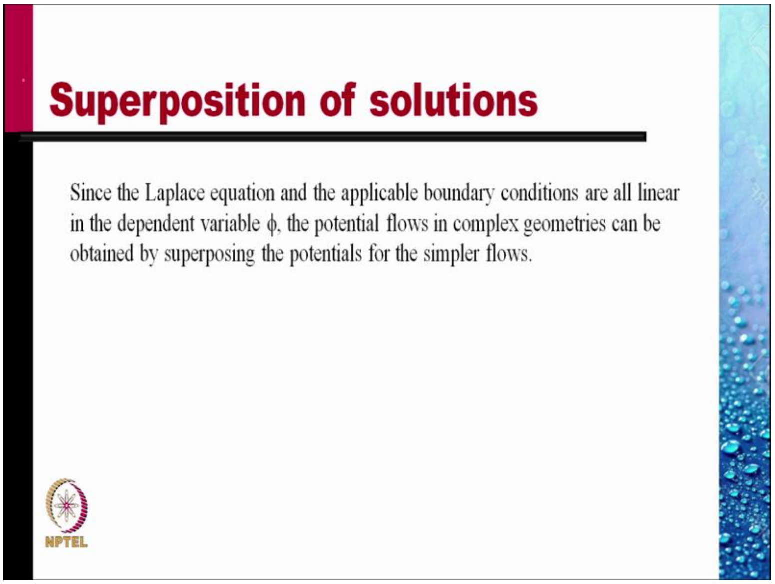
Fluid Mechanics and its Application
Professor Vijay Gupta
Indian Institute of Technology, Delhi
Lecture: 25
Fluid Mechanics and its Application

Welcome back.

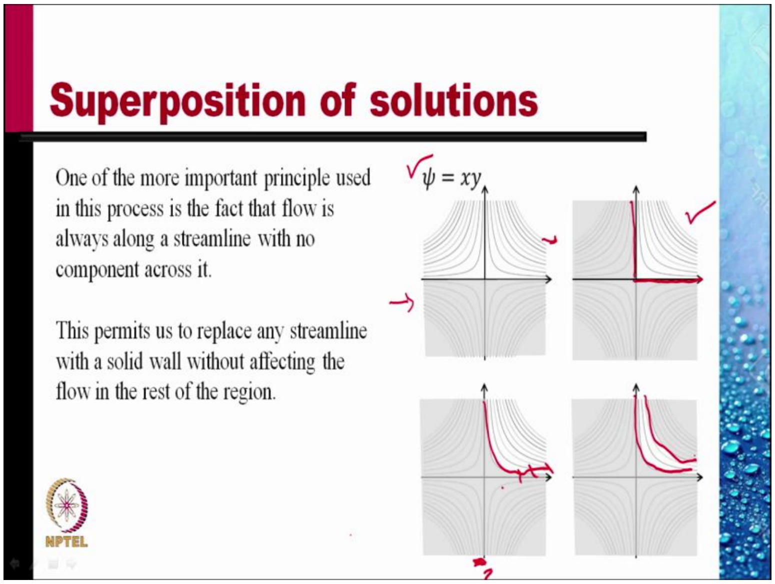
In this lecture, we will cover the development of potential flows by superposition of elementary flows. We had in the last lecture done the basic potential flow solutions for a uniform flow, a source, and a free vortex.

(Refer Slide Time: 00:47)



Since the Laplace equation and the applicable boundary conditions are all linear in the dependent variable ϕ, the potential flows in complex geometries can be obtained by superposing the potentials for the simpler flows.

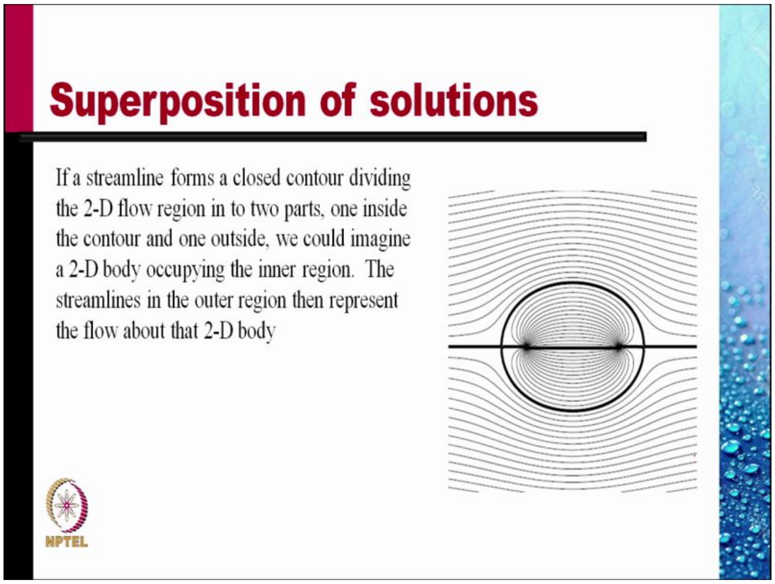
(Refer Slide Time: 01:04)

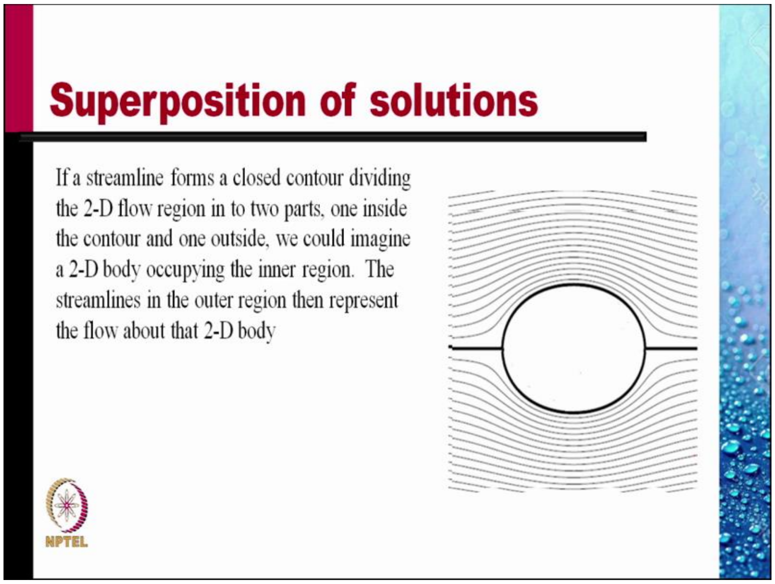


One of the more important principles used in this process is the fact, that flow is always along a streamline with no component across it. This permits us to replace any streamline with a solid wall without affecting the flow in the rest of the region. Thus, if we plot the stream functions, $\phi = xy$, we get rectangular hyperbola in all the four quadrants. Now, in this first picture, we have replaced the x-axis with a wall so, that $\phi = xy$ represents the flow in the upper half-plane, the flow directed towards the wall, perpendicularly.

