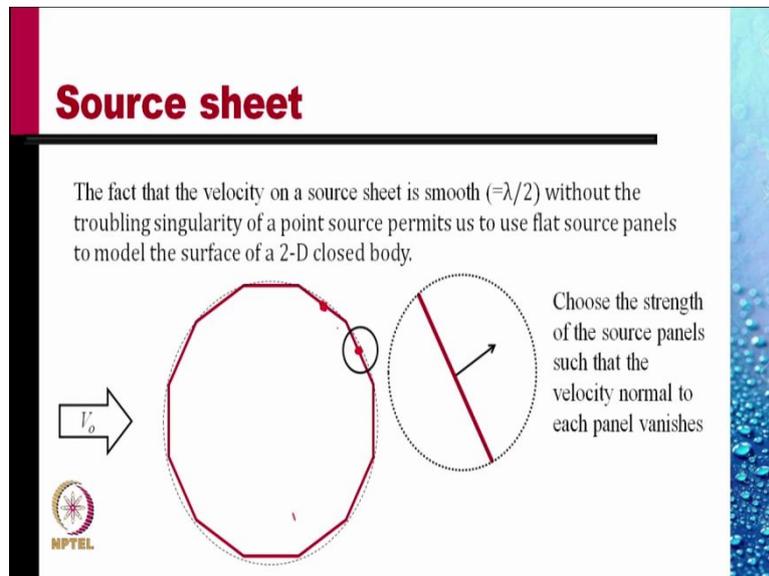


Fluid Mechanics & its applications
Professor Vijay Gupta
Sharda University
Indian Institute of Technology, Delhi
Lecture 26A
Flow over an airfoil

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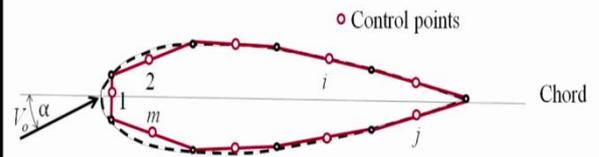
The fact that the velocity of the source sheet is smooth is equal to $\lambda/2$ without the troubling similarity of a point source permits us to use flat plate source panel to model the surface of a 2-D closed body. So, we extend the use of this source panel to flow about an airfoil, or even a circular cylinder.

Consider a circular cylinder. We divide this circumference up into a number of panels, flat panels, and for each flat panel we assume a source strength. Once we assume a strength for this panel, we can calculate the effect of those sources from each of these panels to points at the center of the panels, and make the flow velocity tangent. If they are n panels that we use, there would be n control points. So, by making the velocity 0, making the velocity 0 at these n control points, we get n equations in n undetermined source strengths. And this system of equations can be solved to get the strengths required for each of the panels.

We outline here the method applied to an airfoil. We take a free stream at a velocity V_0 , and find out the velocity at a control point here, and we set the velocity 0. And that will give us one equation in n unknown source strengths. This we repeat for every control point. So, we get n equations in n unknown source strengths. Choose the strength of the source panels such that the velocity normal to each panel vanishes.

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Symmetric airfoil



The diagram shows a symmetric airfoil profile with a dashed line representing the mean camber line and a solid line representing the airfoil surface. The airfoil is divided into m source panels, numbered 1, 2, ..., i , ..., j , ..., m . Control points are marked with red circles at the midpoints of these panels. A horizontal line represents the chord. A free stream velocity V_o is shown approaching the airfoil from the left at an angle of attack α . A legend indicates that the red circles represent control points.

- Select m points around the profile; establish m source panels, name them 1, 2, ..., j , ..., m with source strengths λ_j of the j^{th} panel.
- Determination the contribution to the (normal) velocity of each of the panels at each of the control points.
- The sum of these velocities *plus* the contribution of the free stream at the each control point is equated to zero, leading to a system of m equations in m unknown source panel strengths



So, we start with the profile of the airfoil. the flow approaches it with a velocity V_o , angle of attack α . We divide the surface of the airfoil into a number of straight source panels, and we choose the midpoints of these control panels as control points. We number the panels and the control point, 1, 2, 3, i , j up to m . So, there are m panels and m control points were chosen.

We next do the determination of the contribution to the normal velocity of each of the panels at each of the control points. The sum of these velocities, plus the contribution of the free stream at each control point, is equated to 0, leading to a system of m equations in m unknown source panels strengths.

