Fluid Mechanics and Its Applications
Professor Vijay Gupta
Sharda University
Honorary Professor
Indian Institute of Technology, Delhi
Lecture 23
Inviscid Flows

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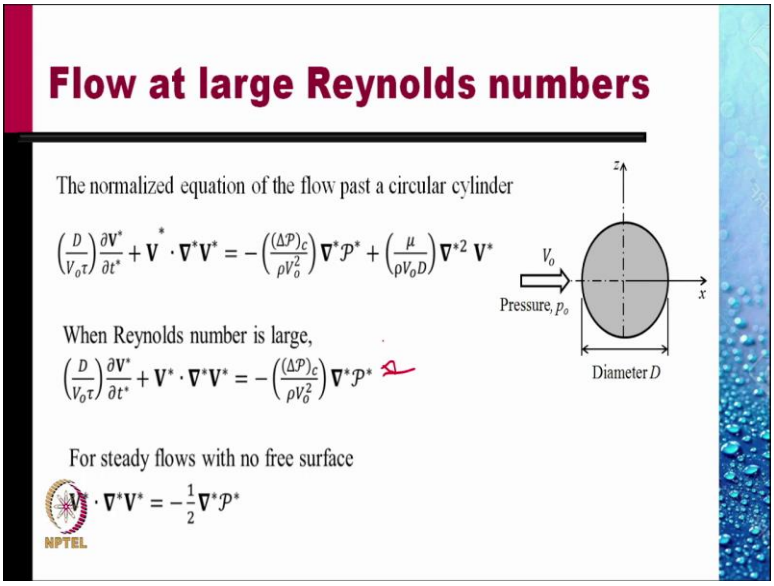

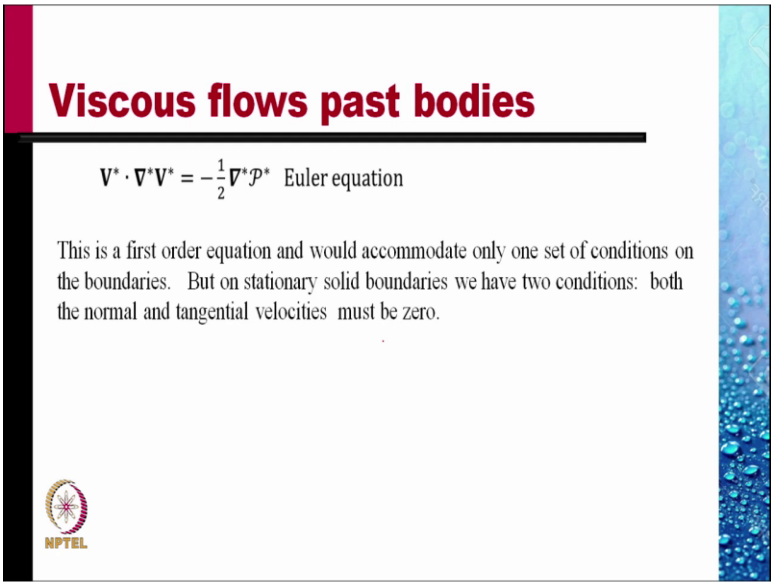

Welcome back.

In the last lecture we discussed the flows at flow at Reynolds number. Today we will deal with flows at large Reynolds number. We start with the normalized equation for the flow past a circular cylinder. Here each variable has been rendered dimensionless by normalizing it with respect to a characteristic quantity. When Reynolds number is large, the viscous terms become small and can be neglected; with this, this is the form of the governing equation. In this we have combined the gravity and the pressure terms to deal with a non-gravitational pressure.

For steady flow with no free surface, the equation reduces to $\boldsymbol{V}^* \cdot \boldsymbol{\nabla}^* \boldsymbol{V}^* = -\frac{1}{2} \boldsymbol{\nabla}^* \mathcal{P}^*$, where $(\Delta \mathcal{P})_c$ the characteristic pressure difference has been taken as $\frac{1}{2} \rho V_o^2$. This equation is known as the Euler equation. This is a first order equation and would accommodate only one set of conditions on the boundaries, but on stationary solid boundaries we have two conditions: both the normal and the tangential velocities must be zero.

