the frictional pressure drop analysis. So, this delta P ftp based on the single phase of slurry what if you multiply it by this phi sl square, you will get this three phase pressure drop. Even if you multiply this phi g square, this is the multiplication factor of the gas phase to this single phase pressure drop of gas; then you can get what should be the total pressure drop of total frictional pressure drop inside the bed.

Another parameter of this Lockhart-Martinelli model is X; what is this X? X is nothing, but this multiplier ratio of this gas to slurry. So, this case it will be equal to root over of delta P fosl by delta P of fog fog. So, in this case the frictional pressure drop due to the slurry and due to the gas, so the X is nothing, but the root over of ratio of this frictional pressure drop of single phase slurry and a single phase gas.

So, from this you can get the total pressure; now this phi s l is the parameter and X is also one parameter these are Lockhart-Martinelli parameter. Now from this Lockhart-Martinelli parameter you will be able to calculate probably the frictional pressure drop. Now how to obtain this phi s l; this multiplication factor or this parameter X? Now different investigators they have developed the different correlation based on the different operating variables from the two phase operation in the bed ah; may be in the bubble column reactor, in the slurry bubble column reactor or any other two phase system; they have actually calculating this frictional pressure drop and they have developed the multiplication factor and the Lockhart-Martinelli model based on their experimental data.

Now, to calculate these multiplication factor of course, you have to move some parameter here; what should be this X parameter that is Lockhart-Martinelli parameter? And for this you have to know the fictional pressure drop of single slurry phase..

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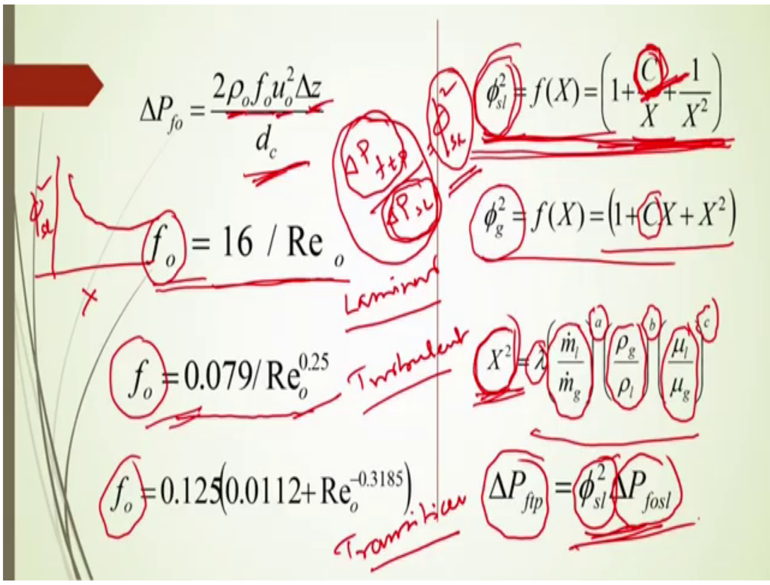


And for these you have to calculate this frictional pressure drop of the single phase by the Fanning equation like this here. So, it will be rho into rho 0; f 0, u 0 square delta z by d c; what is this rho 0? rho 0 is the single phase density f 0 is the single phase frictional friction Fanning friction factor, u 0 is the single phase velocity and delta z is the effective length of the gas liquid mixture inside the bed and this is the column diameter or bed diameter.

And this f 0 is equal to 16 by R e 0; if it is laminar flow, if f 0 is equal to 0.079 by R e 0 to the power 0.5 Reynolds number to the power 0 point 0.25, this Reynolds number will be based on the single phase and it is for turbulent condition.

So, this is for laminar laminar condition and this is turbulent turbulent condition and this is for transition transition; that means, your laminar if Reynolds number of single phase is less than 2300 and if turbulent flow; that means, if Reynolds number is greater than 4000 or the transition flow means here the Reynolds number in between these 2300 to 4000. So, you can calculate this friction factor this if your operating condition is in the laminar flow or in the turbulent flow or in the transition flow.

Once you know this Reynolds number, you be able to calculate this friction factor at different operating conditions. Now this phi s l is shown is actually developed this this phi s l square; that means, here multiplier Lockhart-Martinelli multiplier here or two phase multiplier you can say that this will be is a function of X; X is the Lockhart-

Martinelli parameters, this is nothing, but the root over of ratio of frictional pressure drop of the single phase of liquid to gas or here in this case slurry to gas. And here another one parameter is C; this is C shown parameter C shown parameter.

