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Let us now switch our attention to wind mills. Wind mills are devices that extract energy from the wind. There are various kinds of windmills that are available. But the most promising models that are in operation now are the horizontal-axis wind turbines which have multiple blades which rotate because of the forces created by wind flowing through them, and that rotation is converted through an electrical generator into electric power which is transferred to the grid. In such wind turbines, the size is of consequence. The size of wind turbines are almost constantly increasing.

And now, we have wind turbines of the order of 170 meters in diameters producing as much power as about a 10-megawatt per installation. All the controls including the gearbox and the generator are located at the nacelle, and this nacelle is a fairly large space, the size of a box truck, in which we have mechanisms to make the blade always point in the direction of wind, a gearbox which keeps the speed of the shaft in the generator at the required parameter. It also has mechanisms to change the pitch of the blade to get the maximum power out.

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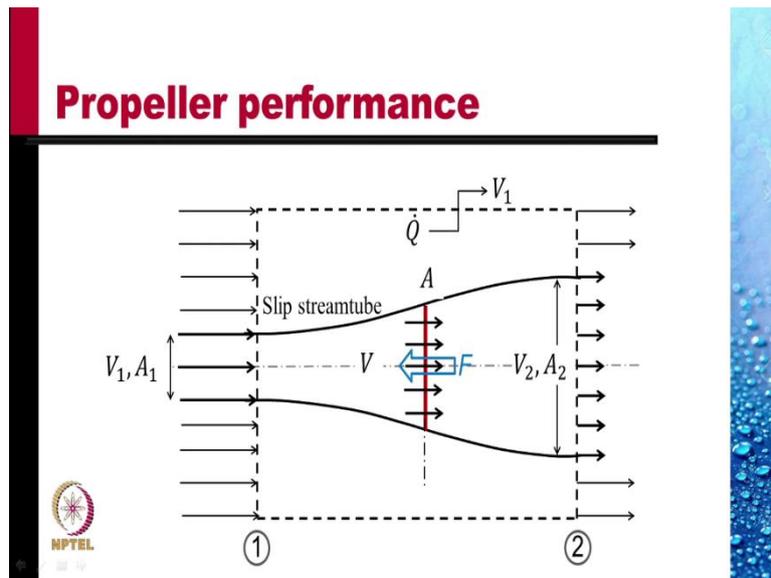
Let us consider the flow past the rotor of a windmill. Let us consider that these two black curvilinear lines represent the slipstream lines. This is the set of stream lines which constitute the slip stream tube. This is the tube in which all of the action is. We assume that outside this, the disturbance in the flow is very little. Whatever is the fluid that enters the stream tube at the left end, leaves at the right end.

Since, we extract energy in a windmill, this means that the velocity must decrease, and therefore, the cross-sectional area A_2 is larger than the cross-sectional area A_1. And that is why the shape of the stream tube is as is. Let the velocity V of the air at the disk of the wind mill be V. It will produce a force F in the forward direction. This is a force that is slowing down the fluid.

Energy Extraction

Power extracted:

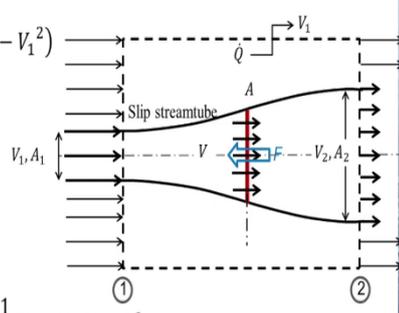$$P = \dot{m}\frac{1}{2}\left(V_2{}^2 - V_1{}^2\right) = \rho S V \frac{1}{2}\left(V_2{}^2 - V_1{}^2\right)$$

$$= \rho S \frac{(V_2+V_1)}{2} \times \frac{1}{2}\left(V_2{}^2 - V_1{}^2\right)$$

$$= \frac{\rho S V_1{}^3}{4}(1+b))(1-b^2)$$
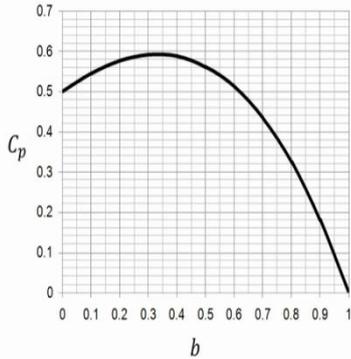
with $b = V_2/V_1$

Performance coefficient:

$$C = \frac{\frac{\rho S V_1{}^3}{4}(1+b))(1-b^2)}{\rho S V_1 \times \frac{V_1{}^2}{2}} = \frac{1}{2}(1+b))(1-b^2)$$

