Fluid Mechanics & its Applications
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Lecture 30
Drag on bodies moving through fluids

Welcome back.

In today's lecture, we will discuss the separation of boundary layer from the surface and about the drag on bodies moving in fluids.

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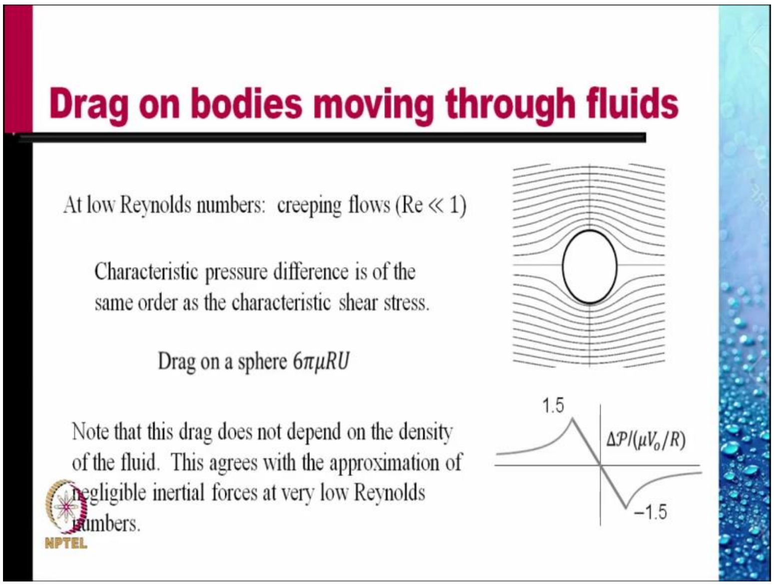

We had seen that in creeping flows, which are flows at Reynolds number much below 1 the characteristic pressure difference is of the same order as the characteristic shears stress. The drag on this sphere is given by $6\pi\mu RU$, Stokes formula, of which $2\pi\mu RU$, is because of the shear stresses on the surface, and $4\pi\mu RU$ is because of the pressure differences.

The pressure is anti-symmetric. The pressure at the nose of the body is positive, the pressure at the tail of the body is negative. Note that the drag on a sphere or any other body in a low Reynolds number flow does not depend on the density of the fluid. This agrees with the approximation of negligible inertial forces at very low Reynolds numbers.

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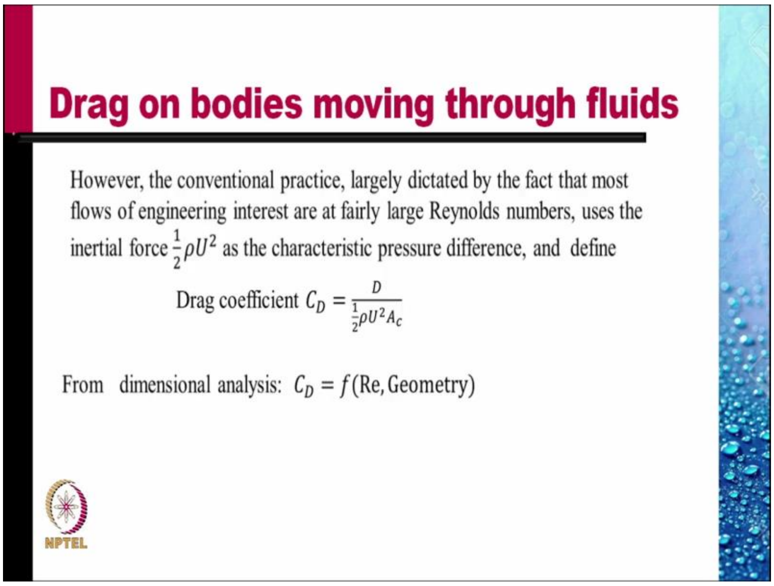

However, the conventional practice, largely dictated by the fact that most flows of engineering interest are at fairly large Reynolds numbers, uses inertial force $\frac{1}{2}\rho U^2$ as the characteristic pressure difference, and defines the drag coefficient $C_D = \frac{D}{\frac{1}{2}\rho U^2 A_c}$. From the dimensional analysis we can get C_D is a function of Reynolds number and the geometry.

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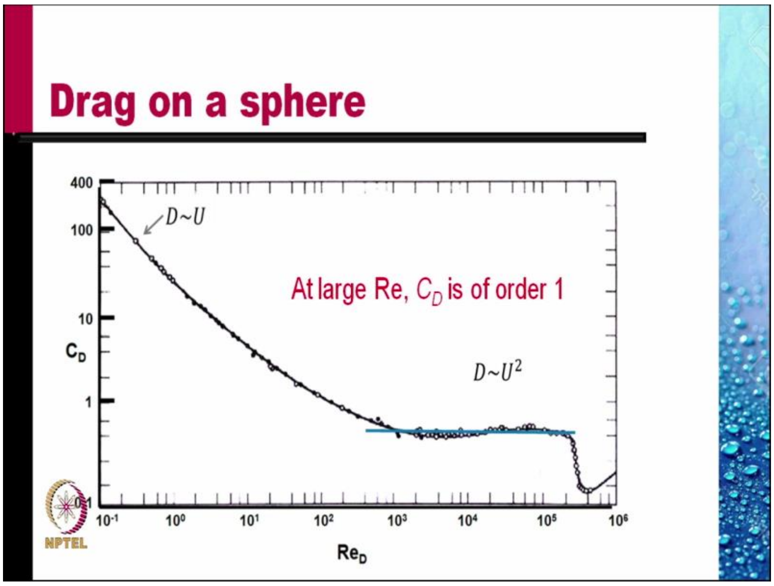

If you plot the drag coefficient versus Reynolds number, we see that the drag coefficient decreases with Reynolds number. But, at very high Reynolds number the drag coefficient is almost constant, of the order one. In fact, its value is about 0.5. Initially, the drag varies linearly with velocity, but for larger values of Reynolds number, the drag varies like U^2, since the drag coefficient is about constant. But the point to notice is that the drag at large Reynolds number, the drag coefficient is order 1. This seems to be in contradiction with the approximations that we learned to make.

