Fluid Mechanics and its Applications
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Lecture: 4A
Surface Tension

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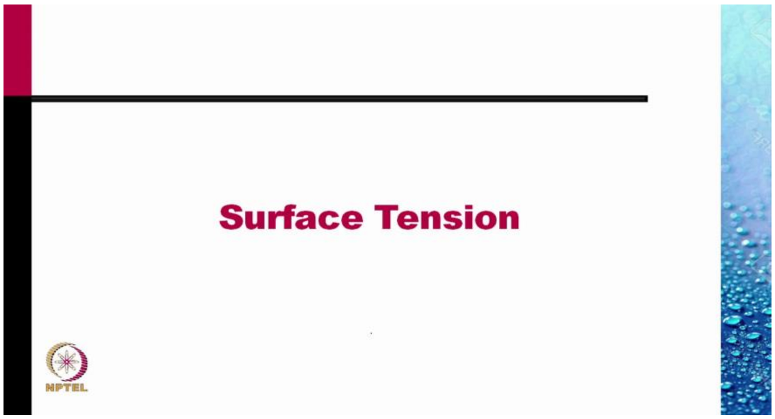

We next cover a phenomenon called surface tension, which is quite prominent in fluids at rest that have an interface with another fluid, most often with a gas.

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You might have seen little insects which seem to be walking on water. There is an insect known as a water strider which is walking on the surface of water without sinking. How is that achieved?

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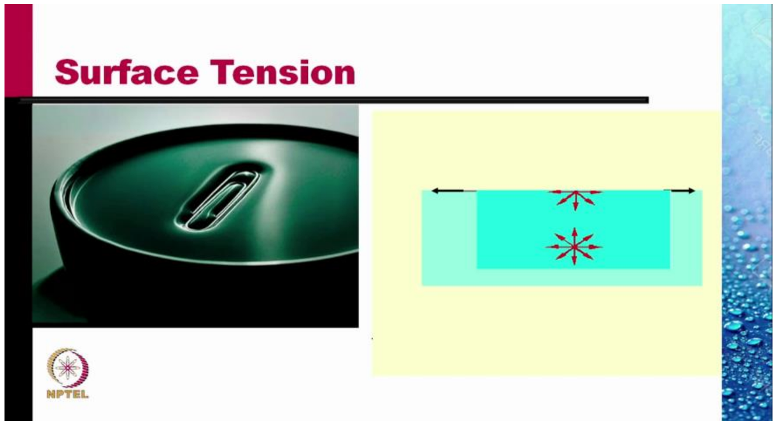

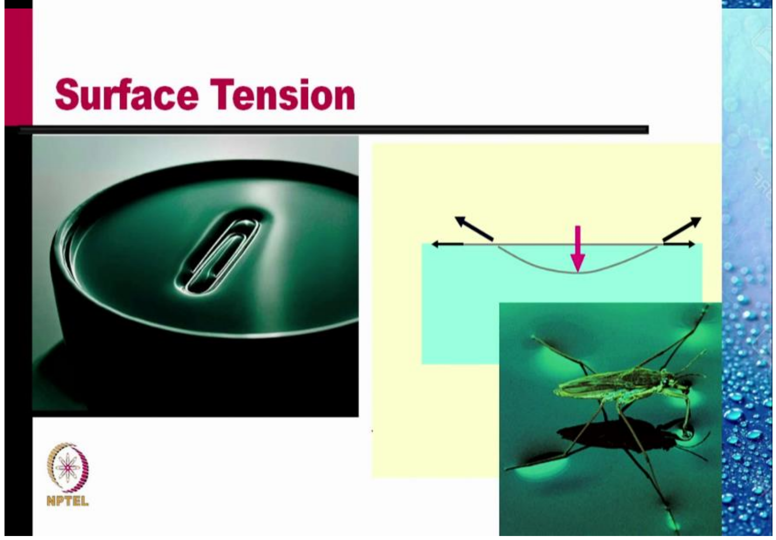

Similarly, in this picture we have a cup of water on the surface of which a paperclip is floating. What is supporting the weight of this paperclip? Clearly, it is not buoyancy because the paperclip is not submerged in water. So, what is going on? The explanation of these two phenomena lies in what is called surfaces tension.