So, this is actually again you have to calculate or you have to obtain from the experimental data; just by fitting these equation by the experimental data. Now how to actually obtain this C here? See if you know this X by this frictional pressure drop of single phase and if you know this phi s l square; how to know this phi s l square? Because you know this total pressure drop delta P of ftp three phase flow and if you know this delta P of s l single phase flow, then this is the ratio now this ratio is defined as phi s l square.

So, this is your experimental data and this is your of course, experimental data. So, from this experimental data or you can get the single phase from the Fanning equation. If you know this and if you divide these two pressure drop or if you are getting this ratio, this will be represented as phi s l square. So, this phi s l square this is known by the experimental data and then X is also known from the Fanning equation and from the pressure drop. And then if you feed this phi sl verses X here maybe this from this graph here if you if you are drawing or if you are plotting this data as the X; then you will get this type of relationship. If you are fitting this you will get this what is that C value from this equation.

So, just by least square method you can fit this equation then you can get what should be the value of constant this C; so you can use either this single phase slurry or you can use gas phase single phase gas phase; from who is also this phi z square you can get or you can relate the phi g square with the X as 1 plus C X plus X square. So, here again the same C X will be obtained; now this C depends on what are these laminar laminar condition, turbulent condition, or transition that depends on how you are calculated this f also and also your operating condition.

Another also see shown they had given or that X square this Lockhart-Martinelli parameter is a function of the ratio of liquid to gas mass flow rate. And then the density ratio of gas to liquid and the viscosity ratio of liquid to gas here and this lambda and a, b, c those are constants that can be obtained from the experimental data.

Now, X you know that X you can calculate from the experiment also; how to get this? If you know, if you do the experiment on the single phase then what should be the pressure frictional pressure drop? And if you do the experiment to the single phase of gas then also you have to have the measurement data other. So, from this measurement you can calculate this, otherwise you can calculate what will be the Fanning equation.

So, you can get it from Fanning equation also this relationship; once you know this X, once you know this C then you will be able to calculate what should be the phi s l square. Once you know this phi s l square then you can easily calculate what should be the total frictional pressure drop just by multiplying this phi s l square to this single phase pressure drop of the slurry.

So, from this equation you will be able to calculate how to or you can analyze what should be the frictional pressure drop of this three phase flow. So, this is one way; this is a Lockhart-Martinelli model based on that you can analyze this here. Now if we know the experimental data; what actually to be known? What actually to be find out?.

You have to find out this C value, if you are just if you are if you are fitting these experimental data with this type of equation, then you have to obtain these parameter C that is one coefficient this is unknown parameter this will be obtained by the experimental data and just by fitting with this experimental.

Otherwise you can directly correlate with the different operating variables by dimensional analysis and you will get different groups, dimensionless groups and then you just correlate with this phi s l square and then you can get the phi s l square; once you know this phi s l you can calculate the total frictional pressure drop here.

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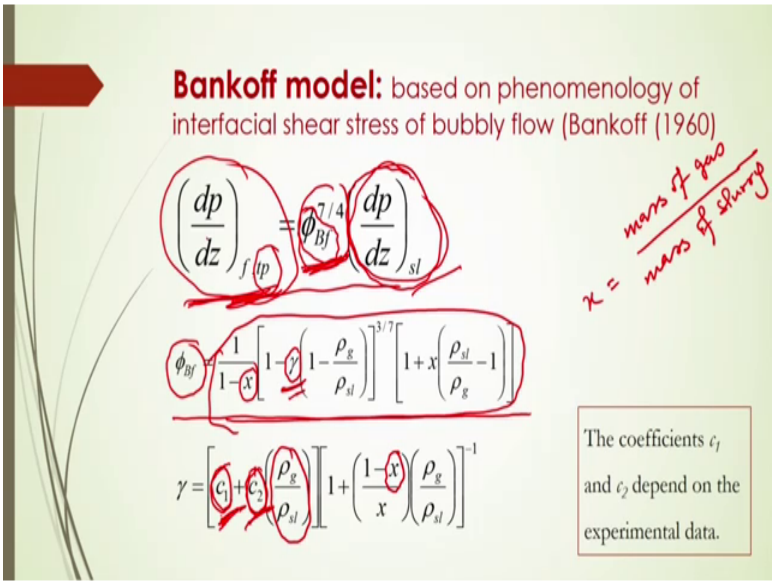


Now, Bankoff 1960; he has developed another correlations based on this Lockhart-Martinelli concept. Here they told that this two phase frictional pressure drop or in this case here three phase frictional pressure drop by just considering the liquid solid has one slurry and gases has another phase.