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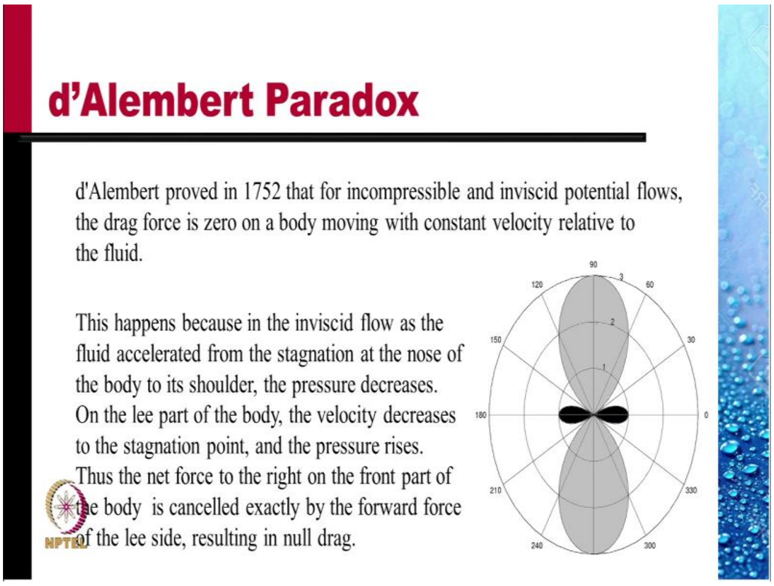

d'Alembert proved that for incompressible inviscid potential flows, the drag force is identically zero on a body moving with constant velocity relative to a fluid. This happens because in inviscid flow as the fluid accelerates from the stagnation point at the nose to shoulder, the pressure decreases. On the lee part of the body, the velocity decreases to the stagnation point, and the pressure rises.

Thus, the net force on the left, on the front part of the body, is being cancelled exactly by the forward force on the lee side, resulting in zero drag, zero pressure drag. Of course, there is no shear stress, so there is no drag. It was proved later, that this drag is zero on all bodies, whatever be the shape, not necessarily cylindrical or spherical.

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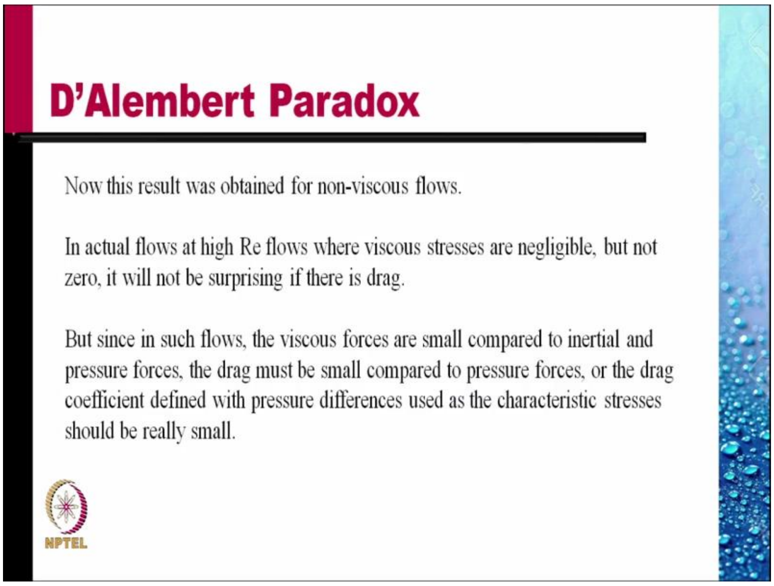

Now, this result was obtained for non-viscous flows. In actual flows at high Reynolds numbers where we have shown that we can neglect viscous stresses, the drag is not zero. And it is not surprising. Drag is not zero, because we are neglecting viscosity. So, in the presence of viscosity, there would be viscous drag, viscous stresses, and hence, the presence of drag would not be surprising.

But, since in such flows, the viscous forces are small compared to inertial and pressure forces, the drag must be small compared to pressure forces, or the drag coefficient defined with pressure difference as the characteristics stress, should really be small.

