Fluid Mechanics and its Application
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Lecture: 24A
Some Simple 2-D Potential Flows

Let us next study some simple two-dimensional potential flows that could be used to construct more complex flows.

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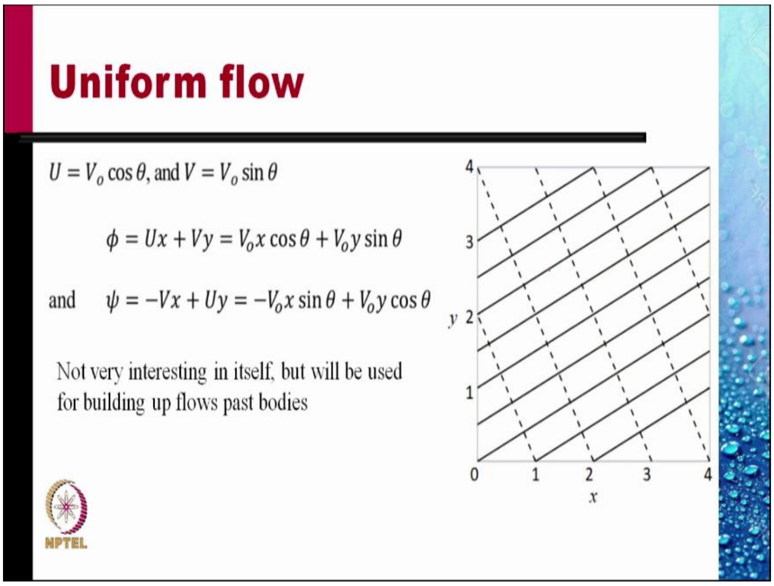

The first flow we consider is a uniform flow where the velocity at every point is the same V_o, and the velocity vector is inclined at an angle θ to the x-axis. So, that the velocity in the x-direction V_x is $V_o \cos\theta$, and we give it a symbol capital U, and the velocity V_y in the y direction is $V_o \sin\theta$ which is given a symbol V.

Then ϕ, the velocity potential is simply $Ux + Vy$, that is $V_o x \cos\theta + V_o y \sin\theta$. And the corresponding stream function is $-V_o x \sin\theta + V_o y \cos\theta$.

This is the flow net. The solid lines represent the streamlines the lines of constant ψ, and the broken lines are the potential lines, the lines are constant ϕ, constant velocity potentials. This is not very interesting in itself, but will be used for building up flows past bodies in 2-dimensions.

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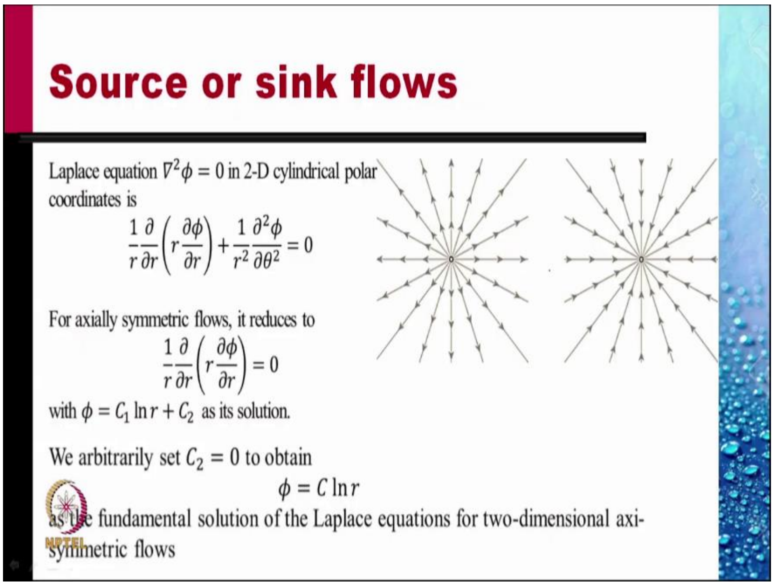

The next flow we take up is the source or the sink flow. Laplace equation $\nabla^2\phi = 0$ in two-dimensional cylindrical polar coordinates is $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}=0$. For axially symmetrical flows, the derivative with respect to θ are 0 and so, this equation reduces to $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right)=0$, which is readily integrated to give a solution $\phi = C_1 \ln r + C_2$, where C_1 and C_2 are constants.