In the second picture, we have replaced these parts of the axis with walls. So, that the same stream function represents the corner flow, flow into one corner. In the third picture, we have replaced this streamline with a wall, and if we do this, the same stream function represents the flow over a body of this shape. In the last picture, we have replaced two streamlines with walls. So, that now, $\phi = xy$ represents the potential flow to this channel, formed by these two walls.

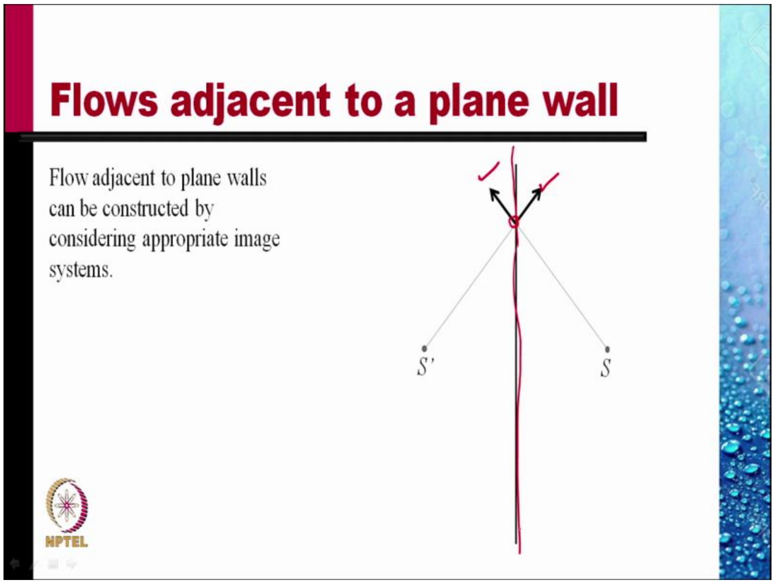
(Refer Slide Time: 03:11)





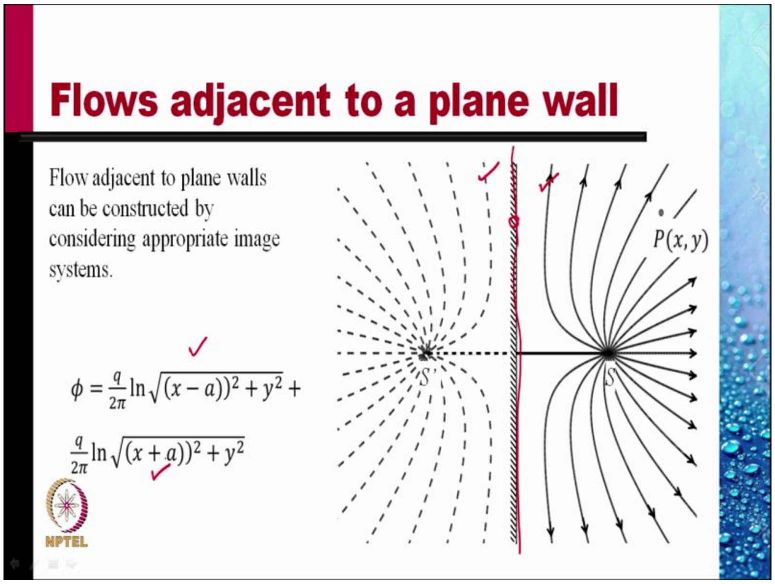
If a streamline forms a closed contour dividing a 2-d flow region into two parts, one inside the contour and one outside, we could imagine a 2-d body occupying the inner region, the streamlines in the outer region then represent the flow about that two-dimensional body. Thus, the internal portion of this could be replaced by a body, and then the same stream function and potential function represents the flow about this body. These are the principles that we use in generating flows through superposition.

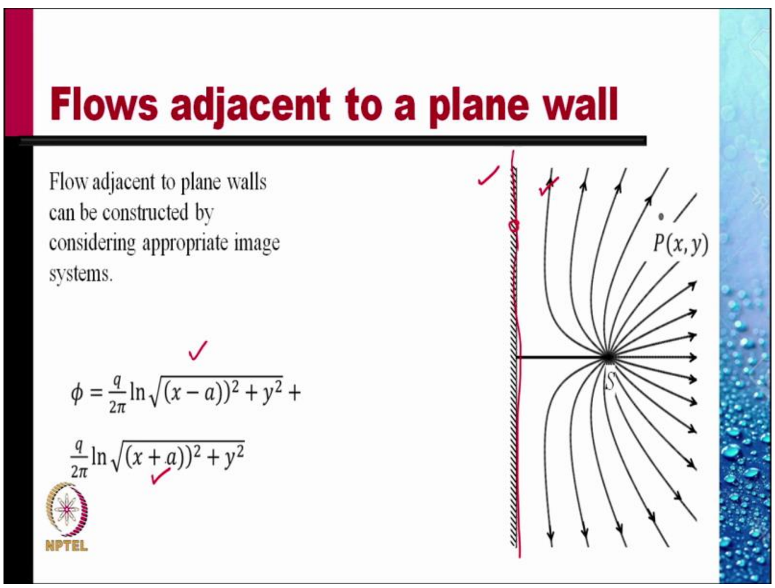
(Refer Slide Time: 03:57)



If we have flow adjacent to plane walls, then these can be constructed by considering appropriate image system. Thus, if we have a source close to a wall, then we can conceive an image source, S', the original source would induce a velocity like this at this point, while the image source would induce a velocity like this. The horizontal components of these two velocities cancel, resulting in the straight line of this wall, the vertical line being a streamline, and this streamline could be taken as a wall. So, then the resulting potential function or the stream function would represent the flow due to a source situated very close to a wall.