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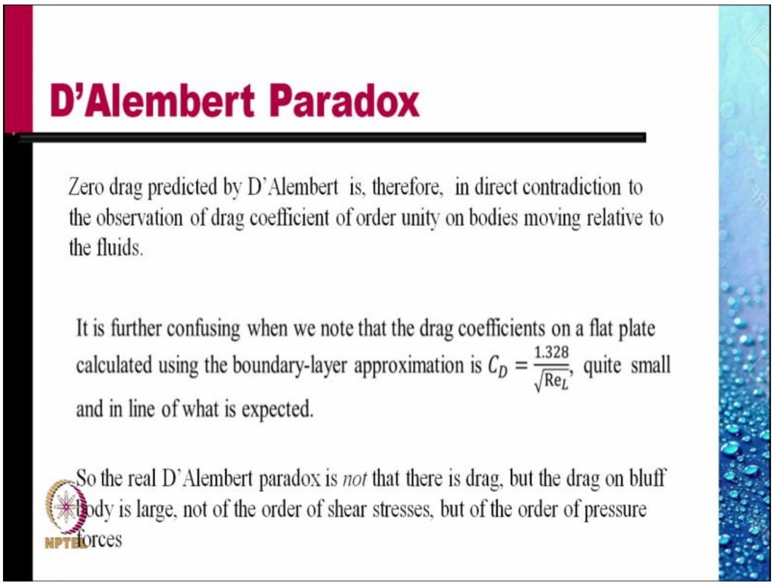

Zero drag predicted by D'Alembert is, therefore, in direct contradiction to this observation of drag coefficient of order one on bodies moving relative to the fluid. It is further confusing when we note that the drag coefficient on a flat plate, derived with the boundary-layer approximation, is about $C_D = \frac{1.328}{\sqrt{\mathrm{Re}_L}}$, which again is quite small, and in line of what is expected because of the approximation. So, the real D'Alembert paradox is not that there is a drag, but that the drag on a bluff body is large, not of the order of shear stresses, but of the order pressure forces.

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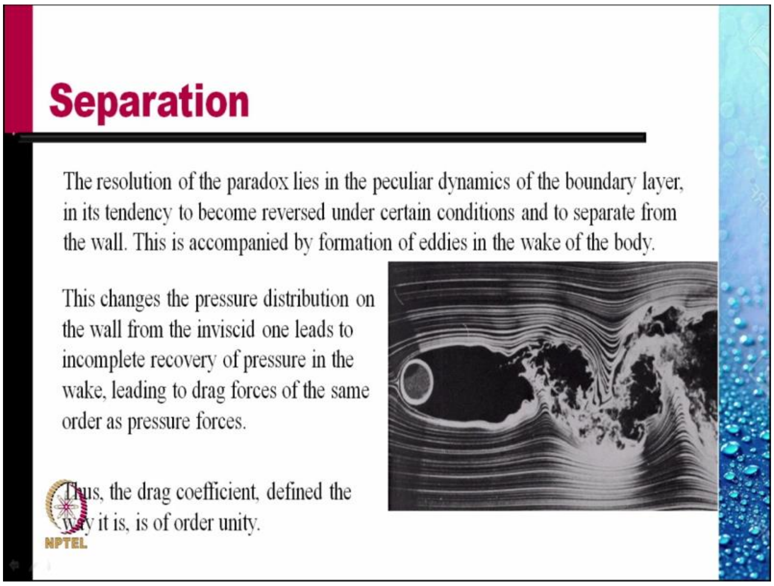

The resolution of this paradox lies in the peculiar dynamics of the boundary layer, that is, in its tendency to become reversed under certain conditions, and to separate from the wall. This is accompanied by formation eddies in the wake of the body. We have seen the flow behind a cylinder looks like this. This changes the pressure distribution on the wall from the inviscid one, and leads to incomplete recovery of pressure in the wake, leading to drag forces of the same order as pressure forces.

Thus, the separation of the boundary layer on the surface of a body makes the flow disturbed, and the pressure does not recover back to its original level in the wake of the body. And thus, a pressure drag results. Thus, the drag coefficient defined the way it is, is of order 1.

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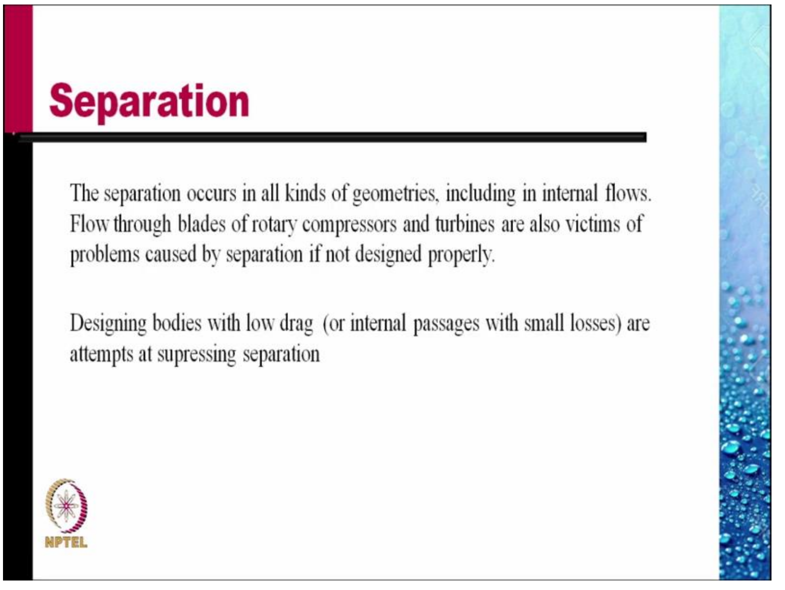

The separation occurs in all kinds of geometries including internal flows. Flows through blades of rotary compressors and turbines are also victims of the problems caused by separation if not designed properly. Designing bodies with low drag or internal passages with small losses are attempts at suppressing separation.

