Fluid Mechanics and its Applications

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Lecture 25A

Flow Past A Circular Cylinder

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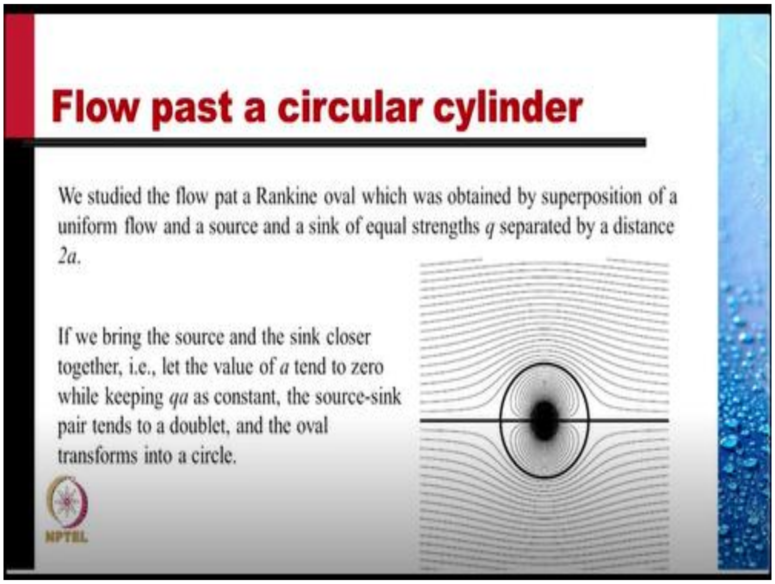

This is the Rankin oval that we obtained, when we had a source and sink separated by distance 2a. As a tends to zero, that is, we bring the source and the sink close together, that is, we let the value of a tend to zero while keeping qa as constant. The source-sink pair tends to a doublet, and the oval transforms into a circle.

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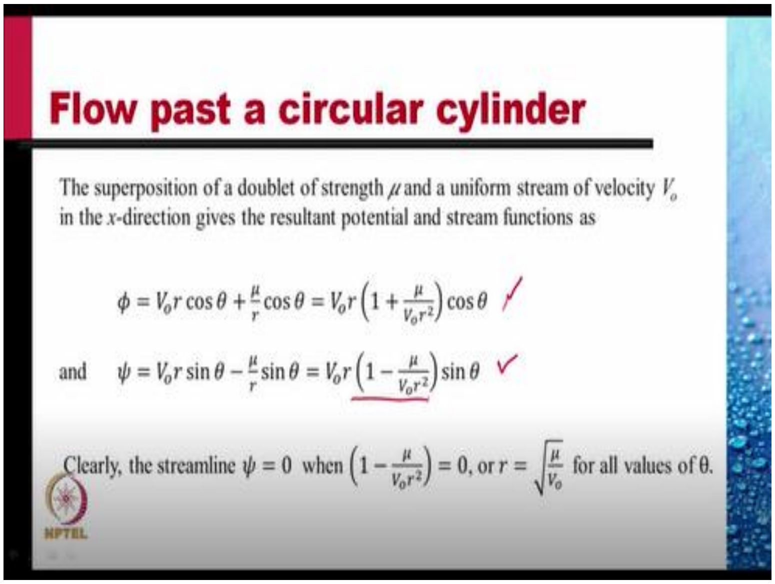

The superposition of a doublet of strength mu and the uniform stream of velocity $V_o$in the x direction gives the resultant potential and stream functions as this and this. Clearly this streamline ψ is equal to 0. When this factor is 0, that is $\left(1-\frac{\mu}{V_o r^2}\right)$ is equal to 0, that is at $r = \sqrt{\frac{\mu}{V_o}}$, ψ is 0, or $r = \sqrt{\frac{\mu}{V_o}}$t is a streamline. That is a circle.