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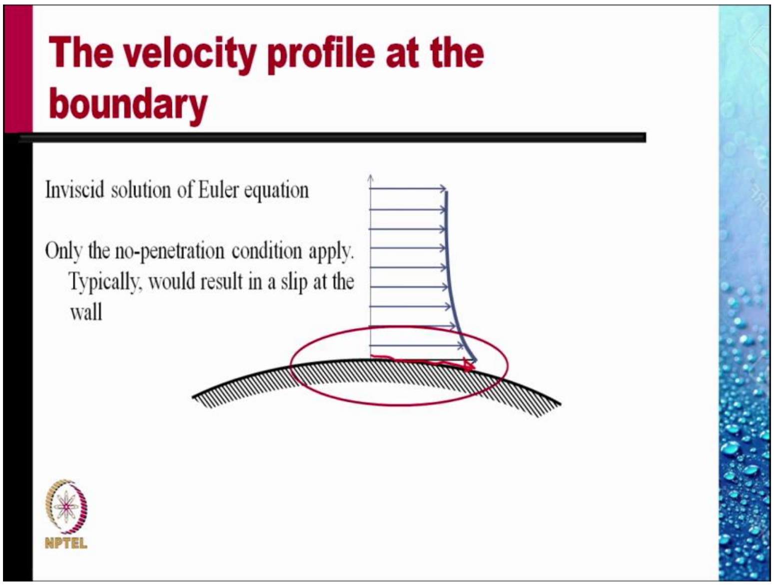

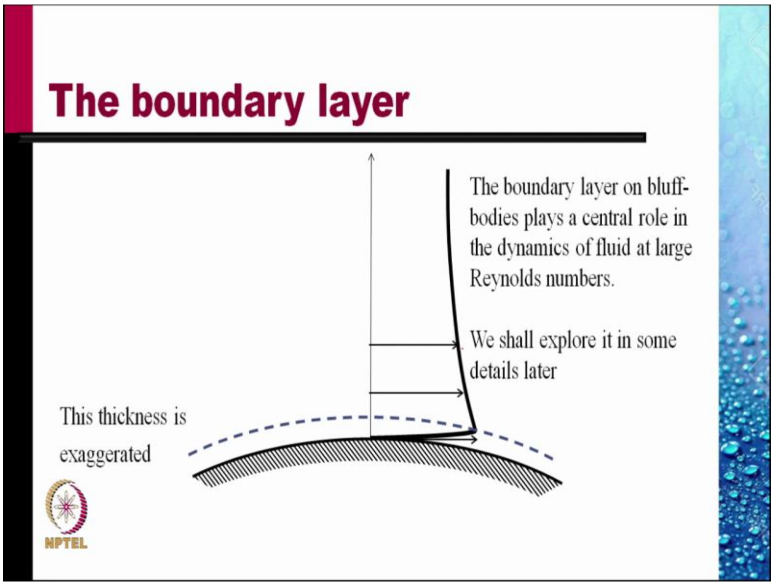

If we solve the Euler equation and obtain the inviscid solutions and apply the no penetration condition, that is, the normal component of the velocity at a solid stationary surface is zero, we would get a velocity profile that varies something like as shown. Clearly, this would result in a slip at the wall, the velocity at the wall is this, rather than zero. This is clearly unacceptable, and as we discussed a few lectures ago when we were discussing the process of approximations, that this difficulty is resolved by assuming a thin boundary layer near the surface.

In this thin region, the viscous effects are not negligible, and the no slip condition is applicable at the wall. We had shown that the velocity would rise very sharply from zero at the wall to a value very close to what the value would be in the inviscid flow at the wall. The two parts of the velocity profile would merge to give a smooth velocity profile.

The thickness that we showed here is exaggerated. The boundary layer thickness, we would learn later, is of the order of $1/\sqrt{\mathrm{Re}}$, so for a typical Reynolds number of 10^4 or 10^5, the boundary layer thickness is of the order of 10^{-2} of the characteristic dimension of the body. The boundary layer on a bluff body plays a central role in the dynamics of fluid at large Reynolds number. We will explore it in some details in a later lecture.