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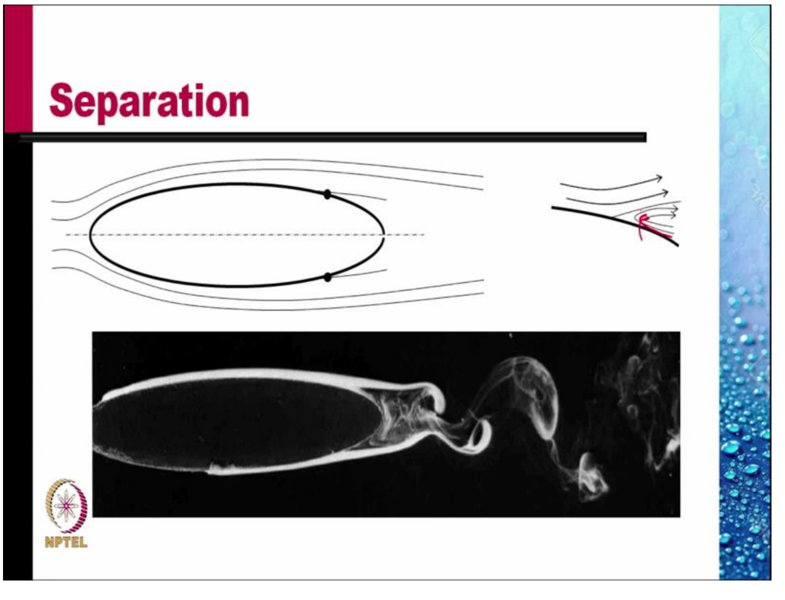

Consider this two dimensional body. The flow about this separates at these two points. And this separation is because of a flow reversal on the surface of the cylinder. And this causes the formation of eddies in the wake. The actual flow about such a body looks like this, separating from the surface.

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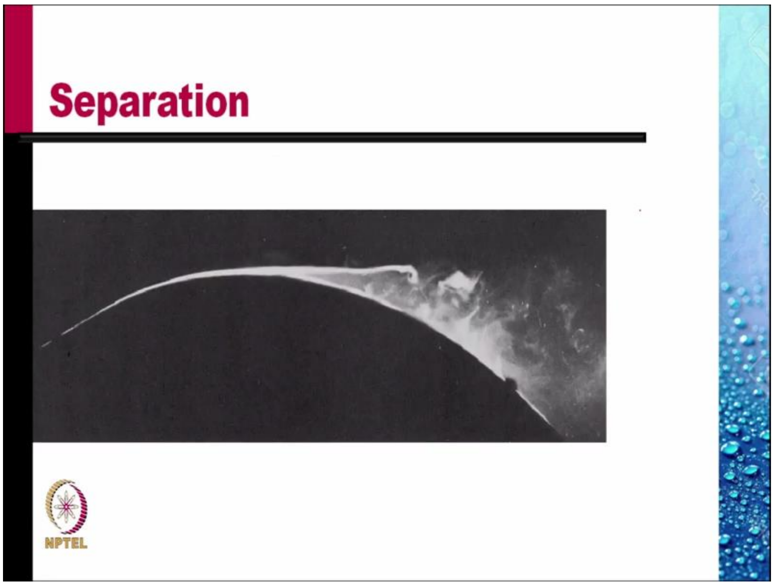

This is another picture of a boundary layer separating from a body. This flow is made visible by applying a coat or titanium tetrachloride on the surface of the body. And when the air flows past this, it produces a smoke like vapour, and we can see how the vapour separates from the body and creates vortices in the wake.

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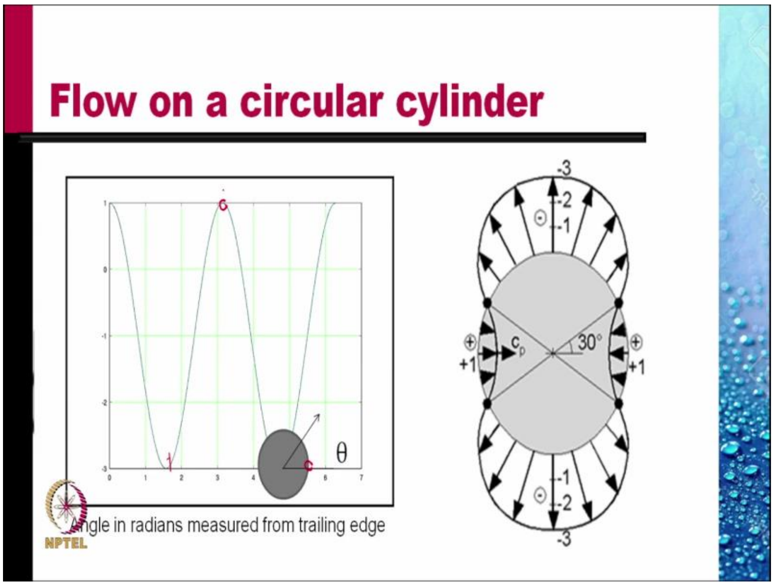

