Fluid Mechanics and its Applications
Professor Vijay Gupta
Department of Chemical Engineering
Sharda University
Indian Institute of Technology, Delhi
Lecture: 3
Fluid Statics and Pressure

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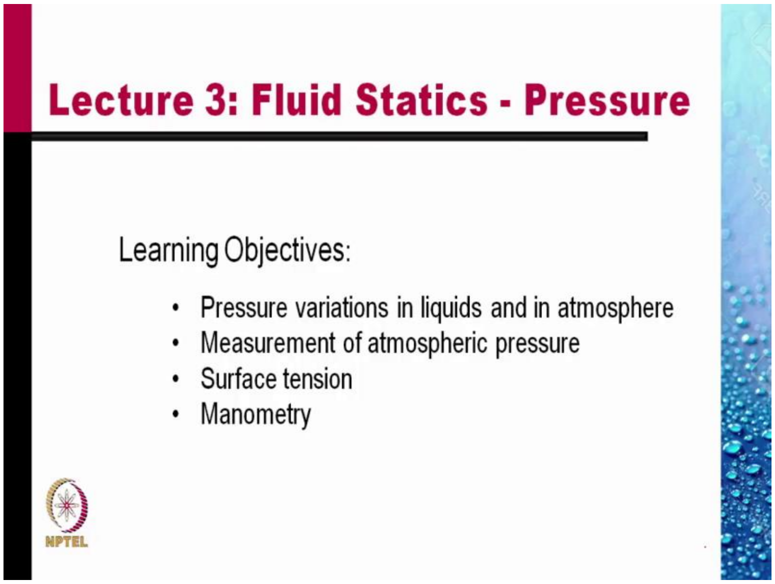

Welcome back. Today we will discuss fluid statics and how the pressure within a fluid changes.

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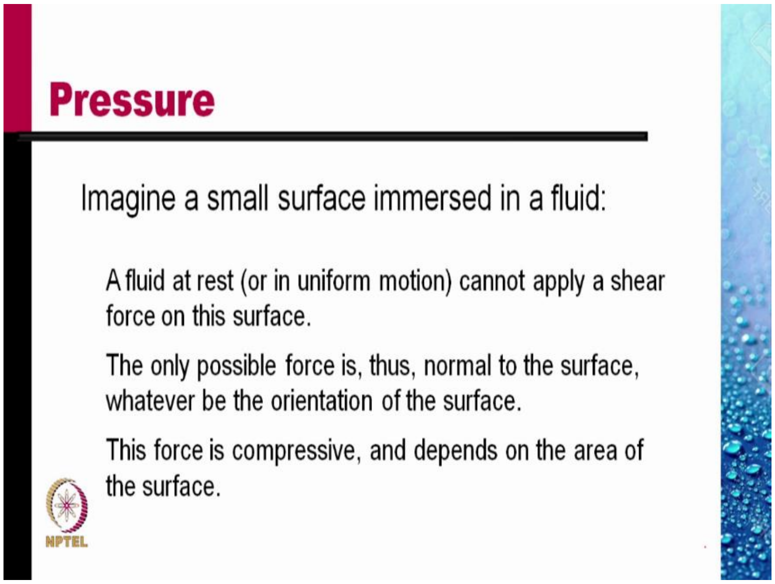

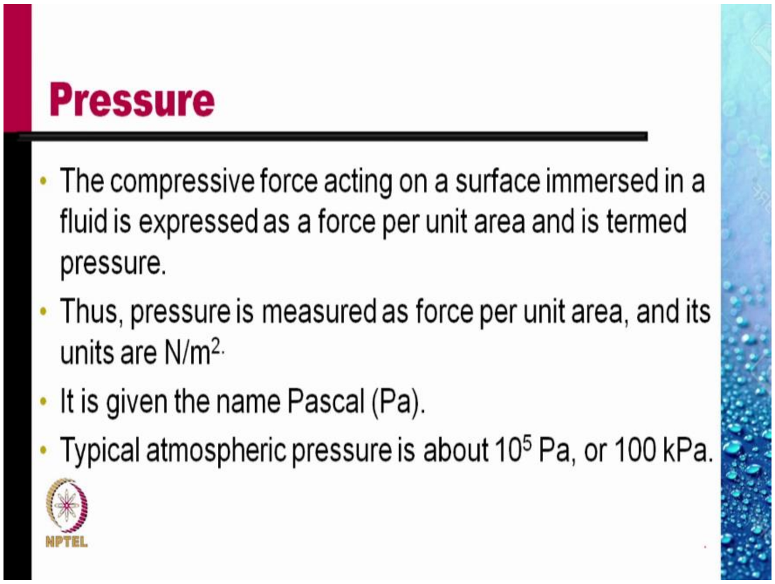

We imagine a small surface immersed in a fluid. We have seen in the first lecture that the fluid at rest or in uniform motion cannot apply a shear force on a surface. The only possible force is, thus, normal to the surface, whatever be the orientation of the surface within the fluid. This force is compressive, and depends upon the area of the surface. This force is termed as pressure. The pressure is measured as force per unit area, and its units are N/m^2. This unit is given the name Pascal, and abbreviated as Pa. The typical atmospheric pressure on earth is about 10^5 Pascal, or 100 kPa.