Performance coefficient

$$C_p = \frac{1}{2}(1+b))(1-b^2) \quad \text{with } b = V_2/V_1$$

$$C_{p,max} = 0.5926 \text{ at } b = 0.33$$

Betz Limit

The mass balance through this stream tube gives $\rho A_1 V_1 = \rho A_2 V_2$, and we denoted with $\dot{m}$. We have the momentum equation, $-F$, the force that is applied on the fluid, is $\rho A_2 V_2{}^2 - \rho A_1 V_1{}^2$, and which can be written as $\dot{m}(V_2 - V_1)$. We also use the energy equation in which $\dot{w}_s$, the energy extracted per unit mass throughput, is $= \frac{1}{2}\left(V_2{}^2 - V_1{}^2\right)$. And since the energy extracted must be $-FV$, we multiply F by V, and equate it to $\dot{m}\frac{1}{2}\left(V_2{}^2 - V_1{}^2\right)$.

This gives us the value of velocity at the disk to be the average of the velocity far upstream and far downstream. The power extracted would be simply mass flow rate times the energy extracted per unit mass throughput. And that is shown to be equal to $= \frac{\rho S V_1{}^3}{4}(1+b))(1-b^2)$

with $b = V_2/V_1$. We define the performance coefficient of the windmill as the power extracted divided by what was the power available in the incoming stream in the slip stream tube, and that is $\rho S V_1 \times \frac{V_1{}^2}{2}$.

And so, the power coefficient value $\frac{1}{2}(1 + b))(1 - b^2)$. It is an interesting result, as V changes, one parameter goes up, the other goes down. And so, there is a maximum value of the C_p. This maximum value of C_p, which occurs at a value of b equal to 0.33, is known as the Betz limit, $C_{p,max}$ is 0.5926 at b is equal to 0.33. And this gives you the optimum performance.

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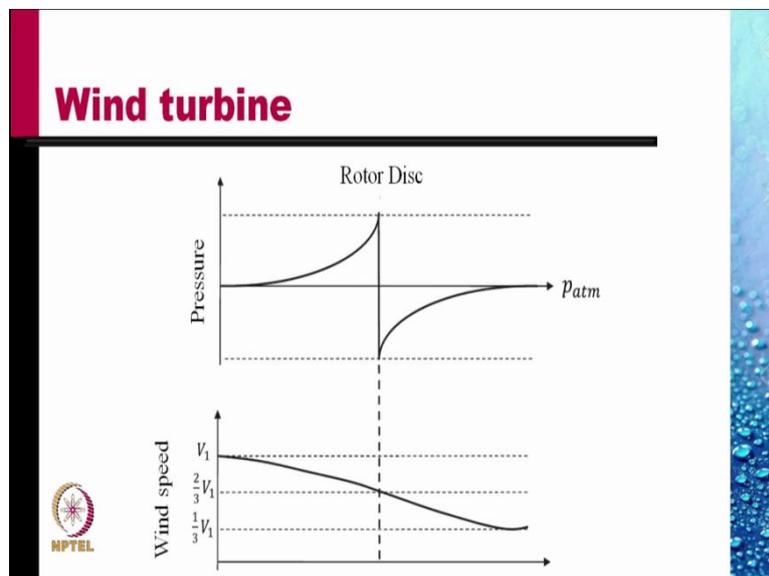
These curves show how the pressure vary in the slip stream tube, increasing from upstream to the rotor disk, decreasing abruptly at the rotor disk, and then increasing back up to the atmospheric pressure after the rotor disk. The wind speed is decreasing from the initial velocity V_1 to the final velocity which is one-third V_1, with the velocity at the disc being the average of the two velocities, two-thirds V_1.
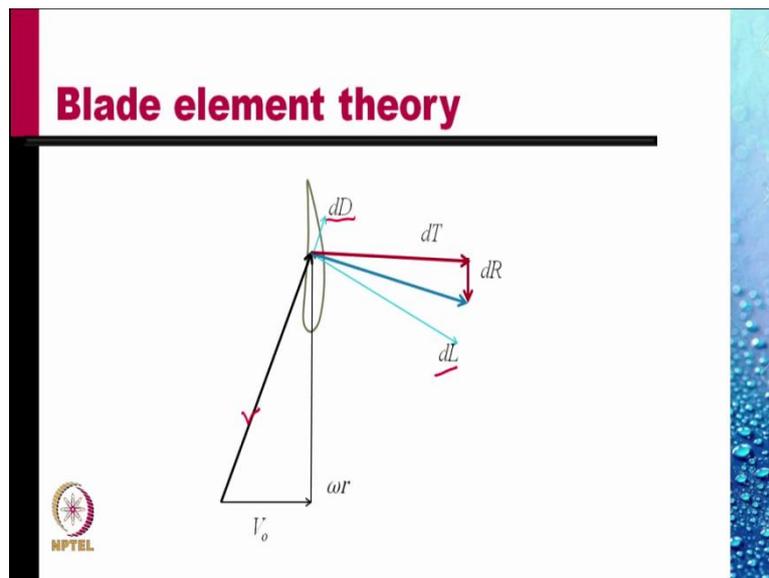
The windmills can also be analyzed by a blade element theory. We consider here the section of a blade at a radius r. This is going down. So, there is a velocity V_0 coming on to this blade horizontally, and since the blade is moving down, the relative wind speed because the rotation is ωr upwards, so that this dark black arrow represents the relative velocity. This relative velocity results in a lift force. So, elemental lift because of a small element at a radius r is dL, which would be like this. And the drag force dD, so that the dark blue arrow shows the net resultant force on this blade element.