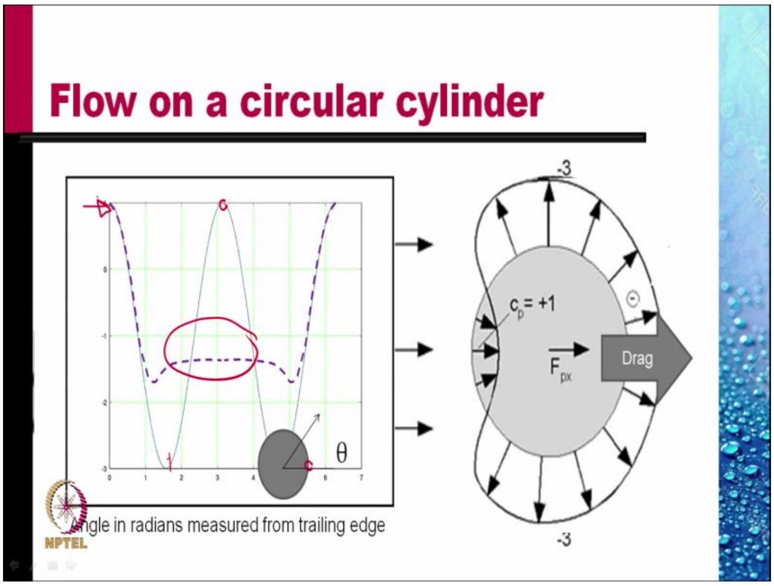

We have seen earlier that in inviscid flows, the pressure distribution is given by this curve. The pressure coefficient is 1 at the nose, it goes down to a value of $+3$ at $\theta = \pi/2$, which is at the top of the cylinder. And in the rear of the cylinder, the pressure coefficient is again 1. The pressure distribution is completely symmetrical fore and aft, and so, the total drag is zero. But because the boundary layer that forms on the cylinder separates from the body, this pressure distribution changes in the rear, though in the front of the cylinder, it remains as if the flow is inviscid.

So, in an actual flow with high Reynolds numbers, the pressure distribution is more like this. On the graph it is like this. So, pressure in the wake has not recovered to the value in front of the cylinder, and this results in a considerable drag. Since this drag force is because of pressure forces, when the drag coefficient is normalized with respect to the pressure forces, we get a value of the order 1.

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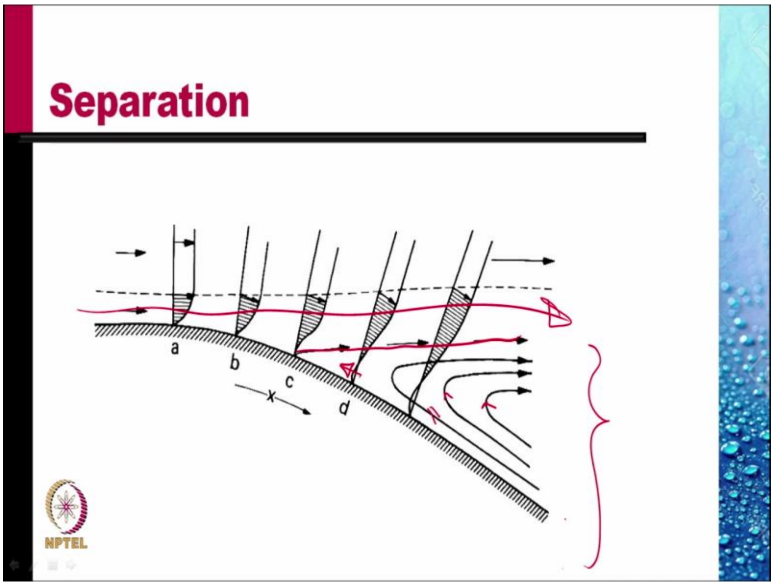

We show here, how the velocity profile changes down a surface where the pressure is increasing in the stream-wise direction. Because the flow in the boundary layer has slowed down, so the kinetic energy of the flow here, within the boundary layer, is much less than the kinetic energy in the main flow, that is, in the flow outside the boundary layer.

And if the flow outside the boundary layer has sufficient energy to negotiate the adverse pressure gradient, the flow within the boundary layer does not have sufficient energy, and so, the flow within the boundary layer near the wall slows down much more. A little later, the flow velocity near the surface has a zero slope, du by dn tends to zero at the surface.

The fluid in the boundary layer close to the plate, not exactly at the plate but close to the plate, is turning to be about zero, and a little later, because of more increasing pressure, the flow reverses. The flow starts flowing in the backward direction near the plate. Because of this separation occurs, and we get this as the line of separation. There is reversed flow within the boundary layer, or close to the surface. So, the boundary layer that was developing here, now separate into this.

This whole region here now is disturbed. The pressure here is no longer equal to the pressure that would have been if the flow was truly inviscid, and no boundary layer was forming. Since an increasing pressure, that is dp/dx positive, leads to separation, which is not a desirable condition, such a pressure difference is called an adverse pressure gradient. And wherever the pressure gradient is negative, that is, pressure is decreasing in the stream-wise direction, we call it the favourable pressure gradient.

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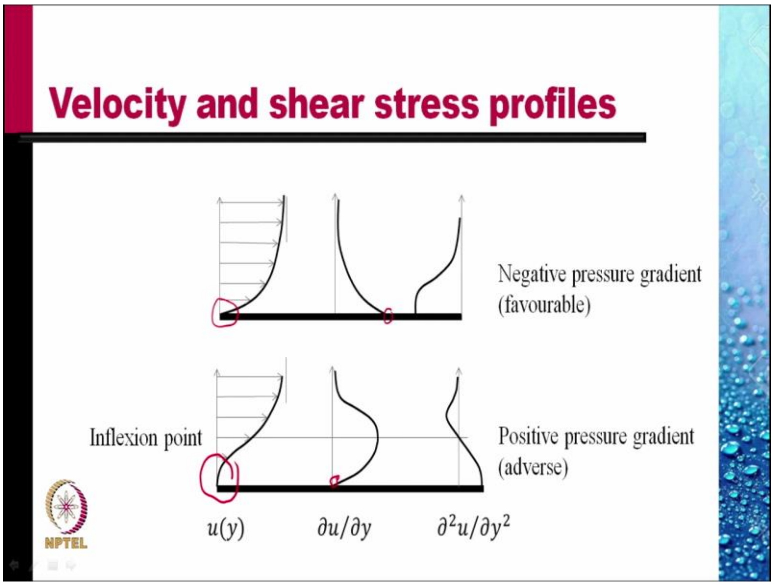

We show here the typical velocity profiles and the shear stress profiles in flows with negative pressure gradient, that is, favourable pressure gradient, and in adverse pressure gradient. The shears stress here becomes zero, because du/dy is zero. In this case, du/dy is positive, there is positive shear stress.

