Fluid Mechanics & its applications
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Lecture 27

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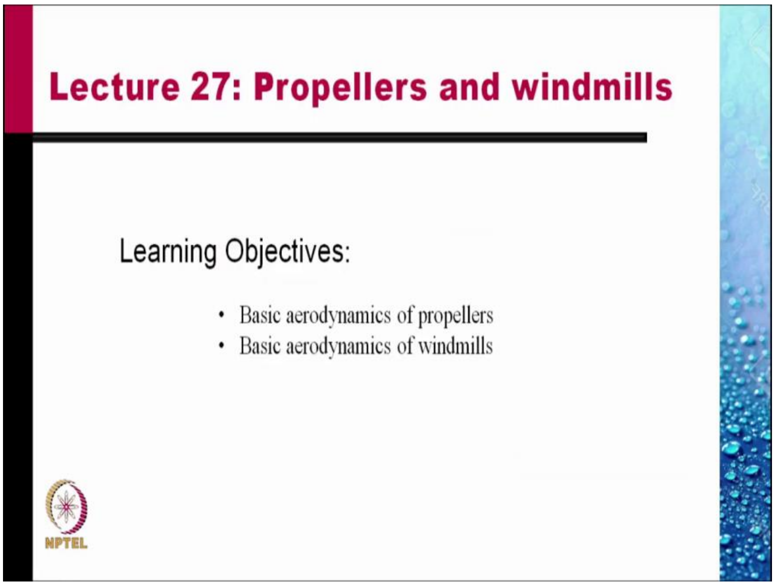

Welcome back.

In today's lecture, we will cover the aerodynamics of propellers and windmills.

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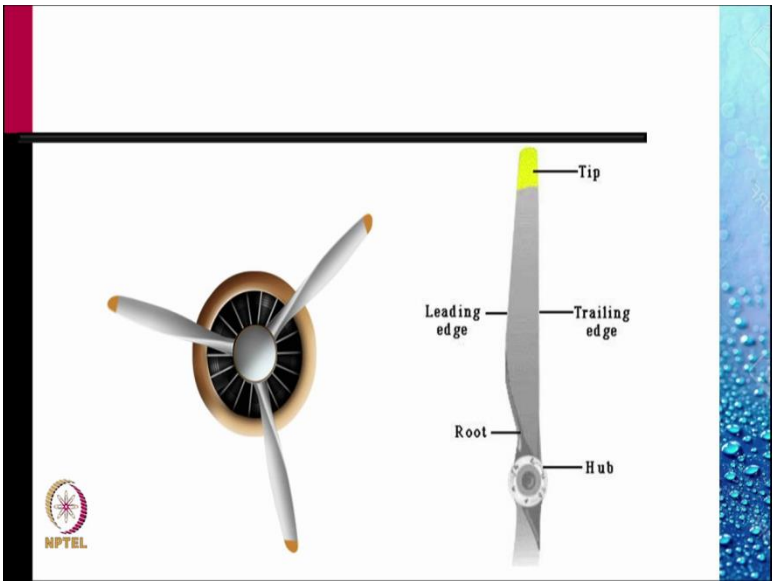

First the propeller. A propeller is a screw like rotating device which rotates to produce thrust. It has multiple blades. Each blade is an airfoil with the leading edge, a trailing edge, a tip, and at the root it is connected to the hub.

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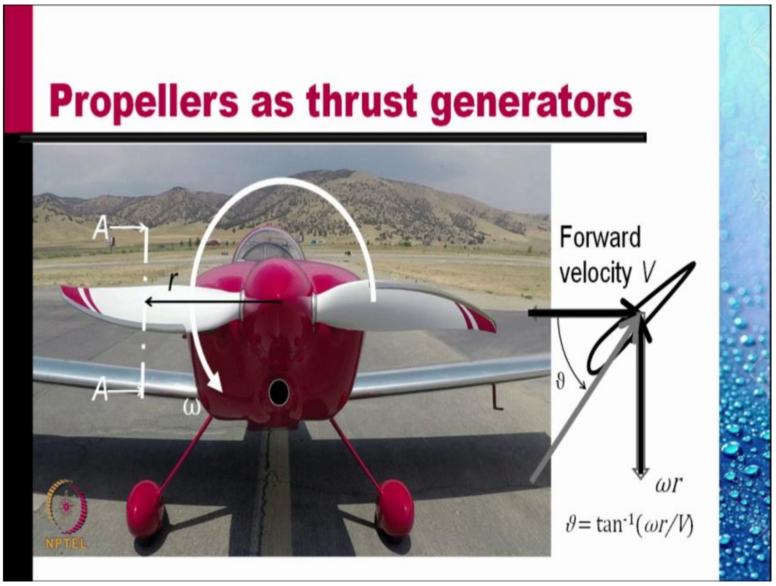

