Fluid Mechanics & its applications
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Lecture 26A
Flow over an airfoil

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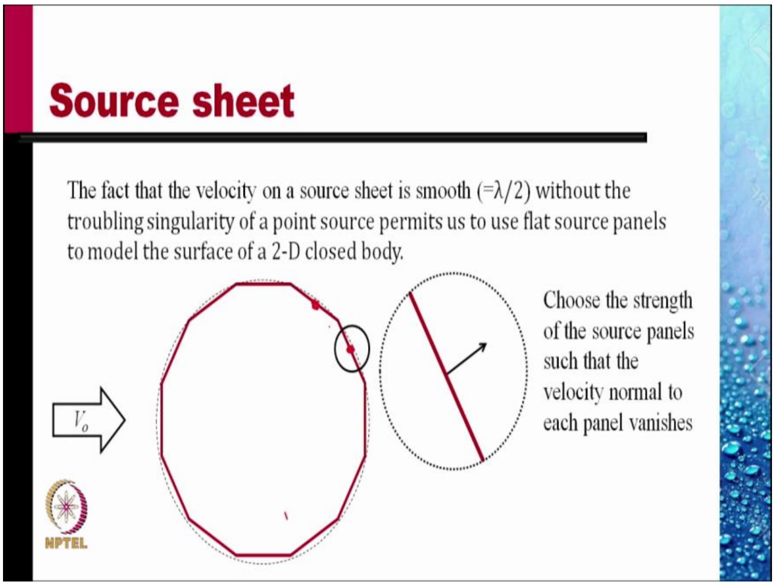

The fact that the velocity of the source sheet is smooth is equal to λ/2 without the troubling similarity of a point source permits us to use flat pate source panel to model the surface of a 2-D closed body. So, we extend the use of this source panel to flow about an airfoil, or even a circular cylinder.

Consider a circular cylinder. We divide this circumference up into a number of panels, flat panels, and for each flat panel we assume a source strength. Once we assume a strength for this panel, we can calculate the effect of those sources from each of these panels to points at the center of the panels, and make the flow velocity tangent. If they are n panels that we use, there would be n control points. So, by making the velocity 0, making the velocity 0 at these n control points, we get n equations in n undetermined source strengths. And this system of equations can be solved to get the strengths required for each of the panels.

We outline here the method applied to an airfoil. We take a free stream at a velocity V_o, and find out the velocity at a control point here, and we set the velocity 0. And that will give us one equation in n unknown source strengths. This we repeat for every control point. So, we get n equations in n unknown source strengths. Choose the strength of the source panels such that the velocity normal to each panel vanishes.

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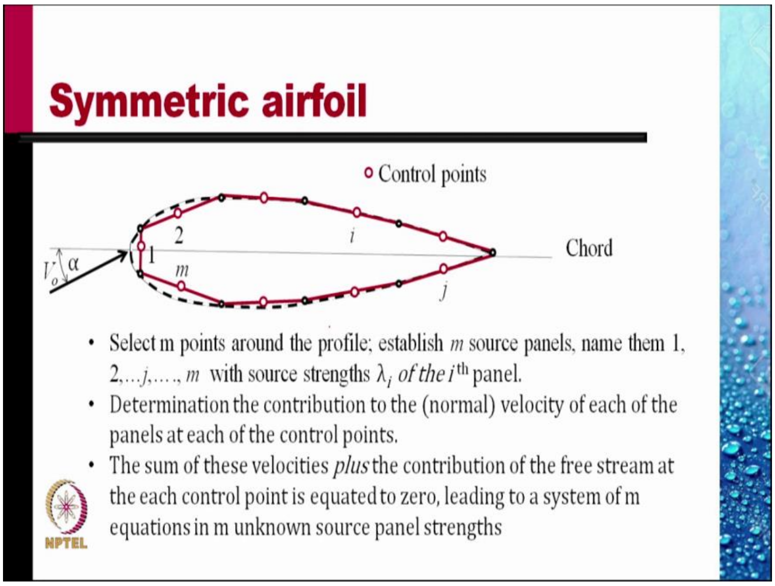

So, we start with the profile of the airfoil. the flow approaches it with a velocity V_o, angle of attack α. We divide the surface of the airfoil into a number of straight source panels, and we choose the midpoints of these control panels as control points. We number the panels and the control point, 1, 2, 3, i, j up to m. So, there are m panels and m control points were chosen.

