Fluid Mechanics & its Application
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Lecture 21A
Oscillating Boundary Problem

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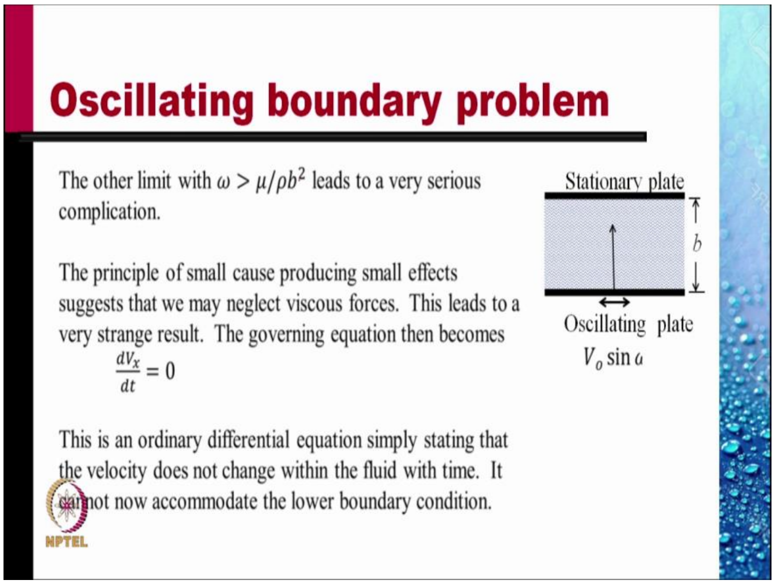

The other limit where the frequency is large, $\omega > \mu/\rho b^2$ from the same formulation, leads to a very serious complication. The principle of small cause producing small effect suggests that we may neglect viscous forces if $\omega > \mu/\rho b^2$.

This leads to a very strange result, because, if we neglect the viscous forces, the only term left in the governing equation is the unsteady term which is $\frac{dV_x}{dt}$, and that would become $\frac{dV_x}{dt} = 0$, because all other terms have been neglected. This is an ordinary differential equation, simply stating the velocity does not change within the fluid with time, and if the velocity does not change with time, how can we accommodate the lower no-slip boundary condition that the velocity of the fluid must be the same as the velocity of the oscillating plate at any given instant. It cannot. So, something breaks down.

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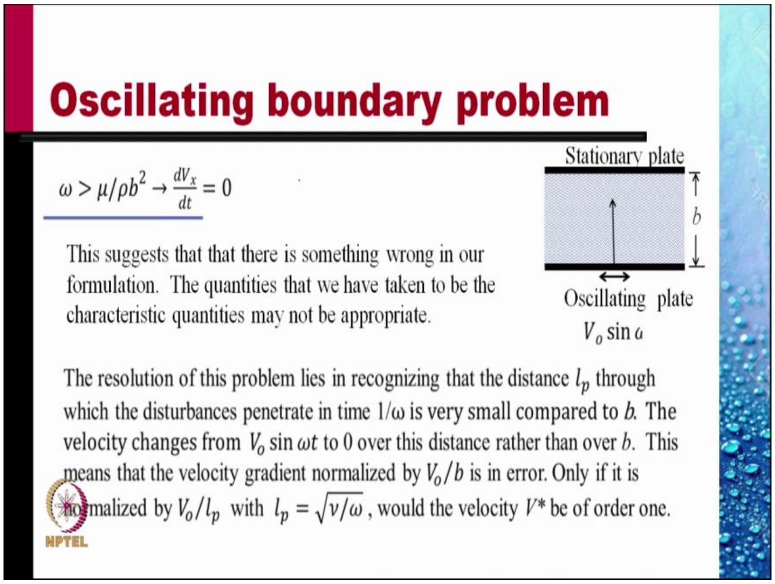

This suggests that there is something wrong in our formulation. The quantities that we have taken to be the characteristic quantities may not be appropriate for our situation. The resolution of this problem lies in recognizing that the distance L_p through which the disturbance penetrates in time 1/ω is very small compared to b, the gap.

The velocity changes from $V_o \sin \omega t$ to 0 over this distance rather than over the whole gap b. This means that the velocity gradient normalized by V_o/b is in error. Only if it is normalized by VV_o/L_p, with the penetration depth, L_p given as $\sqrt{\nu/\omega}$ would the velocity V* be of order one.