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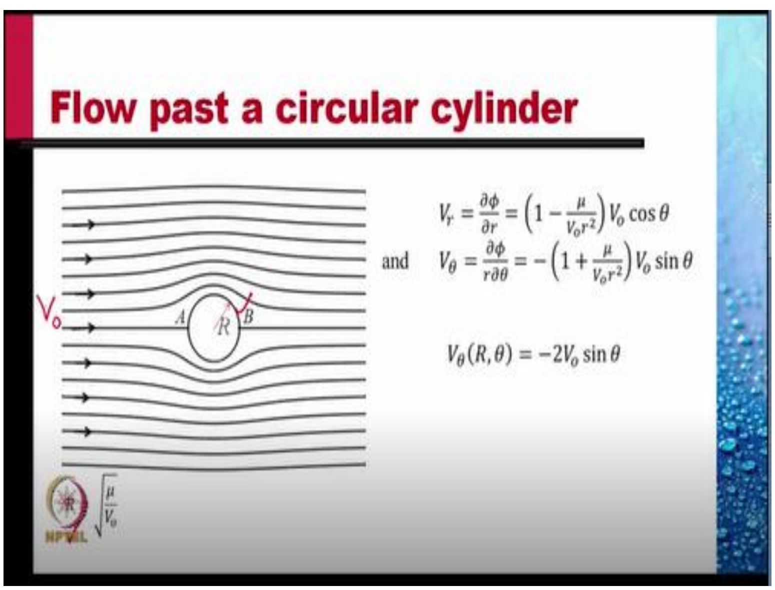

That is what is shown in this picture. This is the streamline ψ is equal to 0, and since this is closed the inside of this can be replaced by a body, a circular cylinder of radius $R = \sqrt{\frac{\mu}{V_o}}$, where μ is the strength of the doublet and V_o is the free stream velocity, the uniform velocity approaching the cylinder. In the resulting r component of velocity and θ component of velocity are given by these formulae. They are obtained by differentiating the velocity potential ψ with respect to r and with respect to θ.

Then the θ velocity, on (R, θ), that is on the surface of the cylinder, is nothing but $-2V_o \sin\theta$. Recall that V_θ is taken positive when it is counter clockwise. Of course, the r component of velocity on this circular surface is zero, this being a streamline on which there should be no radial component of velocity.

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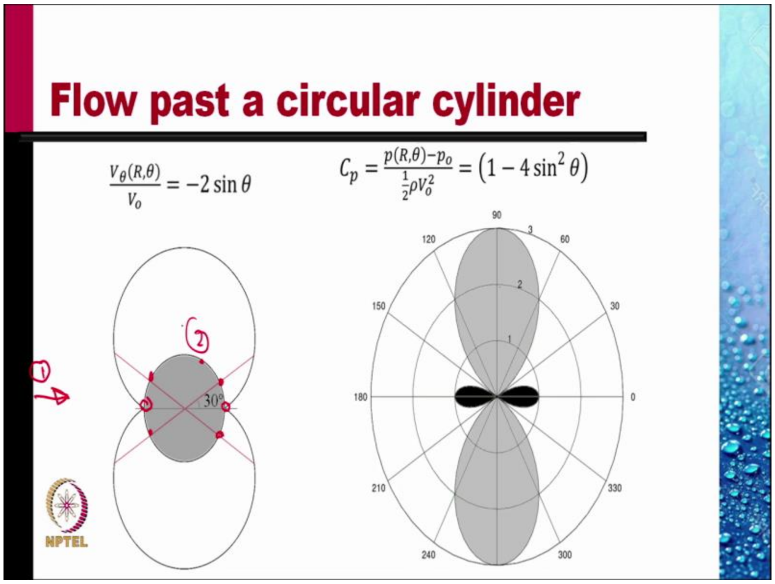

We plot $\frac{V_\theta(R,\theta)}{V_o}$ on the surface of this cylinder of radius R, that is, we plot $-2\sin\theta$ in a polar plot, and this is what we get. The velocity is zero at the horizontal diameter. These being the stagnation points. At 30 degrees, that is when θ is 30 degrees, $\sin\theta$ is 1/2. So, the value of V_θ is -1, that is, we have recovered at these points the value of velocity in the free stream, that is, the value of the uniform stream which is approaching this body.