At the interface between a liquid and a gas the upper surface is under tension. A molecule which is not at the surface is being pulled on each side by molecules of the liquid all around it. But a molecule on the surface is pulled down. But it cannot go down so there is a tension created on that surface. If we apply a force vertically on the surface, the surface bends and produces a small dip, So that at the edge of the dip, the surface is inclined. And the vertical component of these inclined forces balance the vertical force which is shown by a red arrow. This is what is happening to the clip. You can see that the water around this clip is showing a depression.

So, it is like a membrane which is stretched, and it supports the weight of the clip. Similarly, you notice in details that under each leg of the water strider there is a cup like formation of the water surface. The surface is curved and the tension on the surface is now providing an upward force to each leg.

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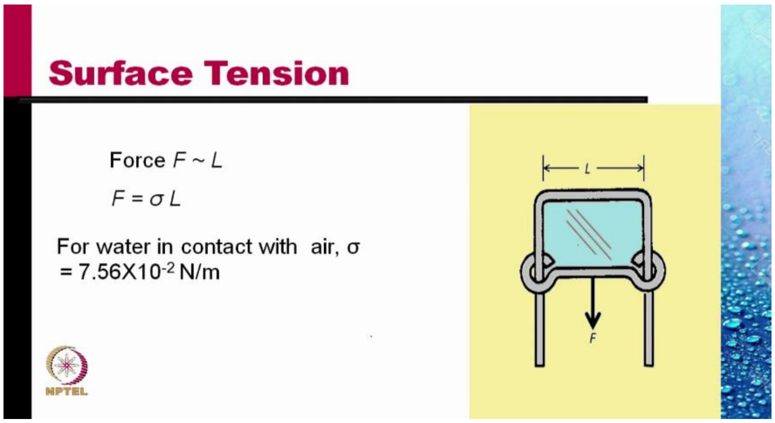

The surface tension in liquid can be measured by a small setup. We have a U-shaped frame on which we have a slider. If we create a film of liquid on this, that film a liquid can support a small force F. This small force F depends upon the length L of the film in the horizontal direction as shown in this. Force F is equal to σL, where σ is termed as the coefficient of surface tension for the liquid-gas pair. It depends upon the liquid and the gas both. For water in contact with air, the value of sigma, the surface tension, is 7.56×10^{2} N/m. It is a very small quantity, but can become significant in many cases, the two of which we have already discussed.

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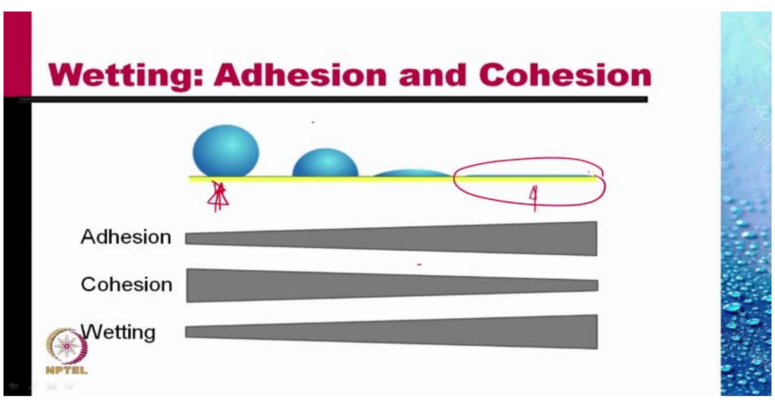

The force of surface tension is also responsible for formation of bubbles. When a droplet of water is in contact with the surface there it is the three-surface interface: the solid, air and liquid. As the adhesion increases to the right, the cohesion decreases to the right, and the wetting increases. We say that at this location the wetting is the minimum. And at that location, the wetting is the maximum.

You would see that if I have a waxed paper, a droplet of water on the waxed paper would almost look like this on the left. But on an ordinary paper that droplet of water would spread out like this, because the water wets an ordinary paper by does not wet a waxed paper. On a waxed paper, the adhesion with the paper is low, cohesion within the water molecules is high.