Let us consider a propeller in which the blade is in the horizontal position, and is rotating at an angular speed of ω, counter clockwise. Let us take the section of a propeller at the radius r from the axis. This looks like an airfoil. It has a downward velocity, and it has a forward velocity V, that is, the velocity of the aircraft.

Now, in a frame of reference fixed with the propeller, we can consider the velocities to be in the reverse direction. So that the resultant velocity vector to the airfoil is like this. Now, this means that we have an airfoil that is flying into a wind at a small angle of attack. This angle ϑ at which the wind is coming is more than the angle the cord makes with the horizontal direction.

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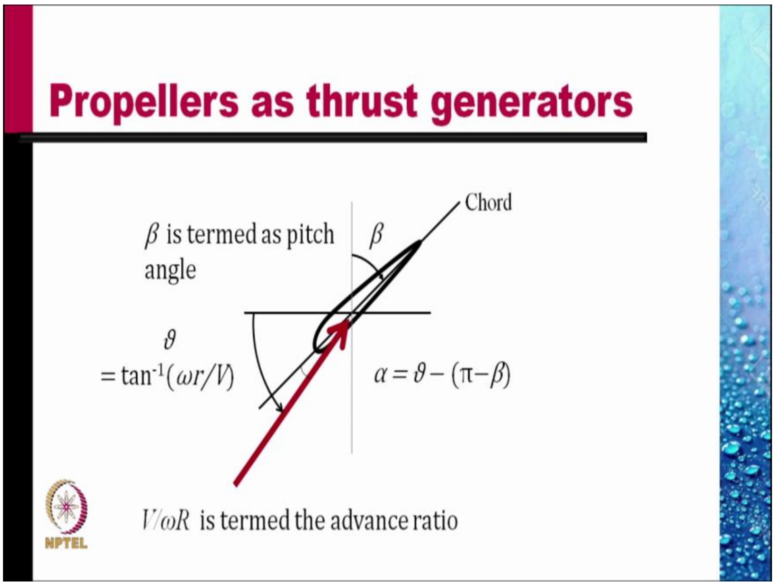

And so, there is an effective angle of attack. We call the angle that the chord makes with the vertical as the pitch angle. This is the resultant velocity onto the blade, this is angle θ which would be $\tan^{-1}(\omega r/V)$, ωr being the vertical component of velocity due to the rotational speed of the blade, and V is the forward velocity of the aircraft. The angle of attack, α, then is $\theta - (\pi - \beta)$. $V/\omega R$ is termed as the advance ratio. This is the ratio of the distance moved by the aircraft in one rotation divided by the distance moved by the tip of the propeller in one rotation. Here R is the radius of the rotor blades.

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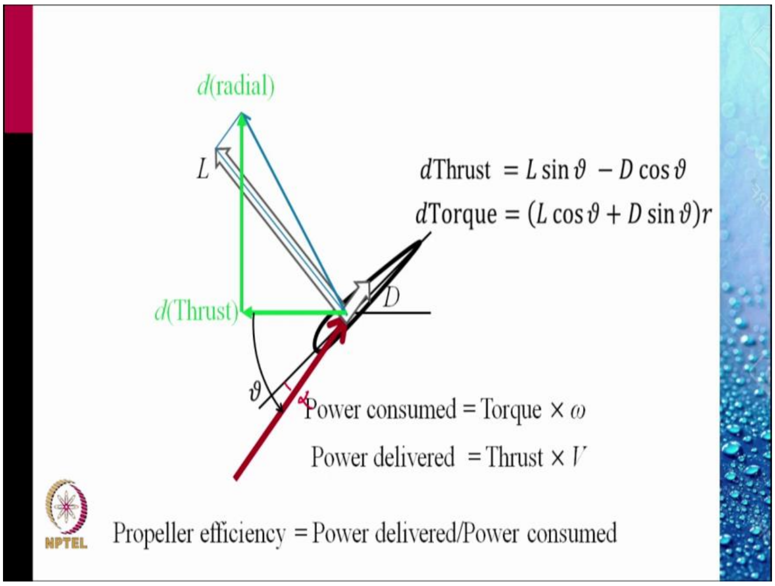