We next do the determination of the contribution to the normal velocity of each of the panels at each of the control points. The sum of these velocities, plus the contribution of the free steam at each control point, is equated to 0, leading to a system of m equations in m unknown source panels strengths.

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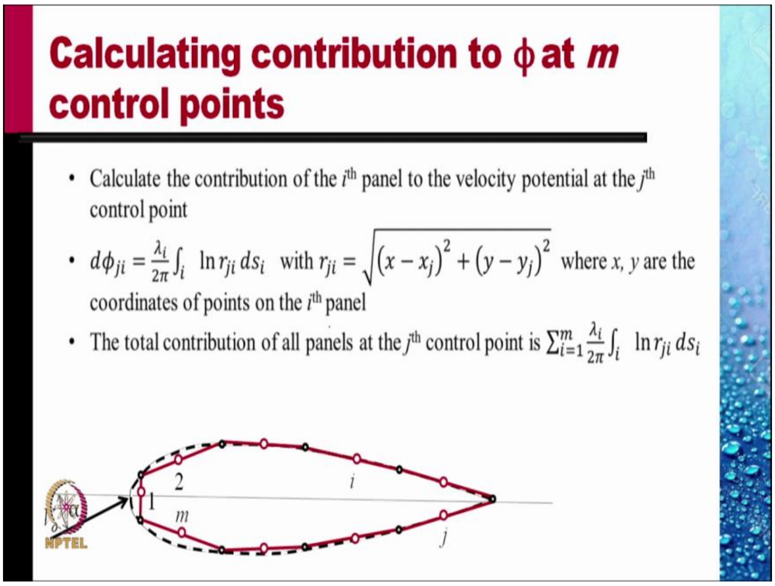

And how do we do this. We calculate the contribution of the i[th] panel to the velocity potential at the j[th] control point. That is, we find out $d\phi_{ji}$, at the j[th] control point because of i is equal to $= \frac{\lambda_i}{2\pi}\int_i \ln r_{ji}\, ds_i$ with $r_{ji} = \sqrt{(x-x_j)^2 + (y-y_j)^2}$, where x and y could be x_i and y_i. The total contribution of all the panels at the j[th] control point is this summation for all the i's.

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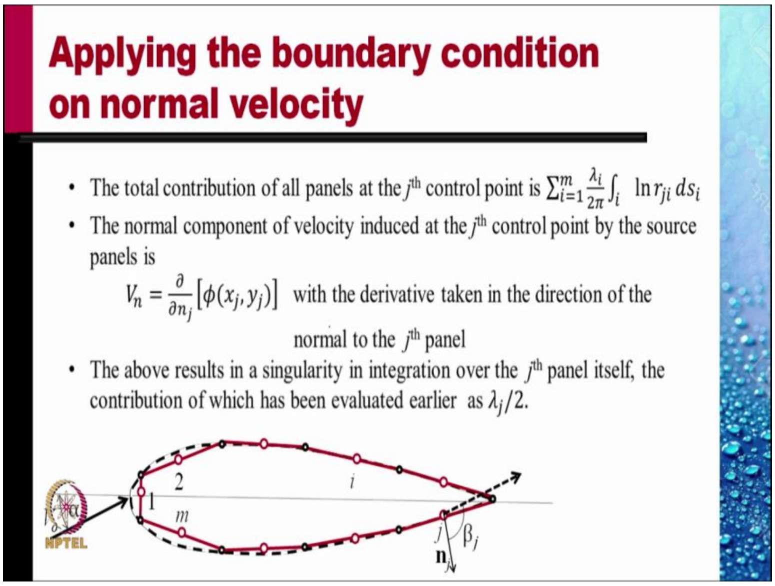

The total contribution of all panels at the j[th] control point was obtained as $\sum_{i=1}^{m} \frac{\lambda_i}{2\pi}\int_i \ln r_{ji}\, ds_i$. The normal component of velocity induced at the j[th] control point by the source panel is then obtained by taking the derivative of this potential with respect to n_j. The above results in a singularity the integration of the j[th] panel itself, the contribution of which has been evaluated earlier as $\lambda_j/2$. So, we eliminate the j[th] panel from this sum and add $\lambda_j/2$ separately.