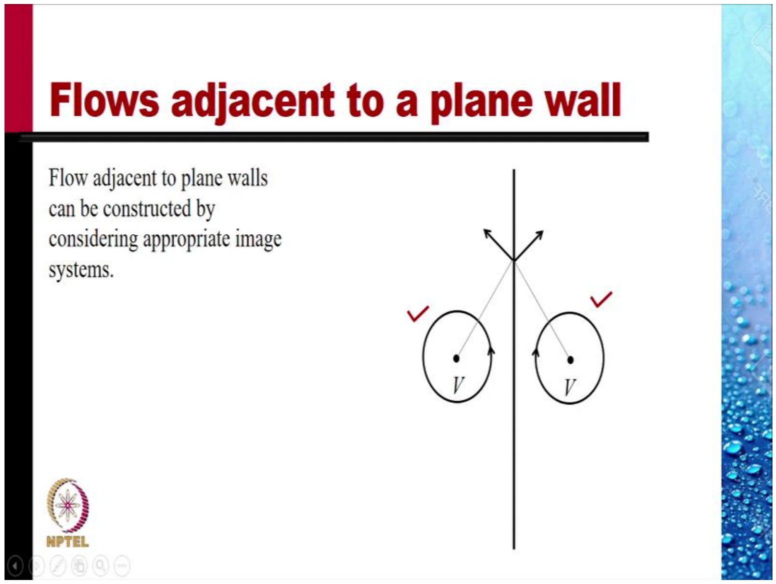
(Refer Slide Time: 05:12)

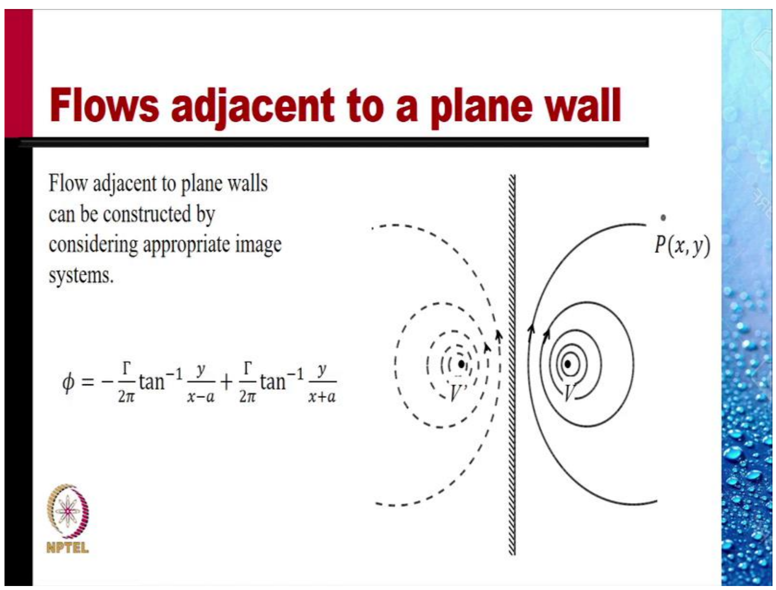


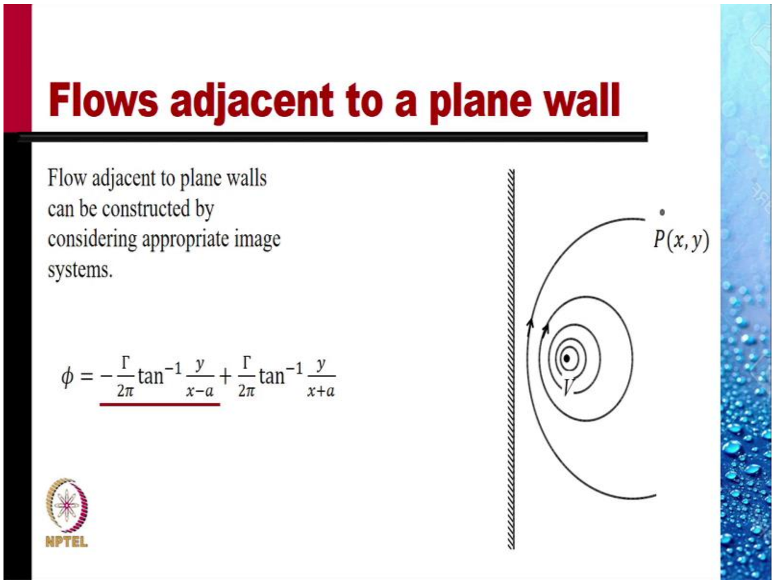


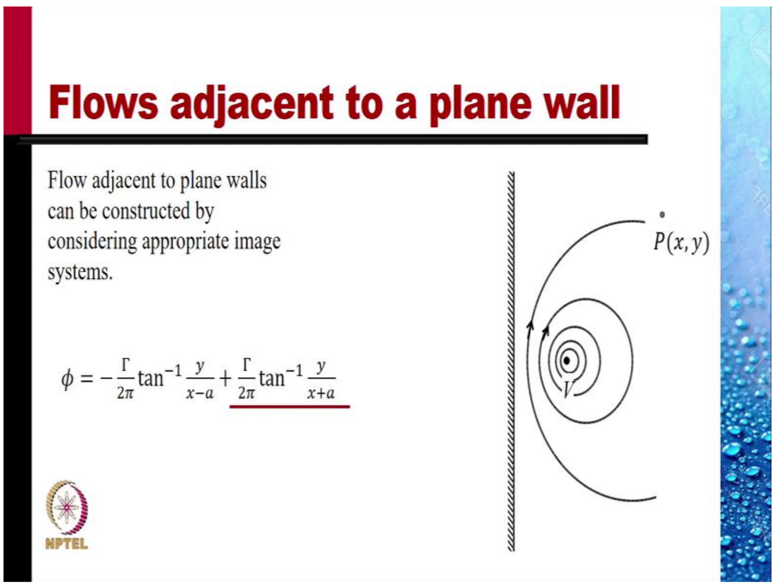
This would be the pattern of streamlines for a source and its image and we could cover the left portion to result in a flow due to a source near a wall. The potential at point P can be written as the potential due to source at $x = a$ and $y = 0$, and at $x = -a$ and $y = 0$, combination of the two potentials. This is the resulting flow due to a source situated very close to a wall.