(Refer Slide Time: 04:23)

Calculating contribution to ϕ at m control points

- Calculate the contribution of the i^{th} panel to the velocity potential at the j^{th} control point
- $d\phi_{ji} = \frac{\lambda_i}{2\pi} \int_i \ln r_{ji} ds_i$ with $r_{ji} = \sqrt{(x - x_j)^2 + (y - y_j)^2}$ where x, y are the coordinates of points on the i^{th} panel
- The total contribution of all panels at the j^{th} control point is $\sum_{i=1}^m \frac{\lambda_i}{2\pi} \int_i \ln r_{ji} ds_i$



And how do we do this. We calculate the contribution of the i^{th} panel to the velocity potential at the j^{th} control point. That is, we find out $d\phi_{ji}$, at the j^{th} control point because of i is equal to

$$= \frac{\lambda_i}{2\pi} \int_i \ln r_{ji} ds_i \quad \text{with } r_{ji} = \sqrt{(x - x_j)^2 + (y - y_j)^2}, \text{ where } x \text{ and } y \text{ could be } x_i \text{ and } y_i.$$

The total contribution of all the panels at the j^{th} control point is this summation for all the i 's.

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Applying the boundary condition on normal velocity

- The total contribution of all panels at the j^{th} control point is $\sum_{i=1}^m \frac{\lambda_i}{2\pi} \int_i \ln r_{ji} ds_i$
- The normal component of velocity induced at the j^{th} control point by the source panels is

$$V_n = \frac{\partial}{\partial n_j} [\phi(x_j, y_j)]$$
 with the derivative taken in the direction of the normal to the j^{th} panel
- The above results in a singularity in integration over the j^{th} panel itself, the contribution of which has been evaluated earlier as $\lambda_j/2$.



The total contribution of all panels at the j^{th} control point was obtained as $\sum_{i=1}^m \frac{\lambda_i}{2\pi} \int_i \ln r_{ji} ds_i$. The normal component of velocity induced at the j^{th} control point by the source panel is then obtained by taking the derivative of this potential with respect to n_j . The above results in a singularity the integration of the j^{th} panel itself, the contribution of which

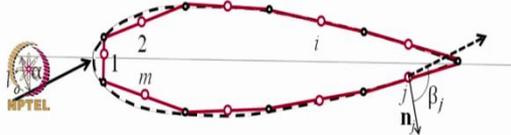
has been evaluated earlier as $\lambda_j/2$. So, we eliminate the j^{th} panel from this sum and add $\lambda_j/2$ separately.

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Applying the boundary condition on normal velocity

- The boundary condition requiring vanishing of normal component of the velocity at the j^{th} panel become

$$\frac{\lambda_j}{2} + \sum_{\substack{i=1 \\ i \neq j}}^m \frac{\lambda_i}{2\pi} \int \ln r_{ji} ds_i + V_o \cos \beta_i = 0$$
- This is a linear equation with m unknown source-panel strengths. There are a total of m such equations, one for each control point.
- These can be solved simultaneously.



The boundary condition requiring vanishing of normal components of the velocity of the j^{th} panel becomes $\lambda_j/2$, the contribution of j^{th} panel, plus the contribution all other panels except i is equal to j , plus the contribution of the free stream $V_o \cos \beta_i$, should be 0, the normal velocity should be 0. This is a linear equation with m unknown source panel strengths. There are a total of m such equations, one for each control point, and these can be solved simultaneously.

(Refer Slide Time: 7:00)

Requirement for a body to be a closed body

- For a closed body

$$\sum_{i=1}^m \lambda_i = 0$$

This can be used as an independent check on the accuracy of calculations



Now, as was said earlier, the net source strength for a closed body must be 0. So, $\sum_{i=1}^m \lambda_i = 0$. We have not used this equation earlier. So, this can be used as an independent check on the accuracy of the calculations performed.