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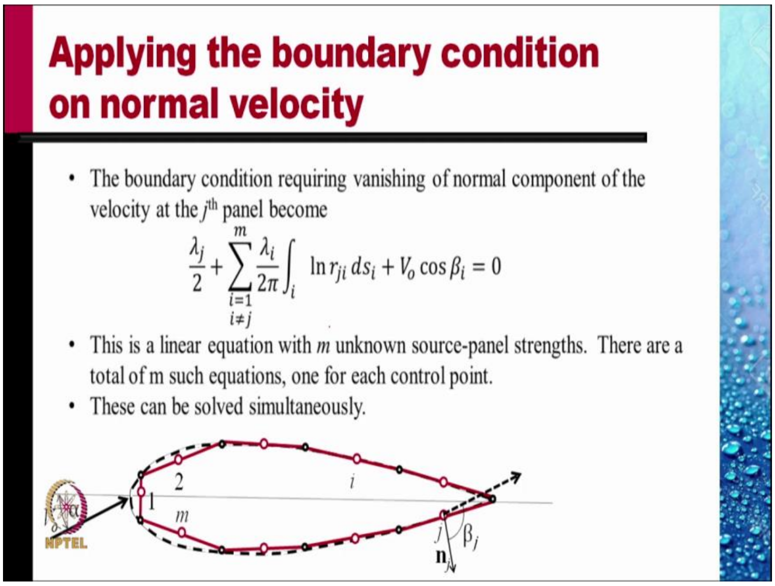

The bounday condition requiring vanishing of normal components of the velocity of the j[th] panel becomes $\lambda_j/2$, the contribution of j[th] panel, plus the contribution all other panels except i is equal to j, plus the contribution of the free stream $V_o \cos\beta_i$, should be 0, the normal velocity should be 0. This is a linear equation with m unknown source panel strengths. There are a total of m such equations, one for each control point, and these can be solved simultaneously.

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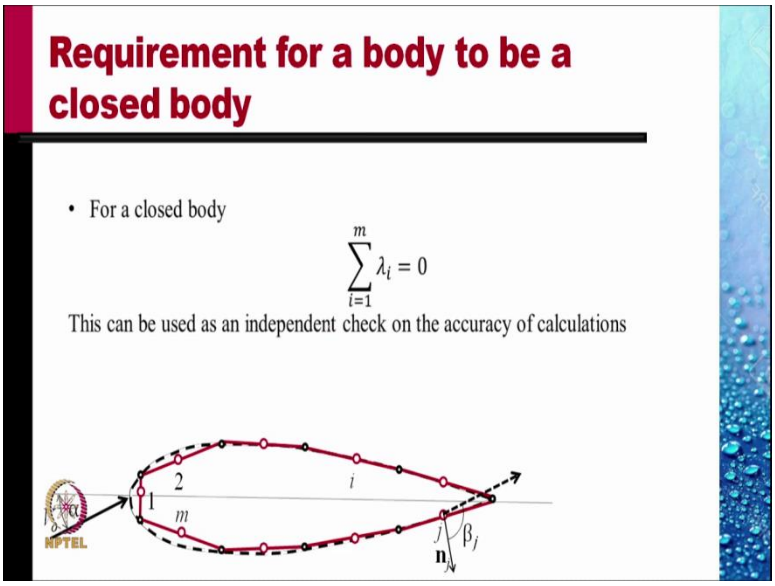

Now, as was said earlier, the net source strength for a closed body must be 0. So, $\sum_{i=1}^{m} \lambda_i = 0$. We have not used this equation earlier. So, this can be used as an independent check on the accuracy of the calculations performed.

