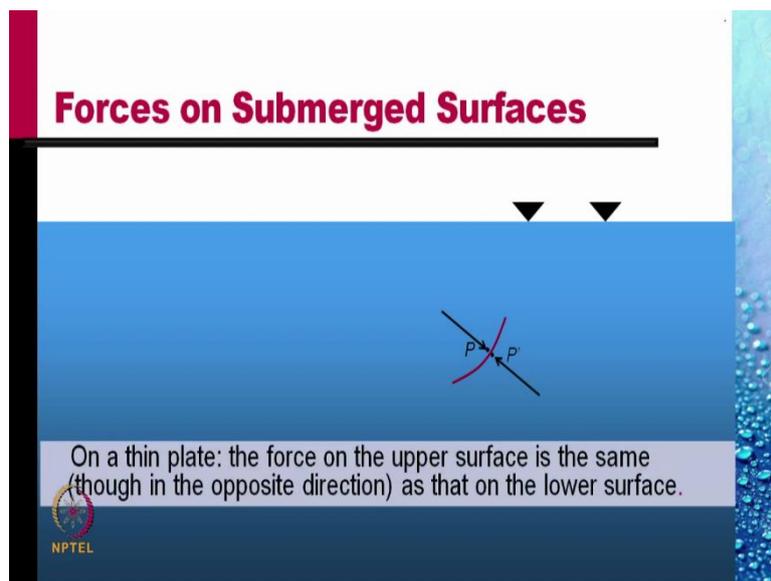


Fluid Mechanics and Its Applications
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Lecture 5
Fluid Mechanics and Its Applications

Welcome back. In this presentation, we will be discussing the pressure forces on submerged surfaces.

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Consider a thin curved plate, shown in red submerged in the liquid. On the upper surface, there would be a pressure. Let the pressure be p at a point. On a point directly below that point on the lower surface, there will again be a pressure p prime. Both the pressures would be equal to ρgh , where h is the depth from the free surface of the liquid. The two pressure will be equal. Therefore, if we need to calculate the pressure force on the lower surface, which will be equal to the pressure force on the upper surface of the thin plate. The force on the upper surface is the same (though in the opposite direction) as that on the lower surface.

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Hydrostatic Force on a Vertical Surface

horizontal force =

$$\int_{h_1}^{h_2} \rho g h (bdh)$$

$$= \frac{1}{2}(\rho g h_1 + \rho g h_2) \cdot (h_2 - h_1) \cdot b$$

$$= \frac{1}{2} \rho g b (h_2^2 - h_1^2)$$

Pressure prism

Let us consider next the hydrostatic force on a vertical surface. We have seen earlier that within a liquid the force varies linearly with the depth of the point. So, the pressure on this plate which was shown in red, is as shown. Clearly, if we consider the width of the plate as shown, there would be a pressure prism on that surface, which would look like this. The magnitude of the force on the upper surface would be $\rho g h_1$, where h_1 is a depth of the upper surface from the free surface of water, and the pressure at the lowest end of this plate would be $\rho g h_2$.

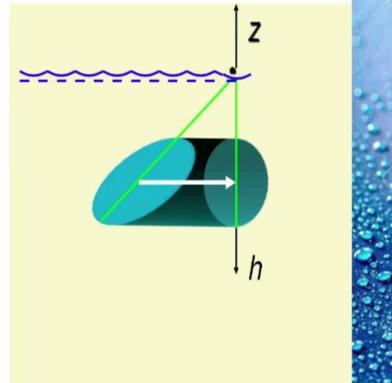
The total force would be acting at some point in between the two ends of the plate. And the total force would be found out by integrating over a small area dA . Let us consider an element of height dh at a depth, depth h , from this free surface. The pressure on that little element would be $\rho g h$ and so, the force on the element would be $\rho g h \times bdh$, where bdh is the area of that element to find the total force on the plate. We have to integrate this over h from h equal to h_1 , the depth of the upper point of the plate to h_2 , the depth of the lower point of the plate. And if we do a simple integration assuming ρ , b and g constant, we get the total horizontal forces $\frac{1}{2} \rho g b (h_2^2 - h_1^2)$.

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Hydrostatic Force on a Vertical Surface

$$\begin{aligned}
 F &= \iint_A \rho g h dA \\
 &= \rho g \iint_A h dA \\
 &= \rho g h_c A
 \end{aligned}$$

Pressure at Centroid \times
Area of surface



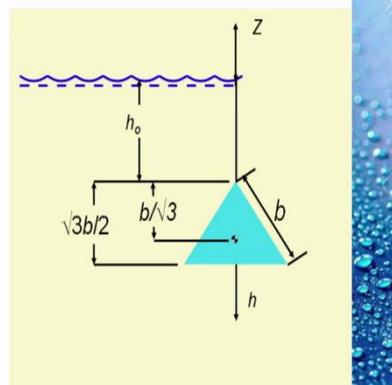
Let us consider another geometry: a circular plate. The pressure prism on the circular plate will look like this. This is the hydrostatic pressure variation and the force would be $F = \iint_A \rho g h dA$ over the whole area A , which is $\times \iint_A h dA$, integral $\iint_A h dA$ can be written as the depth of the centroid: $\rho g h_c A$. $\rho g h_c$ is recognized as the pressure at the centroid of the plate. So, that the total hydrostatic force on this vertical plate is seen as the pressure at the centroid times the area of the plate. F is pressure at the centroid times the area of the plate. This is the relation that we use in calculating pressure on the plate.