So, in that case he has proposed one correlations that this single phase pressure drop; if you multiply it by this a factor of phi B f at to the power 7 by 4; then you can get this total frictional pressure drop inside the bed. So, this phi B f is nothing, but the multiplier a Bankoff; so, Bankoff multiplier. So, it is called if you are calculating these Bankoff multiplier from the experimental data and if you are correlating this experimental data with this model of this phi B f; here this phi is the function of X; X is nothing, but what is that? Mass quality of the phases; this mass quality of the phases here X will be is equal to here mass of mass of gas by, mass of slurry; slurry means here liquid and solid mixture..

So, X and here gamma is one parameter this gamma again is a function of this density ratio of gas to slurry and also this mass quality for the gas of gas slurry mixture. So, here you have to calculate now you have to you have to find out now what should be the gamma value or what should be the c 1 and c 2 coefficients for this gamma. This c 1 and c 2 are the coefficients which will be found out from the experimental data. So, this coefficients c 1 and c 2 are unknown parameter here; if you fit these parameters and by optimization method or by least square method, if you feed this data with the experimental data then you will be obtain you will you will get this c 1 and c 2 of parameters.

Now how to do this? Only thing this one is known to you, this is experimental and this is also experimental then this divided by this; you will get this phi B f. So, phi B f you can get you can you can correlate this phi B f to this model here and X is known to you; only gamma gamma is nothing, but c 1, c 2 and this c 1 and c 2 you have to obtain from the just by fitting with experimental data.

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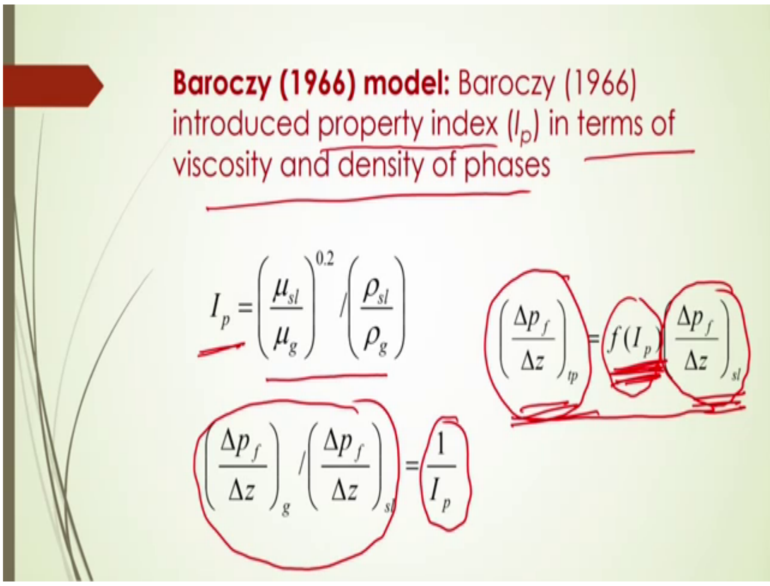


Another model is Baroczy Baroczy 1966 model; here in this case they have introduced property index like I p in terms of viscosity and density of the phases. So, they have defined these property index as the ratio of viscosity of slurry to the gas here and also the ratio of density of slurry to gas. So, here they have defined this as like this mu s l by mu g to the power 0.2 by rho s l by rho g.

Now, based on their concept they proposed that these three phase pressure drop or two phase pressure drop will be is a function of this property index. and the function of single phase a pressure drop; that means, if you multiply the single phase pressure drop with this function of property index then their then two phase or three phase frictional pressure drop you can get.

Now, then 1 by I p will be then from this you can get it just this divided by this divided by this; this is single phase pressure drop to the (Refer Time: 18:49) So, 1 by I p is related to this here single phase pressure drop of gas to the single phase pressure drop of slurry. So, again you have to find out what should be the function of this property index from the experimental data.

So, this one is your experiment; this one is your experiment and then what should be the f I p? And then again this f I p that you have to model that you have to make one correlations for this I p in terms of different operating variables; now this operating variables will be may be what is that geometric variables, may be physical properties, maybe are some other what is that thermodynamic conditions there are different operating conditions that will give you the change of this I p.