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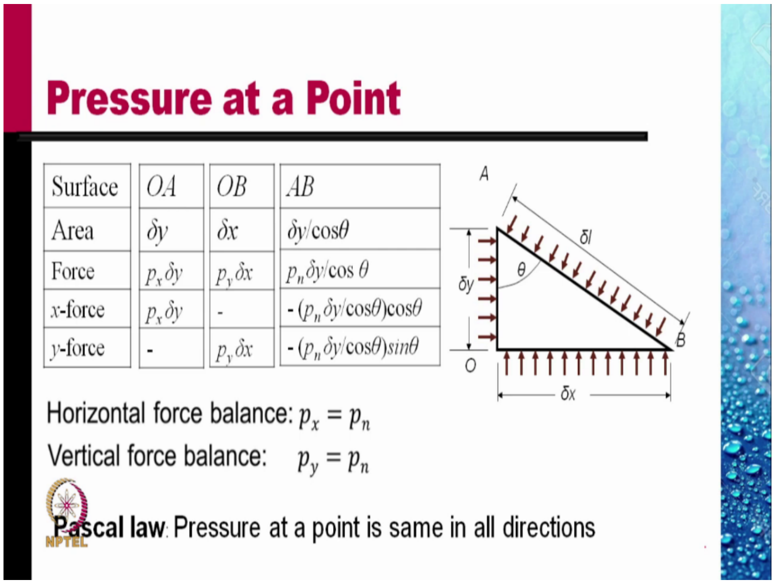

Surface	OA	OB	AB
Area	δy	δx	$\delta y/\cos\theta$
Force	$p_x \delta y$	$p_y \delta x$	$p_n \delta y/\cos\theta$
x-force	$p_x \delta y$	-	$-(p_n \delta y/\cos\theta)\cos\theta$
y-force	-	$p_y \delta x$	$-(p_n \delta y/\cos\theta) sin\theta$

Horizontal force balance: $p_x = p_n$

Vertical force balance: $p_y = p_n$

Pascal law: Pressure at a point is same in all directions

Let us consider the concept of pressure at a point. Imagine a small triangular element as shown. This is a fluid element. We draw a free body of this element A O B. On this element, on this vertical surface AO, the pressure forces are acting horizontally all over this area. On the surface OB, the pressure forces are vertical, and on surface AB they are normal to the surface AB.

Let us consider the equilibrium of this element AOB of the fluid. This table lists, for the three surfaces, the areas: δx, δy, and $\delta y/\cos\theta$ for the inclined surface AB. It could also be written as $\delta x/\sin\theta$. If the pressure on the surface AO is p_x horizontally, and on surface OB is p_y vertically, and on surface AB it is p_n normal to the surface, then the x component of forces on the three surfaces are $p_x\delta y$, zero, and $-\left(\frac{p_n\delta y}{\cos\theta}\right)\cos\theta$, which is the total force multiplied by $\cos\theta$, the horizontal component. Similarly, the y component of forces on the three surfaces are 0, $p_y\delta x$, and $-\left(\frac{p_n\delta y}{\cos\theta}\right)\sin\theta$, as shown.

If we write the equilibrium equation for this: sum the x-direction forces equated to 0, and sum the y-direction forces equated to 0, we get from the horizontal force balance, p_x is equal to p_n. And vertical force balance p_y is equal to p_n. Thus, pressure in the x, the y, and in the n directions are the same. Since the n direction was taken arbitrarily, the angle theta was not fixed first, this works for all angles theta. We can state that pressure at a point is same in all directions. This is known as Pascal's law. This is a very useful result and we will use throughout this chapter.

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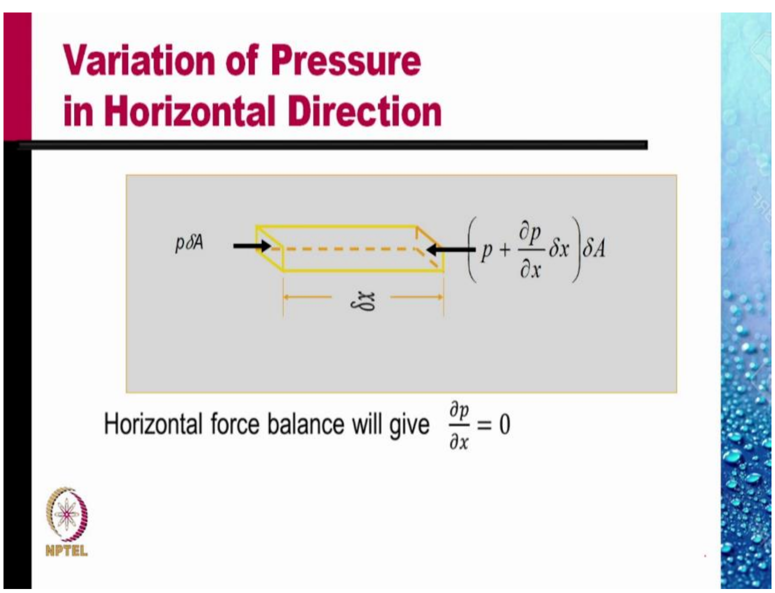

Again, let us consider a fluid element shown by yellow lines within a fluid. This is of length δx. On the left end of this, the horizontal force is $p\delta A$, where δA is the cross sectional area of this fluid element. On the right hand side, by Taylor's expansion, the pressure becomes $p + \frac{\partial p}{\partial x}\delta x$ and the total force should be that multiplied by δA.