Just like in stream functions, in velocity potential we can add an arbitrary constant to any of the velocity potentials and stream functions, because it is only the difference of the stream functions or the gradient of velocity potential that matter. So, we can set C_2 as 0, and then, the velocity potential is constant times ln r. This is the fundamental solution of Laplace equation for two-dimensional axi-symmetric flows.

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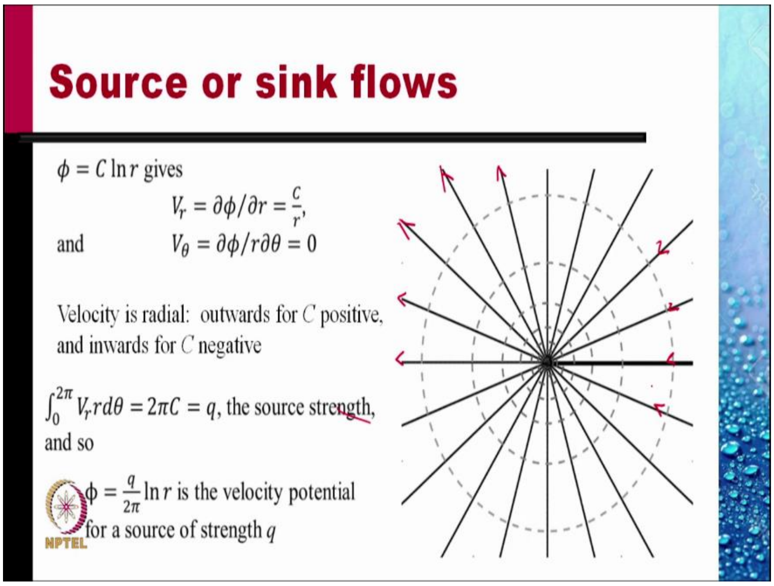

Now, this potential function gives you the r velocity component as $V_r = \partial\phi/\partial r = \frac{C}{r}$, varying inversely as radius, and $V_\theta = \partial\phi/r\partial\theta = 0$. So, velocity is radial, outwards for C positive, and inwards for C negative. So, the velocity vectors are like these. For source flow when C is positive and inwards if C is negative.

If we find out the total flow, total volume flow, crossing a circle of radius r, we can obtain it by integrating the r component of velocity multiplied by $rd\theta$over θ from 0 to 2π, and we get this value as $2\pi C$. We can treat this total volume flowing out as the source strength q, such that C becomes $\frac{q}{2\pi}$, and the velocity potential phi can be given as $\frac{q}{2\pi}\ln r$ as the velocity potential for a source of strength q per unit depth for two-dimensional flows.

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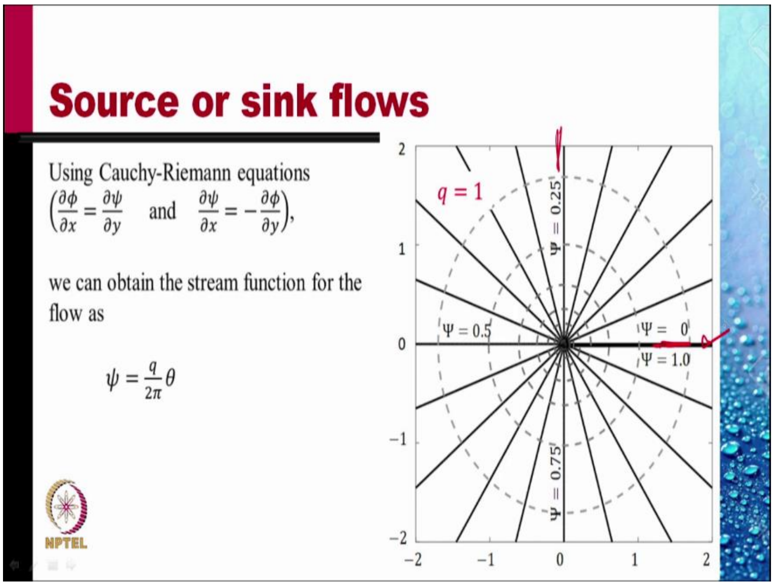

using Cauchy-Riemann conditions, that is $\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$ and $\frac{\partial \psi}{\partial x} = -\frac{\partial \phi}{\partial y}$, we can obtain the stream function for the flow as $\psi = \frac{q}{2\pi}\theta$. These are the streamlines that are plotted for q is equal to 1. That is for unit source strength.