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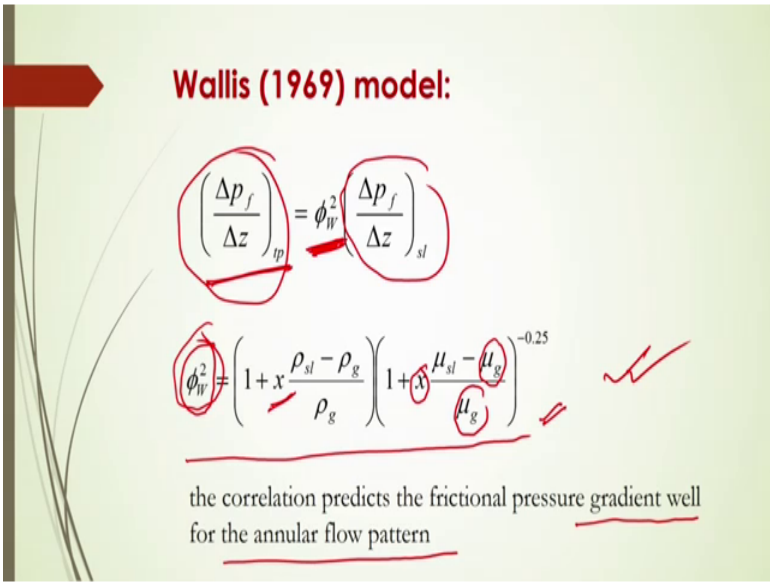


Another is model is Wallis 19696 model what happened here; they have also developed one correlation based on that Lockhart-Martinelli concept again this multiplier here phi w is one multiplier concept; this phi w is the multiplier of the single phase pressure drop.

Then you will get this two three phase or two phase pressure drop here. Then you have to correlate this phi; w square that is here of multiplier as a function of this mass quality of the gas and also density of the slurry, density of the gas viscosity of the gas; so in this way this correlations. So, if you know this slurry density; if you know the slurry viscosity, if you know the mass quality of the gas and other physical properties then you can directly obtain this equation..

And then if you substitute these, you will see if you multiply the single phase slurry frictional pressure drop and multiplied this multiplier then you will get the three phase pressure drop. Now, this this this correlations actually predicts the frictional pressure gradient well for the annular flow pattern. So, if is there any annual fluidized bed, you can apply this Wallis model to predict, to analyze this frictional pressure drop of this three phase flow inside the bed.

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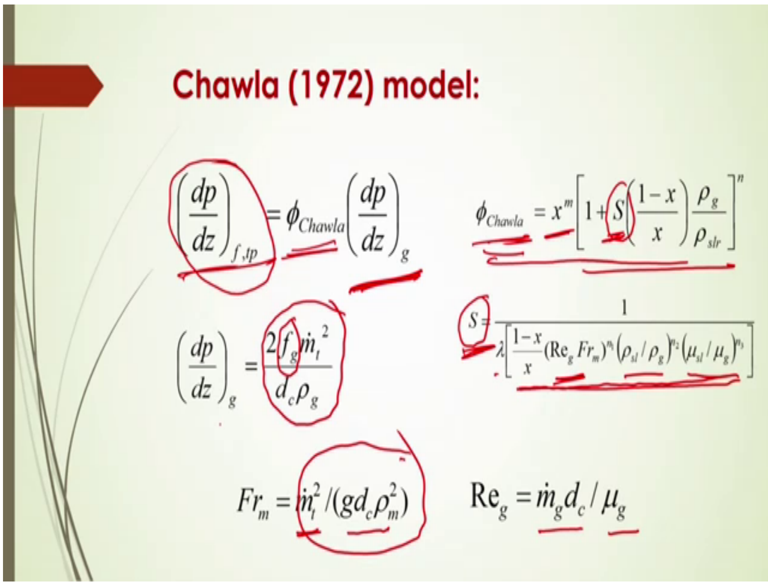


Chawla 1973 she has developed the same concept of this frictional pressure drop by considering the multiplier has phi Chawla. And in this case they have correlated this phi Chawla as this mass quality and one parameter is called here this S that is the sleep parameter this sleep parameter is defined as 1 by lambda into this is a function of Reynolds number of gas, Froude number of mixture and the density ratio of slurry to gas and the viscosity ratio of slurry to gas.