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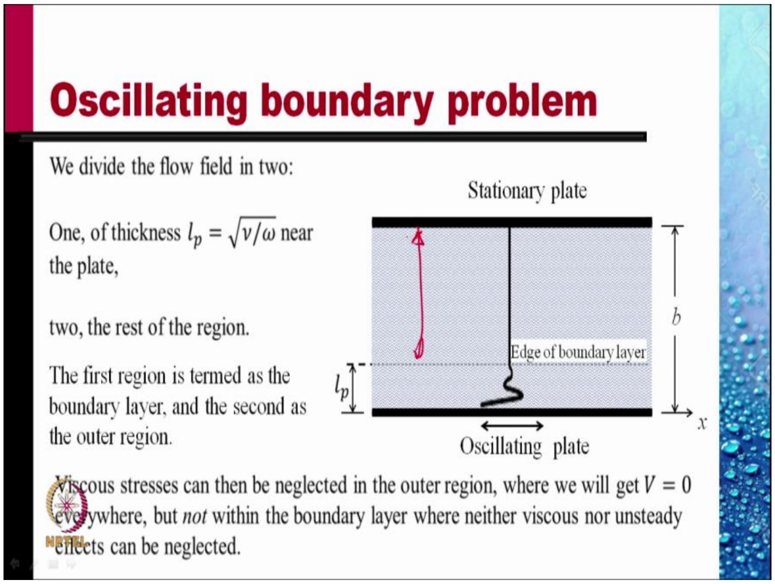

And so, we divide the flow field in two regions. One, of thickness $l_p = \sqrt{\nu/\omega}$, near the plate, and the rest of the region as the second region. The first region is termed as the boundary layer, and the second region as the outer region. Viscous stresses can then be neglected in the outer region, as we are shown in the last slide, where we will get V is equal to 0 everywhere as a solution, but they cannot be neglected within the boundary layer, where neither viscous nor steady effects can be neglected.

So, the flow solution looks something like this. The unsteady solution is confined to this thin boundary layer of thickness L_p, and a steady solution in the upper part, the outer flow. The main region of the flow, the outer region of the flow does not even know that there is a lower plate which is oscillating. Before the effect of the oscillating plate penetrates into this region, the direction of the plate changes and so, the effect felt in the outer region is negligible.

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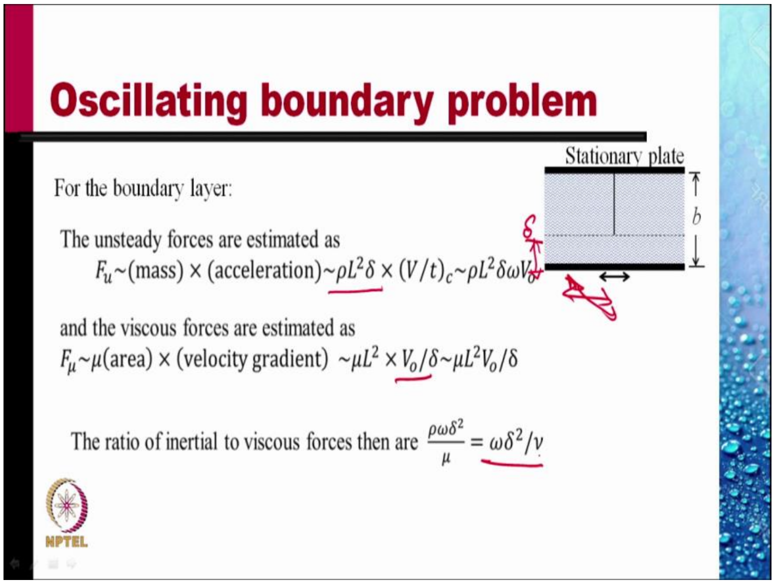

For the boundary layer region, the unsteady forces are now estimated as mass times acceleration, but the mass now is rho L squared delta, delta is taken to be the thickness of the boundary layer, L is a dimension in this or in the cross direction. So, the volume of the element would have two dimensions of L and one dimensional of delta.

So, the mass can be taken as $\rho L^2 \delta$, and the acceleration would be $(V/t)_c$, and t_c is like 1/ω. So that the unsteady forces are estimated as $\rho L^2 \delta \omega V_o$. Earlier, it was estimated as $\rho L^3 \omega V_o$. And the viscous forces are now estimated as μ times the area, which is like L^2, and the velocity gradient which is like V_o/δ instead of V_o/L.

So, that the estimate for viscous forces $\mu L^2 V_o/\delta$. δ is as yet unknown. The ratio of inertial to viscous forces, then are, the ratio of these two which gives you $\omega\delta^2/\nu$. Now, within this layer both the initial forces as well as viscous forces should be significant. So, this should be of order 1, and that gives us an estimate of δ.