And to convert that into the torque at the shaft, we resolve this into horizontal and vertical components. dT is then a component in the axial direction. This would need to be resisted by the thrust bearing of the rotor. At the radial component dR, which would produce a torque dR times the radius, and which would tend to rotate the blade in the required direction. This is the torque that overcomes friction and produces the power. This is the energy produced by this is now absorbed by the generator in producing power.
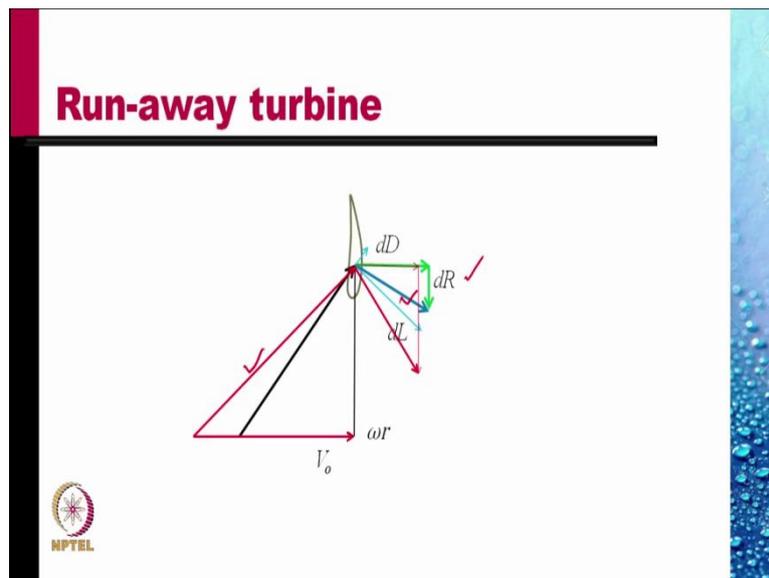
As the velocity V increases, the velocity of the wind as it increases, the angle of the resultant force is such that it is pointed more in the direction, downward direction. And so dR increases. The radial component of the force increases. This increase in the radial component of force increases the rotational speed, or tends to increase the rotational speed of the rotor.

If the wind is too fast, then the rotational speed could become very large and could damage the structure. That is why we use feathering and brakes to keep the speed in control. We have shown here that if the velocity of the wind horizontally increases, the resultant velocity increase, and it is at a lower inclination and so, the lift force is more pointing downwards, and that results in a much larger radial force.
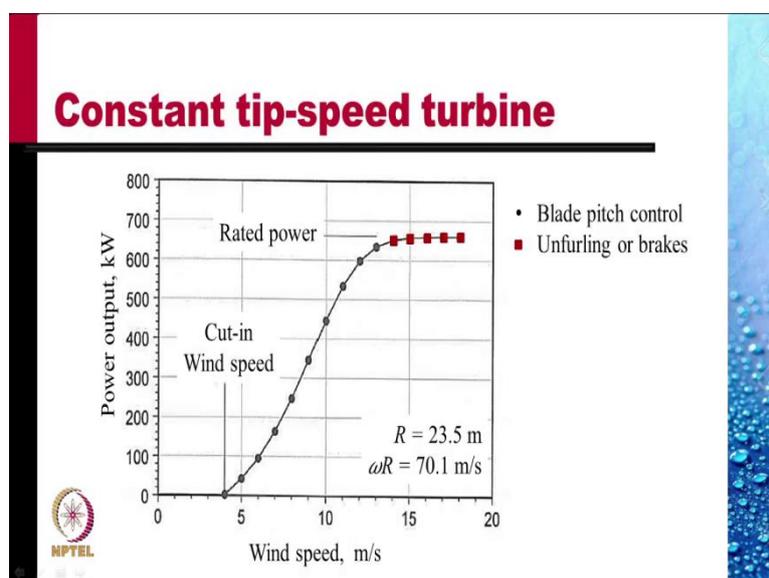
This is the typical power output curve against wind speed. There is a cut in wind speed. We arrange the electronics such that the brakes are applied till the wind reaches a minimal level because below this the speed of the rotor would be too low, and we will not have efficient production of electricity, or the frequency of the power produced will be too low. And therefore, there is a minimum speed at which we cut-in the rotation of the blades, and then when it reaches rated power, we apply the brakes, and we unfurl the pitch of the blades, so that the speed does not increase any further. Otherwise, the blade would run away and break down.