We have shown here only the forces in x-direction. Summing these forces and equating them to 0, because the fluid is in equilibrium, we get $\frac{\partial p}{\partial x} = 0$. If we did the same thing in the y-direction, the direction normal to the screen, we would get $\frac{\partial p}{\partial x} = 0$. So, there are no variations a pressure in the horizontal directions within a fluid. The pressure at two points which are on the same horizontal level must be same. Let us see what is the variation the vertical direction.

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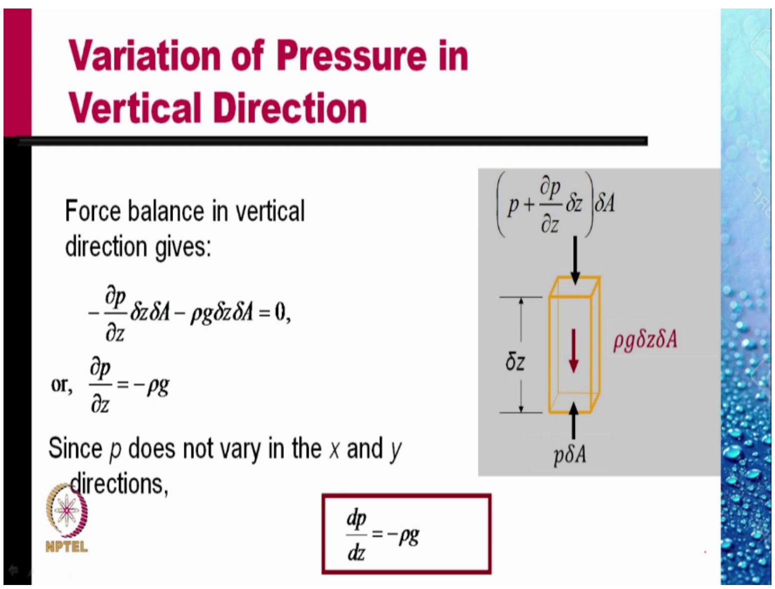

Let us consider this element in a stationary fluid in the vertical direction. Let the length of this element be delta z, and the two pressure forces are as shown. On the lower face, it is p delta A upwards, and on the upper face it is downwards, and we use Taylor's expansion to increase the pressure force.

But in the vertical direction, there is one more force on the fluid element, that is, the weight of the fluid element itself. And that force is the volume of the fluid element $\delta A\delta z \times \rho$ gives you the mass of the fluid element, and g, the acceleration due to gravity. So, that the total force on this fluid element because of weight in the vertical direction is $\rho g\delta A\delta z$.

If the fluid is stationary, then the vertical force balance gives us $\frac{\partial p}{\partial z} = -\rho g$, that is, the pressure variation in the vertical direction is like $-\rho g$, where ρ is the density of the fluid, and g, the acceleration due to gravity. Even when the density of the fluid is not constant, the same equation is applicable because we have taken a very small element. So, $\frac{\partial p}{\partial z} = -\rho g$ where ρ is the density at the location we are calculating the pressure gradient at. Since p does not vary in the x and y directions, we can write $\frac{dp}{dz} = -\rho g$. Z is the only direction in which there is a pressure gradient.

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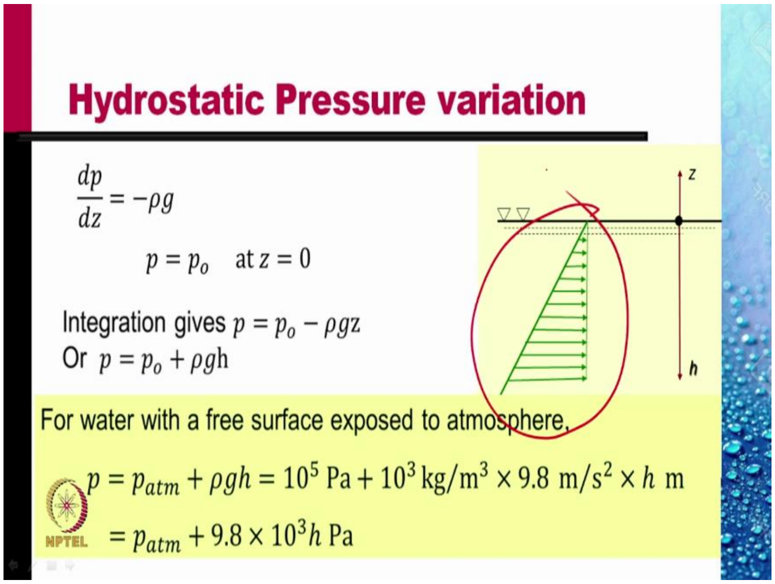

Let us now look at the variation of pressure in a fluid such as water, where the density is assumed constant throughout. This is obtained by simply integrating the equation that we obtained in the last slide, $\frac{dp}{dz} = -\rho g$. And we integrate this with the condition that p is equal to p_o at $z = 0$.