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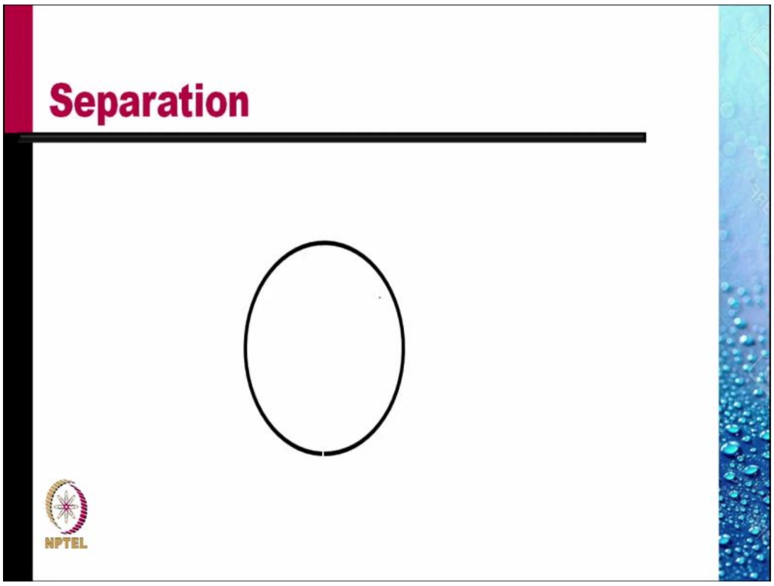

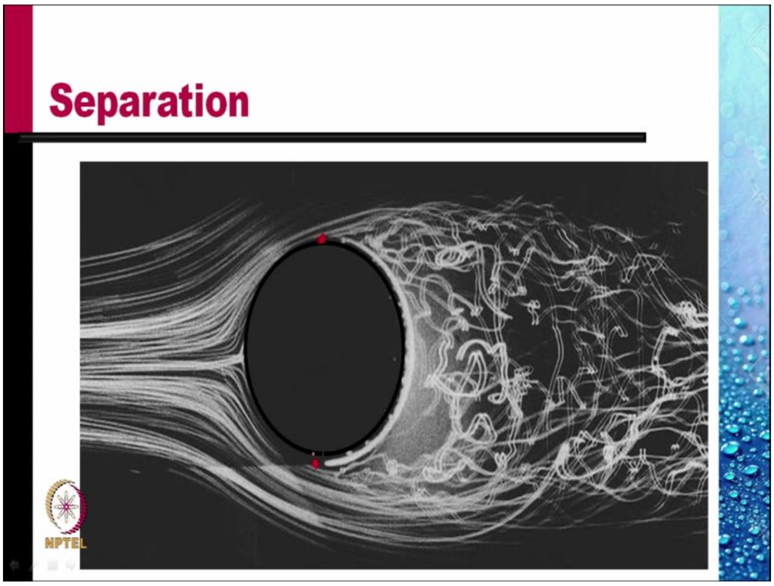

Separation occurs wherever there is an adverse pressure created. If we have a cylinder and the flow past this, there is separation here, because, if the flow was inviscid, the pressure would have been increasing after that. And so, the separation takes place a little before the shoulder points.

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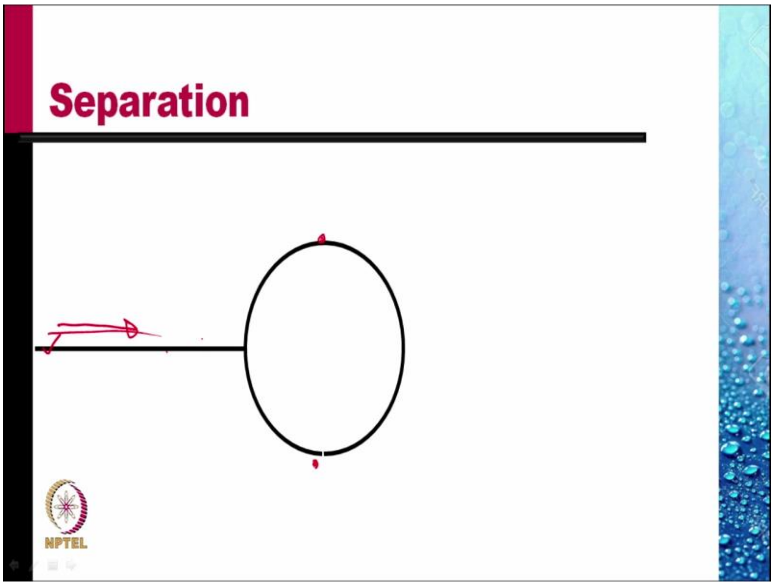