Let us consider the forces that result. The air is coming to the airfoil at a small angle α. Because of this, a lift results which is perpendicular to the direction flow, and a drag force results which is in the direction of the relative wind. Because of this, the resultant force on this propeller element is shown by this blue arrow. We resolve this resultant force into a component which is thrust-like. It is in the forward direction. And a component which is radial. This component of thrust can be determined from the lift component and the drag component.

The radial component will produce a torque that will need to be overcome by the engine that is revolving the propeller. The contribution to thrust, from this figure, we can see is $L\,sin\,\vartheta\ -D\,cos\,\vartheta$, while the contribution to torque would be the radial component of the force which is $= (L\,cos\,\vartheta + D\,sin\,\vartheta)r$. The total thrust and the total torque are obtained by integrating these elemental thrust and elemental torque over the entire r from the root to the tip of the rotor blades.

The L and D values will depend upon the velocity V, the location r along the blade, and on the density of air. The power consumed by engine would be the torque time the angular velocity ω of the rotor, and the power delivered would be the thrust produced times the velocity. So that, the propeller efficiency is power delivered divided by power consumed.

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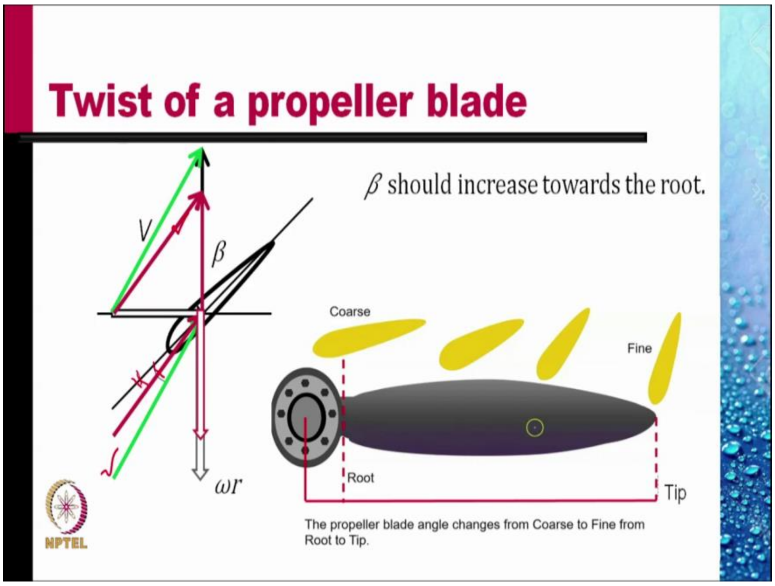

We will establish here the need for twist on a propeller blade. Shown here is the relative speed of air with respect to the propeller, and this is shown by this green arrow. The angle that this velocity makes with the chord of the blade is the angle of attack given here. Now suppose, we take a section of the blade at a different value of r. Let us take it at a smaller r than this picture that we have drawn. The forward velocity is the same, and because of this

this red arrow gives you now, the relative velocity of wind at this section of the blade, which is at the location inwards of the earlier section at which the relative velocity is V.

And so, when we transfer it to the blade, this is what the relative velocity is, and you see clearly the effective angle of attack has decreased. So, the blade would act differently than it will be acting at that r. For a blade to be acting at its most efficiency, we need to keep the L/D ratio, that is lift to drag ratio, or C_L/C_D ratio fixed at the optimum, and when we keep it at optimum, there is an optimum angle of attack that must be maintained throughout the length of the blade.