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Hydrostatic Force on a Vertical Surface- Example

Total hydrostatic force on the triangle =
Pressure at centroid \times area of the triangle =

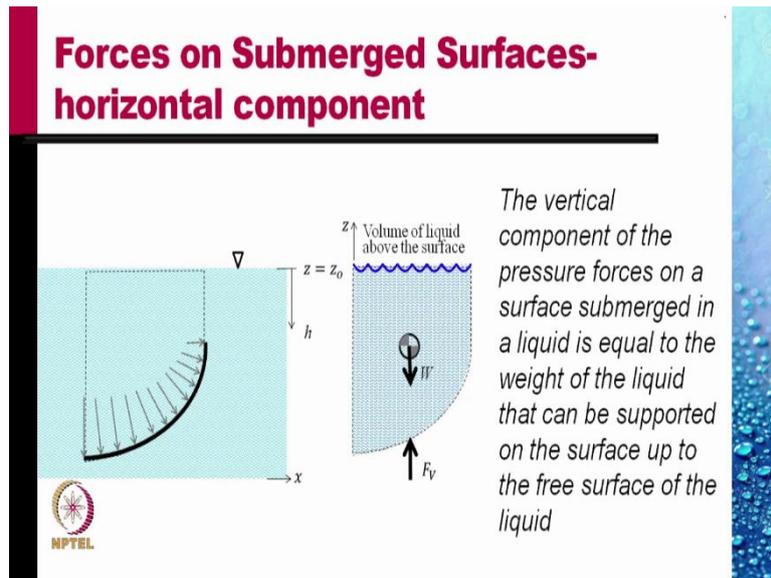
$$\rho g (h_o + b/\sqrt{3}) \times (b \times \sqrt{3}b/2) / 2$$



Let us do an example: We have an equilateral triangular plate with each side equal to b . It is immersed in a liquid of density ρ to a depth such that the vertex of the triangular plate is at

depth h_o . We have to find the location of the centroid. We know that the depth of the centroid from the tip of the plate is two-thirds of the height of the plate, and the height of the plate is $\sqrt{3}b/2$. So, that the centroid is at a depth of $b/\sqrt{3}$. The total depth of the centroid is $h_o + b/\sqrt{3}$ and so, the total force is the pressure at the centroid, which would be $\rho g(h_o + b/\sqrt{3})A$. The area A of the plate being $(b \times \frac{\sqrt{3}b}{2})/2$.

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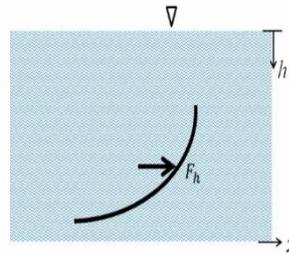


Now, let us find out the forces on curved surfaces. Let us consider a two-dimensional plate curved as shown in this figure and immersed in a liquid of density ρ . We measure h downwards from the free surface. The pressure distribution on upper plate would be as shown. The pressure acting normal to the plate everywhere and increasing with depth. Let us consider the volume of the liquid above the plate, let us isolate this as a free body if F_v is the vertical force acting on the plate because of the liquid.

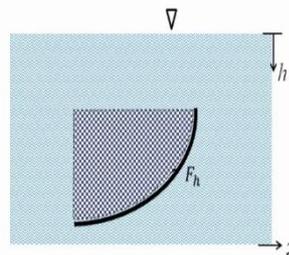
Then on this free body of liquid there is a force due to plate equal to a F_v acting upwards, the reaction to the force on the plate by the liquid. The only other vertical force on this free body is the weight of that liquid resting above the plate up to the free surface. So, the vertical component of pressure force on the surface submerged in liquid is equal to the weight of the liquid that can be supported on the surface up to the free surface of the liquid.

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Forces on Submerged Surfaces- horizontal component



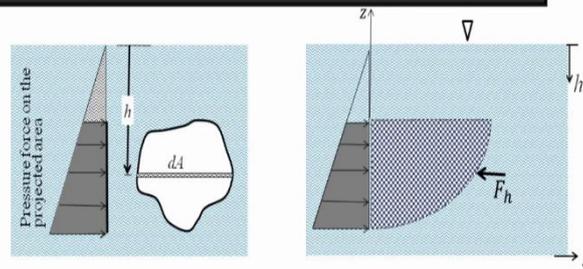
Forces on Submerged Surfaces- horizontal component



Let us next consider the horizontal force on this submerged surface, same surface. Let the horizontal force because of liquid on the surface be F_h , as shown. Let us consider this body of fluid and draw a free body diagram of this body of fluid. Let us draw the horizontal forces on this body of fluid.