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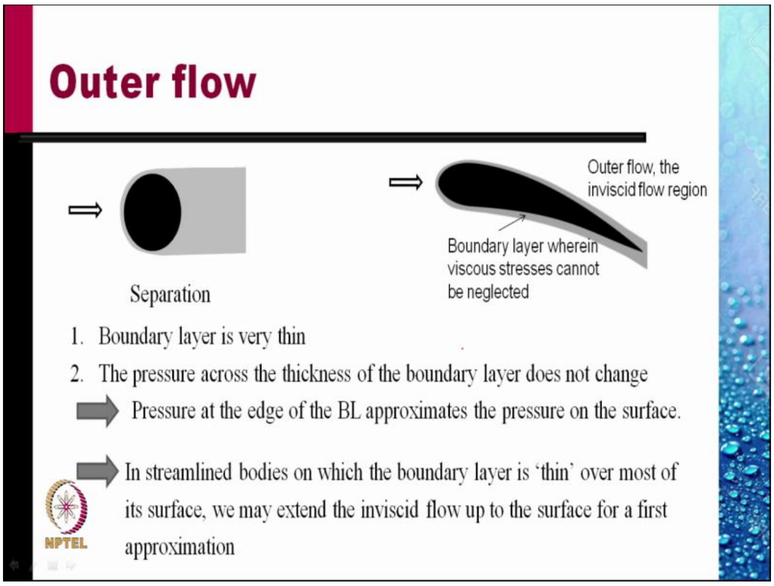

Let us consider the flow of a fluid at a very large Reynolds number. In that case the boundary layer in the front half of the body would be thin, but this boundary layer would separate near the shoulder or slightly ahead of the shoulder, and the flow behind the cylinder and in the wake of the body would be disturbed, would have viscosity playing important part.

So, separation is the culprit in this case. If however, we have a streamlined body like an aerofoil, the boundary layer develops in the same manner, but the separation is delayed, and so, the wake is much smaller. Most of the flow around the aerofoil is an inviscid flow region.

It is for this reason that the theory of aerodynamics is built largely as an inviscid flow in the first approximation.

There are a couple of reasons why this holds. One is the boundary layer is very thin. Second, as we shall show later, the pressure across the thickness of the boundary layer does not change, that is del p by del n, normal to the surface within the boundary layer is approximately zero. Therefore, the pressure at the edge of the boundary layer approximates the pressure that impresses on the surface itself.

Since the boundary layer is thin over most of a surface in a streamlined body, we may extend the inviscid flow right up to the surface as a first approximation. That is why in aerodynamics we neglect the boundary layer completely, solve for the inviscid flow right up to the surface of the aerofoil, calculate the pressure from the inviscid flow, and assume that that is the actual pressure distribution on the surface of the airfoil.

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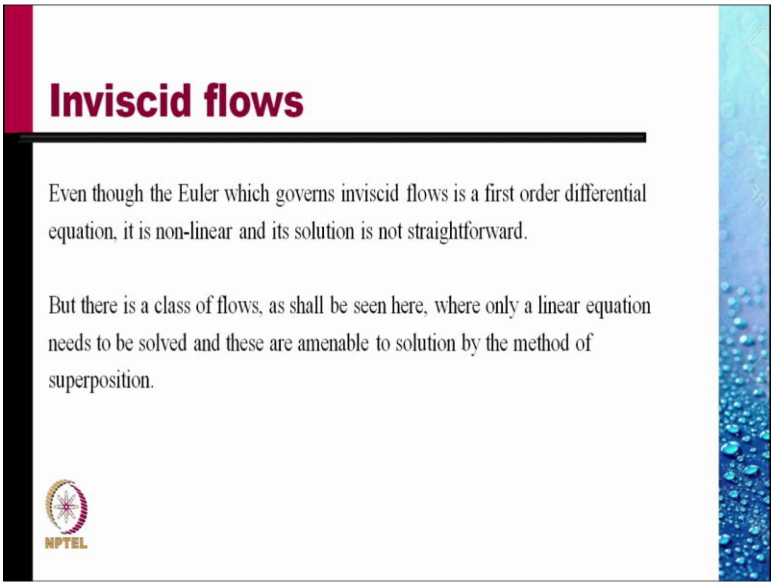