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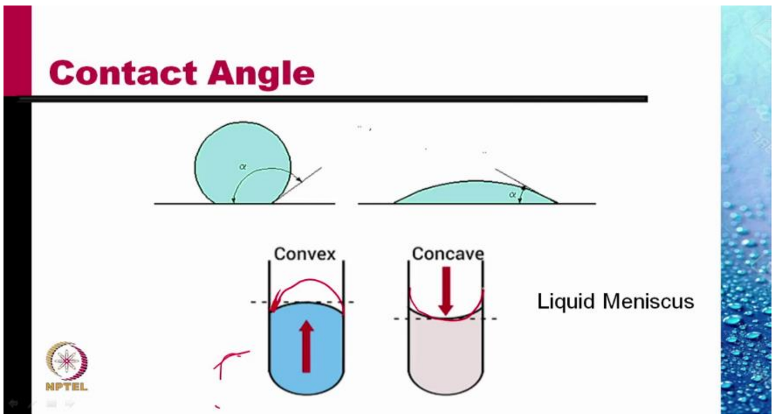

We introduce here a term *contact angle*. Contact angle is the angle that the droplet makes with the surface. So, here the contact angle is large. Here the contact angle is small. These are surfaces that are wetted surfaces. The contact angle is low on glass. Water has a very low contact angle, almost zero. Water wets the glass almost completely.

But if there was mercury, the mercury does not wet glass, and mercury would be sitting on glass with almost spherical bubbles so that the contact angle alpha is 180 degrees.

This wetting results in a liquid meniscus as shown on the left. In a fluid like mercury which does not wet the tube material we have a convex meniscus. On the right is a liquid that wets the wall of the tube. So we have a concave meniscus. Water in a glass tube forms a concave meniscus, mercury in a glass tube forms a convex meniscus. The meniscus of mercury in a glass would be much more pronounced than that and it would be like this. The contact angle would be almost 180 degrees there, and the contact angle in water would be 0 degrees. So, meniscus would be much sharper.

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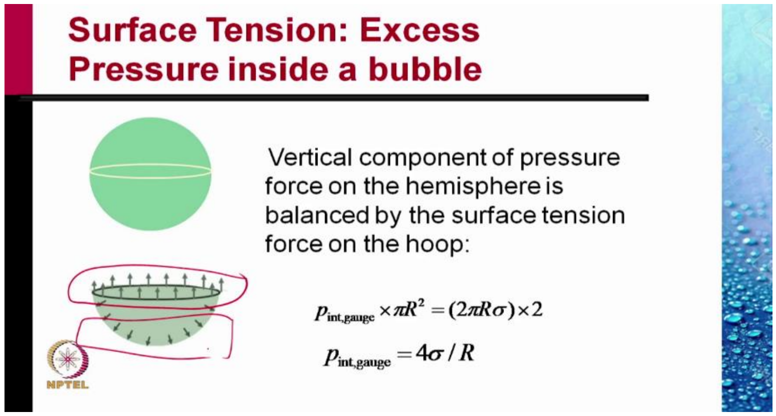

We next calculate the excess pressure inside a bubble like a soap bubble. Let us consider a soap bubble of radius R. The pressure inside is a little more than the pressure outside. How do we relate the pressure difference to the surface tension?

Consider half the bubble below the equator. The free-body diagram of this bubble shows the forces in this bubble. There are two kinds of forces. This set of forces are the forces that act on the surface of the bubble, and they are because of surface tension of the liquid surface and the air. These are all around the circumference of the cut that we have created. And these forces are the pressure forces that act on the hemispherical surface because of the excess pressure inside. If the excess pressure is p, then we can show that the component in the vertical downward direction, is $p\pi R^2$, where $\pi R^2$is the area of the diametrical plane of this bubble.

In the surface tension forces upwards are sigma times the length of the equatorial cut, which is $2\pi R$, where R is the radius of the bubble. But that is only on one side. A bubble would have two sides of the surface: one the outside in contact with the outside air, and one inside in contact with the inside air.

So, the total surface tension force would be $2\sigma \times 2\pi R$. The equation would be the $p_{int,gauge} \times \pi R^2$ is equal to $2\pi R\sigma$, which is the force on one side of the hoop into 2, because there are two sides to the hoop.