The free surface of water, or the free surface of the liquid. Simple integration gives $p = p_o - \rho gz$, where z is the distance measured from the location where the pressure is p_o, in the vertically upward direction. Usually, in the case of liquids, it is convenient to measure the depth, not the height, from the surface. And the depth from the surface is denoted here as h, and so that we can write $p = p_o\rho gh$.

For water with free surface exposed to atmosphere, $p_o = p_{atm}$, which is about 10^5 Pascal, and we plug in the value of ρ, which is 10^3 kg/m^3, and of g. Then the equation becomes $p = p_{atm} + 9.8 \times 10^3 \times h$ in Pascal. This is the pressure distribution shown here, which is linear with depth. This all, you must be familiar with, from your high school.

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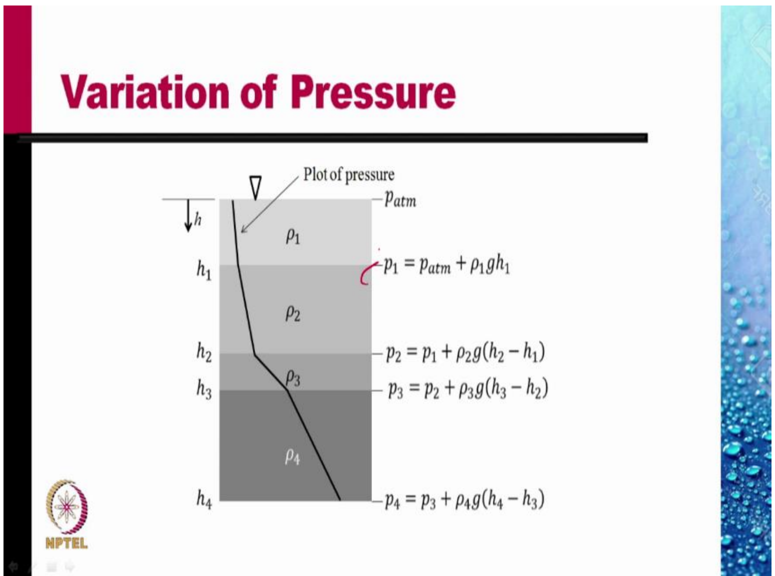

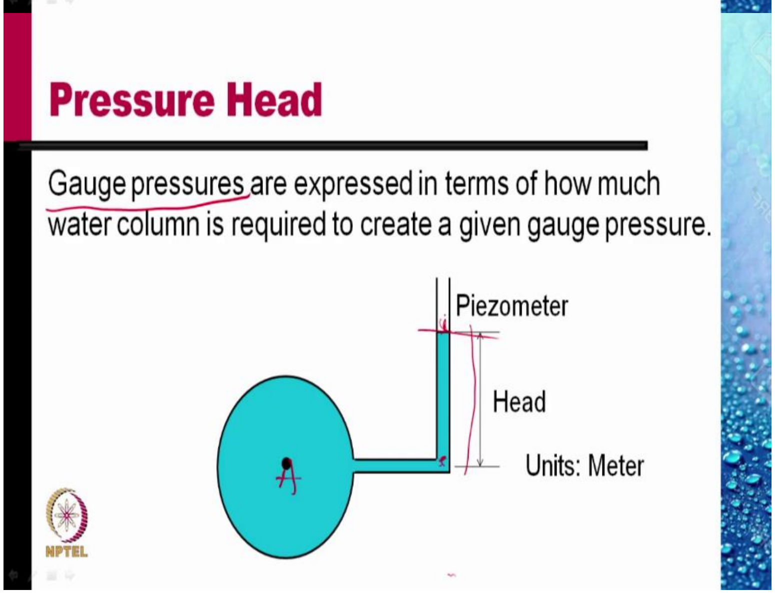

If we have combination of fluids arranged in stratified layers with various densities $\rho_1, \rho_2, \rho_3, and\ \rho_4$, then the plot of pressure variation would be as shown. At the top-most layer, it will start with p_{atm}. Then at the interface with a second layer, a height h_1 down, it would become $p_{atm} + \rho_1 g h_1$.

Then we go down further in a layer with the density ρ_2 up to depth h_2. The p_2 there would be p_1, the pressure at this interface plusρ_2, the density of the second fluid, times g times h_2 minus h_1, the depth of the second layer. Similarly, the third and the fourth layer.

One common method of specifying pressures is called the pressure head. We use the term gauge pressure as the pressure above the atmospheric pressure. The name gauge pressure comes from the fact that ordinary pressure gauges measure pressure above the atmosphere. The gauge pressures are usually expressed in terms of how much water column is required to create a given gauge pressure.

Consider this bulb containing water, and if the water rises into this tube, attached tube which is called a Piezometer, up to this height, then we call that this is the head of pressure within this bulb, the pressure at point A is the same as pressure at this point. Why? Because these are at the same horizontal level. So, if we specify head, then we know that the pressure is rho g h. While specifying the pressure as head, we have to specify what fluid. So, if the fluid is water, this is h meter of water: that is the head.