So, in this case here this parameters is very important this parameter depends on the different operating conditions. And here this Reynolds number of gas is defined as m dot gdc by mu g where as Froude number is this total mixture flow rate square divided by gdc into rho m square. So, why this this Froude number is defined; if you know this Froude number, you can calculate and if you substitute here and if you are making correlations to experimental data with this S; then you will be able to calculate what should be the total of frictional pressure drop.

Now, what is that here? In this case this phi Chawla to be obtained from the experimental data. And then just fitting with this model with this experimental value of phi Chawla and you can get this value of S and which is correlated directly with the physical properties and geometrical properties and other operating conditions..

And this of course, this del p; del p by del z or dp by d z of gas single phase it is very important in this case Chawla has taken a single phase pressure drop of gas and that can be obtained by experiment either or you can calculate it from this Fanning equation of this. For that f g is required that f g is a function again Reynolds number, it is greater than 2300; of course, it will be transition if it is greater than 4000, it will be turbulent and if it is less than 2300; of course, you have to consider that laminar flow design and based on which you have to calculate this gas phase pressure drop.

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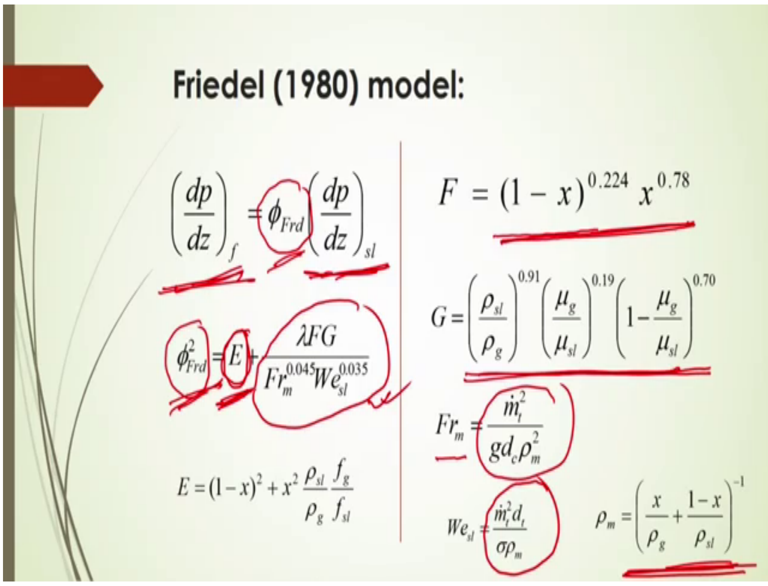


Now, Friedel 1980 he has also developed the based on the same Lockhart-Martinelli concept; here again he has taken this frictional pressure drop of single phase pressure drop. And he has multiplied these multiplier of phi Friedel and to get this total frictional pressure drop. Now this the multiplier; he has proposed this correlation of this this multiplier into two components like is one E another is the you like this lambda into FG by Froude number and Weber number to the power this.

So, according to this what should be this E and lambda; E and lambda are both are two parameters here in this case E is directly related to this ratio of the slurry to the gas density and the friction factor of gas to slurry and also it is the mass quality of the gas. It is seen that this E value is increases if if increases E value is increases; if mass quality is decreases; mass quality is decreases.

Whereas if it increases; so, for the same mass quality if increase where the density of the slurry then of course, this value will increase. So, this E is the one component which will be contributed by this a property of the a fluid and what is that friction factor in the Fanning. Another important is that here lambda into FG; F is defined as 1 minus X to the power here. This F is directly related to this mass quality of the gas and G; G is the function of ratio of slurry to gas density, ratio of gas to slurry density and the viscosity of the viscosity ratio of gas to slurry here.

So, G can be calculated from this correlation, F will be calculated from this correlation and F r m that is Froude number of the mixture will be defined as Weber number is defined as m dot t square to d t by sigma rho m. And rho m will be defined as this X by rho g plus 1 minus x by rho s l to the power minus 1; so this will be the mixture density of gas and slurry. So, once you know this E, F, G and Froude number and Weber number of the slurry you; will be able to calculate what should be the multiplier factor of this Friedel, then multiply this multiplier with this single phase slurry pressure drop you will be able to calculate what should be the frictional pressure drop.

Then you have to compare with experimental data of own experiment with this friction factor. Then how much deviation from you experiment to this model you will be able to calculate. If it is any modification of this model is required based on your experimental data, you can also do just by fitting these unknown parameter with your experimental data or you can directly correlate this F r d that is phi F r d to your in terms of your different operating variables just by dimensional analysis.