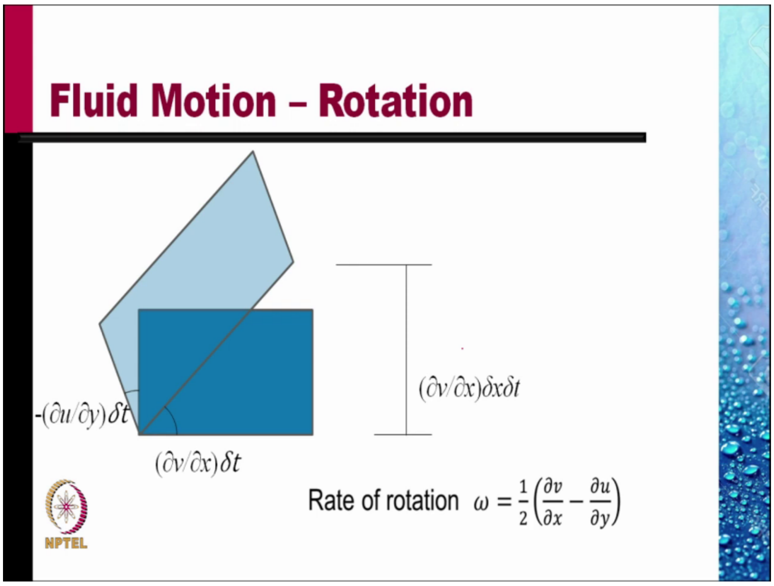

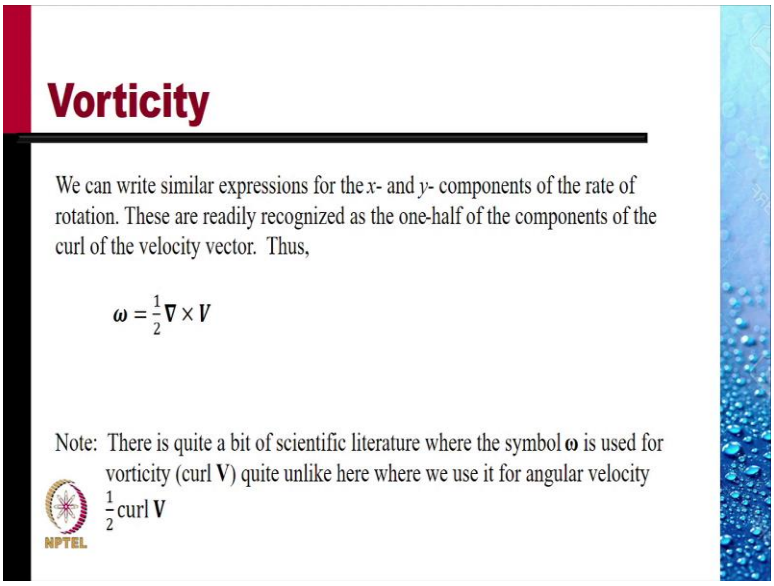

Even though the Euler equation which governs inviscid flow is a first order differential equation, it is non-linear and its solution is not straight forward. But there is a class of flows, as shall be seen here, where only linear equation needs to be solved, and these are amenable to solutions by the method of superposition. This class consists of what are termed as irrotational flows.

We had seen in an earlier lecture that the rotation of the fluid element can be measured by averaging the rotation of two perpendicular lines, and that is obtained as rate as ω is equal to $\frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$, where u and v are the component of velocity in the x and y directions.

We can write similar expressions for the x and y components of the rate of rotation., These are readily recognized as one half of the components of the curl of the velocity vector. Thus, the vector ω, which is the rate of rotation vector with three components, ω_x, ω_y and ω_z, is

$\frac{1}{2}\boldsymbol{\nabla} \times \boldsymbol{V}$. We may note that there is quite a bit of scientific literature where the symbol ω is used for the vorticity, that is, for the $\boldsymbol{\nabla} \times \boldsymbol{V}$ itself, quite unlike here where we use it for the angular velocity there is $\frac{1}{2}\boldsymbol{\nabla} \times \boldsymbol{V}$. So be careful about what notation we follow.