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Forces on Submerged Surfaces- horizontal component



$$F_h = \int_{A_v} \rho g h dA = \rho g \int_{A_v} h dA = \rho g \bar{h} A_v = p_{\bar{h}} A_v$$



Then, on this free-body diagram, there are two horizontal forces one the F_h , the reaction of the force on the plate acting to the left, and a horizontal force acting on the flat surface to the left of this body. This would be the hydrostatic force varying linearly from the top to the bottom. This is the force we have already calculated. This surface is flat. So, by the relation that we derived for the horizontal force on a flat vertical surface, this force is equal to the pressure at the centroid of this plate times the area of this vertical flat plate.

To evaluate the force on this vertical flat plate, let the white area represent the projection of the original plate on to vertical plane. The pressure distribution on this is as shown. At depth h let us consider an area dA . The pressure on this area dA is $\rho g h$ and the force is $\rho g h dA$. We can find the total force by integrating this over A and so, that we get the horizontal force as $\rho g \bar{h} A_v$, where $\rho g \bar{h}$ is the pressure at the centroid of the projected area and A_v is the vertically projected area. This is then the relation for the horizontal force: the pressure at the centroid of the projected area multiplied by the projected area of the surface.

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Forces on Submerged Surfaces

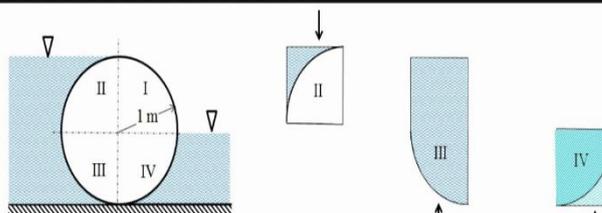
- the horizontal component of the pressure forces is equal to the pressure at the centroid of the vertically projected area times the projected area
- the vertical component of the pressure forces is equal to the weight of the liquid that can be supported on the surface up to the free surface of the liquid



So, the two rules finding the horizontal and the vertical component of pressure forces are that the horizontal component of the pressure forces is equal to the pressure at the centroid of the vertical project area times the projected area, and the vertical component of the pressure force is equal to the weight of the liquid that can be supported on the surface up to the free surface of the liquid.

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Example



$$F_v = 10^3 \frac{\text{kg}}{\text{m}^3} \times 9.8 \frac{\text{N}}{\text{kg}} \times 0.43 \text{ m}^3 = 4.21 \text{ kN} \downarrow$$

$$F_v = 10^3 \frac{\text{kg}}{\text{m}^3} \times 9.8 \frac{\text{N}}{\text{kg}} \times 3.57 \text{ m}^3 = 34.99 \text{ kN} \uparrow$$

$$F_v = 10^3 \frac{\text{kg}}{\text{m}^3} \times 9.8 \frac{\text{N}}{\text{kg}} \times 1.57 \text{ m}^3 = 15.26 \text{ kN} \uparrow$$

Let us do an example. Let us consider a cylindrical body that is immersed in a liquid as shown. On the left, the free surface of the liquid extend up to the topmost point of the cylinder and on the right the free surface of the liquid is only up to the diameter at the middle. If the radius of this cylinder is 1 meter, the depth of liquid on the left is 2 meters and on the right is 1 meter.

To determine the forces of this, let us consider the force on each of the four quadrant of the circular cylindrical surface. On quadrant one, there are no forces. Let us first look at the vertical forces. On the quadrant two, the vertical force is equal to the weight of the water up to the free surface which is supported on that surface, which is shown as the blue area on this picture. On quadrant three, the weight of the liquid that will be supported on quadrant three would be as shown.

This would be a weight which is downward. But from the very first slide in this lecture, we has shown that the force that would be on the upper surface would be same as force on the bottom surface. So, the vertical force which is upward on this surface in quadrant three would be equal to the weight of this area of the liquid shown.

On the fourth quadrant the force would be the weight of the blue, the darker blue area of the liquid shown. So for unit depth, we can evaluate these areas and find out the total force. On quadrant two, the force is downward, while on quadrant three and quadrant four the force is upward. On quadrant one the force is 4.21 kN downwards, and on third and fourth quadrant, they are 34.99 kN upwards and 15.26 kN upward. We can sum up the three and find out the total vertical force on this circular cylinder per unit depth into the paper.

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Example

$$F_h = \left(10^3 \frac{\text{kg}}{\text{m}^3} \times 9.8 \frac{\text{N}}{\text{kg}} \times 0.5 \text{ m} \right) \times 2 \text{ m}^2 = 9.8 \text{ kN}$$

$$F_h = \left(10^3 \frac{\text{kg}}{\text{m}^3} \times 9.8 \frac{\text{N}}{\text{kg}} \times 1.50 \text{ m} \right) \times 2 \text{ m}^2 = 29.4 \text{ kN}$$

$$F_{\text{net}} = 10^3 \frac{\text{kg}}{\text{m}^3} \times 9.8 \frac{\text{N}}{\text{kg}} \times 0.5 \text{ m} \times 2 \text{ m}^2 = 9.8 \text{ kN}$$

Let us next find the horizontal forces. Again, there will be no force on the first quadrant. On the second quadrant, it would be a triangular pressure variation over 1 meter depth, the net force would be the pressure at the centroid of the projected area times the vertically projected area. The vertically projected area is of course a rectangle of depth 1 meter. So, the depth of

the centroid would be one-half meter on quadrant three, the gray pressure prism is the force on quadrant three.