(Refer Slide Time: 06:00)



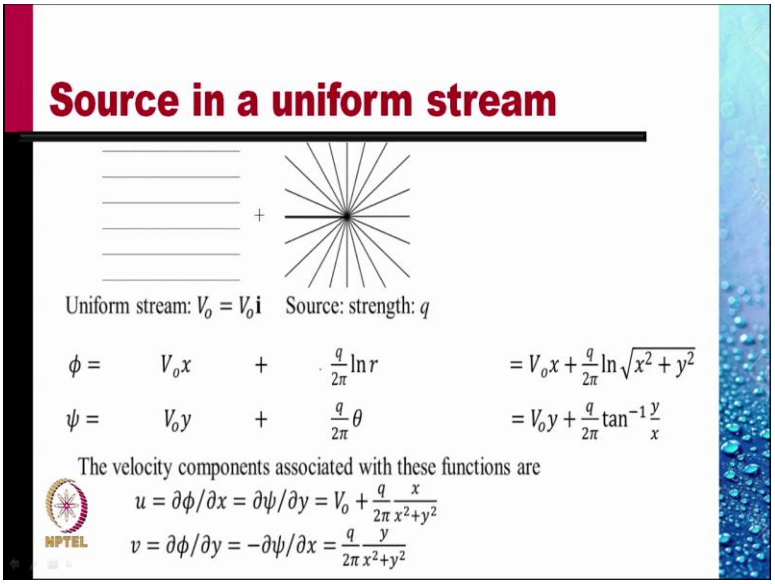






Similarly, we might have a vortex sitting close to a wall. To construct the potential for this, we imagine an image vortex situated on the other side of the wall at equal distance. The induced velocity because of this vortex is this, while the induced velocity because of this vortex is this. No normal component of velocity. So, the wall is a streamline and we could suppress one portion to obtain a flow like this. This is the potential because of a clockwise vortex at $x = a$, and this is the potential to do a counterclockwise vortex at $x = -a$.

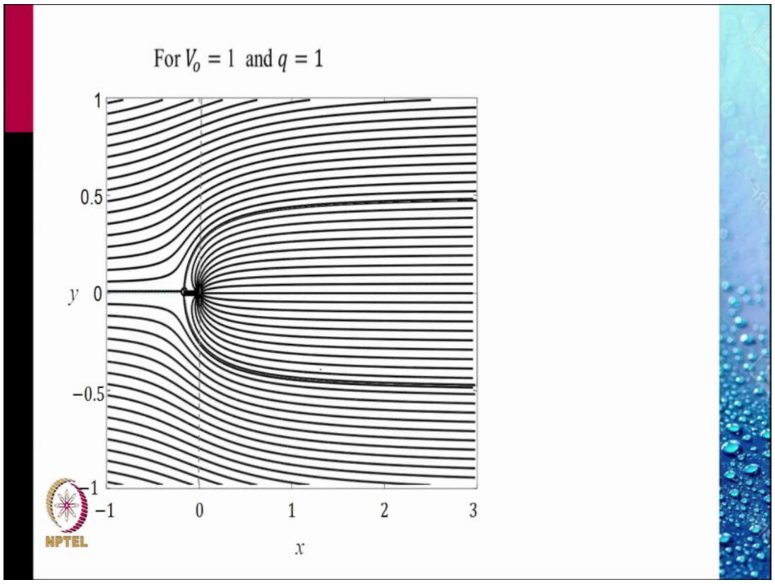
(Refer Slide Time: 07:16)

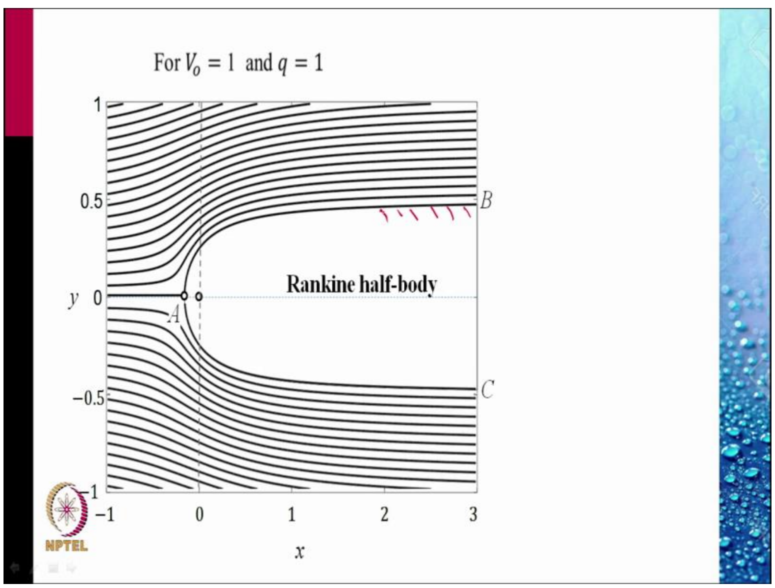


Now, we combine a uniform stream V_o is equal to $V_o i$. This is a uniform stream in i direction, and a source of strength Q. We take the potential of the uniform stream and the potential of a source, and add them together. Thus, $\phi = V_o x$, because of the uniform stream, plus $\frac{q}{2\pi}\ln r$ because of the source, resulting in $\phi = V_o x + \frac{q}{2\pi}\ln\sqrt{x^2+y^2}$ as the potential of a flow which is a combination of a streamline and a source.