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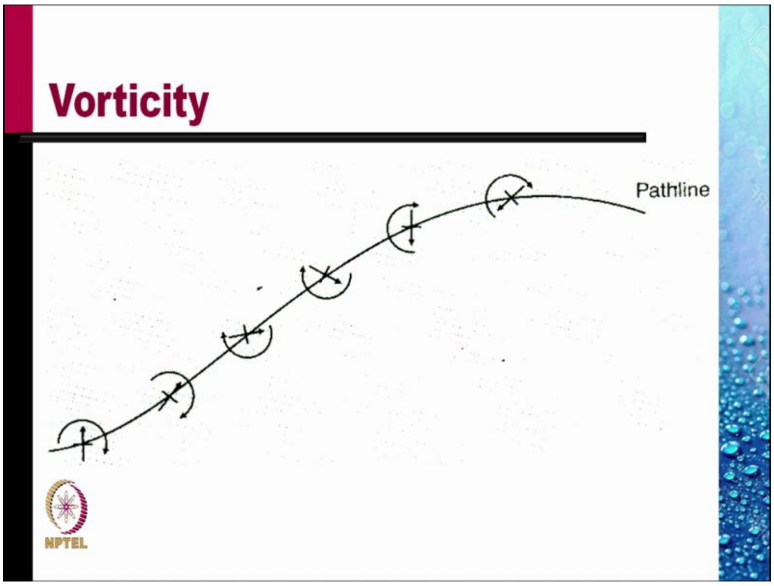

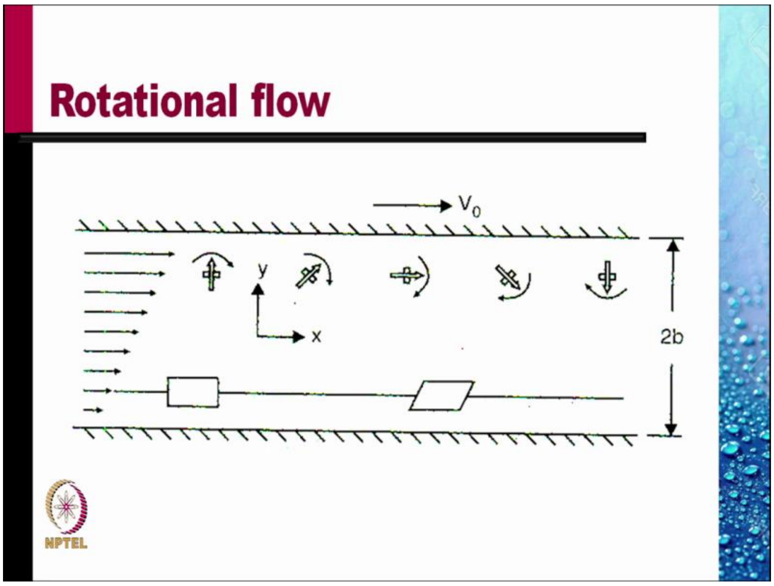

If this is a pathline on which a particle is moving a rotational flow is one that if we could imagine a paddle wheel moving along the pathline, the paddle wheel would rotate. That rotation is largely because of velocity gradients.

A rotational flow does not need to have a curvilinear streamline. For example, this flow within a channel made up of two walls; lower wall stationary and the upper wall moving with the velocity V_o there is a velocity variation from zero at the lower wall to the velocity V_o at the upper wall in the steady flow. A paddle wheel floating down this channel would move in a straight line, but since the velocity near the upper wall is larger than velocity down below, this pedal wheel would have a clockwise rotation as shown. So this definitely is a rotational flow.

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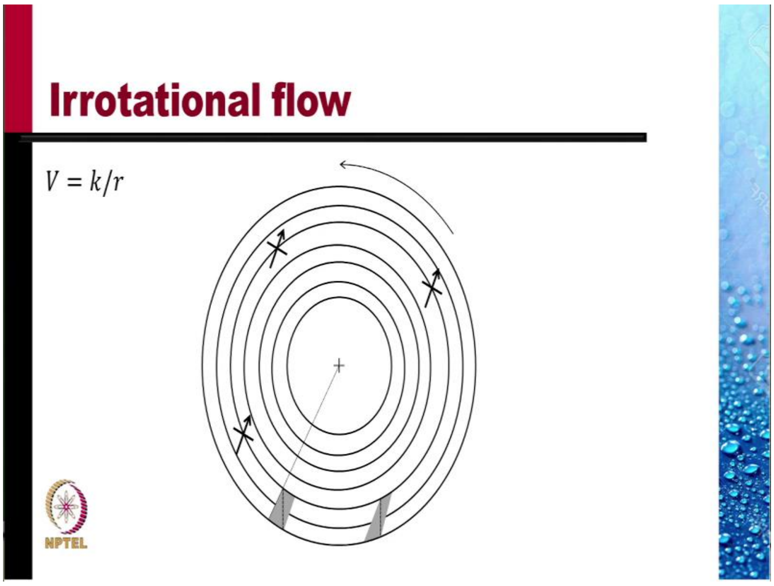