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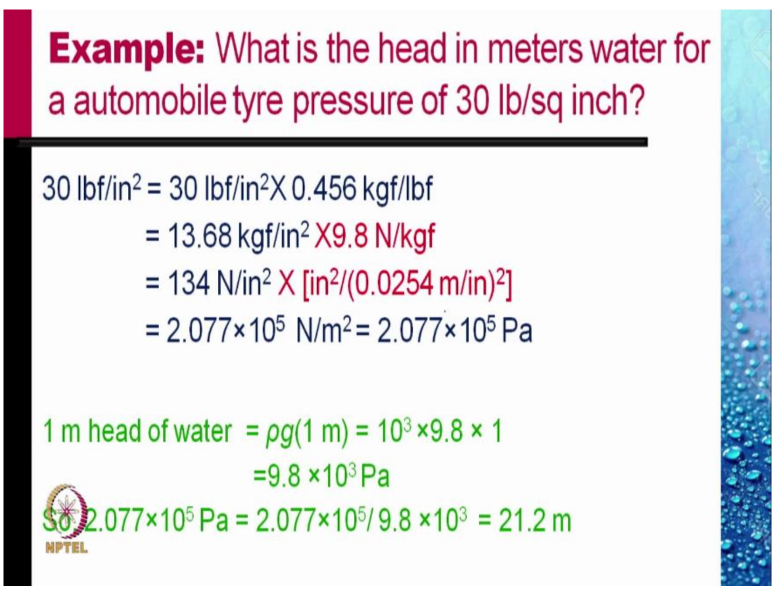

Let us do an example of conversion of units. What is a head in meters of water for an automobile tyre pressure of 30 pounds per square inch? We do the conversion: 30-psi is 30-lb_f /in^2 × 0.456-kg_f,/lb_f which gives 13.6-kg_f /per in^2.

We want to change the kilogram force into Newton. So, multiply this by 9.8 N/kg_f to get 134 N/in^2. And to convert /in^2 into /m^2, we multiply this by in^2 divided by 0.0254(/m/in). And if we do this, we get 2.077×10^5 N/m^2. And N/m^2 is termed as Pascal. So, this is 2.077×10^5 N/m^2, or 207.7 kilo Pascal, a more often used unit of pressure. Now, 1 meter head of water is ρ of water times g into 1 meter which gives you 9.8×10^5 Pascal.

So, that 2.077×10^5 Pa is 21.2 m of water. So, this is the head of water that will create the same pressure as 30 psi: 21.2 meters. Compare this to the atmospheric pressure, which is about 10 meters of water, a little less at 10 meters of water. In fact, about 9.8 meters of water is atmospheric pressure head. This 21.2 is above the atmospheric pressure. This is the gauge pressure.

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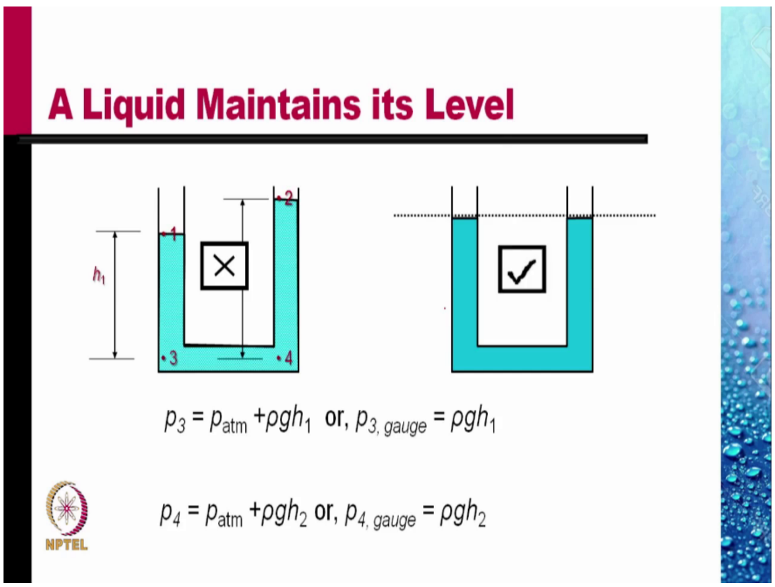

You must have seen, or you must have heard that a liquid maintains its level. If I have a U-tube in which I put water, then this arrangement where the level in limb 1 is different from that in limb 2 is not sustainable. How do we show it? We show it using the fact that we have already obtained: that the pressure at the horizontal level are same in a liquid.

Point 3: the pressure at point 3 is p_{atm}, which is the $p_1 + \rho g h_1$, using the variation in vertical direction, the hydrostatic pressure law, which we derived earlier. So, $p_{3,gauge}$, the pressure above atmosphere, is $\rho g h_1$. Pressure at point 4, similarly, is $\rho g h_2$. but point 3 and point 4 are at the same horizontal level. So, pressure at 3 must be the same as pressure at 4. If this is so, then h_1 must be equal to h_2; that is, the level in the two limbs must be equal for equilibrium. This does not work. So this is the correct configuration.