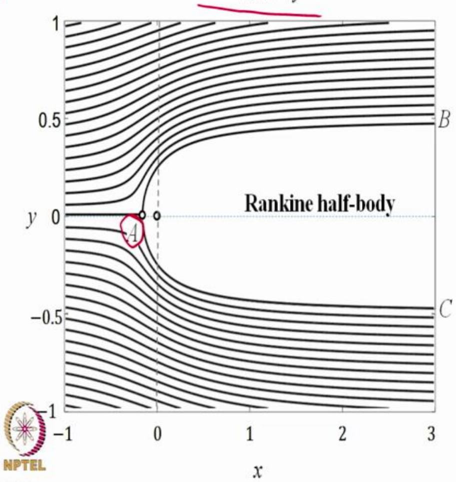
The stream function similarly, could be written in the same fashion. The velocity component associated with these functions are $u = \partial\phi/\partial x = \partial\psi/\partial y$, and they are obtained as $V_o + \frac{q}{2\pi}\frac{x}{x^2+y^2}$.

(Refer Slide Time: 08:44)





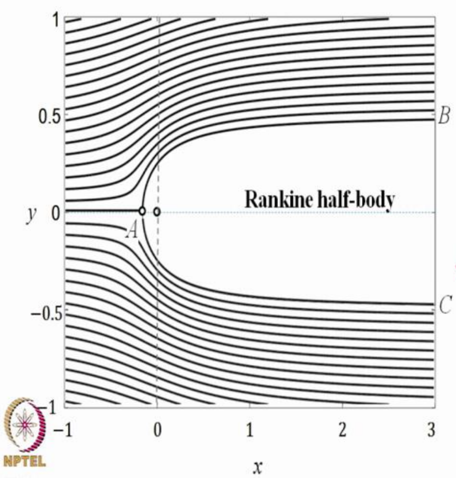
$u = \partial\phi/\partial x = \partial\psi/\partial y = V_o + \frac{q}{2\pi}\frac{x}{x^2+y^2}$; $v = \partial\phi/\partial y = -\partial\psi/\partial x = \frac{q}{2\pi}\frac{y}{x^2+y^2}$



The point A where a the streamline bifurcates must necessarily be a stagnation point.

The coordinates of point A are $(-q/2\pi V_o, 0)$, so that the x-coordinate of the stagnation point is $-1/2\pi$

$\psi = V_o y + \frac{q}{2\pi}\tan^{-1}\frac{y}{x} = V_o r\sin\theta + \frac{q\theta}{2\pi}$



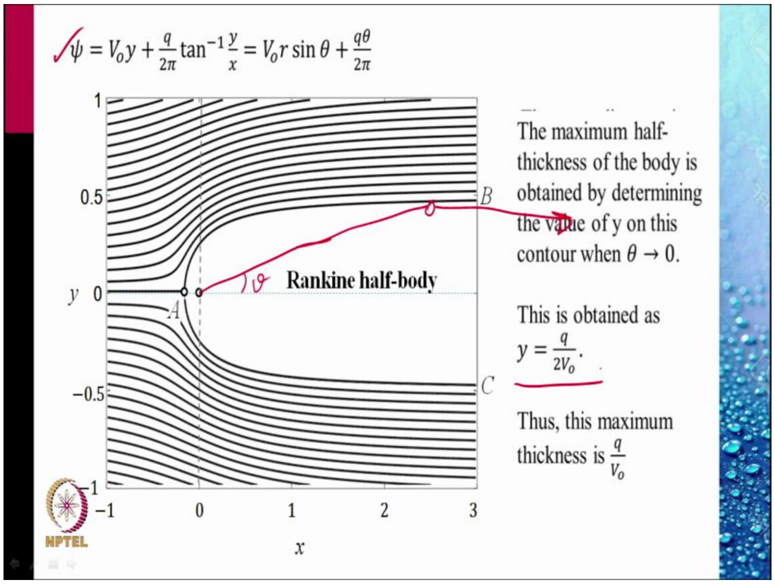
The streamline passing through A is then given by

$V_o r\sin\theta + \frac{q\theta}{2\pi} = q/2$

or,

$V_o y + \frac{q}{2\pi}\tan^{-1}\frac{y}{x} = q/2$

This is the equation of the body contour



Similarly, the v component. If we take this stream function and plot we get this shape, for $V_o = 1$, and $q = 1$. Now, this streamline can be taken as the wall of this half-body. Half-body, since it extends to infinity on the right. And so then, the potential function and the stream function that we obtained in the last slide can be used to represent the potential flow about this Rankine half-body. These are the velocity components u and v that we obtained earlier. The point A, where the streamline bifurcates must necessarily be a stagnation point.

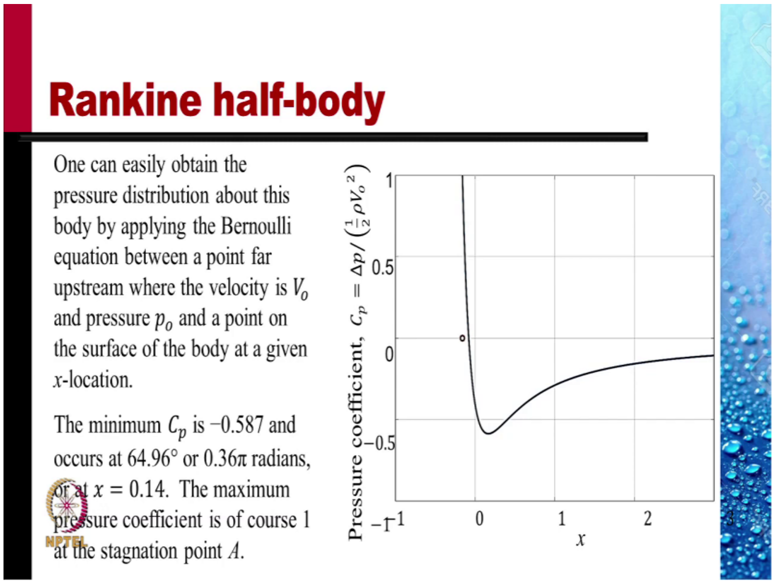
And to locate the point A, we put in the values of u and v, both equals 0, to obtain the values of x and y at which this point occurs. The y coordinate of this point is clearly 0 from this equation, and from this equation for u, the x coordinate is obtained as $-q/2\pi V_o$. Thus, the point A has the coordinate ($-q/2\pi V_o$, 0). For the value of q and V_o, both equal to 1 for this picture. This point is located at ($-1/2\pi$, 0).