And then fitting with experimental data with these different dimensions groups just by having the dimensional analysis and fitting with experimental data with these dimensional groups by regression multiple regression analysis, you can easily obtain the or you can easily predict the multiplier of this Friedel.

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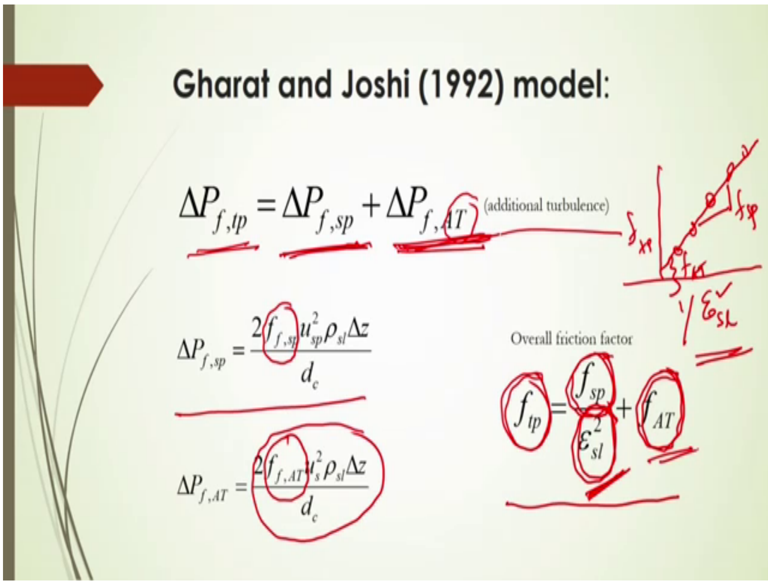


Another module is Gharat and Joshi model actually they have analyzed the frictional pressure drop and they have suggested that this total frictional pressure drop in this three phase system or two phase system based based on the two contribution one is single phase frictional pressure drop. Another is the pressure drop due to the additional turbulence that made by these other phases.

From one single phase and then pressure drop will be developed by turbulence like this; if is there any bubble system. Then of course, this bubbly system bubble will just (Refer Time: 28:30) will enhance the turbulence even some other provisions; if it is made in the fluidized bed some other like intensification of the bed by providing other devices like black (Refer Time: 28:46) or other internal device we are adapt in the bed. Then additional turbulence may produce and because of which this frictional pressure drop will change.

So, they have considered those additional turbulence by this delta P f AT; AT means additional turbulence here. So, this frictional pressure drop will be and the single phase frictional pressure drop without turbulence and then if is there any turbulence made by the different provisions; what will be the frictional pressure drop. Now this single phase frictional pressure drop, you can obtain it from the Fanning equation that is simple; this Fanning equation you have this is Fanning friction factor again to be calculated from the what is the Blasius equation.

And then additional frictional pressure drop that is that by additional turbulence that will be calculated in that. In this case f f AT; that means, friction actor due to this additional turbulence will be function of that is the concentration of the solid, concentration of the slurry inside the bed. And this will be is equal to f t p that will be is equal to f s p by epsilon s l square that is volume fraction of the slurry and this f AT; f AT this is the friction factor due to the additional turbulence.

Now, you have to you have to calculate this overall friction factor from the mixture from the mixture velocity of the fluid inside the bed. Then what will be the overall friction factor from which if you know this single phase friction factor and the slurry concentration, what should be the friction factor by this additional turbulence just by fitting this situation here?

Now, you see that overall friction factor will change here like this; if you are changing this epsilon s l and if you get this overall friction factor as this; then you will see there will be data like this here. So, this type of equation straight line you will get; now from this straight like you will see here 1 by epsilon epsilon s l square; if you are considering then here f s p will be your slope; f s p will be your slope and then f AT will be the intersection that will give you your f AT..

So, from this ah analysis of this overall friction factor from the experimental data, you will be able to calculate what should be the friction factor due to these additional turbulence.