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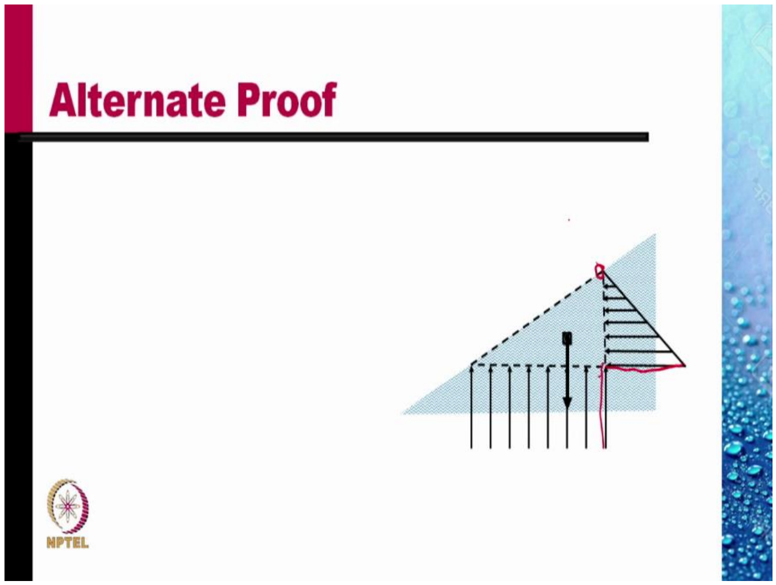

It is interesting to construct an alternate proof for the same thing. Suppose, a liquid does not maintain its level, and at some location in a liquid at rest, the surface is sloping as shown here. If the surface was sloping, we could construct a small element of the fluid of this triangular shape. This element, a small element, is also at rest.

What are the forces acting on this? The forces acting on this are: the pressure variations on the vertical face. These are horizontal forces. Forces on the bottom which are uniform, because the bottom is horizontal at the same level. Clearly, this magnitude of force must be the same as this magnitude of force.

We have shown forces above the atmosphere, the gauge forces, because we started with 0. So, the upper surface, the sloping surface exposed to atmosphere, so pressure is 0 everywhere. What are the other forces? Of course, the weight of this triangular fluid element. Now, we got our result. If the configuration of the fluid element was like this, how would the horizontal forces balance? There are forces in one direction only.

So, the horizontal forces would always be out of balance, if the configuration of the element is anything like this, i.e., if the configuration of element is wedge shaped. And therefore, the only valid configuration is when the surface of the liquid is horizontal, so that we cannot create a wedge shaped element.

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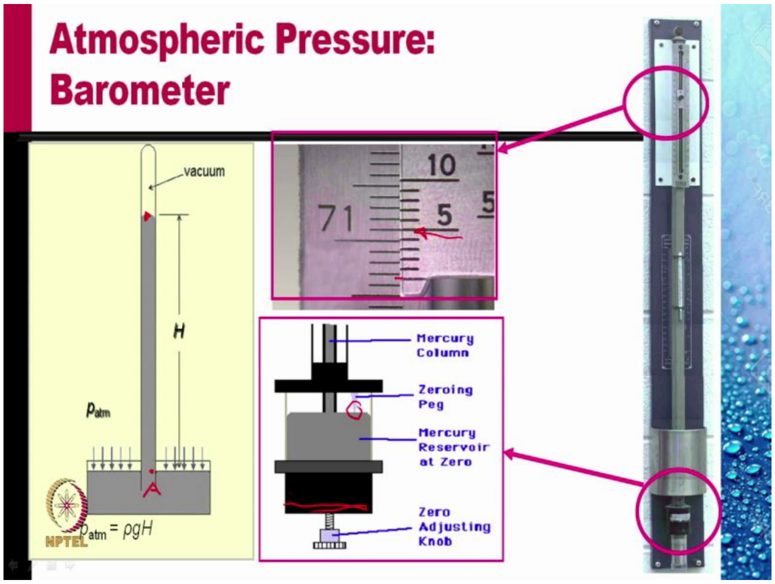

You do know about a barometer, a device to measure the pressure of the atmosphere. A long tube, about a meter long, closed at one end and open at the other, is filled with mercury. It's inserted into a cup of mercury. The level of the mercury decreases slightly, and then it comes to equilibrium, such that a column of height H is held up into this tube which is closed at the top end.

Clearly, this point A, which is at same level as the level of mercury in the open cup, should have the atmospheric pressure. So, this point is at atmospheric pressure. So, this point here should be at a pressure equal to $p_{atm} - \rho gH$, where ρ is the density of the mercury, g is the acceleration due to gravity, and H is the height of the column of mercury.

Now, the pressure at this point should be 0, because it is vacuum. Actually it will not be vacuum. It would be the vapor pressure of mercury at the given temperature. But that vapor pressure is quite low. So, we can neglect this, and this can be treated as the pressure 0, absolute 0, and therefore, the atmospheric pressure must be equal to ρgH, where ρ is the density of the mercury. So, if we know the value of H, we can measure the atmospheric pressure.

This shows a typical construction of a commercial barometer which are in use in the school laboratories. The lower end of this barometer consists of a cup in which mercury is filled up. The bottom of this cup is made of a flexible membrane. There is a screw, a zero-adjusting screw, which we turn to keep the level of mercury in the cup just touching the zeroing peg here.

This establishes the zero of the scale, because this tube that goes up there carries a scale with this, the zero of the scale coincides with the tip of this peg. So, the length of the column standing above the cup level now, when measured against this provided scale gives you the height H of the mercury column.