This line here is for ψ is equal to 0. We go around we reach here, this is for ϕ is equal to 0.25, and we come back, we go the whole circle. We have this line which represents ψ is equal to 0 as well as ψ is equal to 1.0. These are the streamlines for the source flow, velocity outwards for a source, and the velocity inward for a sink, when the value of q would be minus 1.

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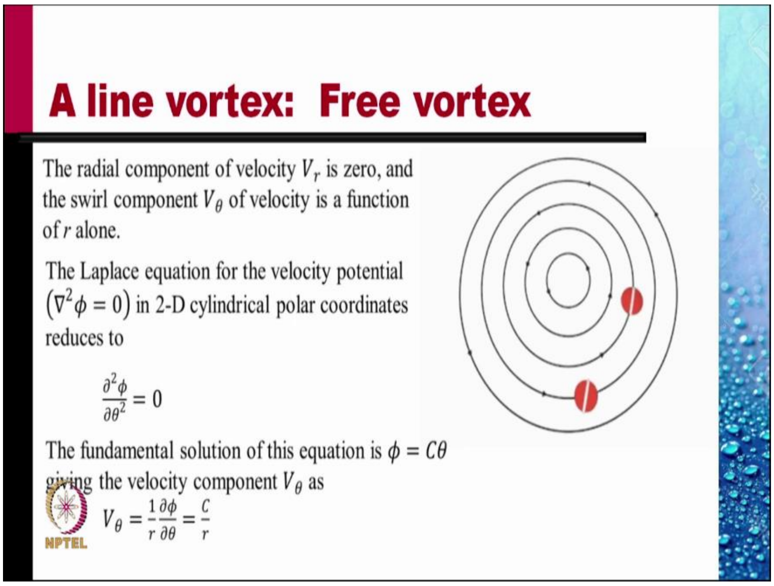

Let us next do what is called a line vortex, or a free vortex. We have seen earlier that free vortex is the name given to a swirling flow in which there is no rotation of the fluid elements. Two particles are shown with same orientation throughout the motion. The particles are not rotating about their own axis, but the different velocities in the two circles, both the angular velocity as well as the linear velocity. The radial component of velocity $V_r = 0$ in this case, and the swirl component V_θ is a function of r alone.

So, swirl component is not a function of θ. The Laplace equation for the velocity potential in 2-d cylindrical polar coordinates reduces $\frac{\partial^2\phi}{\partial\theta^2} = 0$. And the fundamental solution is $\phi = C\theta$, up to an arbitrary constant, and which gives the velocity component $V_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = \frac{C}{r}$. So, this swirl component of velocity varies inversely as r. Fastest is at the centre and slowest outside.

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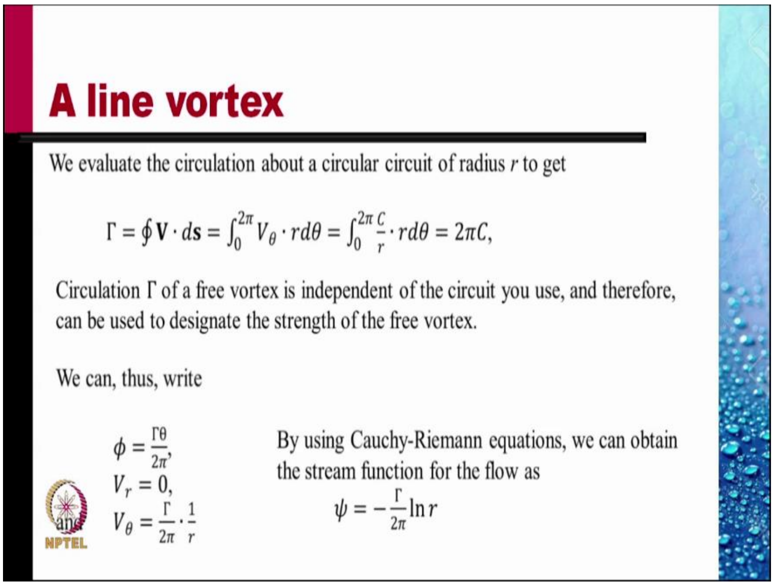

We evaluate the circulation about a circular circuit of radius r in this flow, and we get the circulation is equal to twice pi C, independent of the radius of the circuit. We have chosen circular circuit of radius r and find out that r cancels out, and the circulation gamma is obtained as independent of $2\pi C$.