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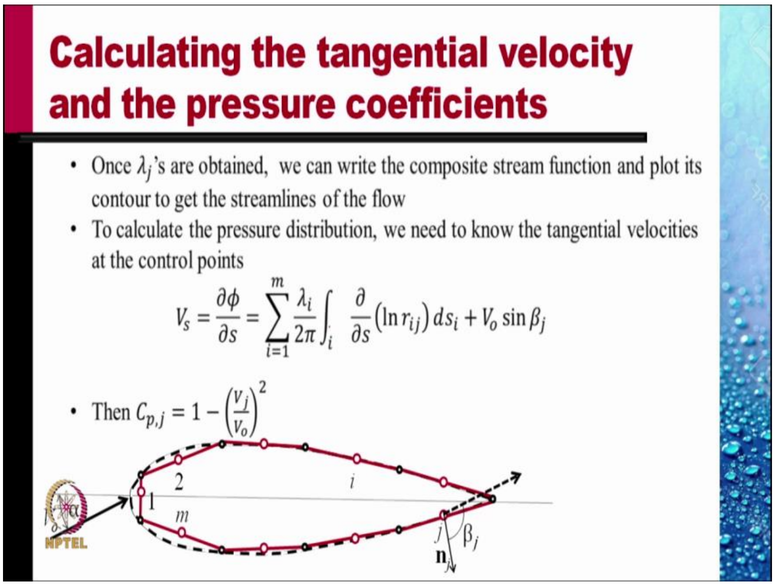

Once λ_j's are obtained, we can write the composite stream function, and plot its contours to get the streamlines of the flow. To calculate the pressure distribution, we need to know the tangential velocities at the control points which are obtained from this expression. Then, the coefficient of pressure at the j[th] control point is nothing but $1 - \left(\frac{V_j}{V_o}\right)^2$.

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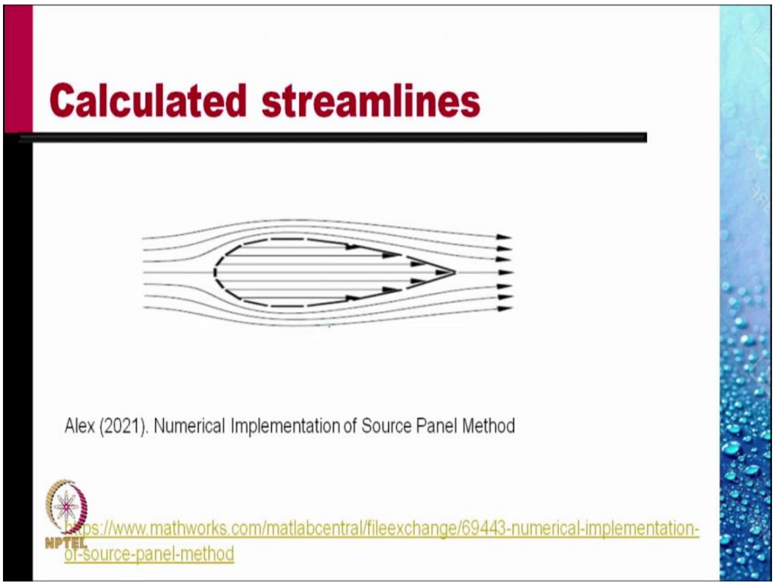

This panel method is used very often, and lots of programs have been developed with this. One such implementation in MATLAB is available at this reference. The viewers are encouraged to visit this site, which is a MATLAB sharing source, and obtain it. A rather small source panel method program, which can use to solve for any closed body.

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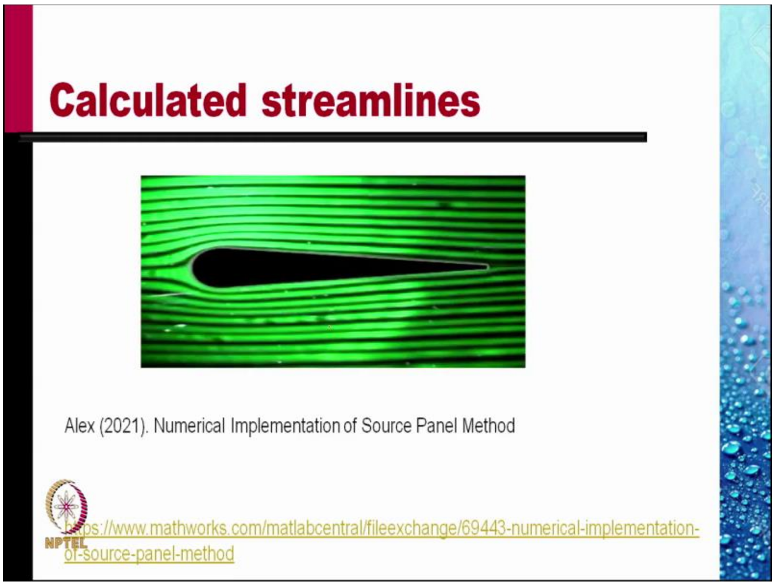

The inside can be replaced by the solid body and the calculated streamlines are very close to what is obtained.