The top end carries a Vernier. And using the rule of reading a Vernier, we can read the pressure. In this case, it is this, and this is what is coinciding here. So, it is 70.65 centimeters of mercury is the pressure head of atmosphere. 70.6, and then 5, centimeters: 70.65 centimeters of mercury. You multiply it with the density of mercury, which is about 13,600 kilogram per meter cube, and g as 9.81, and we can get the pressure.

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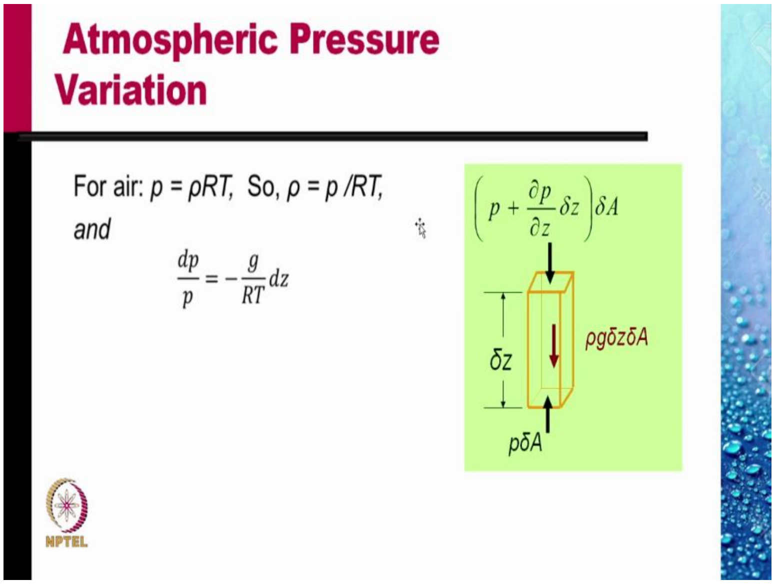

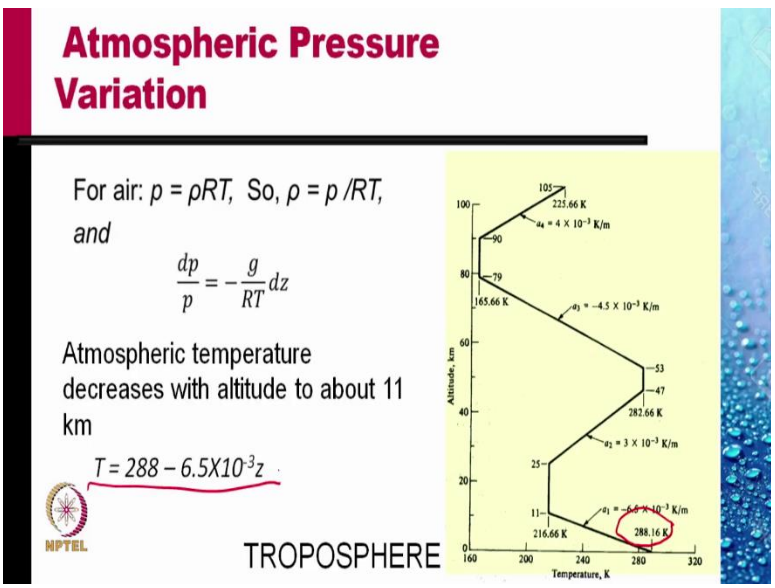

Let us now consider the variation of pressure in atmosphere. You know that as we go up in the atmosphere, the pressure decreases. So, how does the pressure change? We use the same equation: $\frac{dp}{dz} = -\rho g$, but ρ is variable here. The density depends upon the pressure. As the pressure decreases going up, the density too decreases.

So, we cannot integrate like we integrated for a liquid when we derived the hydrostatic pressure variation. So, here we will have to write or express the density by using the gas law. The perfect gas law says $p = \rho RT$. So, ρ is replaced by p/RT. So, the equation for the pressure variation becomes $\frac{dp}{p} = -gRTdz.$ z is measured upwards.

Now, this cannot be integrated easily because the temperature itself varies vertically. There are various reasons why the temperature varies and temperature varies in the atmosphere day to day, moment to moment, and location to location. But, for the purposes of easing the aviation industry, we use the concept of standard atmosphere that organizations of international civil aviation agencies have established: a standard atmosphere in which the temperature variations is seen to be this.

This curve of the temperature variation with the altitude in kilometers is derived from a very large number of measurements in the atmosphere, and people from various countries coming together and deciding that this temperature variation best represents the temperature variations in the atmosphere throughout the year.

In all locations, it starts with a temperature of 288.16 Kelvin at sea level. Then for 11 kilometers this temperature is decreasing at the rate of 6.5 ℃/km, or 6.5×10^{-6} K/m. At 11 kilometers height, the temperature is only 216.66 Kelvin, which is about -58 ℃.

This atmosphere up to 11 kilometers is called troposphere. This is the band of atmosphere in which most of the weather activity is confined. Above that, up to a level of 25 kilometers, the temperature remains constant. Then, the temperature starts increasing. The layer immediately above the troposphere is called the tropopause. Compare the height of 11 kilometers with the height of Mount Everest, which is only about 8 kilometers.