And this is possible only if we twist the blade. We will twist the blade so that it is a higher β towards the root than at the tip. Beta should increase towards the root. If you take a rotor blade, a propeller blade, the propeller blade has a very coarse pitch at the root compared to the pitch at the tip of the blade. The propeller blade angle changes from coarse to fine, from root to tip.

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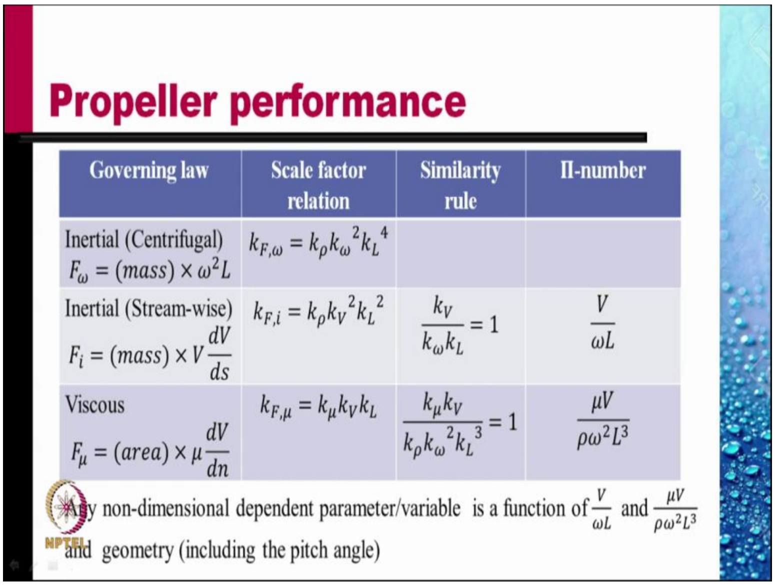

Governing law	Scale factor relation	Similarity rule	Π-number
Inertial (Centrifugal) $F_\omega = (mass) \times \omega^2 L$	$k_{F,\omega} = k_\rho {k_\omega}^2 {k_L}^4$		
Inertial (Stream-wise) $F_i = (mass) \times V \frac{dV}{ds}$	$k_{F,i} = k_\rho {k_V}^2 {k_L}^2$	$\frac{k_V}{k_\omega k_L} = 1$	$\frac{V}{\omega L}$
Viscous $F_\mu = (area) \times \mu \frac{dV}{dn}$	$k_{F,\mu} = k_\mu k_V k_L$	$\frac{k_\mu k_V}{k_\rho {k_\omega}^2 {k_L}^3} = 1$	$\frac{\mu V}{\rho \omega^2 L^3}$

To analyze the propeller performance, we do a similarity analysis. The governing laws are three that are important. The first one is the centrifugal inertial force, which would be $(mass) \times \omega^2 L$. So, the scale factor for this centrifugal initial force will be $k_\rho {k_\omega}^2 {k_L}^4$. The stream wise inertial force is mass times the streamwise acceleration, which is $V\frac{dV}{ds}$. And that gives you a $k_{F,i} = k_\rho {k_V}^2 {k_L}^2$. Similarly, the viscous force is the area times the viscous shear stress, which is $\mu\frac{dV}{dn}$, and that in scale factors terms gives you $k_{F,\mu} = k_\mu k_V k_L$.

From this, we establish the similarity rule that $\frac{k_V}{k_\omega k_L}$ should be 1, and $\frac{k_\mu k_V}{k_\rho {k_\omega}^2 {k_L}^3}$ should 1. From the first one we obtain the pi number $\frac{V}{\omega L}$. We use L as the diameter of the propeller, and V characteristic is the forward velocity of the aircraft.

The second parameter is mu $\frac{\mu V}{\rho \omega^2 L^3}$. And you can see this is like product of Reynolds number and the first parameter that we obtained earlier. So, any non-dimensional parameter or variable which is dependent, should be a function of $\frac{V}{\omega L}$ and $\frac{\mu V}{\rho \omega^2 L^3}$, and the geometry, including the pitch angle.