We can calculate the coefficient of pressure, and we have done this in an earlier lecture by using the Bernoulli equation between a point 1 far upstream and a point 2 on the surface at an angle θ, and we obtain the coefficient of pressure which is $\frac{p(R,\theta)-p_o}{\frac{1}{2}\rho V_o^2}$, , and this is obtained as $(1 - 4\sin^2\theta)$. This, when plotted, looks like this. The two dark lobes are the positive pressures, where the velocity, this is between minus 30 degrees and plus 30 degrees, the velocity is less than the free stream velocity. And so, by Bernoulli equation the pressure is more than the pressure far away.

C_p is positive from 30 degrees to 150 degrees, the velocity is more than V_o, both on the upper part of the cylinder and the lower part of the cylinder, and the pressure coefficient is negative. The pressure picture is completely symmetrical fore and aft and top and bottom, signifying that there is no lift or drag on a circular cylinder in a potential flow.

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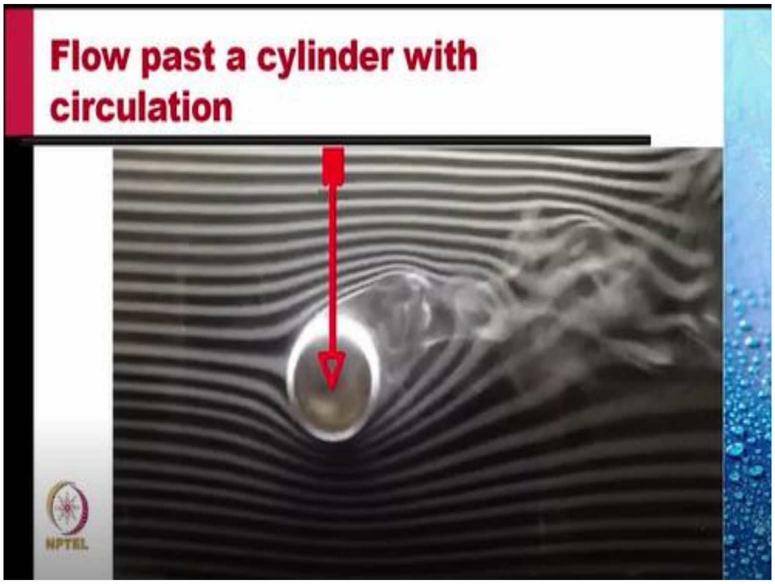

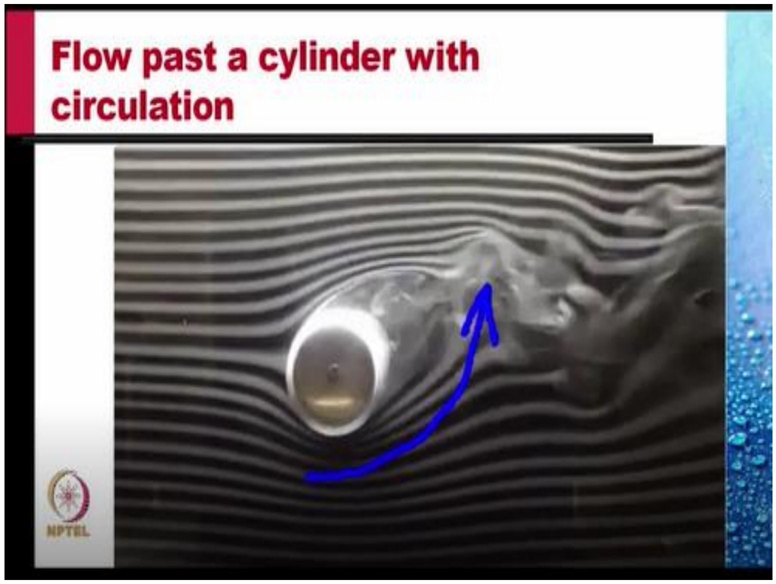

Now, let us consider a flow past a cylinder with circulation. That is, we take the potential of a circular cylinder that we obtained earlier, and superimpose on this a vortex.