So, Troposphere is present there up to the height of Everest and beyond, a couple of kilometers beyond that. Most of the flights take place in the troposphere. We can use the equation here to track the variation of temperature with the altitude. Using this expression of temperature in the expression of dp/p above, with this temperature we can plug in there, and get the pressure variations within the atmosphere.

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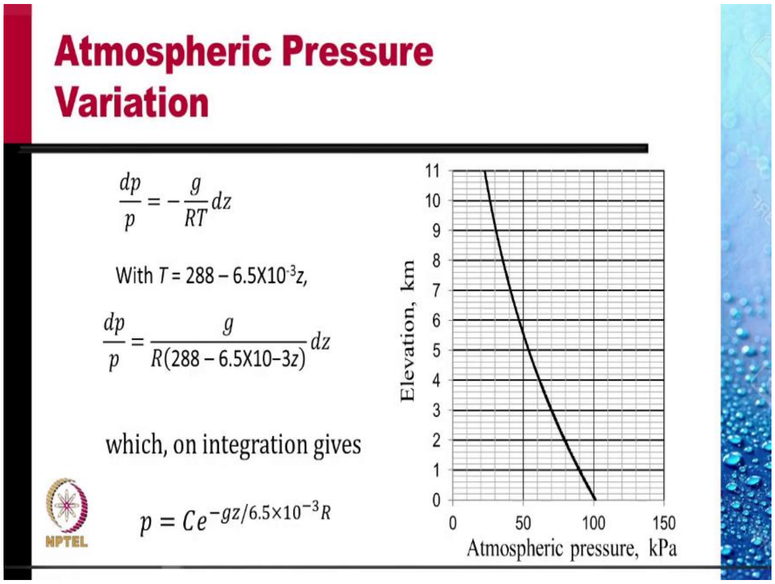

The pressure variations within the atmosphere for the first 11 kilometers is seen like this: about 101 kPa at the mean sea level, and about 22 kPa at the height of 11 km: it decreases by a factor of five.

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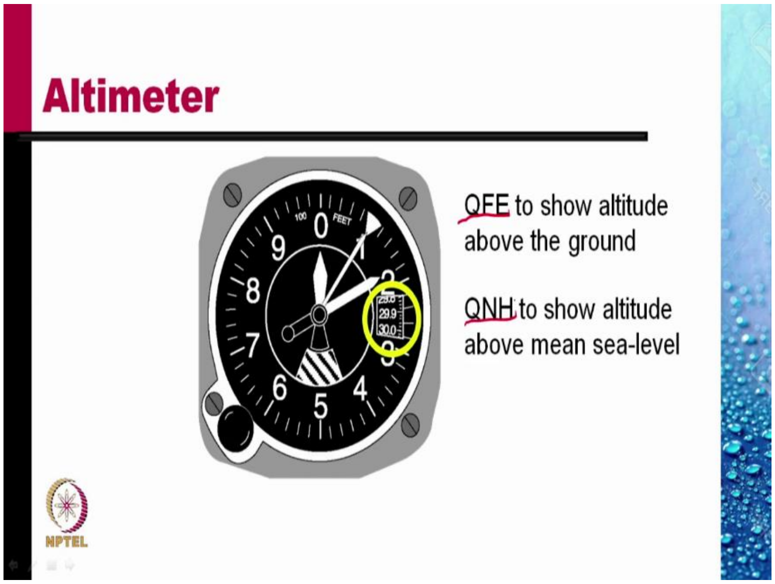

This variation of pressure with altitude is used an instrument called altimeter to measure the altitude of the aircrafts as they fly. This altimeter is nothing but a barometer. It measures the atmospheric pressure, but it is calibrated in altitude in meters, or altitude in feet, by using the equation that we obtained earlier. The three needles will tell you what is the altitude from the given datum.

While flying there are two kinds of altitude that we interested. One is when we are taking off and coming into land. We want to know how high we are above the airfield, because that is important for safe landing. Suppose, your airfield is at 500 meters above mean sea level. And if your altimeter was calibrated only to show the height above the mean sea level, it would read 600 meters when you are only 100 meters above the airfield. This could be quite confusing.

So, it is always advisable to ask the traffic controller the atmospheric pressure at the airfield. The pilot calls the control tower and says what is the pressure? and they ask -- the word used for this is QFE. QFE are the letters that indicate the altitude above the ground, above the air strip at which you are going to land.

So, the pilot asks the air control tower, what is the QFE? And if he gets a reply that it is 29.96, then he sets the reading in this window, the yellow window, 29.96 in inches of mercury. Now, this is an old picture where the pressures are shown in inches of mercury and the altitude in feet. Most altimeters today would show altitude in meters. And this window would show pressures in millimeters of mercury.

So, by changing, by turning a knob, they switch the reading within the window to 29.96, the QFE reported by the tower, and then this would read the altitude above the air field. On the other hand, when you are flying from one city to another over long distances in the airways in between, we set the window to QNH which is 29.92 inches of mercury, the pressure at mean sea level or 760 mm of mercury. Then your altimeter would be reading pressures, reading altitude above the mean sea level assuming that the atmosphere is standard.