Therefore, Γ can be used as a strength of the free vortex. So, that in the expression of the velocity potential that we have $C\theta$, we replace C by $\frac{\Gamma}{2\pi}$, and if we do so, the velocity potential becomes $\frac{\Gamma\theta}{2\pi}$, the radial component of velocity is 0, the swirl component velocity is $\frac{\Gamma}{2\pi} \cdot \frac{1}{r}$, C by r. By using Cauchy Riemann equations, we can obtain the stream function, and stream function $\psi = -\frac{\Gamma}{2\pi} \ln r$.

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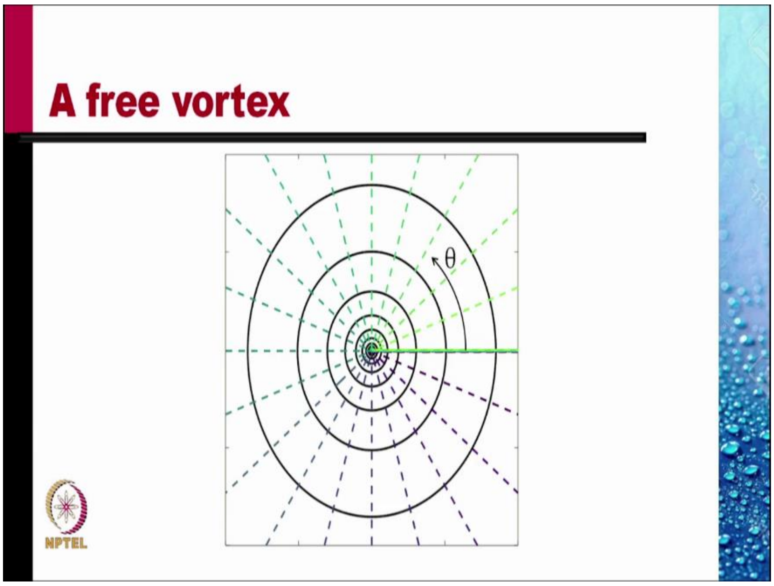

So, these are the streamlines and potential lines for a free vortex. The solid lines are the streamlines. The velocity tangent to the streamlines, counter clockwise for positive value of Γ, and clockwise for the negative values of Γ. The radial lines are the equipotential lines or the potential lines.

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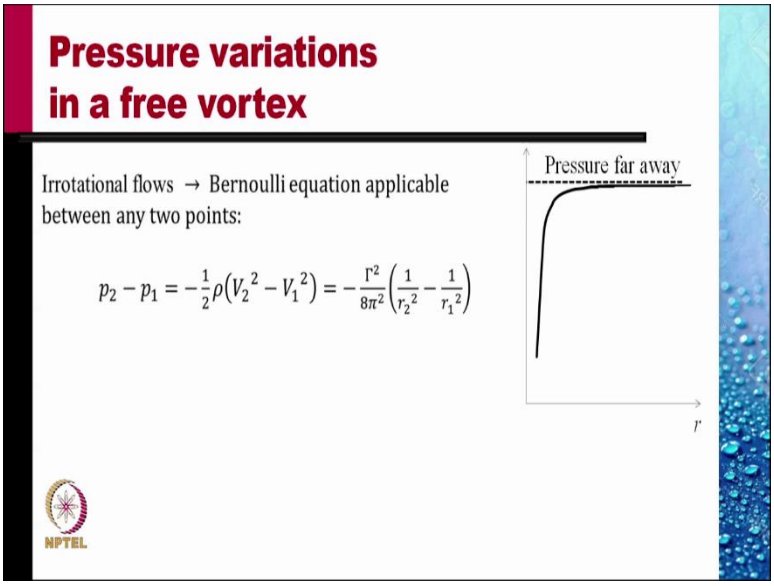

Now, we have seen earlier that in irrotational flows, the Bernoulli equation is applicable between any 2 points in the flow field. And therefore, for a free vortex if we take a point 1 at radius r_1 and a point 2 at radius r_2, then the pressure difference p_2 minus p_1 is obtained as

$$p_2 - p_1 = -\frac{1}{2}\rho\left(V_2^2 - V_1^2\right) = -\frac{\Gamma^2}{8\pi^2}\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right).$$

If r_2 is less than r_1, then this quantity is negative. That means, pressure p_2 at the inner point is much less than pressure at the outer point. This is the shape of the pressure distribution. Pressure far away, at r tending to infinity, is shown by the broken line, and then the pressure decreases as r decreases.