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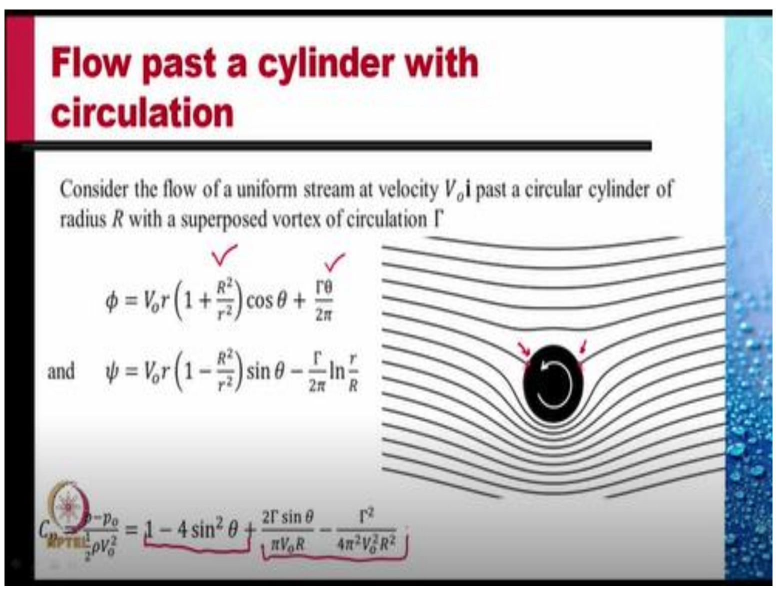

And when we do this, the resulting potential function is nothing but that due to a uniform stream and a doublet plus a free vortex of strength Γ. And similarly, the stream function. If we plot the streamlines, this is the pattern of streamlines that we see. Clearly, it is asymmetrical, top and bottom. On the lower part the streamlines are closer together then on the top part, signifying that velocities on the lower part are more than the velocities on the top part.

The stagnation points which were on the two ends of a horizontal diameter have now moved up to these two points, and the coefficient of pressure around this cylinder is given by this expression. These two terms are the same as when there was no circulation, and these two additional terms are due to circulation.

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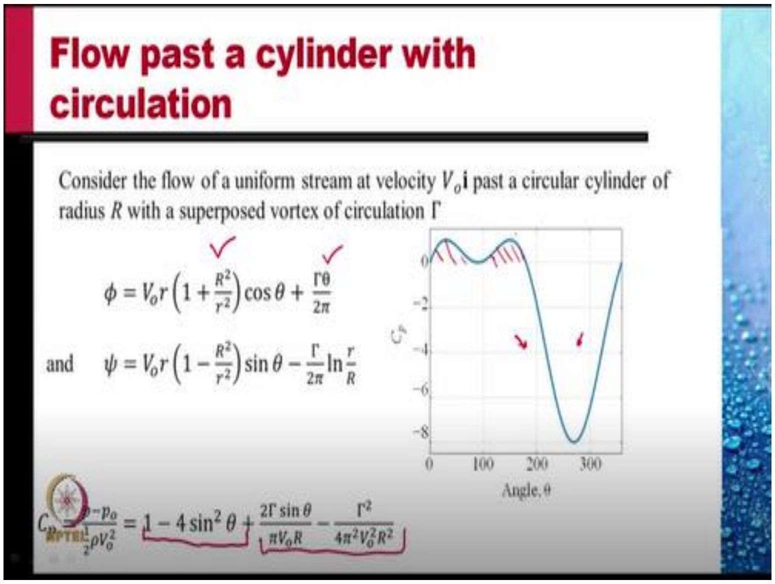

This C_p plot looks like this. Clearly, on this top portion there is positive pressure pushing it down, and in the bottom portion the pressure coefficient is negative, pulling it down. So, both act toward producing a downward force perpendicular to the direction of the uniform flow.

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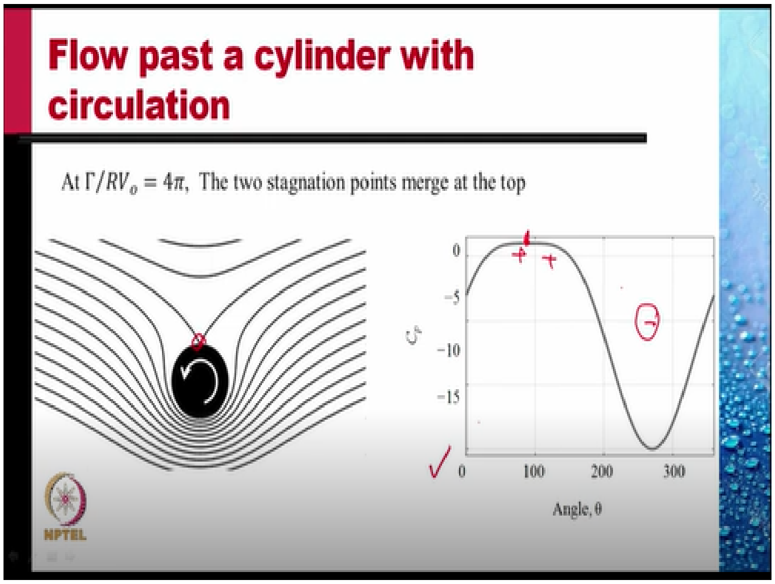