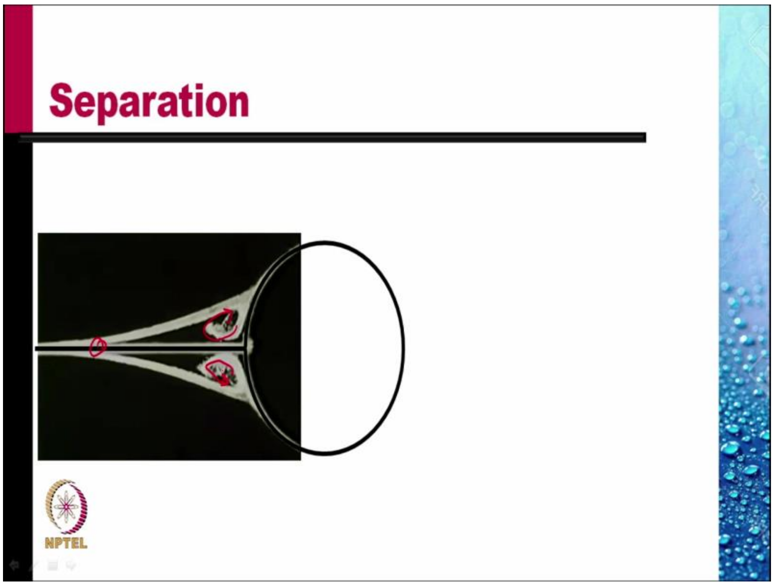

Now, if we attach a splitter plate to the cylinder, a plate in front of the cylinder. Then the flow taking place in that direction, the velocity is decreasing and pressure is increasing. And the presence of this plate causes a boundary layer to develop here, and this boundary layer would separate in the presence of this adverse pressure gradient. Thus, we have the flow which is separating somewhere here on the two sides. Then we have vortices formed here in which the fluid is recirculating on top. This did not happen when there was no plate because though the pressure was increasing, there was no wall on which a boundary layer was developing.

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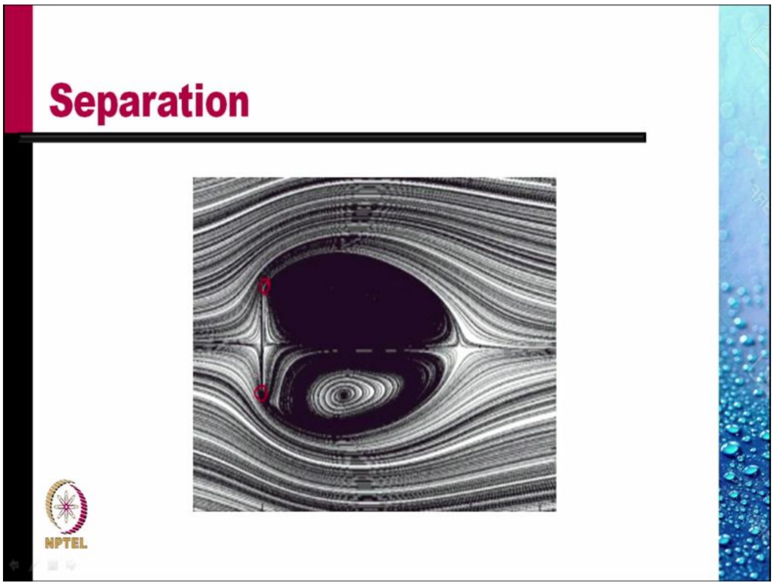

Similarly, we have this plate held perpendicular to the flow. The flow separates at these tips, because if the flow was to proceed down the backside of the plate, the pressure would have been increasing, and the flow separates forming the two large attached vortices because

Reynolds number is not very high. If the Reynolds number was very high, these vortices would have washed down and we would have a region of turbulence in that place.

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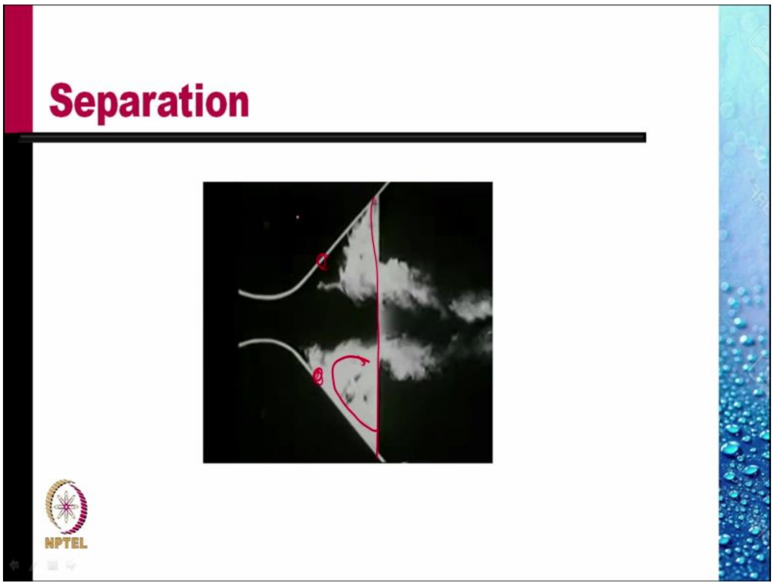

Here we have a flow in a diverging passage. The divergent angle is very large. We have, this is the flow of water through the diverging passage. There is an electrical wire here in which we are passing a current so that the hydrogen bubbles are being formed. You notice that near the wall, the flow is backwards. The hydrogen bubbles formed here are going back, and then being washed down.