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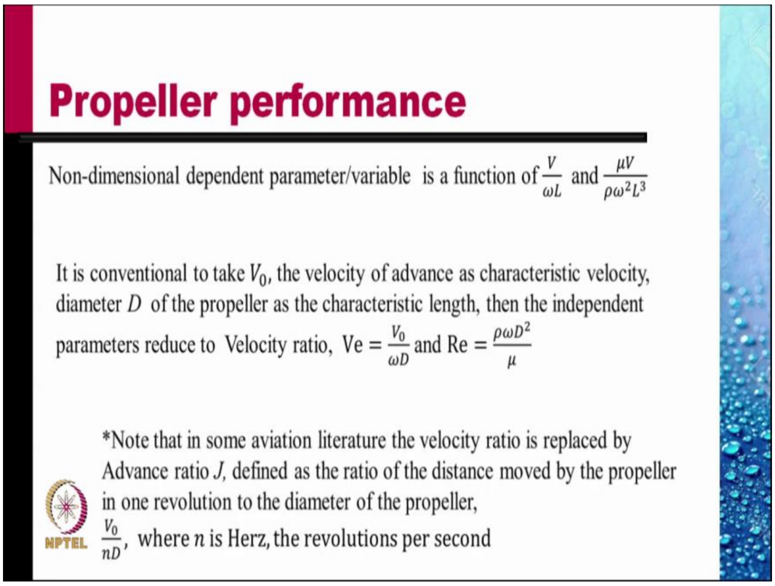

It is conventional that V_0, the velocity of advance as the characteristic velocity, the diameter D of the propeller as the characteristic length, and then the independent parameter is reduced to velocity ratio, $\mathrm{Ve} = \frac{V_0}{\omega D}$ and $\mathrm{Re} = \frac{\rho \omega D^2}{\mu}$. Note that in some aviation literature, the velocity ratio is replaced by the advance ratio J, defined as the ratio of the distance moved by the propeller in one revolution to the diameter of the propeller, $\frac{V_0}{nD}$, where n is in Hertz, the number of revolutions per second.

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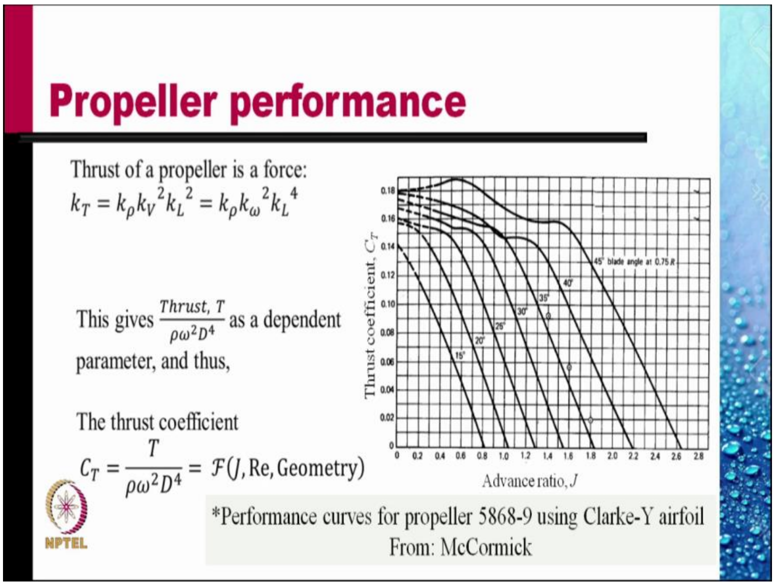

Thrust of the propeller is like a force. So, k thrust is $k_T = k_\rho {k_V}^2 {k_L}^2$, and by replacing k_V by $k_\omega k_L$, we obtain $k_T = k_\rho {k_\omega}^2 {k_L}^4$. From this we obtain the pi number $\frac{Thrust,\ T}{\rho\omega^2 D^4}$ as the dependent pi, the dependent parameter, and thus, the trust coefficient is defined as $C_T = \frac{T}{\rho\omega^2 D^4}$, and it should be a function of the advance ratio, the Reynold number and the geometry.

The dependence on the Reynolds number is rather low for large Reynolds numbers at which most propeller operate, so that the thrust coefficient is largely a function of the advance ratio for a given propeller with a given geometry. The typical performance curves, the thrust coefficient versus advance ratio look like this for different blade angle,s for different pitch that we adjust. These are the propeller curves for a propeller numbered 5868-9 which uses Clarke Y airfoil.