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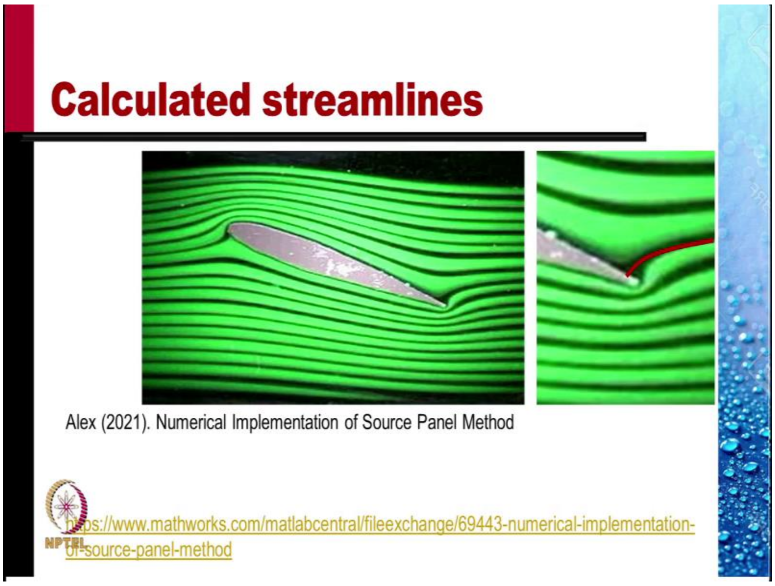

Of course, there is no restriction on α in this method. If you use larger α, we get a flow like this. We will enlarge this. You see here, the stagnation point has moved to a point above the trailing edge of the airfoil. This results in a rather large acceleration at the trailing edge, a situation which is not physical tenable, you cannot have infinite velocity there. So, this breaks down.

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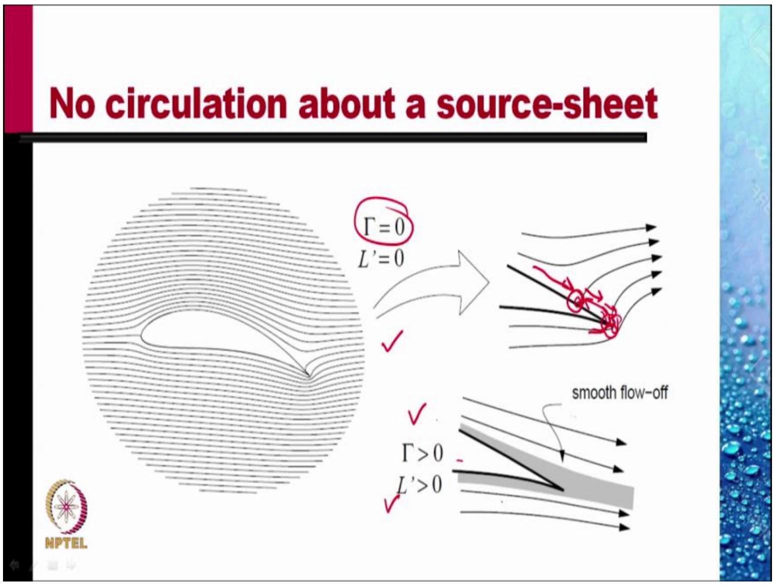

There is no circulation about a source sheet we have calculated earlier. And so, if we calculate the circulation in the flow that we obtain, we will find that circulation 0, and the lift per unit length of the airfoil obtained will be 0, whatever be the angle of attack. So, this method cannot predict the lift by itself. As we have seen the trailing edge is this. The flow goes around the trailing edge acquiring infinite velocity at the real point. The stagnation point moves up. So, there is flow which is slowing down in this region and in this region. This result in separation. A famous scientist by the name of Kutta produced a theorem which says that the actual flow has a circulation which is non zero, which predicts a non-zero lift, and that circulation can be predicted. The actual physical circulation in a real flow can be predicted by imposing on this solution, a vortex strength such that, the stagnation point moves to the trailing edge.