So, the internal pressures is $4\sigma/R$. It is interesting. This pressure, or the excess pressure, varies inversely like R. That means, lower the radius we need more pressure. So, while blowing a soap bubble will need a larger pressure to start the bubble, and then the pressure needs to decrease.

We need more air in there to increase the diameter, to increase the size of the bubble, but the pressure inside must be lower.

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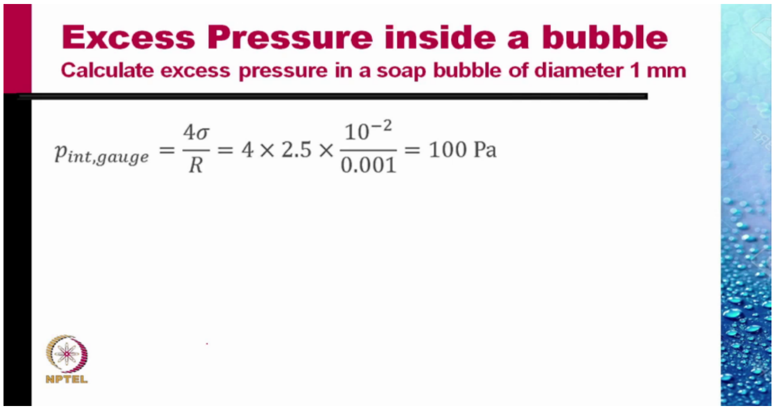

This is an estimate of this internal pressure. We are talking for water with a sigma for 2.5 into 10 raised to minus 2 N/m, and the radius 1 mm, that is, 0.001 m, and this gives you 100 Pa. 100 Pa is a very small excess pressure.

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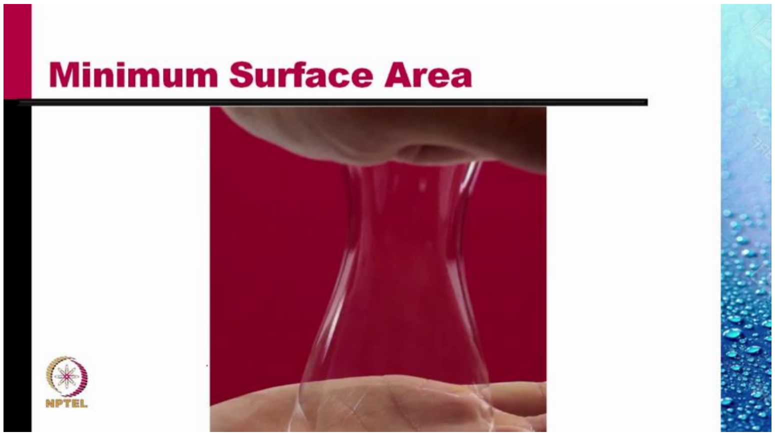
Liquid soap



In this video that we showed, when the two bubbles coalesce together, and we stretch them by taking our hands apart, the bubble surface acquired this shape. This is because surface tension results in stored energy in the film and the surface acquires a shape with the minimum energy, and that minimum energy occurs when the surface area is the minimum. So, this surface is the minimum areas surface that is formed with the amount of air that we had in those two bubbles.

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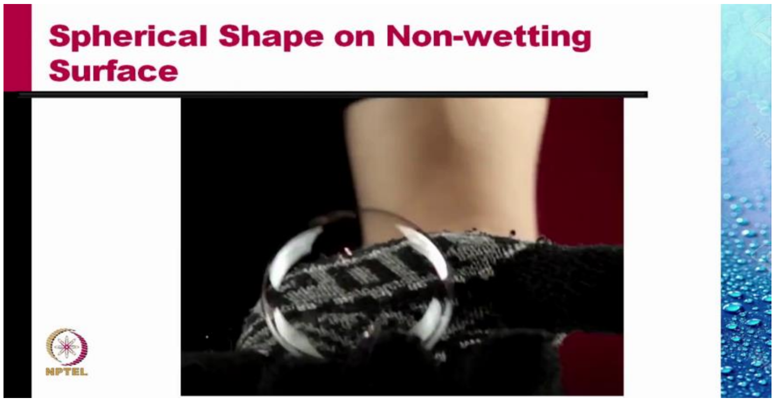

Similarly, we showed that when the experimenter played with a bubble with woolen gloved hands, the bubble were very spherical and they were jumping from hand to hand. This is because the wool is not wetted by the soapy water that the bubbles are made off.