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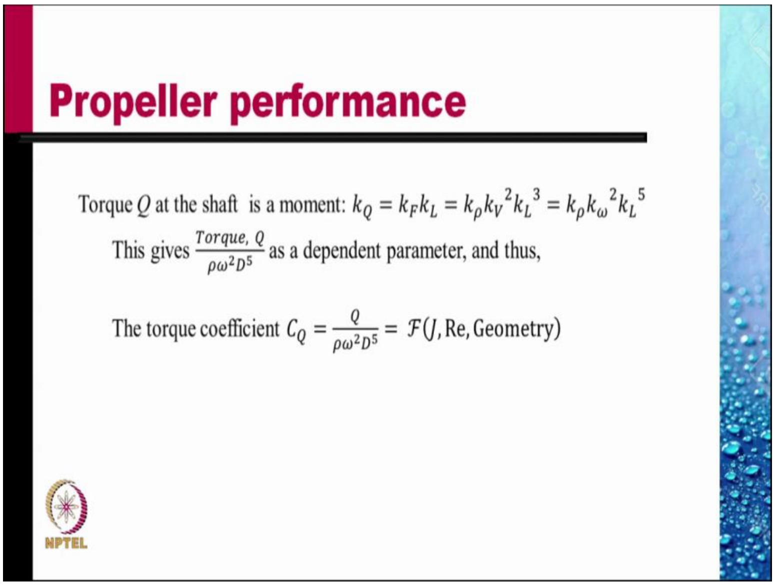

The torque Q at the shaft is a moment. So, k_Q is like $k_F k_L$, and that gives you a $k_\rho {k_\omega}^2 {k_L}^5$. This gives $\frac{Torque,\ Q}{\rho\omega^2 D^5}$, as a dependent parameter, and we define a torque coefficient C_Q as $\frac{Q}{\rho\omega^2 D^5}$, which should be a function of advance ratio, and Reynolds number and geometry.

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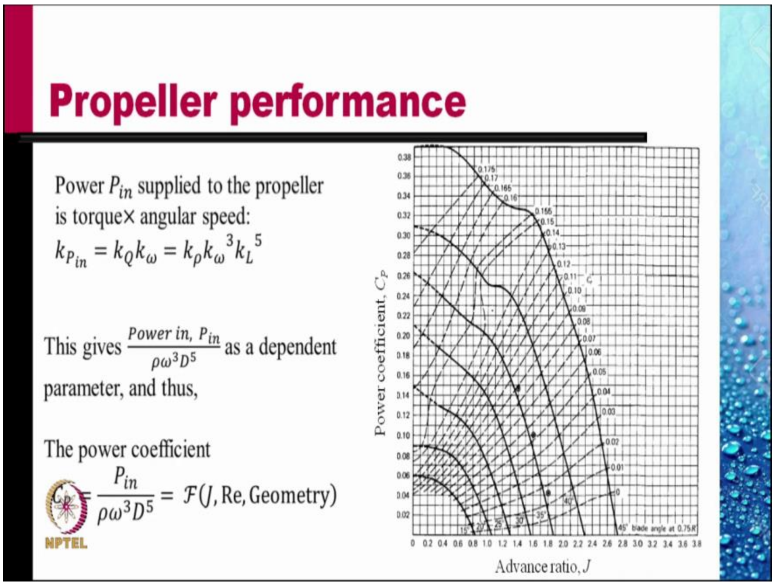

The power supplied to the propeller is torque times the angular speed of the propeller. So, $k_{P_{in}}$ the scale factor for the input power is $k_Q k_\omega$ which is $k_\rho {k_\omega}^3 {k_L}^5$. This gives $\frac{Power\ in,\ \ P_{in}}{\rho\omega^3 D^5}$ defined as a dependent parameter, and it is called the power coefficient $C_P = \frac{P_{in}}{\rho\omega^3 D^5}$. And these are the typical plots for power coefficient and advance ratio for the same propeller for which the torque coefficient curves were given in the earlier slide.