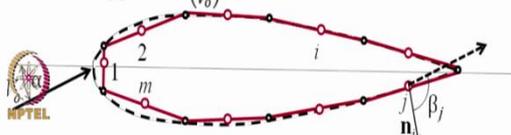
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Calculating the tangential velocity and the pressure coefficients

- Once λ_j 's are obtained, we can write the composite stream function and plot its contour to get the streamlines of the flow
- To calculate the pressure distribution, we need to know the tangential velocities at the control points

$$V_s = \frac{\partial \phi}{\partial s} = \sum_{i=1}^m \frac{\lambda_i}{2\pi} \int_i \frac{\partial}{\partial s} (\ln r_{ij}) ds_i + V_o \sin \beta_j$$

- Then $C_{p,j} = 1 - \left(\frac{V_j}{V_o}\right)^2$

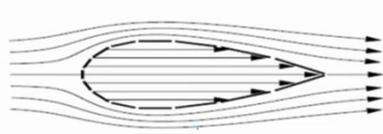


The diagram shows a closed body with several control points labeled 1, 2, i, and m. A normal vector n_j is shown at a point j on the body's surface, with an angle β_j relative to the horizontal flow direction.

Once λ_j 's are obtained, we can write the composite stream function, and plot its contours to get the streamlines of the flow. To calculate the pressure distribution, we need to know the tangential velocities at the control points which are obtained from this expression. Then, the coefficient of pressure at the j^{th} control point is nothing but $1 - \left(\frac{V_j}{V_o}\right)^2$.

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Calculated streamlines



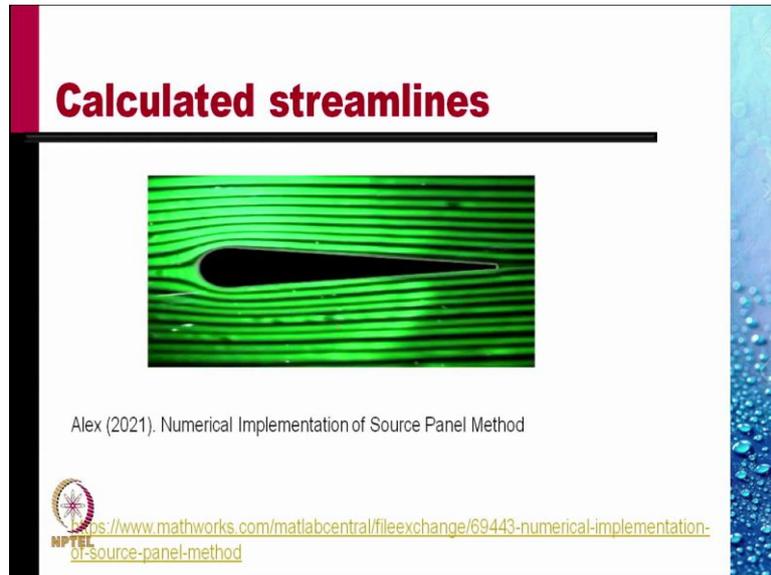
The diagram shows a closed body with several streamlines flowing around it from left to right. The streamlines are smooth and follow the contour of the body.

Alex (2021). Numerical Implementation of Source Panel Method

<https://www.mathworks.com/matlabcentral/fileexchange/69443-numerical-implementation-of-source-panel-method>

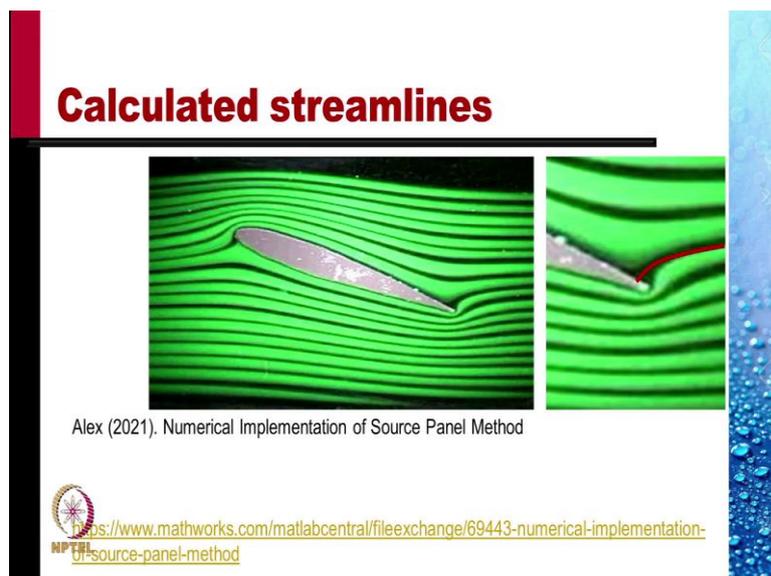
This panel method is used very often, and lots of programs have been developed with this. One such implementation in MATLAB is available at this reference. The viewers are encouraged to visit this site, which is a MATLAB sharing source, and obtain it. A rather small source panel method program, which can use to solve for any closed body.