Now, this pressure distribution depends upon the value of circulation, that is, the strength of the vortex that we have imposed. When $\Gamma/RV_o = 4\pi$, Γ is a circulation RV_o is the product of the radius of the cylinder and the free stream velocity. When this value is 4π, the two stagnation points merge at one point, this stagnation point is right at theta is equal to 90 degrees, and the distribution of the coefficient of pressure is like shown here.

The positive pressure here at 90 degrees, which will be somewhere here, the pressure coefficient is one this being the stagnation point, and through much of the bottom portion and some part of the top portion there is negative C_p. So, this results again in a net force which is downwards.

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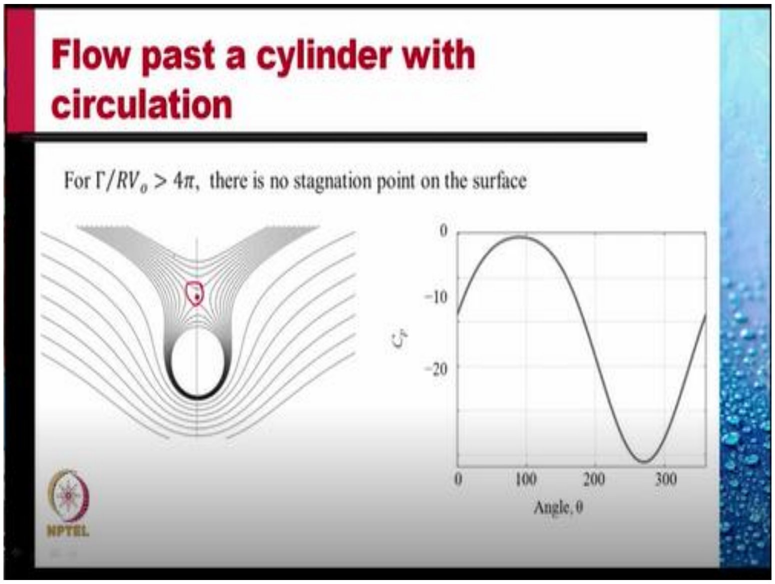

Here I have superimposed on this, a case when $\Gamma/RV_o > 4\pi$, and then there is no stagnation point on the circular streamline. The stagnation point has moved up into this free region. The flow looks like this, and now, there is a negative pressure coefficient throughout the surface of the cylinder resulting is large negative force, force which is perpendicular to the direction of velocity.

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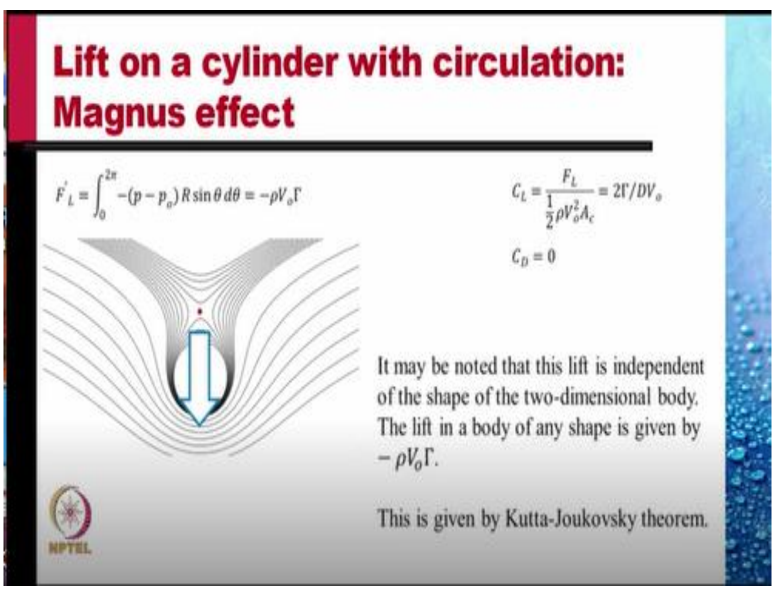