This again is found by the same formula. The projected area here is again a rectangular plate of height 1 meter, the centroid of that plate would be 0.5 meter from the top of this plate and so, the depth of the centroid from this free surface is 1.5 meter. So pressure here would be $\rho g \times 1.5 \text{ m}$ and the force obtained by multiplying it by the area of this projected area.

On the fourth quadrant, the pressure variations is as shown. The depth of the centroid would again be at 0.5 m from the top of this, and therefore, all the three horizontal forces have been determined. They are respectively ρgh , where h is the depth of the centroid for each of the element into area of the plate. If the depth of the cylinder normal to the paper is considered 2 m, then these are the forces. The first two are to right and the last one is to left. We can add up to find out the total force on this circular cylinder gate of 2 m depth normal to the screen.

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Force on the Dome

Dome, $R = 40 \text{ cm}$

Oil

Oil

3.4 m

20 cm

Mercury

NPTIL

This volume is

$$V = \pi \times (0.40 \text{ m})^2 \times (3.4 \text{ m} - 1 \text{ m}) - \frac{2}{3} \pi \times (0.40 \text{ m})^3 = 0.87 \text{ m}^3$$

The net vertical pressure force then is

$$F_z = \rho g V = 0.8 \times 10^3 \frac{\text{kg}}{\text{m}^3} \times 9.81 \frac{\text{N}}{\text{kg}} \times 0.87 \text{ m}^3 = 6.83 \text{ kN, upwards.}$$

Let us consider the next example as a force on a dome of oil tanker. It is a cylindrical oil tanker with a hemispherical dome on it, it contains oil and the pressure of this is measured by a mercury manometer rising to a height of 20 cm. So, first we will find out the pressure at the bottom of the tank. The gauge pressure the bottom of the tank is $\rho_{\text{mercury}} g h_{\text{mercury}}$ which is 20 centimeters. So, this pressure is 26.7 kPa.

This pressure of 26.7 kPa would mean that the pressure in the oil on the dome inside the dome is higher than the atmospheric pressure. Let us find out what is the height of oil that will result

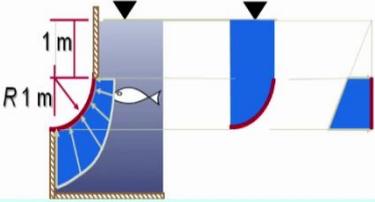
in this pressure of 26.7 kPa gauge at the bottom of the tank: ρgh should be equal to 26.7 kPa. ρ_{oil} is $0.8 \times 10^3 \text{ kg/m}^3$, g is 9.8 and this multiplied by h would give you the pressure 26.7. The result is 3.40 meters from the bottom.

So, a pressure of 26.7 kPa at the bottom of the tank would be obtained if the oil extends up to a height of 3.40 meters in an open tank. So the pressure variations in this vessel is equal to the pressure variations in an open tank in which the oil is standing up to a height of 3.40 m, that is, the pressure variations within the tank are as if the free surface oil is up to 3.4 meters from the bottom.

Clearly then, the vertical force on the hemispherical cap is equal to the weight of oil over and above the dome shown as grey in this picture. We can easily evaluate that volume as here: volume is $\pi \times (0.4 \text{ m})^2$ which is the area of cross section multiplied by $(3.4 \text{ m} - 1 \text{ m})$, that is 2.4 meters. This would be that volume of this cylindrical vessel. From this I have subtracted the volume of the hemisphere where there is no oil, and so, the total volume of this grey portion is 0.87 m^3 . And so, the vertical force is the weight of this oil, which is the density of the oil times g times this volume: 6.83 kN, upwards. Of course, there will be no horizontal force because of the symmetry of the hemisphere.

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Forces in an Aquarium Glass



$$F_H = \rho g h_c A_x = 10^3 \left(\frac{\text{kg}}{\text{m}^3} \right) \times 9.81 \left(\frac{\text{N}}{\text{kg}} \right) \times 1.5(\text{m}) \times 1(\text{m}) = 1.45 \times 10^4 \text{ N/m}$$

$$F_V = \rho g \times (\text{Volume of water that could stand on surface})$$

$$= 10^3 \left(\frac{\text{kg}}{\text{m}^3} \right) \times 9.81 \left(\frac{\text{N}}{\text{kg}} \right) \times \left[1(\text{m}) \times 1(\text{m}) + \frac{\pi}{4} (1\text{m})^2 \right] = 7.7 \times 10^3 \text{ N/m width}$$

Let us do another example. An aquarium has a quarter cylindrical glass plate for viewing the fishes. The glass plate is as shown in red in this figure. What are the forces on this glass plate because of the water?