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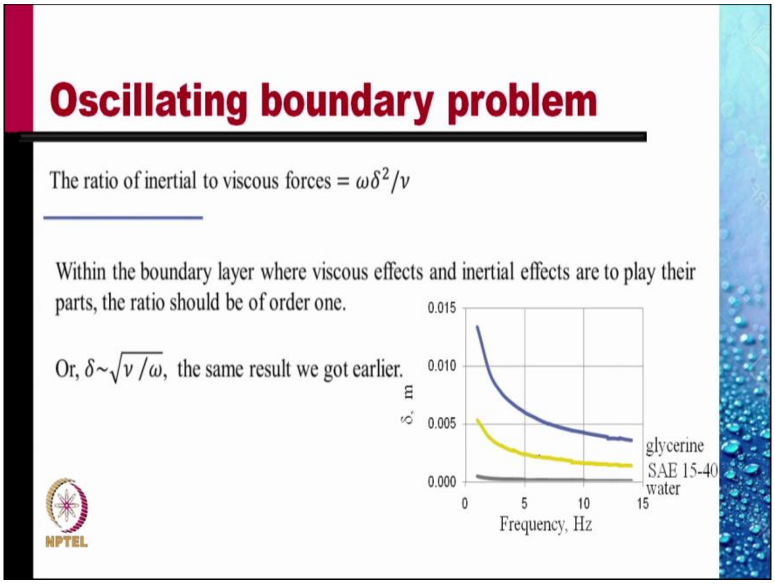

Within the boundary layer where viscous effects and inertial effects are to play their parts, the ratio should be of order 1, so that δ should be order of $\sqrt{\nu/\omega}$. The same result we got earlier. It is like L_p the penetration depth. This curve shows the variation of delta with the frequency in Hertz, and this depth increases as the fluid because more viscous. For water it is less than a mm in this frequency range. For glycerine it is much higher. And this δ increases as frequency comes down, that is when the plate slows down. The vibration of the plate is slower, then it can penetrate to a greater depth.

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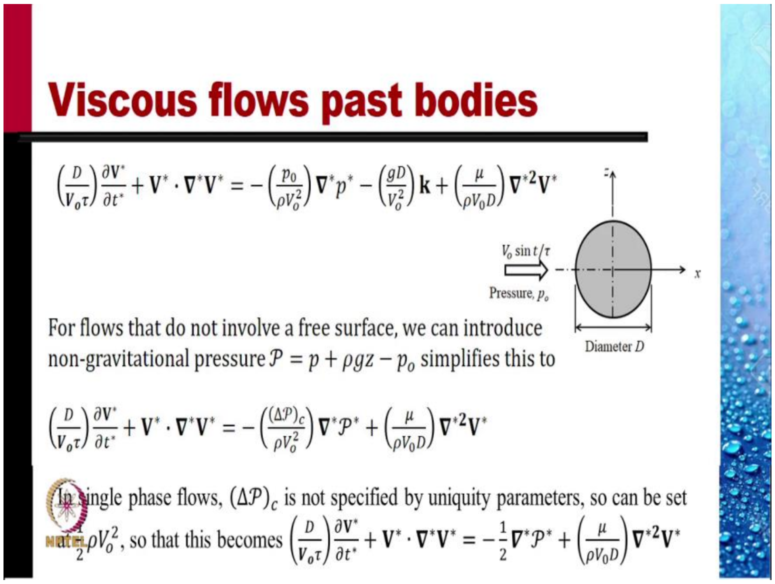

Let us look at now the viscous flow past bodies, and try to apply these concepts that we learned here. The equation that we have derived earlier, the Navier Stokes equation non-dimensionalized is this. For flows that do not involve a free surface, we can introduce a non-gravitational pressure $\mathcal{P} = p + \rho g z - p_o$, and this simplifies the equation by combining the first two terms on the right. Here we use delta p c as the characteristic pressure difference.

In single phase flow, the cavitation is not involved, $(\Delta\mathcal{P})_c$ is not specified by uniquity parameters, because there is only one pressure specified. So, we can set $(\Delta\mathcal{P})_c = \frac{1}{2}\rho V_o^2$ arbitrarily. So, that the equation now has an inertial term which in order 1, a pressure terms with a coefficient one-half in front of this, and a viscous term.