We can find out this force by integrating the pressure difference multiplied by the area, and if we do this integration, not a complicated job and we obtain the pressure force permanent length of the cylinder as $-\rho V_o \Gamma$. It is linearly proportional to V_o and Γ and the density ρ. This force is perpendicular to the direction of flow and we will find out that there is no component of drag, there is no force along x axis, the force is purely downwards.

We define a lift coefficient C_L is equal to this lift force that we obtained divided by $\frac{1}{2}\rho V_o^2$, which is the characteristic pressure difference in such flows, and the area is A_c, a characteristic area. Typically, the characteristic area is taken as the plan-form area, which for a cylinder radius R and length L would be 2RL, that is, DL.

And we make the necessary calculation we get the lift coefficient as $2\Gamma/DV_o$, and the drag coefficient is 0. Recall that the lift is the force perpendicular to the direction of the uniform stream in which the cylinder is situated, and drag is the force in the direction of the stream.

It may be noted that this lift is independent of the shape of two-dimensional body. We may have worked with a Rankin oval or any other closed two-dimensional body, and there is a theorem that says, shows, that the lift on a body of any shape is given by $2\Gamma/DV_o$ per unit length. This theorem is known as Kutta-Joukovsky theorem.

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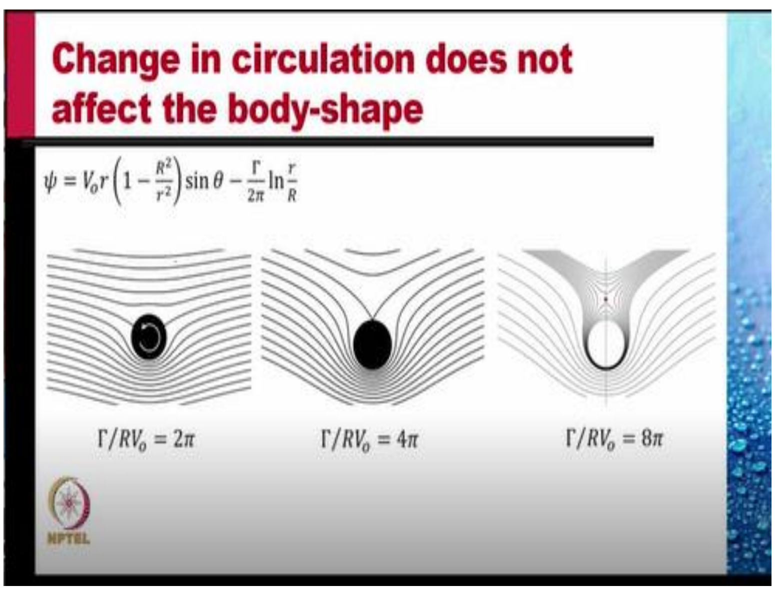

We had obtained stream function is this and we had shown that when we change the value of gamma from 2π to 4π to 8π, the flow changes, but the shape of the closed stream line remains the same, circular, same radius. So, changing Γ, that is changing the circulation, changing the strength of the free vortex, does not change the shape of the body. This result is of great significance in aerodynamics, where we would see that all values of gamma produce the same close body, but there is only one value of Γ that fits the practical situation. We will talk about this in the next lecture.