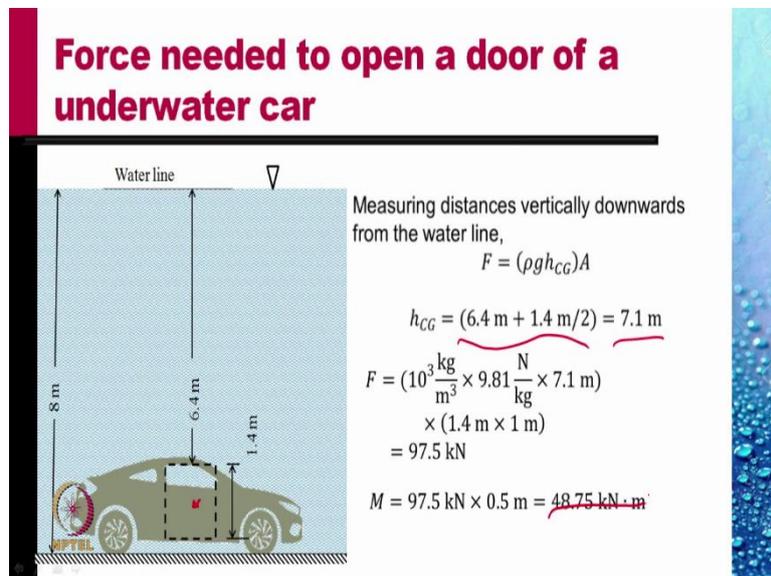
These forces would be because of this pressure distribution, which at the top of this quarter-cylindrical glass plate is because of 1 m depth of water. At the bottom, because of 2 m depth of water. Integration of this pressure force would be quite complicated, but this is simplified by using the two rules that we obtained above.

To determine the vertical force on the bottom of the glass plate, we recognize that this would be equal to the downward vertical force on the top of the ground glass plate, if there was water up to the height of the free surface. So, it is weight of this blue water prism shown. This weight is easy to find, because volume of that water is equal to the volume of the square prism of water, plus a quarter cylindrical prism of water. And the horizontal force would be equal to the horizontal force on the vertically projected area of this window.

Vertically projected area, of course, would be rectangle of height 1 m and so, it would be the pressure at the centroid of this plate, this is projected area which would be 1.5 m from the free surface. So, this pressure would be $\rho g \times 1.5 \text{ m}$ and we multiply by the vertically projected area to find the vertical component of the pressure forces. This is the horizontal force and the vertical force is $\rho g \times V$, where V is the volume of the water that could stand on the surface.

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Force needed to open a door of a underwater car



The diagram shows a car on the floor of a pond, 8 meters below the water line. The car's door is 1.4 meters high and 1 meter wide. The water level is 6.4 meters above the top of the door. The calculation determines the force needed to open the door by finding the weight of the water above it.

Measuring distances vertically downwards from the water line,

$$F = (\rho g h_{CG})A$$

$$h_{CG} = (6.4 \text{ m} + 1.4 \text{ m}/2) = 7.1 \text{ m}$$

$$F = (10^3 \frac{\text{kg}}{\text{m}^3} \times 9.81 \frac{\text{N}}{\text{kg}} \times 7.1 \text{ m}) \times (1.4 \text{ m} \times 1 \text{ m}) = 97.5 \text{ kN}$$

$$M = 97.5 \text{ kN} \times 0.5 \text{ m} = 48.75 \text{ kN} \cdot \text{m}$$

Let us now do one more example, where the force needed to open a door of an underwater car. Suppose, a driver with a car takes a plunge in water, and the car comes to rest at the floor of a pond, 8 meters below the water line. The dimension of the car. If the diver wants to open the door of the car shown in broken lines here, what force would he need to apply?

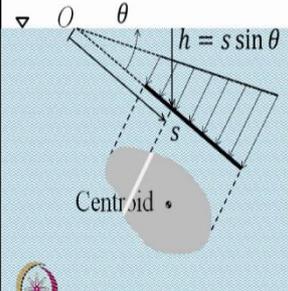
Measuring distance vertically downwards from the water line, the force would be the pressure at the centroid, again, times the area vertical area of the door. The centroid of the door is at the middle of the rectangular door. So, it is at a depth of 6.4 m plus 1.4 m by 2, or 7.5 m. So, the force would be 10^3 kg/m^3 , the density into 9.81 N/kg, the value g into 7.1 meter.

The depth of the centroid multiplied by the area of the door which is $1.4 \text{ m} \times 1 \text{ m}$ or is 97.5 kN. The moment he would need to apply would be this force into 0.5 meters if he applies a force at the middle of the door, 48.75 kNm. This is far beyond the capability of a human being.

So, if this is a situation that exists, the diver cannot get out, cannot open the door. So, is she doomed? No, she could lower the window if that is possible, if the electrical system is not broken down, she could lower the window and as water enters the cabin the forces on the back of the door would balance the forces from outside keeping the door closed. And so, it is possible that before the water inside reaches the top of the cabin, the force may reduce from 97.5 kN to a figure which can be handled by the driver.