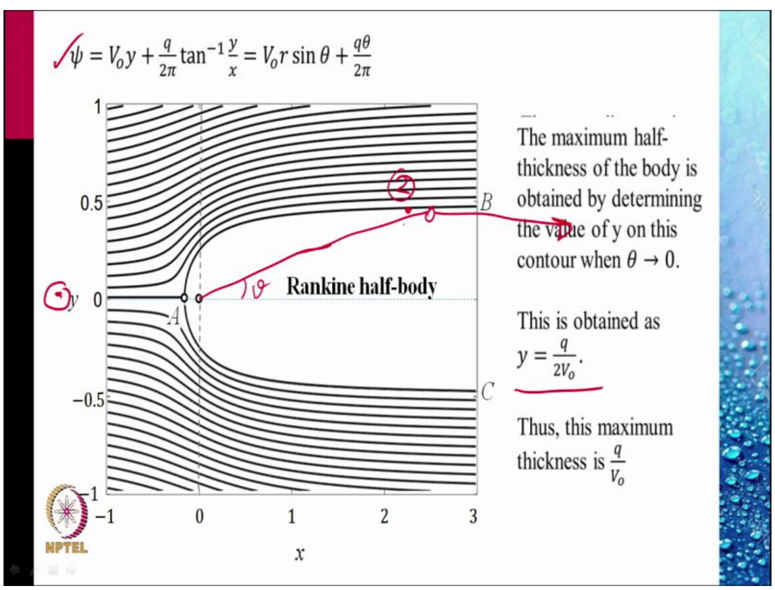
Now, this streamline passing through this point is then given by $V_o r\sin\theta + \frac{q\theta}{2\pi} = q/2$. From this expression of ψ, we put in the value of x and y that we obtained and when we do this, we get $V_o y + \frac{q}{2\pi}\tan^{-1}\frac{y}{x} = q/2$. That is in the x-y coordinates.

The maximum half thickness of the body is obtained by obtaining the value of y on this contour when theta tends to 0. Theta angle is measured from the origin, theta angle is measured from the origin. As theta tends to 0, this the point on the body moves to infinity.

So, the y coordinate as theta tending to 0 represents half-thickness of this Rankine half body at infinity. And this is easily obtained from this equation is $y = \frac{q}{2V_o}$. So, that the total maximum thickness of this half body is $\frac{q}{V_o}$. We can do all kinds of manipulation once we know the stream function psi and the potential function ϕ.

(Refer Slide Time: 13:04)

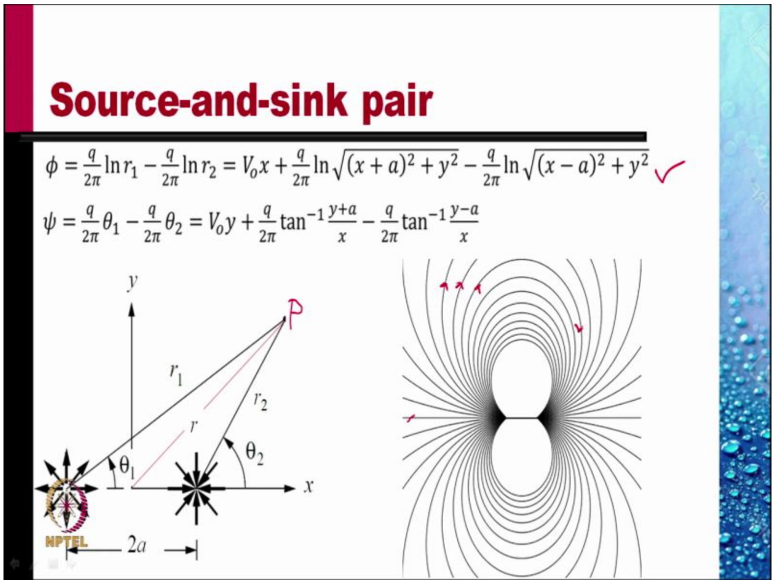




We can easily obtain the pressure distribution about this body by applying the Bernoulli equation between a point far upstream where the velocity is $V_o$and the pressure p_o and the point on the surface of the body at a given x location. Thus, we take a point 1 far upstream, where the velocity is undisturbed V_o, and the pressure is taken as p_o, and the second point we take is a point on the body surface, at the coordinate x.

And when we do this, we obtain this as the curve for pressure variations along the x-axis. The minimum coefficient pressures C_p is minus 0.587, and occurs at theta is equal to 64.96° or 0.36π radians, that is, at x is equal to 0.14 in this picture. The maximum pressure coefficient is of course, 1 and which occurs at the stagnation point A.