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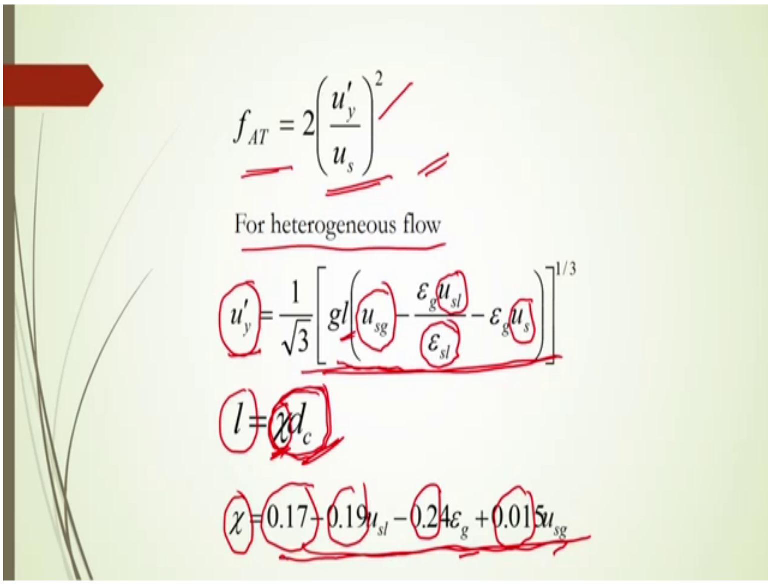


Now, this additional turbulence actually depends on the velocity fluctuation inside the bed. Now that velocity fluctuation maybe in the y direction, z direction or x direction; if you are if you if you if you are if your are fluidized bed in the in such a way that your y directional velocity fluctuations is the dominant one, then this f AT due to this is dominant velocity fluctuation of this y direction, it will be u dash y by u s. What is this u s? u s is nothing, but u s is nothing, but the solid velocity and here this is the fluctuating velocity at the y direction.

Now, for heterogeneous flow you see this u y dash that is turbulent velocity or you can say velocity fluctuation due to this turbulence and it will be is equal to u y dash will be 1 by root over 3 into g l into u s g minus epsilon g u s l by epsilon s l minus epsilon g u s to the power 1 by 3. This is actually in this heterogeneous flow this fluctuation velocity will you obtain from the energy balance.

So, this energy balance will give you this direct this fluctuating velocity now this see this fluctuation velocity again is a function of a u s g; that means, superficial gas velocity and superficial liquid velocity. Of course, there will be if it is slurry system; you have to have the concentration of the liquid inside the bed. And also what would be the solid velocity inside the bed; and then this l is called this mixing length inside the bed the within which range of this fluctuation will be there and then it will be defined as this some lambda into d c depends on this column diameter or there is bed diameter.

This lambda is a parameter which is the function of again this slurry velocity and then the volume fraction of the gas and the gas velocity. So, from this correlation you will be able to calculate this psi value and from which you will calculate this mixing length inside the bed and from this mixing length you will be able to calculate what will be the fluctuating velocity or friction velocity you can say sometimes it is called friction velocity; sometimes it is called fluctuating velocity, sometimes it is a turbulent velocity.

And then from this turbulent velocity and the solid velocity inside that bed, you will be able to calculate what will be the friction factor due to this additional turbulence additional turbulence. Now, additional turbulence if you calculate from this here; the overall friction factor and if you are messing with these if it is unknown parameter here lambda, you can lambda you can directly obtain from this overall friction factor..

And if you can correlate again with this liquid velocity or slurry velocity or gas velocity here, you will be the deviation from this. If there is a significant deviation then you have to modify this model by just fitting the experimental data and having this coefficient different from modification. So, in this way you can analyze what should be the friction factor inside the bed due to this fluctuating velocity and also additional turbulence.

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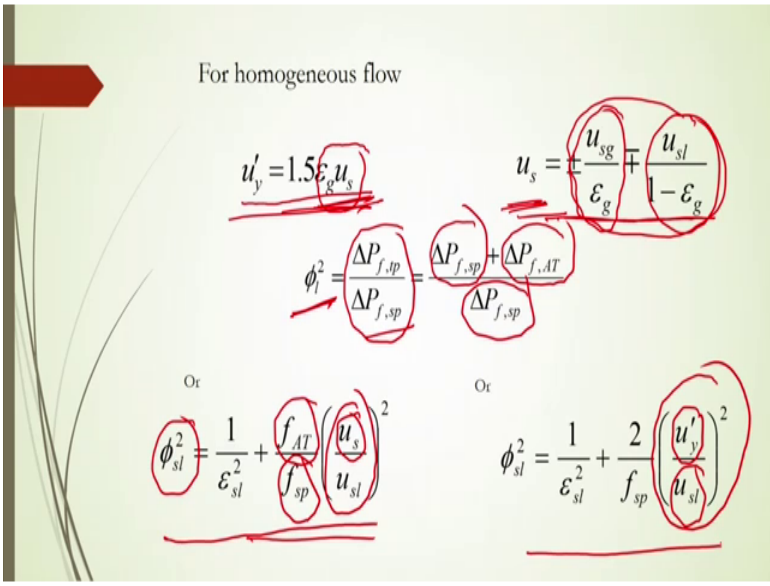