As we increase the value of Γ, the circulation, this stagnation point shifts downward. At one particular value of Γ, this stagnation point is sitting at the trailing edge. And the Kutta condition states that the actual flow is one in which Γ is exactly equal to the required circulation to make the rear stagnation point at the trailing edge.

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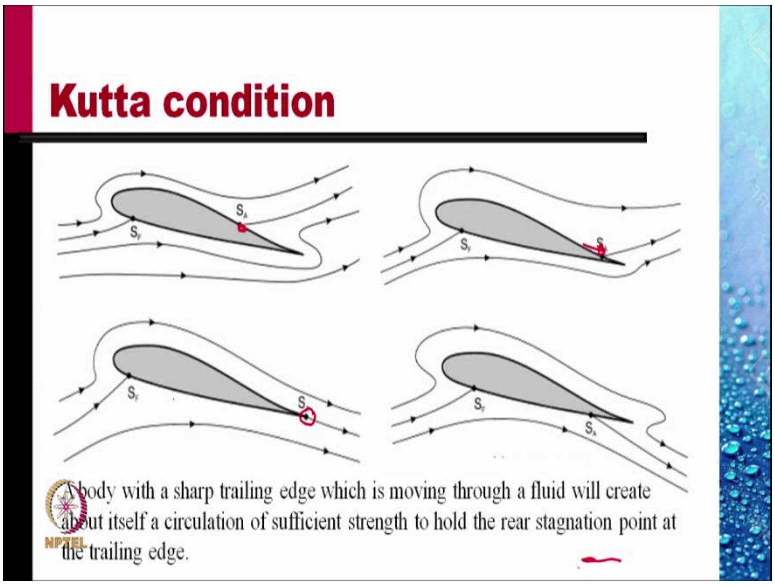

To repeat: the stagnation point here with zero circulation. I change the value of circulation and this point starts moving down towards the trailing edge. At one specific value of circulation superimposed, that is, on the potential flow that we obtained without the circulation, we keep on adding a vortex, and when the vortex strength reaches a certain value, the stagnation point is right at the trailing edge. If I add more of circulation, the stagnation point will move down the bottom surface. A body with a sharp trailing edge which is moving through a fluid will create about itself a circulation of sufficient strength to hold the rear stagnation point at the trailing edge. This is the Kutta condition.

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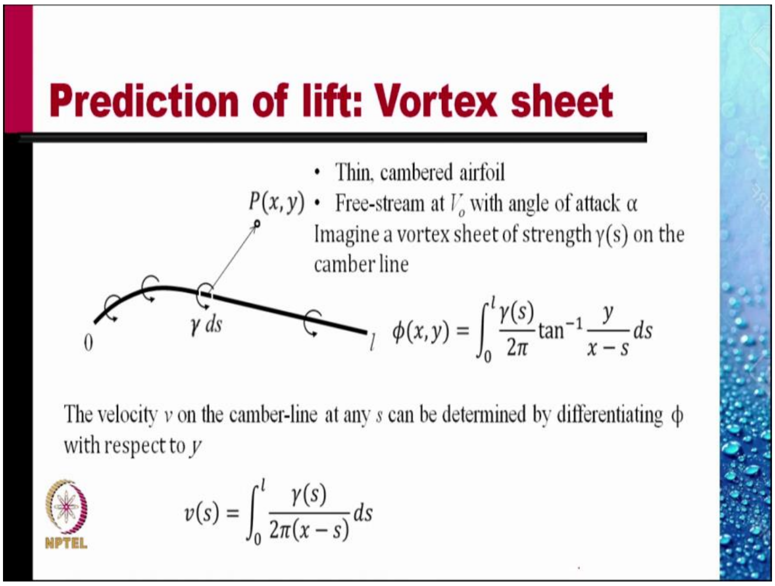

So, what we do is, we predict the lift by introducing a vortex sheet in much the similar manner as we did with the source sheet. This vortex sheet is now laid along the camber line of the airfoil. The thickness of the airfoil does not contribute to the lift. So, to the first approximation of the flow, we can forget about the thickness, and worry only about the camber line. We imagine a vortex sheet of strength γ(s) on the camber line.

Calculate the potential at point (x, y) because of this vortex sheet of strength that varies like γ(s). The velocity u on any location s can be determined but differentiating ϕ with respect to x. There is a discontinuity at $y = 0$, and we get u the horizontal velocity slightly about the camber line is $-\frac{\gamma}{2}$, and slightly below s $+\frac{\gamma}{2}$. And this gives us the velocity on the camber line.