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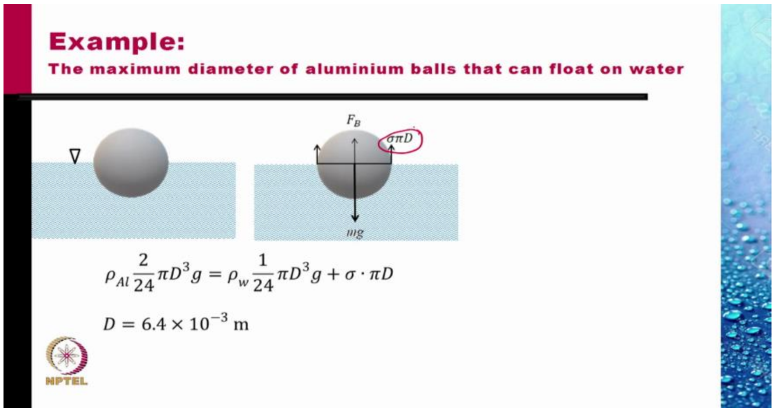

In the context of surface tension, let us do an example where we calculate the maximum diameter of an aluminum ball that can float on water. There is a small aluminum ball which is floating on water. We have to find out the maximum diameter that this ball can have. If the diameter increases, the ball would sink, that is, the surface tension force would not be enough to keep the ball a float.

It stand to reason that is at the maximum size of the ball is when the weight of the ball would be maximum and the surface tension force would be maximum. And the maximum surface tension force would occur when the ball is immersed up to its equator, that is, half the ball is inside and half the ball is outside in air.

So that the length of the water-air interface at the location of the ball is maximum equal to πD. Now, we draw a free-body diagram of the ball. It has three forces. One is the weight of the ball, mg. Other is the buoyancy force on the ball. The buoyancy as you studied in high school, is related to the weight of the liquid displaced.

Here the volume of the liquid displaced is equal to half the volume of the sphere $\frac{2}{3}\pi R^3$. And so, the buoyancy force would be $\rho_{water} g \left(\frac{2}{3}\pi R^3\right)$. The weight would be $\rho_{aluminium} g \left(\frac{4}{3}\pi R^3\right)$. And the third force is the surface tension force, which acts over a length πD. So, the force is $\sigma\pi D$.

If we put all the forces together, we get that the weight should be equal to the buoyancy force upward plus the surface tension force upward. Unlike in the case of bubbles, there is only one interface here. So, the length we have taken is πD. And plugging in the values for various quantities, we get D is equal to 6.4×10^{-3} m or 6.4 mm, not a very small diameter. So, aluminum spheres of diameter 6.4 can float on water if they are carefully launched.

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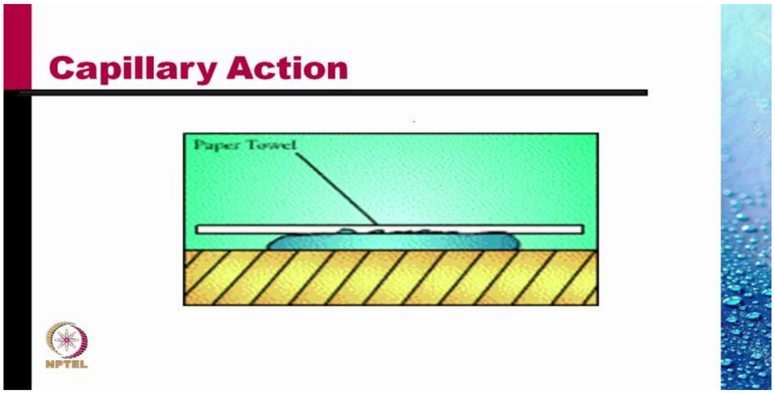