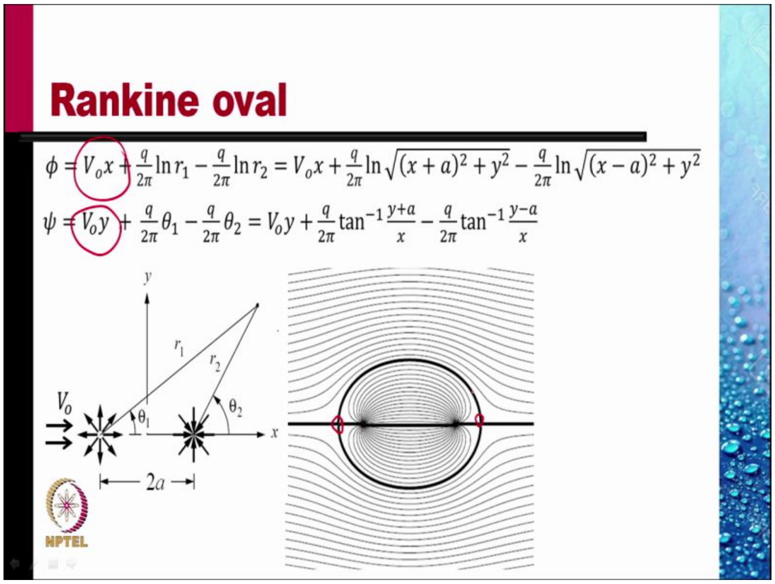
(Refer Slide Time: 14:38)

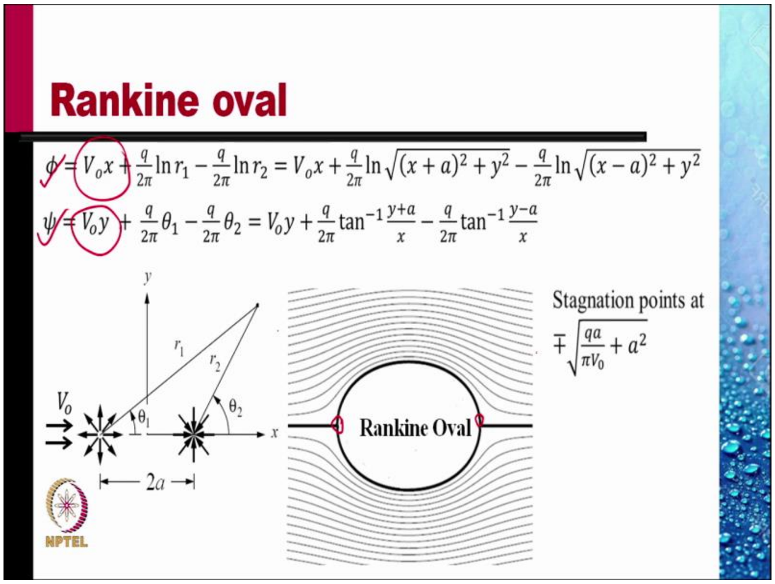


Let us next consider the flow due to a source and a sink pair. Imagine a source situated on the x-axis at $-a$ and a sink at $x = +a$, both of the same strength q. Positive q for the source, and minus q for the sink. Then let us write the potential of this combined flow at the point P at location (r, θ).

This is obtained simply as $\phi = \frac{q}{2\pi}\ln r_1 - \frac{q}{2\pi}\ln r_2$, r_1 is the distance of the source point from point P, and we use plus q because it is a source. And r_2 is a distance of the sink from point P, and we use minus q because it is a sink. This is the expression in the x-y coordinates. Similarly, we write the stream function. If you plot the streamlines, this is what we get. In this, the flow velocity would be directed like this.

(Refer Slide Time: 16:29)

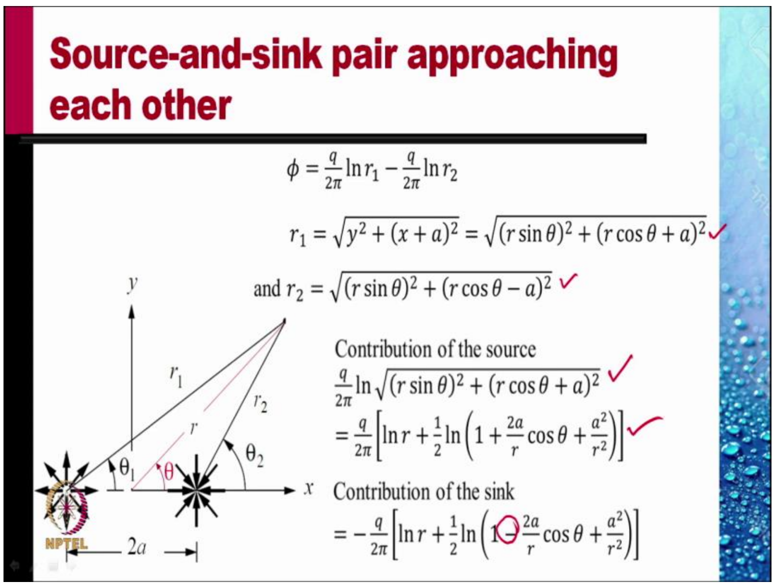


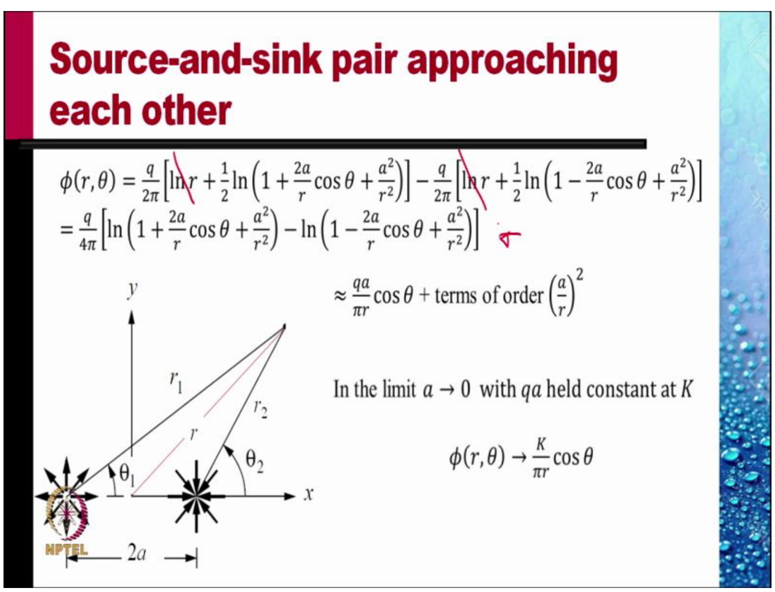


Let us superpose on this source and sink pair, a free stream velocity V_o, a uniform flow V_o in the x-direction. Then the potential function and the stream functions are like this. We have added the potential due to the free stream and the stream function due to the free stream in the expressions that we had in the last slide. If you plot the streamlines, the streamlines are seen like this. This streamline, which is bifurcating at two points must have stagnation points there. And so, we can replace the inside of this streamline with a solid body this shape is known as a Rankine oval. And so, this potential function and this stream function represent the flow past this Rankine oval. We can do all the algebra, we can find the locations of the streamlines, we can find out the pressure coefficient about this body. The stagnation points are at $\mp\sqrt{\frac{qa}{\pi V_0}+a^2}$.