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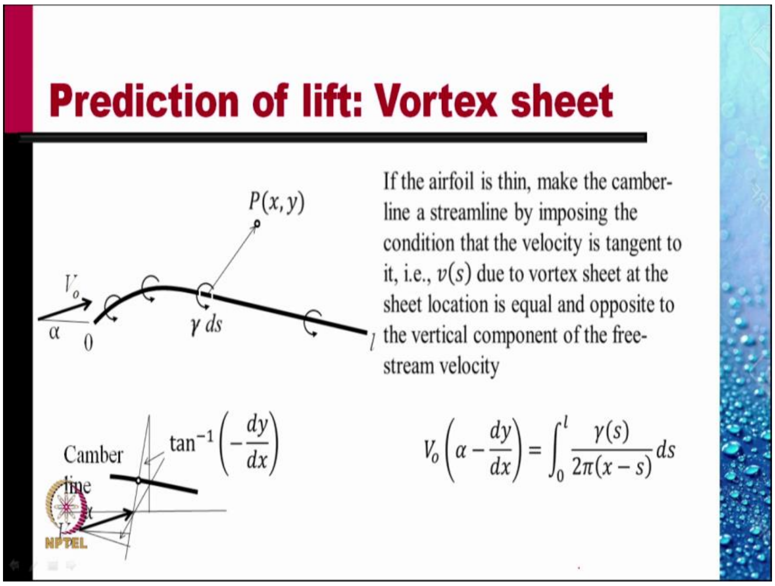

If the airfoil is thin, make the camber line a streamline by imposing the condition that the velocity is tangent to it that is $v(s)$ due to vortex sheet at the sheet location is equal and opposite to the vertical component of the freestream velocity. The vertical component of the freestream velocity. So, that $V_0\left(\alpha - \frac{dy}{dx}\right)$ is the vertical velocity predicted by this vortex sheet.

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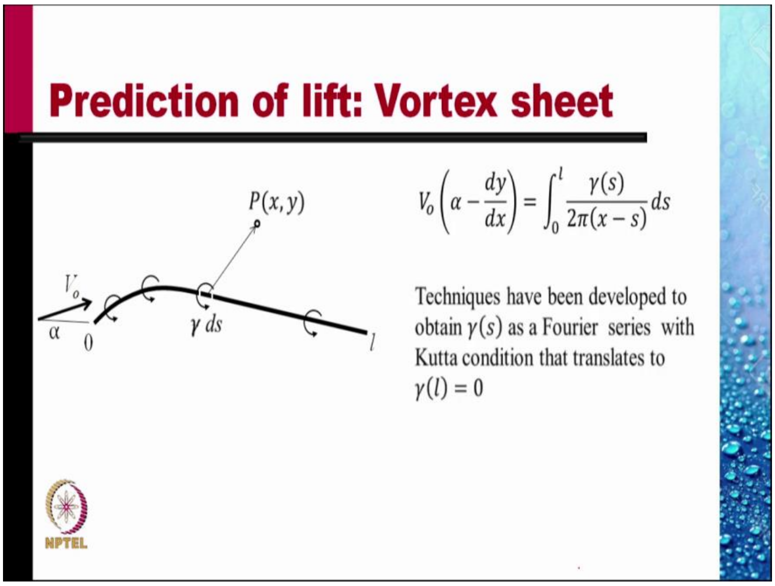

Techniques have been developed to obtain γ(s) as a Fourier series with Kutta condition that translates to $\gamma(l) = 0$.