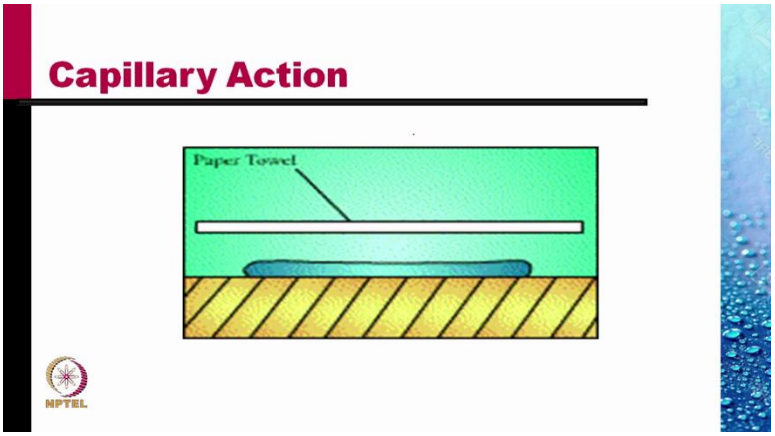

Another consequence of surface tension is the capillary action. If you see there was a droplet of water on a table-top, and if a paper towel is brought in contact with this, the water is soaked up by the paper towel. What pulls up this water? It is the force of surface tension.

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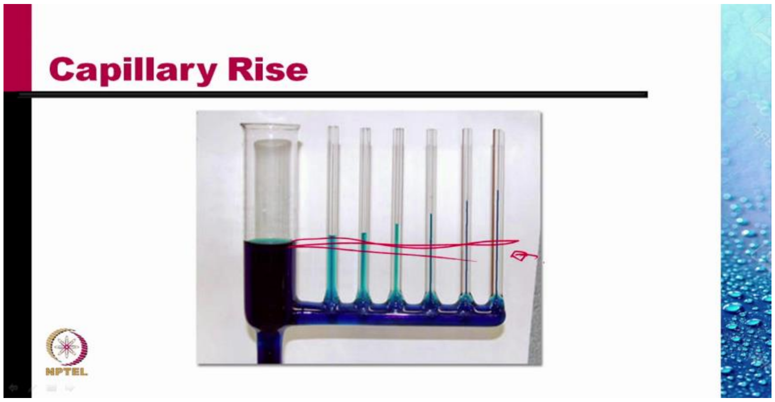

In this picture is shown a container where there is a blue fluid in a number of tubes with varying bores. The left most tube has the largest bore, and the bore size decreases as you move to the right. So, this tube has the smallest bore and you see the blue liquid rises up in this tube beyond the level contained in this tube. The liquid is not maintaining its level. The liquid is rising up, and rising more in tubes with a smaller bores. Tubes with their very small bores are called capillaries, and so the phenomenon is termed as the capillary rise or capillarity.

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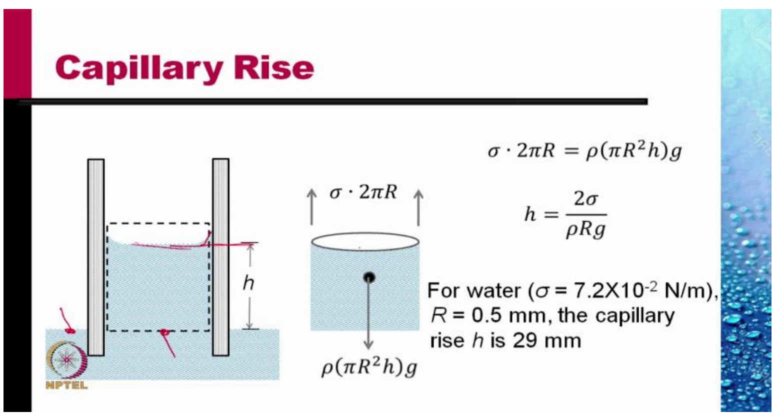

Another example of capillarity is the colored water from the cup with a higher level of water flowing into this over the edge through a rag.

To analyze this capillary rise, consider a capillary in which the water or the liquid has risen up to a level *h*. Of course, we ignore the meniscus and measure only the height in the main tube.

We can draw the free body of this fluid. The free body is shown here. Clearly, the pressure at this point is the same as pressure at this point: same level in a liquid connected with the same fluid. And the pressure here is atmospheric. So, the pressure there is atmospheric. So, in this the pressure on the top is atmospheric pressure the bottom is atmospheric.