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The inside can be replaced by the solid body and the calculated streamlines are very close to what is obtained.

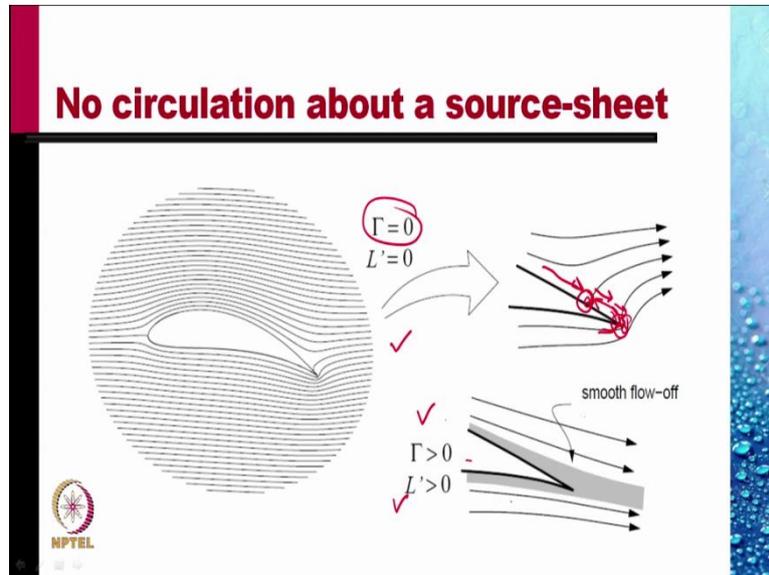
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Of course, there is no restriction on α in this method. If you use larger α , we get a flow like this. We will enlarge this. You see here, the stagnation point has moved to a point above the

trailing edge of the airfoil. This results in a rather large acceleration at the trailing edge, a situation which is not physical tenable, you cannot have infinite velocity there. So, this breaks down.

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There is no circulation about a source sheet we have calculated earlier. And so, if we calculate the circulation in the flow that we obtain, we will find that circulation 0, and the lift per unit length of the airfoil obtained will be 0, whatever be the angle of attack. So, this method cannot predict the lift by itself. As we have seen the trailing edge is this. The flow goes around the trailing edge acquiring infinite velocity at the real point. The stagnation point moves up. So, there is flow which is slowing down in this region and in this region. This result in separation. A famous scientist by the name of Kutta produced a theorem which says that the actual flow has a circulation which is non zero, which predicts a non-zero lift, and that circulation can be predicted. The actual physical circulation in a real flow can be predicted by imposing on this solution, a vortex strength such that, the stagnation point moves to the trailing edge.

As we increase the value of Γ , the circulation, this stagnation point shifts downward. At one particular value of Γ , this stagnation point is sitting at the trailing edge. And the Kutta condition states that the actual flow is one in which Γ is exactly equal to the required circulation to make the rear stagnation point at the trailing edge.

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Kutta condition

A body with a sharp trailing edge which is moving through a fluid will create about itself a circulation of sufficient strength to hold the rear stagnation point at the trailing edge.

To repeat: the stagnation point here with zero circulation. I change the value of circulation and this point starts moving down towards the trailing edge. At one specific value of circulation superimposed, that is, on the potential flow that we obtained without the circulation, we keep on adding a vortex, and when the vortex strength reaches a certain value, the stagnation point is right at the trailing edge. If I add more of circulation, the stagnation point will move down the bottom surface. A body with a sharp trailing edge which is moving through a fluid will create about itself a circulation of sufficient strength to hold the rear stagnation point at the trailing edge. This is the Kutta condition.

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Prediction of lift: Vortex sheet

- Thin, cambered airfoil
- Free-stream at V_∞ with angle of attack α

Imagine a vortex sheet of strength $\gamma(s)$ on the camber line

$$\phi(x, y) = \int_0^l \frac{\gamma(s)}{2\pi} \tan^{-1} \frac{y}{x-s} ds$$

The velocity v on the camber-line at any s can be determined by differentiating ϕ with respect to y

$$v(s) = \int_0^l \frac{\gamma(s)}{2\pi(x-s)} ds$$

So, what we do is, we predict the lift by introducing a vortex sheet in much the similar manner as we did with the source sheet. This vortex sheet is now laid along the camber line of the airfoil. The thickness of the airfoil does not contribute to the lift. So, to the first approximation of the flow, we can forget about the thickness, and worry only about the camber line. We imagine a vortex sheet of strength $\gamma(s)$ on the camber line.