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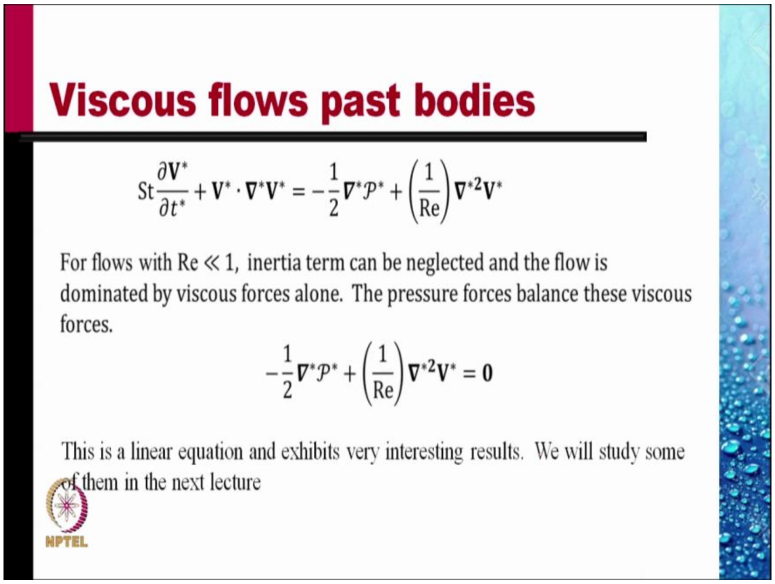

The first coefficient is recognized as a Strouhal number, and the last coefficient as 1/ Reynolds number. For flows with a Reynolds number much less than 1, inertia term can be neglected, and the flow is dominated by viscous forces alone. The pressure forces balance these viscous forces and we get an equation $-\frac{1}{2}\nabla^* \mathcal{P}^* + \left(\frac{1}{\mathrm{Re}}\right)\nabla^{*2}\mathbf{V}^* = \mathbf{0}$. This is a linear equation and exhibits very interesting result. We will study some of them in the next lecture.

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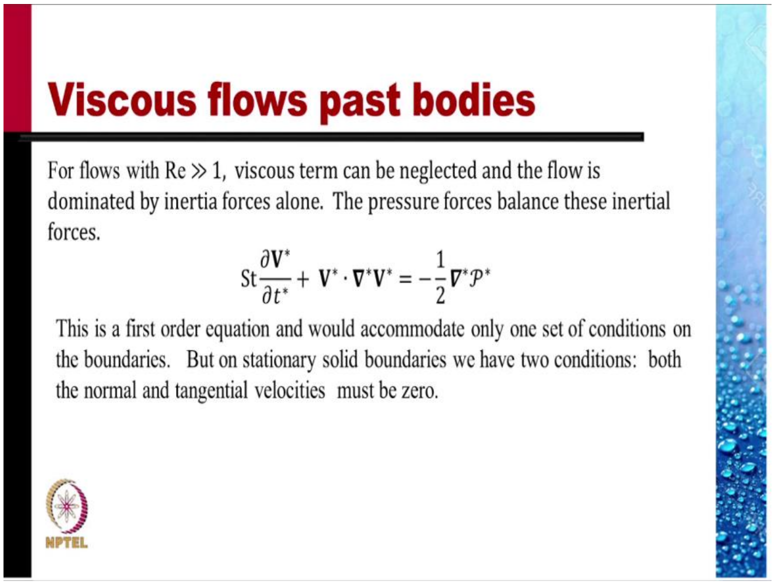

For flows with Reynolds number much greater than 1, viscous term can be neglected, and the flow is dominated by inertial forces alone. The pressure forces balance these inertial forces. The equation is $\mathrm{St}\frac{\partial \boldsymbol{V}^*}{\partial t^*} + \boldsymbol{V}^* \cdot \boldsymbol{\nabla}^* \boldsymbol{V}^* = -\frac{1}{2}\boldsymbol{\nabla}^* \mathcal{P}^*$. This is a first order equation in velocity and would accommodate only one set of boundary conditions.

But on stationary solid boundaries, we have two conditions, that both the normal and tangential velocity must be 0. So, obviously, this equation cannot satisfy both the boundary conditions. You can see that the problem is developing in the same manner as the problem of the oscillating plate done just now with ω large. There also we could not accommodate both the boundary conditions. Here too, we cannot accommodate both the boundary conditions.