Other forces: the weight of this column of liquid which would be $\rho(\pi R^2)hg$. And this is balanced by the surface tension force $\sigma \times 2\pi R$, assuming that the contact angle is 0. This force is vertically upward. Water in a glass tube would have the contact angle equal to 0.

So, this force will be s $\sigma \times 2\pi R$. Clearly, at equilibrium, the two forces must balance, and therefore, we can get a capillary rise h equal $\frac{2\sigma}{\pi R g}$. The larger the value of R, the less is the capillary rise. Let us estimate this capillary rise for water in a capillary of diameter 1 mm, that is, of radius 0.5 mm. And we plug in the value of h the capillary rise is 29 mm, or almost 3 cm in a tube with a bore of 1 mm.

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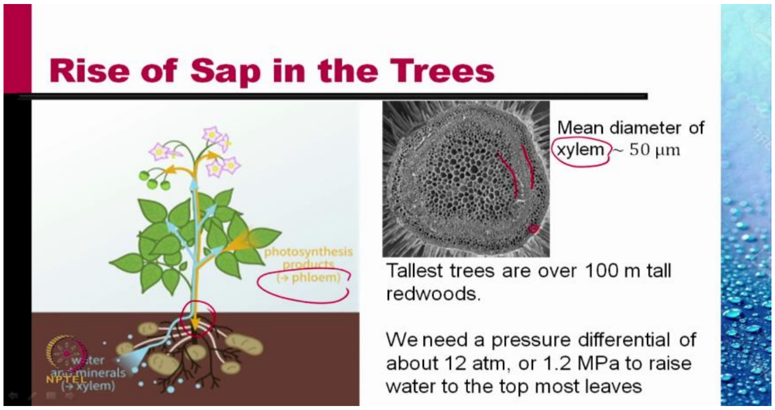

All trees and vegetation on earth need water for growth. The water is sucked up through the roots through a system called xylem. And as it goes up, the product of synthesis travel down through a system called phloem shown here with yellow arrows flowing down.

Now, the tallest trees on earth over 100-meter-tall, the redwoods of California USA. For over 100 meters rise, we need a pressure differential of about 12 atmosphere or 1.2 MPa to raise water to the top most leaves. There are many ways by which this kind of pressure differences are created, not the least of which is the capillarity.

The capillaries of the sap wood of redwood trees, which is in this region around the tree trunks, is a foam like wood with a mean diameter of xylem at about 50 μm. Of course, the phloem are just inside the bark of the tree, and they produce a flow downwards.

In fact, it is argued that this flow of fluid of the product of photosynthesis downwards also helps in raising the water up because it creates a kind of vacuum up there when the water flows down. But you remember that even the absolute vacuum produces only about 10 m of water rise in a barometric tube. So, this is only a very small portion. It is a complicated phenomena and we cannot explain it fully here, just to give an indication of that the capillarity is part of the answer.

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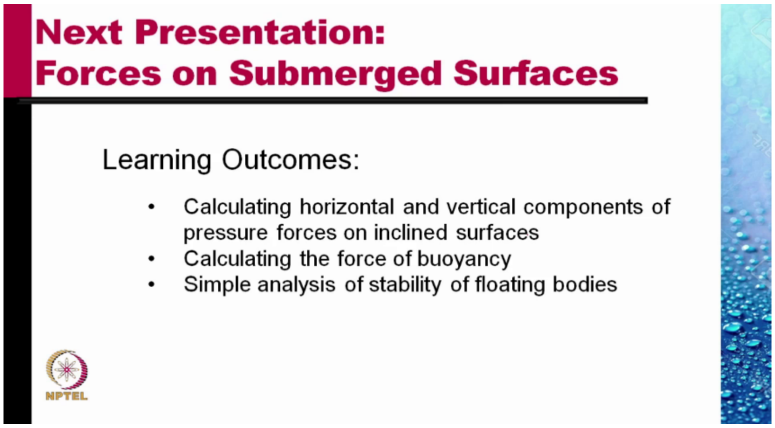

That brings us to the end of this lecture. In the next lecture we will do forces or submerged surfaces. Thank you.