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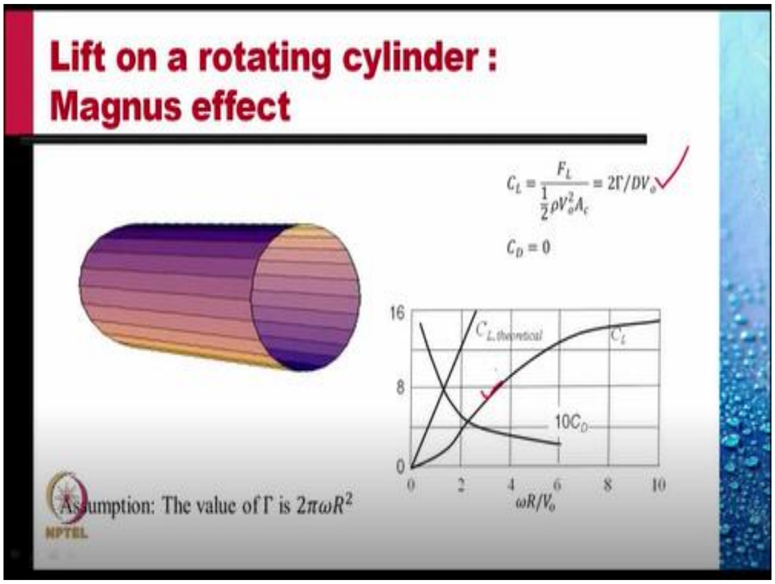

If we have a rotating cylinder the flow about this starts rotating because of the no slip boundary conditions, and we wait long enough there would be a theta velocity which would vary like 1/r. On the surface of this cylinder, the velocity would be ωR. The analysis that we did just now, where we had circulation about a cylinder is sometimes used to model this flow.

Clearly, there is a conflict, because this rotating cylinder would not set up a free vortex in a flow which is non-viscous, and it is the non-viscous flow, the potential flow, in which we obtain the earlier results. But we see that there is, in effect, and as this cylinder rotates in a real fluid, and there is a uniform flow past the cylinder, we have a force which is like lift.

And this force is of the same order as predicted by the formula in the last slide, that is C_L is like $2\Gamma/DV_o$ obtained earlier, and the drag coefficient 0. Plotted, this is the C_L obtained by the formula above, where ωR is the surface velocity of this rotating cylinder and V_o is the stream velocity of flow approaching it.

And this curve we brought is a straight line, C_L theoretical, but the measured values are like this. Clearly, there is large scale deviation, but the forces are of the same order. The drag coefficient should have been 0, but we obtain a drag coefficient that looks like this. This is not of the same order as C_L. This order one-tenth of C_D of C_L, so, C_D is much less than C_L but the predicted C_D is 0 if we assume the flow to be invested.

This effect is given the name Magnus effect, and we use this as the formula for the Magnus effect though the actual lift force is an order of magnitude less than this. A lot of experiments have been done in wind tunnels, and now with digital computers calculations have been made for real fluid, and we get a C_L curve which is very close to this, without invoking the effect of the circulation.

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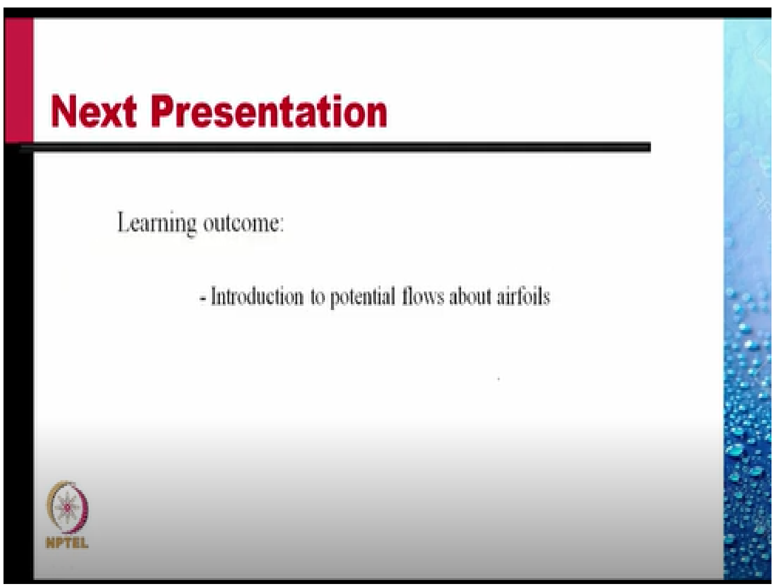

In the next lecture, we will introduce the potential flow about airfoils.

Thank you.