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Centre of Pressure on a Submerged Surface



$$dF = \rho g s \sin \theta dA$$

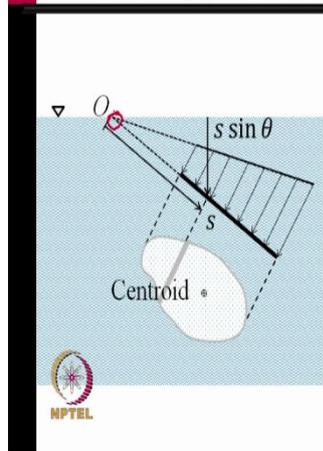
$$F = \int_A \rho g s \sin \theta dA = \rho g \sin \theta \int_A s dA$$

$$\int_A s dA = s_{CG} A$$

$$F = \rho g h_{CG} A = p_{CG, g} \times \text{plate area}$$



Centre of Pressure on a Submerged Surface



$$dM_o = -s \cdot \rho g s \sin \theta dA$$

$$M_o = \int_A s \cdot \rho g s \sin \theta dA$$

$$= \rho g \sin \theta \int_A s^2 dA$$

$$= \rho g \sin \theta I_{yy,0}$$

$$s_{CP} \times \rho g s_{CG} A \sin \theta = \rho g \sin \theta I_{yy,0}$$

$$s_{CP} = \frac{I_{yy,0}}{s_{CG} A}$$

Let us next consider the center of pressure on a submerged surface. For this, we will deal only with plain flat surfaces. Let us consider an inclined surface at an angle theta as shown. The grey area shows the plan form of that surface. There are pressure forces acting on the surface. The pressure variation, of course, would be linear $\rho g h$, where h is the vertical depth from the free surface. If, we measure s , the distance from the free surface along the slope of the plate at any location s , the depth from the free surfaces $s \sin \theta$, and the force on a strip of width ds on the surface would be $\rho g s \sin \theta$ into area dA of that strip.

So, this can be written as: we can take $\rho g \sin \theta$ outside the integral, as on this force $\rho g \sin \theta$ integral $s dA$. Integral $s dA$, over the whole area is nothing but the distance s of the centroid, s_{CG} times the area. So, that the force is pressure at the centroid times the plate area. The same result as we obtained above. This the force.

But, where is the center of pressure?

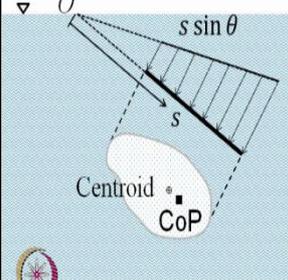
Center of pressure is obtained by keeping this total force at a location such that the moment of this total force at that location is equal to the moment of the distributed force. The moment of the distributed force is obtained like this: the moment of the force on this element dA , at distance s about origin O is $-s \cdot \rho g s \sin \theta dA$. This is negative because this moment is clockwise.

Now, to find the total moment we integrate over the area. This $s^2 dA$, this term, integral over A , is nothing but the second moment of area about an axis perpendicular to this screen passing through O , so that the moment now is: $\rho g \sin \theta I_{yy,0}$.

This moment should be equal to this: distance s of the center of pressure s_{CP} multiplied by the total force which is determined in the last slide as the pressure at the centroid $\rho g s_{CG} A \sin \theta$. So, that the s_{CG} is obtained as $I_{yy,O} / s_{CG} A$. This is the distance of the center of pressure measured along s from the free surface. But this is in terms of $I_{yy,O}$ which is not easy to find, it will vary with a whole lot of things like an angle theta.

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Centre of Pressure on a Submerged Surface



$\rightarrow s_{CP} = \frac{I_{yy,O}}{s_{CG} A}$

Using parallel-axis theorem

$$I_{yy,O} = s_{CG}^2 A + I_{yy,CG}$$

We get

$$\rightarrow s_{CP} = s_{CG} + \frac{I_{yy,CG}}{s_{CG} A}$$

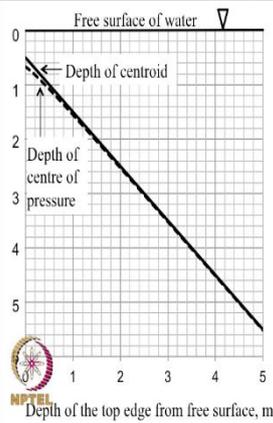
Below the centroid

So, let us find out $I_{yy,O}$ in terms of I_{yy} about the centroid of the plate, and for this we use the parallel-axis theorem. The parallel-axis theorem says the second moment of area about any arbitrary point O is equal to the distance of that point O from the centroid squared, i.e., s^2 times the area of the plate plus the second moment of area about the centroid. Second moment area about the centroid is the property of the area.

Using this in the first relation, we get s_{CP} , the location of this center pressure as the location of the CG: s_{CG} plus $I_{yy,CG} /$ divided by $s_{CG} A$. So clearly, the center of pressure is always below the center of gravity. But, this distance between centroid and center of gravity, decreases as s_{CG} decreases.