So, the flow is separated at somewhere these locations. The passage is diverging, area is increasing, velocity is decreasing, pressures is increasing, adverse pressure gradient, presence of wall, boundary layer is developing. So the boundary layer separates in the presence of adverse pressure gradient. This is why the divergent portions of nozzles, known as diffusers, should be designed with very small divergence angles, so that the adverse pressure gradient is not too much, not that causes separation.

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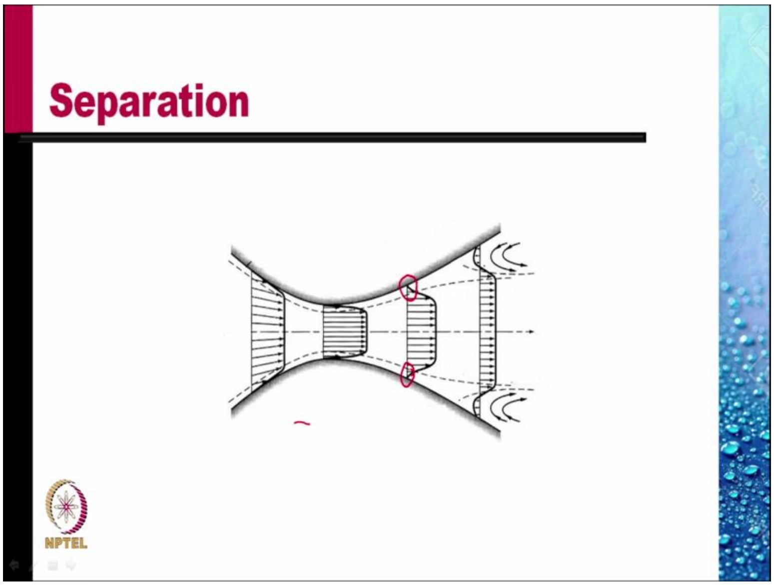

This sketch illustrates the separation that we are talking of. At about this point, the velocity gradient normal to the wall decreases to zero, and the flow separates. We have, thus, separation at these two locations, and there is a reverse flow near the wall as you can see. We have recirculating eddies in this region.

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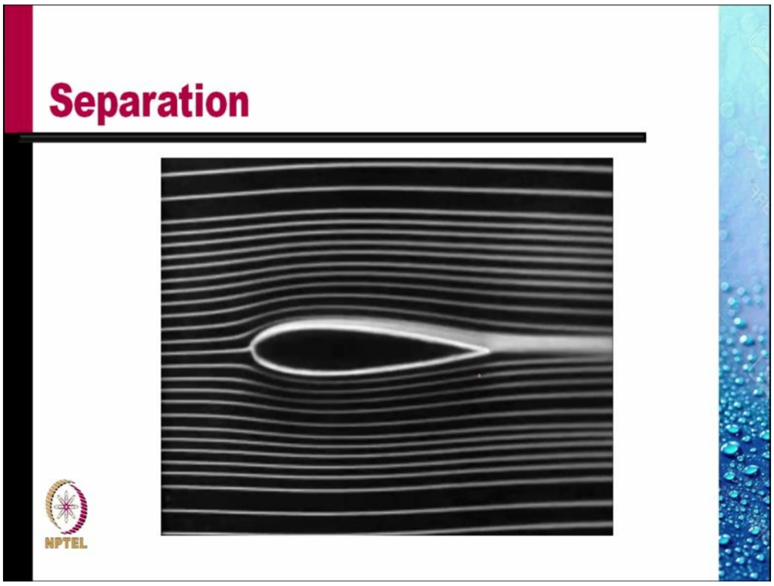

Separation also takes place on airfoils, if the airfoils are at large angles of attack. In this case, this shows a small angle of attack, a boundary layer that is developing on the upper surface is shown here by smoke streak lines.

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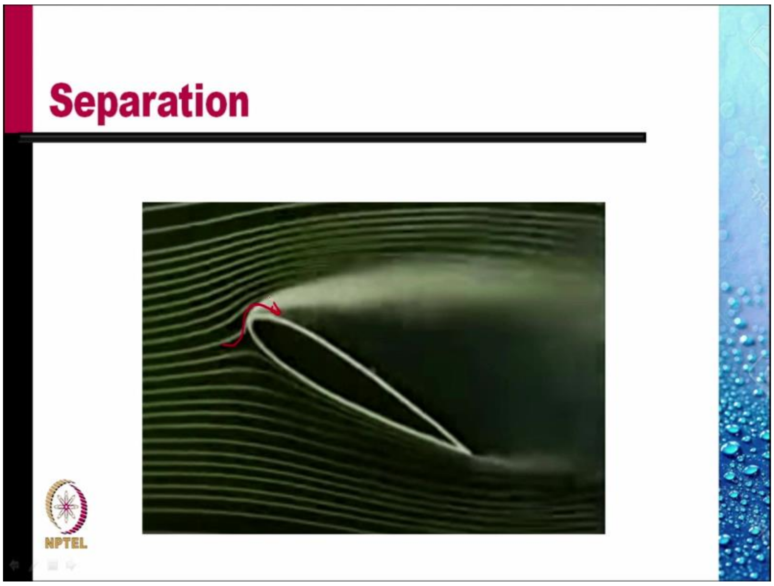

When we increase the angle, if the flow was to go around this, then the pressure would increase and so, it creates an adverse pressure gradient very close to the nose, and the flow separates itself from the wall at the nose itself. This is stall in the language of aeronautics, and needs to be avoided. This leads to a sudden decrease in lift and a sharp increase in drag. A condition quite harmful for the aircraft.