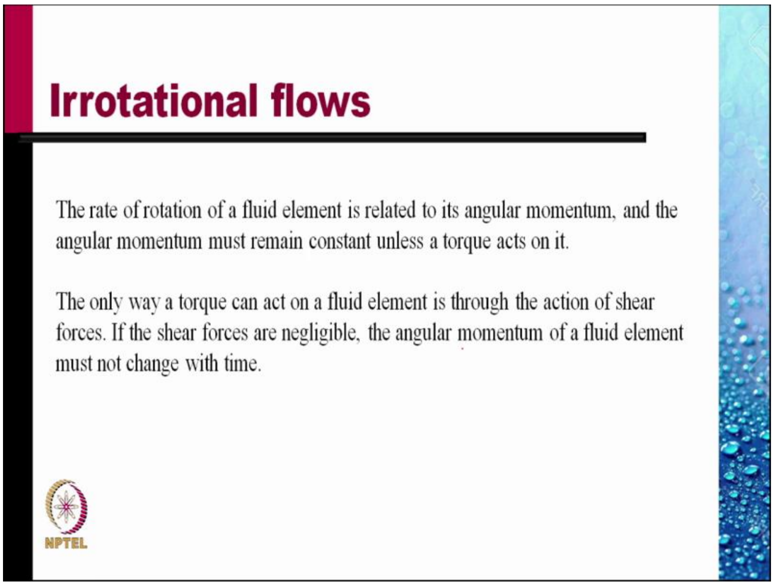

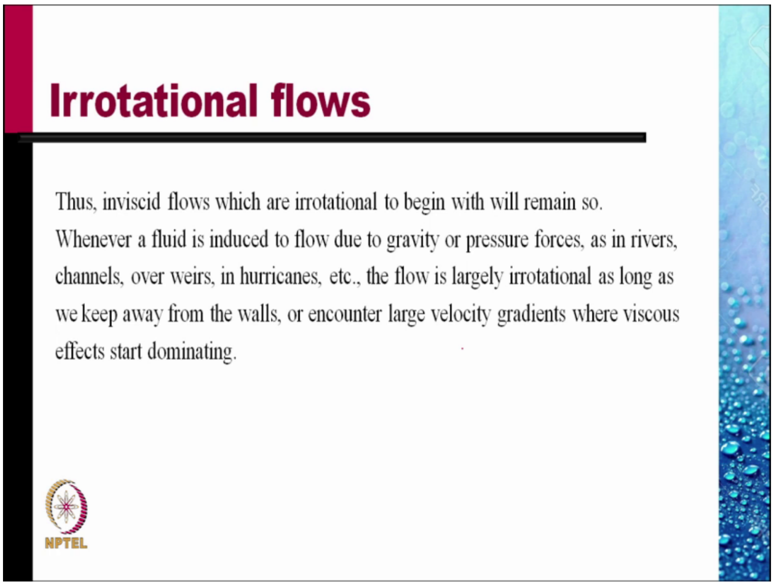

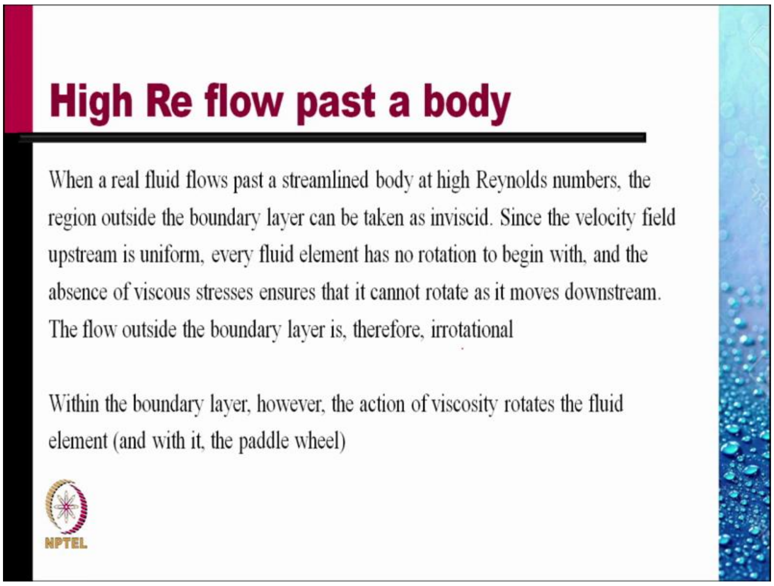

On the other hand, even if the streamlines are curved, it is not necessary that the flow be rotational. If I have a flow in which the velocity varies like k/r, $V_\theta = k/r$, inversely with radius; the flow is irrotational. A paddle wheel moving along the fluid path would not be rotating about its own axis, and so the flow is irrotational.

The rate of rotation of a fluid element is related to its angular momentum, and the angular momentum must remain constant unless a torque acts on it, second law of Newton. The only way a torque can act on a fluid element is through the action of shear forces. If the shear forces are negligible, then there is no torque acting on a fluid element, and the angular momentum of the fluid element must not change with time.

Thus, inviscid flows which are irrotational to begin with will remain so. Whenever a fluid is induced to flow due to gravity or pressure forces, as in rivers, channels, over weirs, in hurricanes, etc., the flow is largely irrotational as long as we keep away from the walls, or encounter large velocity gradients where viscous effects start dominating.