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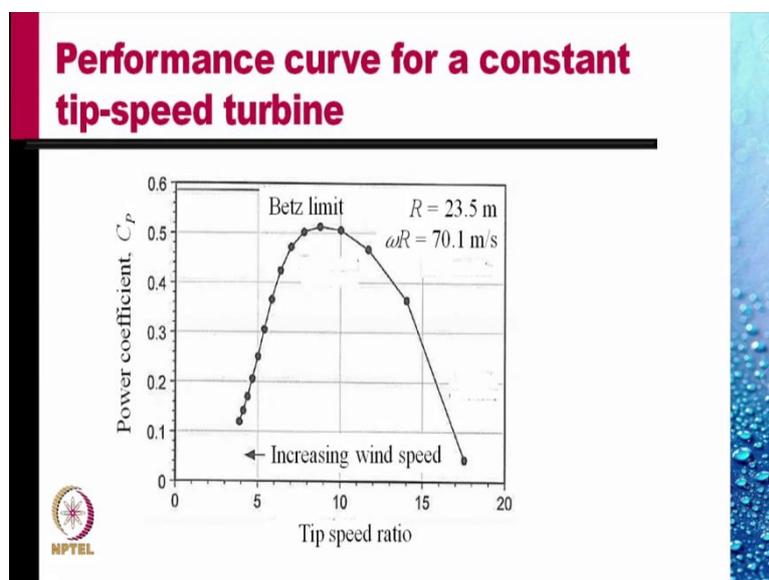
There are two kinds of machines that are operation. One, is in which we maintain the constant speed, that is, we maintain a constant rate of rotation. The advantage of constant rate or rotation is that then the power produced is at a constant frequency, because the frequency of AC power that is produced in the generator depends upon the rotational speed and the number of poles. So, if the speed changes, the frequency changes. The other operation is when we do not take the speed as constant, then the AC power that is generated needs to be converted into DC power by rectification, and the DC power is then converted back into an AC power of the desired frequency.

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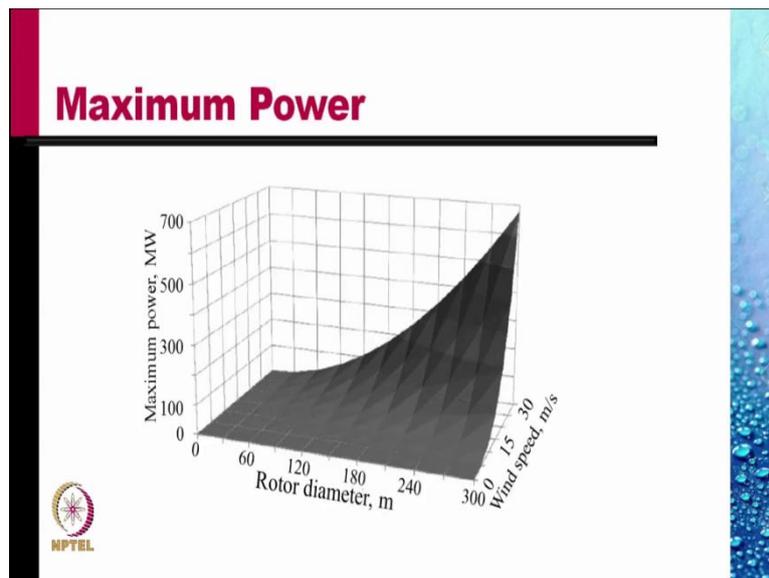
This three dimensional curve shows the maximum power that is produced as a function of rotor diameter and wind speed.
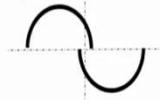
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There are other kinds of wind mills rotors. One is the Savonius rotor. The schematic of is a Savonius rotor is shown here. It is a vertical axis windmill. Most anemometers that measure the wind speed are Savonius turbines. These turbines are not very efficient, but are very reliable and very inexpensive So, in anemometer where reliability is more important than the power output, we use Savonius rotors. Much larger Savonius turbines have been used to generate power on ocean buoys where again, the reliability and the need for service and maintenance should be low.