Calculate the potential at point (x, y) because of this vortex sheet of strength that varies like $\gamma(s)$. The velocity u on any location s can be determined but differentiating ϕ with respect to x . There is a discontinuity at $y = 0$, and we get u the horizontal velocity slightly above the camber line is $-\frac{\gamma}{2}$, and slightly below $s + \frac{\gamma}{2}$. And this gives us the velocity on the camber line.

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Prediction of lift: Vortex sheet

If the airfoil is thin, make the camber-line a streamline by imposing the condition that the velocity is tangent to it, i.e., $v(s)$ due to vortex sheet at the sheet location is equal and opposite to the vertical component of the free-stream velocity

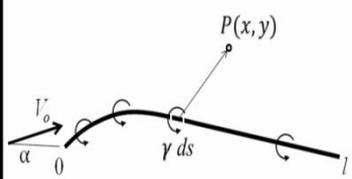
$V_0 \left(\alpha - \frac{dy}{dx} \right) = \int_0^l \frac{\gamma(s)}{2\pi(x-s)} ds$

Camber line $\tan^{-1} \left(-\frac{dy}{dx} \right)$

If the airfoil is thin, make the camber line a streamline by imposing the condition that the velocity is tangent to it that is $v(s)$ due to vortex sheet at the sheet location is equal and opposite to the vertical component of the freestream velocity. The vertical component of the freestream velocity. So, that $V_0 \left(\alpha - \frac{dy}{dx} \right)$ is the vertical velocity predicted by this vortex sheet.

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Prediction of lift: Vortex sheet


$$V_0 \left(\alpha - \frac{dy}{dx} \right) = \int_0^l \frac{\gamma(s)}{2\pi(x-s)} ds$$

Techniques have been developed to obtain $\gamma(s)$ as a Fourier series with Kutta condition that translates to $\gamma(l) = 0$



Techniques have been developed to obtain $\gamma(s)$ as a Fourier series with Kutta condition that translates to $\gamma(l) = 0$.

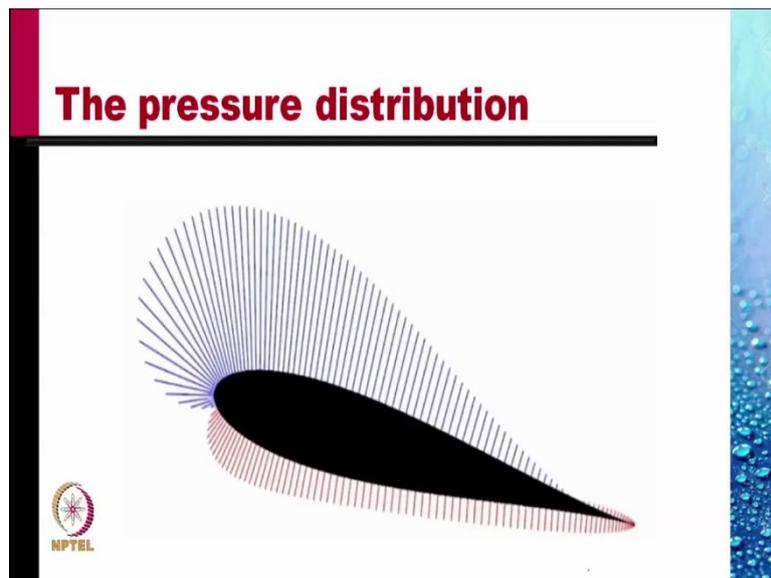
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Cambered airfoil - streamlines



With these we can find out the flow about a cambered airfoil. And the lift on such an airfoil is $-\rho V_0 \Gamma$, the total sheet strength.

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The pressure distribution can be calculated there. This shows the pressure distribution about one cambered airfoil.

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The slide is titled "Next Presentation" in red. Below the title, it lists "Learning outcomes:" followed by two bullet points: "• Understanding the physics of propellers" and "• Understanding wind turbines". The slide has a red vertical bar on the left and a blue vertical bar with a water droplet pattern on the right. The NPTEL logo is in the bottom left corner.

Thank you very much.