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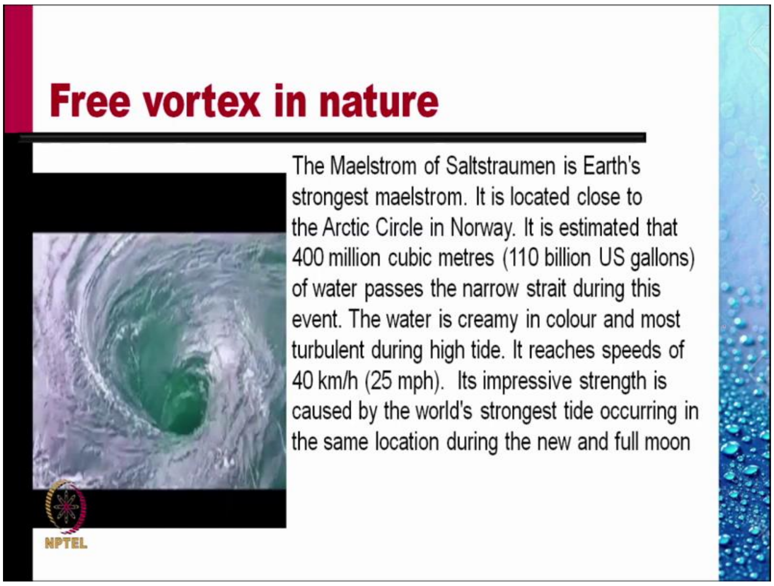

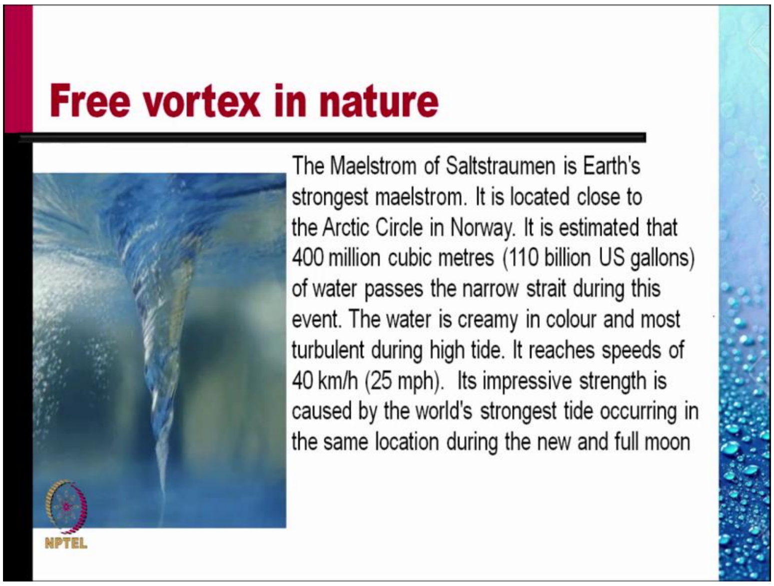

The vortices are very common in nature. They are found in oceans as whirlpools, in the atmosphere the tornadoes. They are huge vertices in the ocean, one of the largest one, is located close to the Arctic Circle in Norway. It is estimated that 400 million cubic meters of water passes in a narrow state during this event. This occurs regularly because of the lower pressure towards the centre of this vortex, the water level decreases. So, it produces a kind of a hole.

This is an underwater view of the same vortex. What you see here is an air cone that seems to be pointing downwards towards the ocean floor.

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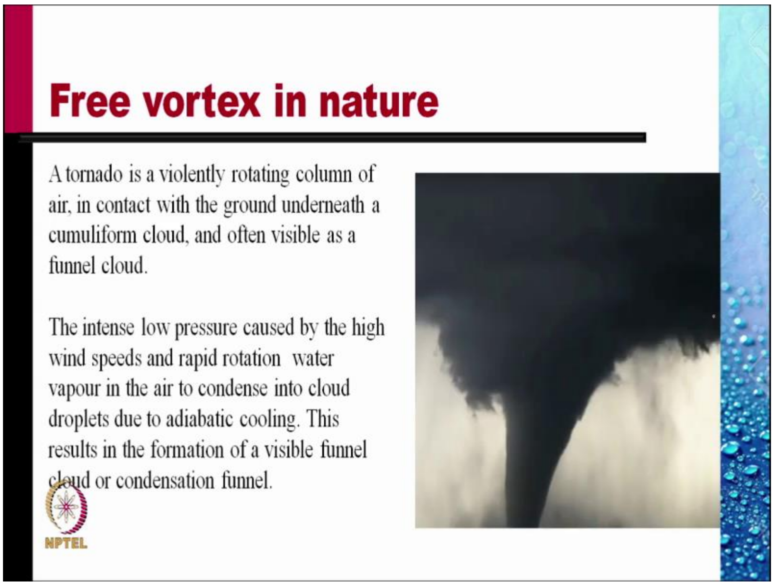