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Centre of Pressure on a Submerged Surface



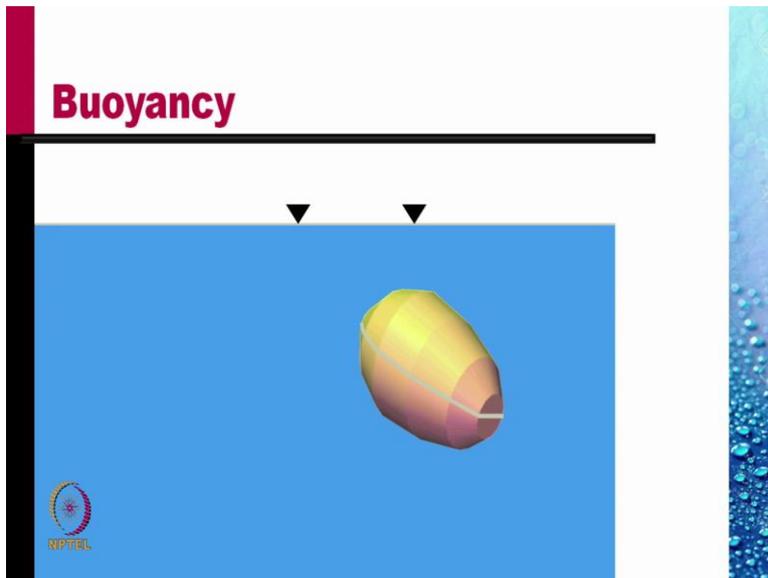
Movement of the centre of pressure on a flat plate (1 m × 1 m) as it moves down from just immersed in the liquid

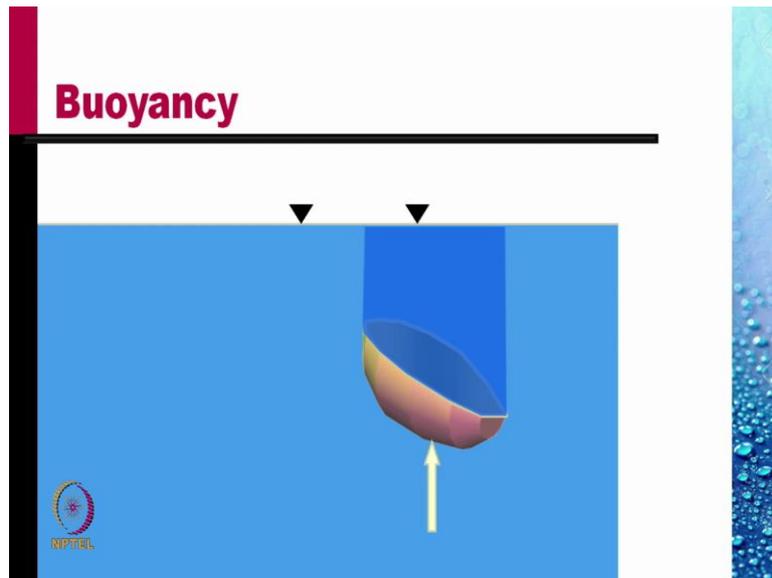


For a depth about 2 meters, so the depth of the center of pressure is almost at the same location as depth of the centroid, if the top edge of the surface is more than two meters from the free surface of water.

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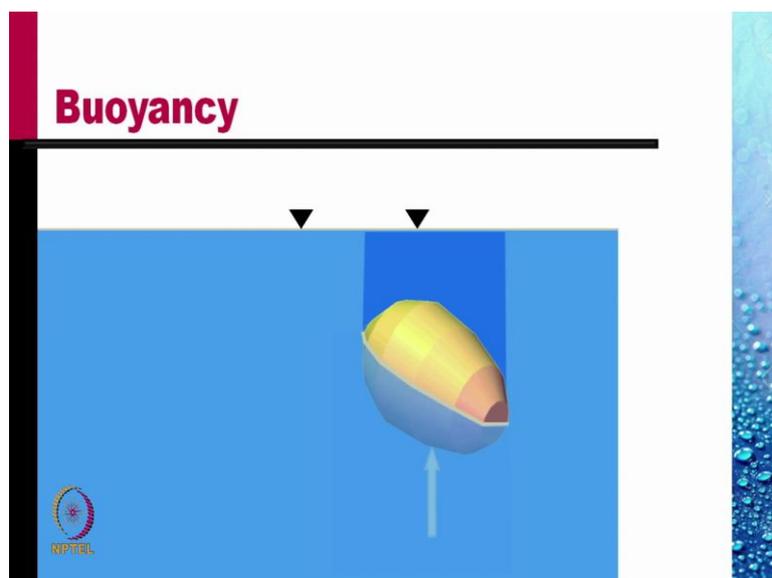
Buoyancy