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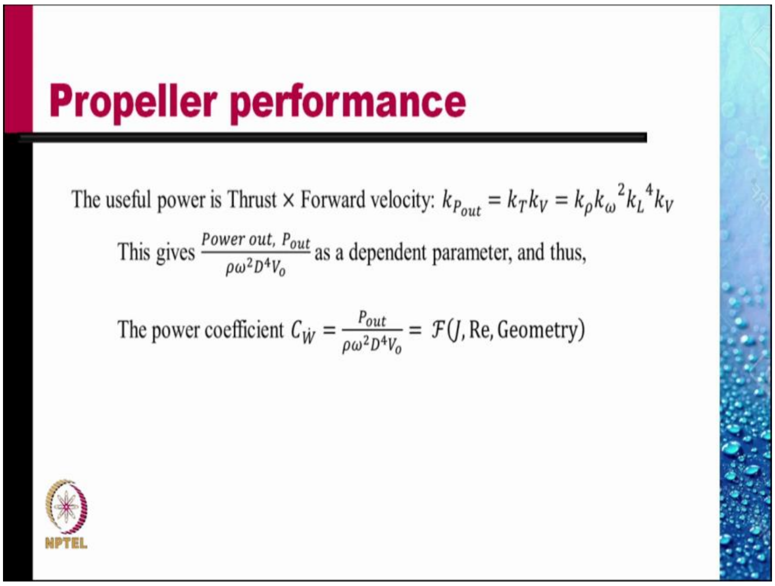

The useful power from the propeller, the payoff delivered, is thrust times the forward velocity. So, $k_{P_{out}}$, the scale factor for power out, is $k_T k_V$, and from this we get a power coefficient $C_{\dot{W}} = \frac{P_{out}}{\rho\omega^2 D^4 V_o}$, which again should be a function only of advance ratio, Reynolds number and the geometry, including the pitch angle.

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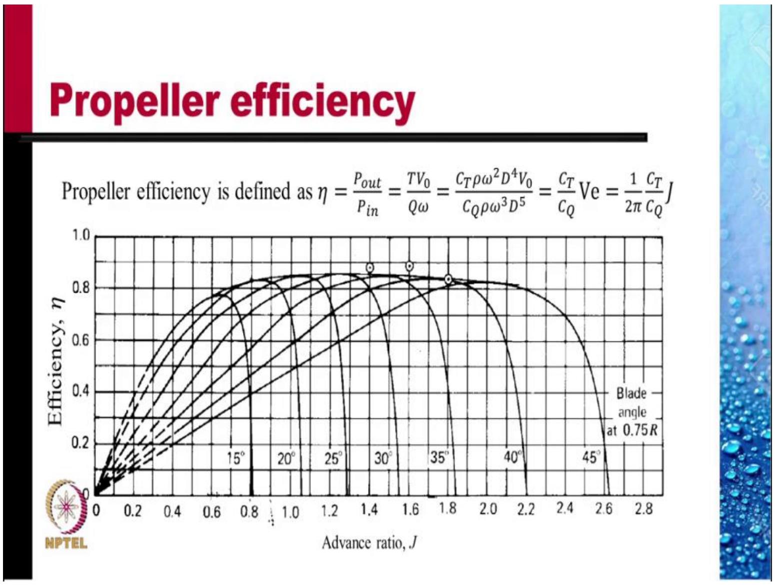

The propeller efficiency is defined as $= \frac{P_{out}}{P_{in}}$, and if I put for $P_{out,}$ TV_0, and for P_{in}, $Q\omega$, then we get this as $\frac{C_T}{C_Q}$Ve, or $\frac{1}{2\pi}\frac{C_T}{C_Q}J$. And for the propeller, we plot the propeller efficiency versus J, This is the kind of curve that we get for various β, various blade angles, various pitch angles. All have about the same maximum efficiency, but the maximum efficiency is obtained at a higher advance ratio for a blade with a higher pitch.