In the atmosphere, these whirlpools appear as tornadoes. A tornado is a voluntary rotating column of air in contact with the ground underneath a cumuliform cloud, and is often visible as a funnel cloud. The pressures are very low in this funnel, and so, it sucks the debris, cars and human beings up. The intense low pressure caused by the high speed wind and the rapid rotation, the water vapour in the air condenses into cloud droplets due to adiabatic cooling. This results in the formation of a visible funnel cloud or a condensation funnel.

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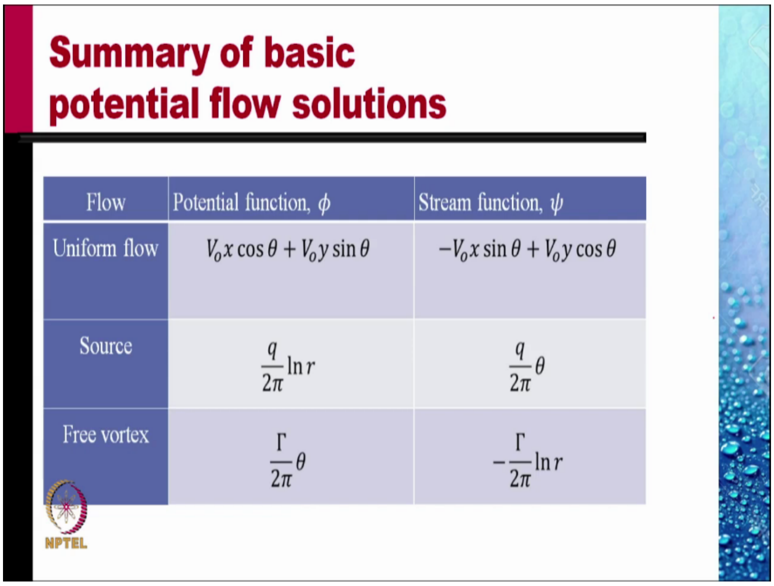

So, let us summarize the three basic potential flow solutions that we studied so far. A uniform flow, a source flow, and a free vortex. We will build up the solutions of more complex flows from these basic flows in the next lectures.

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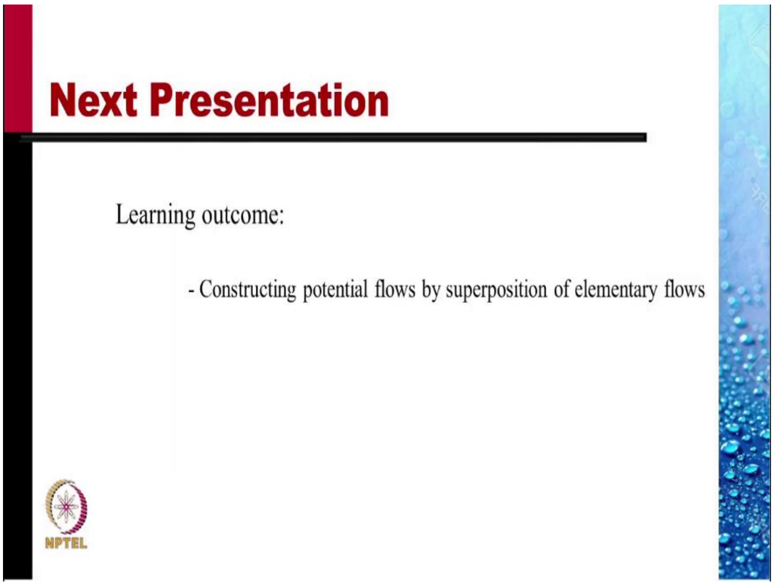

Thank you.