Unlike with the horizontal axis wind turbine, no pointing mechanism is required to allow for shifting wind directions. Savonius and other vertical axis machines are good at pumping water and other high torque but low RPM applications, and are not usually connect to electrical power grids.
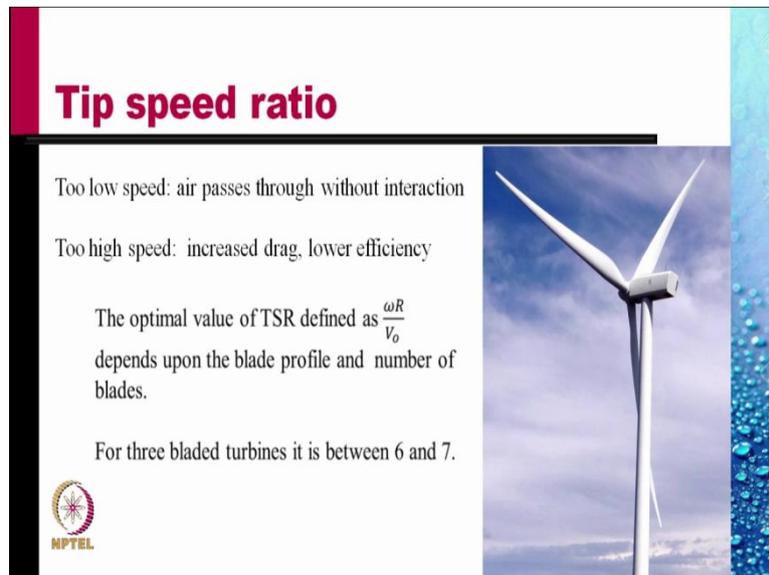
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Tip speed ratio is a very important parameter in horizontal axis wind tunnels. Too low tip speeds, the air passes through without much interaction, and too high tip speed results in increased drag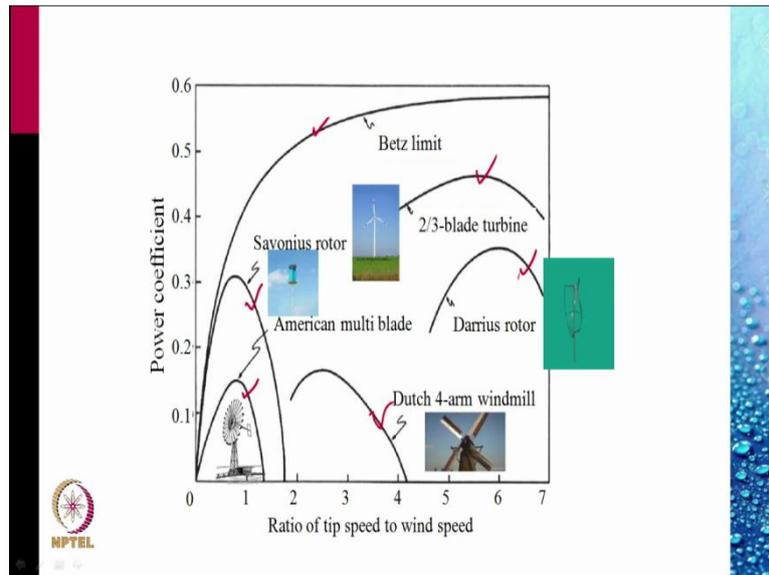, and therefore, lower efficiency. The optimal value of the tip speed ratio which is defined as $\frac{\omega R}{V_o}$, depends upon the blade profile and the number of blades used. For three bladed turbines, which are very common commercially, the optimal value of TSR is between 6 and 7.

This diagram shows you the power coefficients plotted against the tip speed ratio, the ratio of the tip speed to wind speed. The top curve is the Betz limit, the maximum efficiency that you can obtain. The Dutch armed, Dutch 4-armed windmills have curves like this. The American multi-blade turbine have is a curve like this.

Savonius rotor is not very efficient and works with lower tip speed, would give a curve like this. And 2 to 3 blades horizontal axis turbines that give you the best efficiency at relatively high tip-speed ratios of 6 to 7. And this is the Darrius rotor in which again no pointing mechanism is required. These are a number of airfoils which are arranged to rotate about a vertical axis.

Thank you very much.