Now, for homogenous flow this u dash y; that means, here fluctuating velocity is directly related to this effective velocity of the solid. So, it is seen that it is 1.5 times of this effective velocity of the solid. And this u s; this us solid velocity will be the this is called sometimes sleeve velocity and it will be represented as this u s g plus minus here u s l. This u s g is the u s g by epsilon this is the actual velocity and us l by 1 minus epsilon g is called actual liquid velocity..

So, from which you will be able to calculate this u s value or relative velocity or sleeping velocity, sleeve velocity; this sleeve velocity will give you the fluctuating velocity in the homogenous mixture. And then phi l square to be calculated here this two phase friction factor to the single phase friction factor and this two phase friction factor will be is equal to single phase friction factor and this friction factor due to the additional turbulence..

And then this is what this friction factor due to this single phase or you can calculate this ah multiplier of the single phase slurry phase that is in terms of the concentration of the slurry and the friction factor of this additional turbulence to this friction factor of a single phase flow inside the bed. And also what should be the ratio of slip to the single phase slurry phase.

So, you can directly obtain these this multiplier of this slurry by this correlation. Another way to calculate this in terms of these fluctuating velocity like this here; here one terms is the u s by u s l instead of u s l by u s l here it will come only the fluctuating velocity to the slurry velocity this is fluctuating velocity instead of u s here; so, you can calculate this phi s l.

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So, for more details you can get this different frictional pressure drop to analyze and to predict from your experimental data from this two books from this books here this Kunii and O. Levenspiel books, you can follow. And also this Fluidization and Fluid particle Systems that is by Yang 2003, you can get more information about the different correlations models given by me that is your Hydrodynamics and Transport Processes of Inverse Bubbly Flow; here there it is the extensive actually of analysis extensive way that is represented of different fictional pressure drop correlations that you can get more information from this relationship.

So, we can obtained from this lecture that how to analyze the frictional pressure drop by Lockhart-Martinelli model, how to analyze this frictional pressure drop in the fluidized bed by Friedel model by different other models also. In this case, you have to take care of taking the concept of this Lockhart-Martinelli which is actually basically for two phase flow. Here we can apply these two phase flow for three phase operations only just by taking this liquid and solid as one phase and gases is another phase, because this Lockhart-Martinelli concept basically was from the; flow of horizontal horizontal flow of gas and liquid.

So, here instead of liquid you are considering this slurry of liquid and solid whereas, in the horizontal flow we are considering the vertical flow. Of course, all the parameters whatever this Lockhart-Martinelli parameters, multiplication or multiplier Lockhart-Martinelli you can say or two phase multiplier or three phase multiplier and different investigators, they are naming in different way. So, these parameters will be obtained from the experimental data of data of three phase floor.

Then once you know this three phase flow and three phase frictional pressure drop just by analyzing with this concept of Lockhart-Martinelli model; you will be able to predict the three phase frictional pressure drop at different operating condition also. By this frictional pressure drop correlations you can scale of the system of this fluidized bed ah with different operating variables.

Of course, this variables or you can say parameters Lockhart-Martinelli parameters that is fix that is the ratio of frictional pressure drop of that is slurry to the gas and ah, but the multiplier that is obtained that you have to make a correlations in terms of different operating parameters; it will be better just by regression analysis with the different of my experimental data and making a general correlations for this. And then you can apply it with different operating condition, even after without doing experiment you can calculate what should be the frictional pressure drop by that model.

So, I think it will be helpful to calculate this two phase and three phase frictional pressure drop by different models and ah. So, up to this we have a discussion this liquid solid frictional pressure drop, gas solid frictional pressure drop, even gas liquid solid frictional pressure drop in fluidized bed. So, next lecture onward we will be discussing what should be the actually mechanism of distribution of gas inside the bed that will be discussed, and how this distributor actually play important role for different hydrodynamics parameters.

Thank you for this lecture.