This is obtained by differentiating ϕ with respect to x and with respect to y to obtain the expressions for the velocities u and v, and equating those two velocities as 0. We will obtain y is equal to 0 for both, the stagnation points in x given by this expression.

(Refer Slide Time: 19:01)



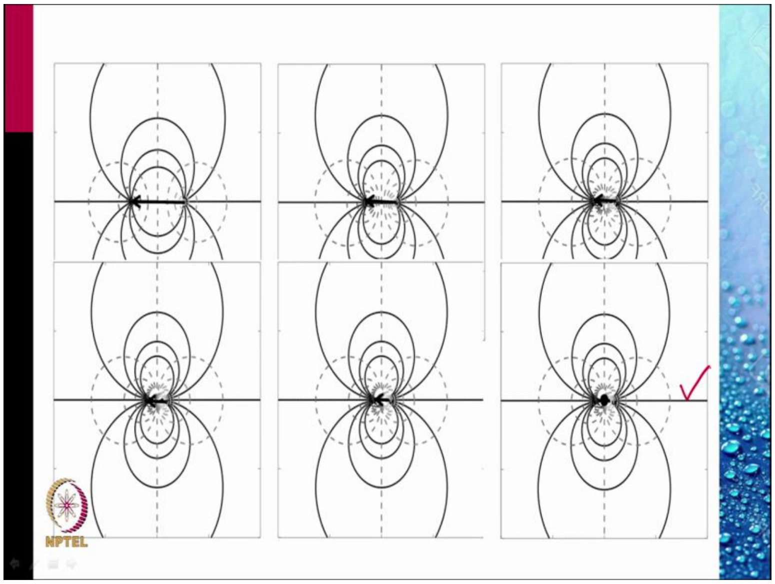


An interesting case occurs when the source and sink pair approach each other. In the same picture as before, the velocity potential ϕ is given by this expression, the r_1 is $\sqrt{y^2 + (x + a)^2}$, because now the distance is measured from this point, and r_2 would, similarly, be $\sqrt{y^2 + (x - a)^2}$. And in r and θ coordinates the expressions are obtained like these.

So, the contribution of the source, which is $\frac{q}{2\pi} \ln r_1$ becomes this, and from the radical side, we take r^2 out which becomes r, and then inside it is $\sin^2 \theta + \cos^2 \theta$ plus a^2 plus $2ar \cos \theta$. And this can be simplified as this. And similarly, the contribution of the sink to the potential function can be written down, and it is similar to that of the source, except for this sign change.

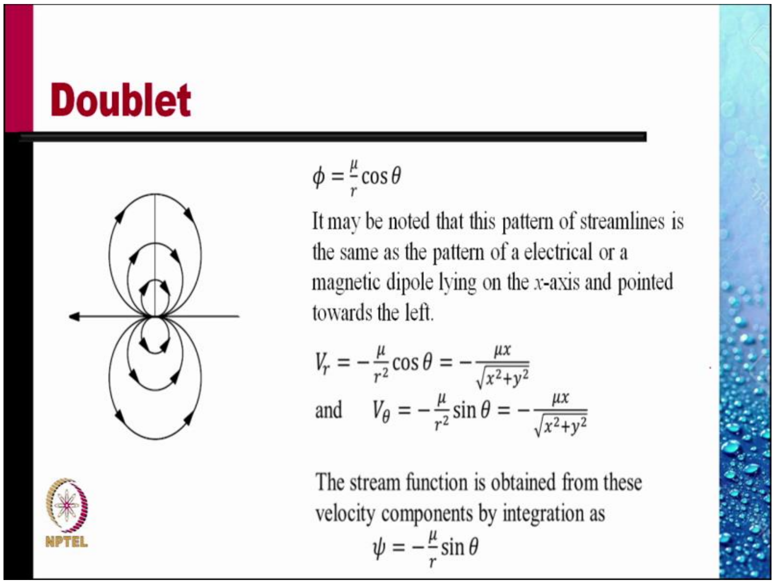
The two together give this, and if we look carefully, this $\ln r$ cancels with this $\ln r$, and we have this is the result, which is, approximately, $\frac{qa}{\pi r} \cos \theta$ plus terms of order $\left(\frac{a}{r}\right)^2$. So, if a is small compared to r, this approaches $\frac{qa}{\pi r} \cos \theta$. In the limit, as a tends to 0 with q a held constant at a value K, the potential function phi at r and theta becomes $\frac{K}{\pi r} \cos \theta$.

(Refer Slide Time: 21:52)



In this slide, I have shown six different configurations in which the value of K that is q times a is held constant, but, the value of q is increasing and the value of a is decreasing. And this is the final shape when a has become is equal to 0 with q a held constant at K.

(Refer Slide Time: 22:31)



This flow is known as the doublet flow and the potential for this is simply $\frac{\mu}{r}\cos\theta$. It may be noted that this pattern of stream lines is the same as the pattern of electrical or a magnetic dipole lying on the x-axis and pointing towards the left. So, a doublet is like a dipole, a positive pole, source, and a negative pole, the sink.

The r component of velocity is obtained as $V_r = -\frac{\mu}{r^2}\cos\theta$, and the theta component of velocity is obtained as $-\frac{\mu}{r^2}\sin\theta$. The stream function obtained from these velocity components by integration is $-\frac{\mu}{r}\sin\theta$. Now, we are interested in doublet because if we superimpose on this, a uniform flow we get the flow past a circular cylinder.