When a real fluid flows past a streamlined body at high Reynolds number, the region outside the boundary layer can be taken as inviscid, and since the velocity field upstream is uniform, every fluid element has no rotation to begin with, and with the absence of the viscous stresses outside boundary layer, this ensures that it cannot rotate as it moves downstream.

The flow outside the boundary layer is therefore irrotational. Within the boundary layer, however, the actual viscosity rotates the fluid element and with it the paddle wheel, if a paddle wheel was present.

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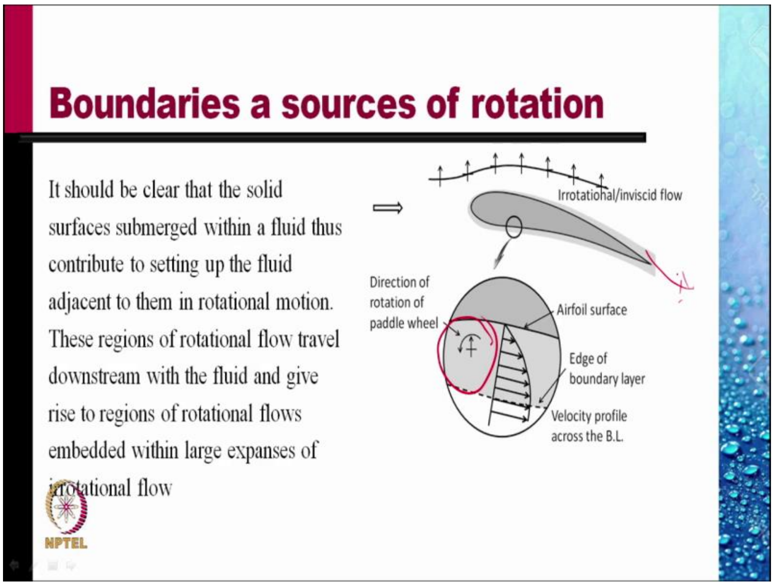

You should be clear that the solid surfaces submerged within a fluid contribute to setting up the fluid adjacent to them in rotational motion. These regions of rotational flow travel downstream with the fluid and gives rise to the regions of rotational flows embedded within the large expanses of irrotational flows. Thus, within the boundary layer, rotational flow develops, and this is a source of rotation. Ultimately, this rotational flow is shed in the wake. But for streamlined bodies the wake is thin. So, this region of rotational flow is limited.