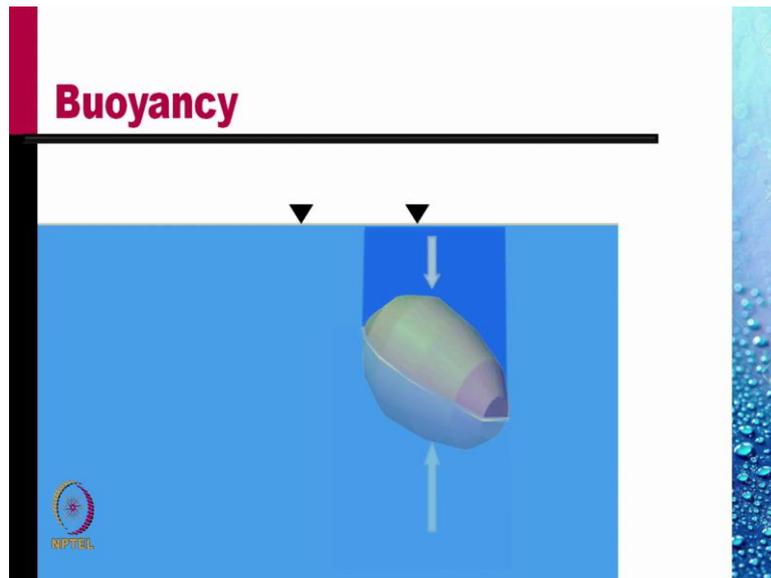




Let us determine what is the force of buoyancy. From whatever we have done so far. Let us consider a body immersed within a liquid the free surface is marked. Let us split the body in two halves about its meridian. The vertical force acting on the bottom half which will act upwards at the bottom is equal to the weight of the water supported, or weight of the liquid supported by this. So, weight of the darker blue column of water up to the lower half surface. This is of course, acting downwards on the top, and so it is acting upwards on the surface shown.

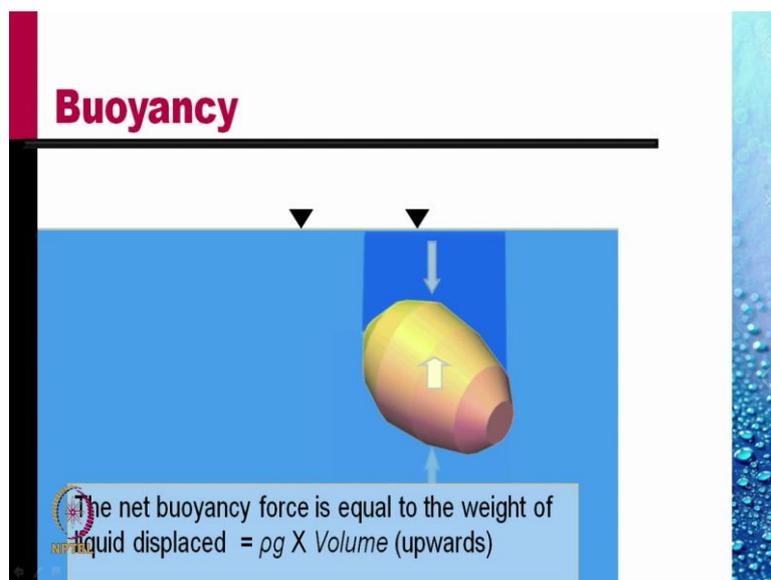
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Now, the other half of the surface supports this column of water above it. So, the pressure force on this upper half of the body is downwards equal to the weight of this column water.

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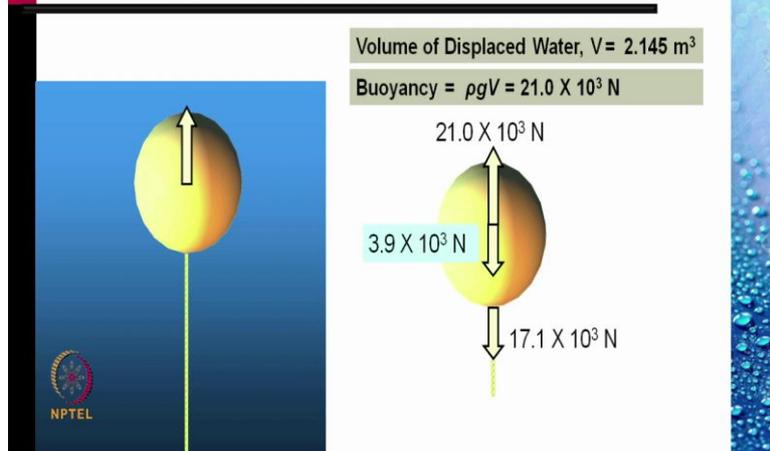


The difference of the two obviously is the weight of this volume of water, and this would be upwards. So, the buoyancy force is equal to the weight of water equal to the volume of the body, or weight of the water displaced by the body. This is what you studied as Archimedes principle in your high school. The buoyancy force is equal to the weight of the liquid displaced: ρ times g times the volume of the body itself, and it is upward. This is of course, independent of the depth to which the body is submerged, if the density of the fluid does not change with depth.

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Example:

A 400 kg buoy with a 0.8 m radius is tethered to the bed of a lake. What is the tensile force in the cable?



Let us do a very simple example. A 400-kg buoy with a 0.8-m radius is tethered to the bed of a lake. What is the tensile force of the cable? For this of course, we will draw a free-body diagram of the buoy. On this free-body diagram, we would have the buoyancy force and the tension in the string. So, the tension on this, and the weight of the buoy itself.

So, the buoyancy must be balancing the weight of the buoy plus the tension, and the tension will be equal to the buoyancy minus the weight of the buoy. To find the buoyancy, we find out the volume of the displaced water, which is spherical of 0.8-m radius, which is 2.145-m^3 . And so, the buoyancy is $\rho g V$ which is $21 \times 10^3 \text{ N}$. This is the free body. $21 \times 10^3 \text{ N}$ and upwards, and the weight for 8 kg, which is $3.9 \times 10^3 \text{ N}$ downwards, and the difference must be balanced by tension which would give you $17.1 \times 10^3 \text{ N}$.