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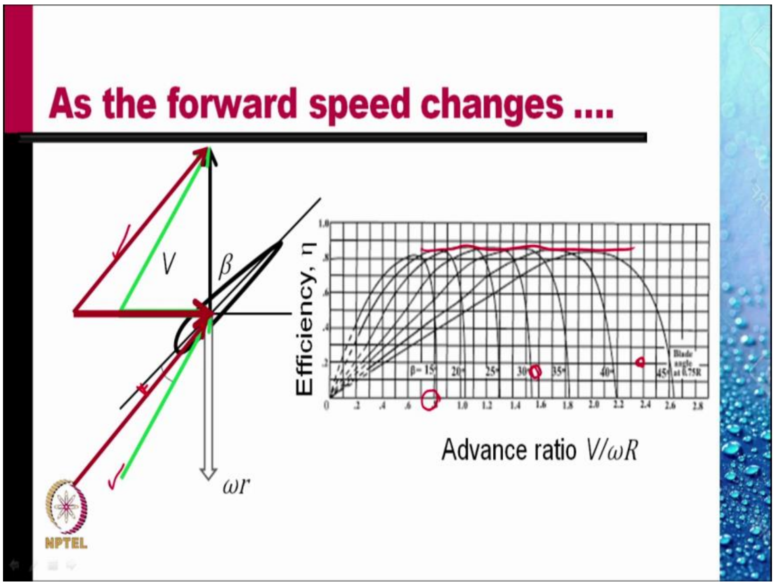

Now, let us see what happens when the forward speed of a propeller changes, that is, when the aircraft moves faster, the rotational speed of the propeller remaining constant. Here the green arrows show the velocity vector with respect to the chord for the given forward speed V, and a given lateral rotation ω at a radius r from the hub. Now, as the forward speed increases, the resultant velocity given by this arrow is now inclined at a different angle. When I transfer it to the airfoil, the velocity becomes this.

This results in a lower angle of attack, if no changes are made in the blade geometry. If the earlier value of angle of attack was optimum, then we must change the pitch of the propeller, pitch of the blade, such that, so the angle β increases so, α gets back to its optimum value. This is the curve that we see saw for efficiency versus advance ratio.

And we like to remain here as the advance ratio changes. That means, as the velocity increases, the advance ratio increases too. And so, the value of β must increase, as was established just now. By how much is seen from this curve. At advance ratio of 0.8, the β at three fourths of the radius should be 15 degrees for this propeller, but for twice the speed, that is at 1.6, β should be more than 30 degrees, 3 times the speed, that is at an advance ratio of 2.4, it should be something like 43 degrees.

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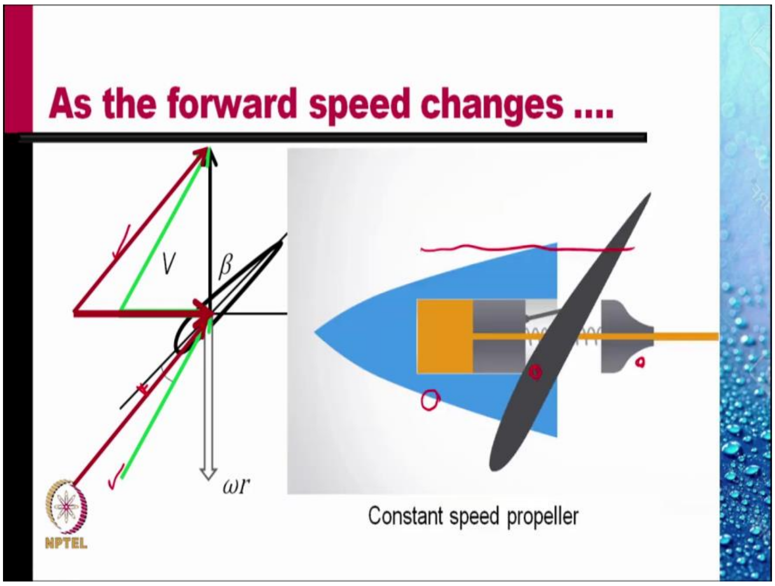

So, β needs to change with advance ratio. This usually is accomplished by a mechanism, an automated mechanism, which changes β as the advance ratio increases. This is called a constant-speed propeller in which the rate of rotation of the propeller is kept constant, that is, to get the engine running at the best possible speed throughout the journey, whatever be the forward speed, and so, we have a constant speed propeller, but we changed the pitch depending upon the forward speed of the aircraft.