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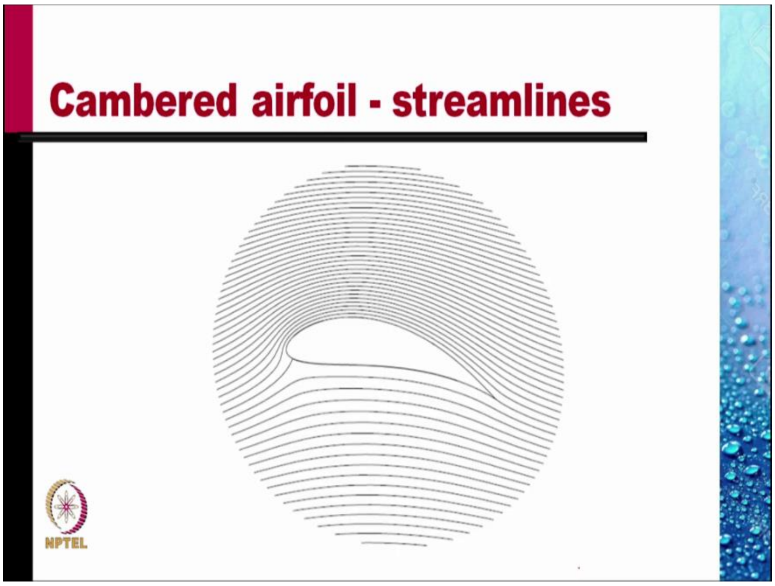

With these we can find out the flow about a cambered airfoil. And the lift on such an airfoil is $-\rho V_o \Gamma$, the total sheet strength.

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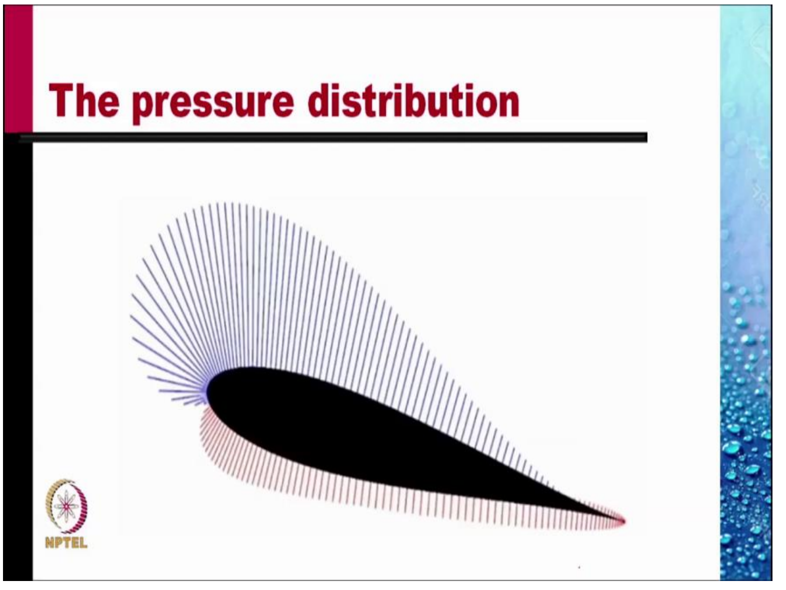

The pressure distribution can be calculated there. This shows the pressure distribution about one cambered airfoil.

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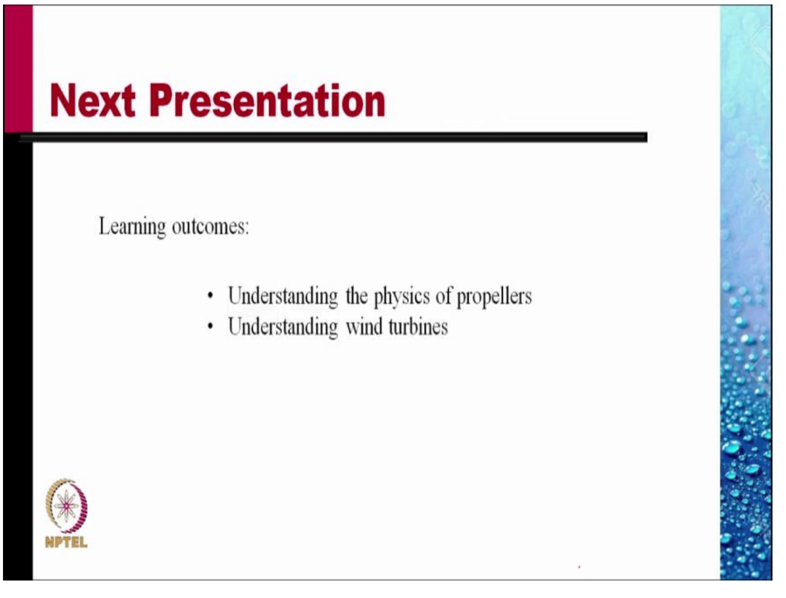

Thank you very much.