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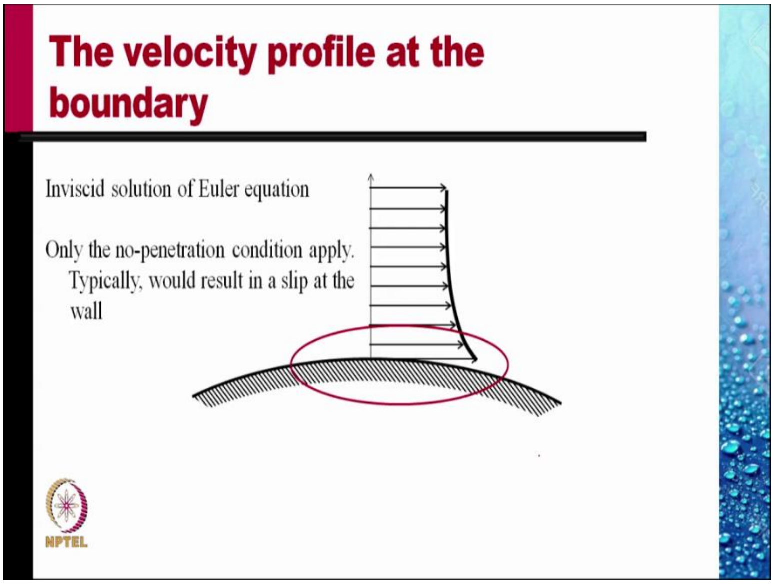

The inviscid solution of the Euler equation, only the no penetration condition applies, and typically would result in a slip at the wall, as shown. But this would be that there is something wrong at the wall. Because the no slip condition must also be satisfied at the wall, and this can happen only if the velocity is changing too fast, too quickly, in a very thin region near the wall such that the velocity gradients are large. And so, for the velocity gradient, we can not take the dimension of the body to the characteristic length. But the distance over which the velocity changes from 0 to the inviscid value should be taken as the characteristic length in the normal direction.

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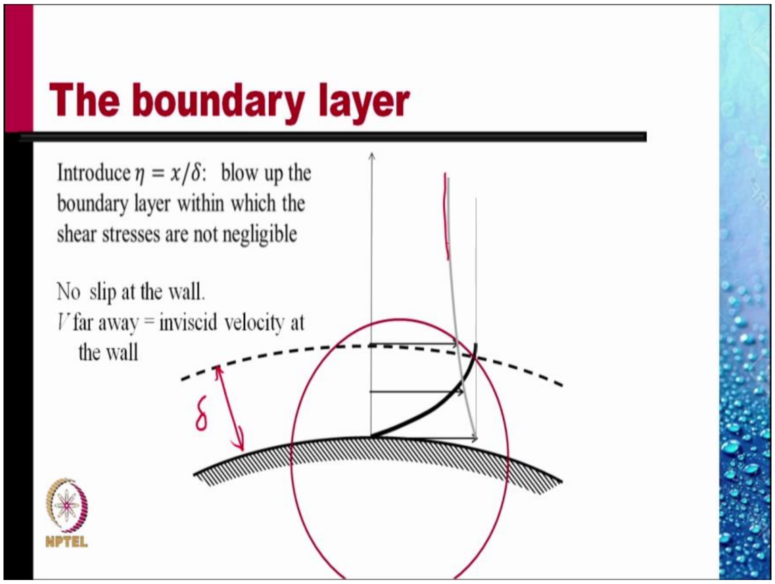

We introduce a non-dimensional distance variable $\eta = x/\delta$ and this blows up the boundary layer within which these shear stresses are not negligible. We consider a layer of thickness delta. This is an inviscid profile. The viscosity produces a sharp velocity gradient within the boundary layer, and we get a solution that looks like this.

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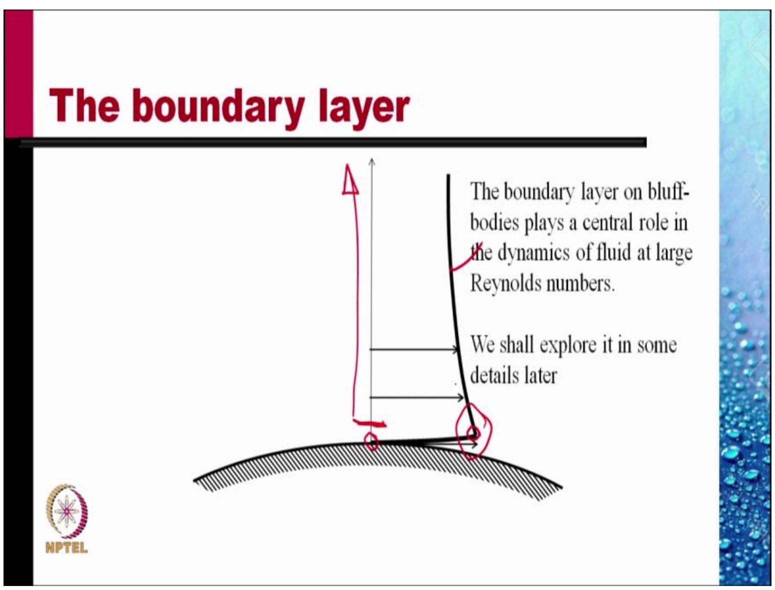

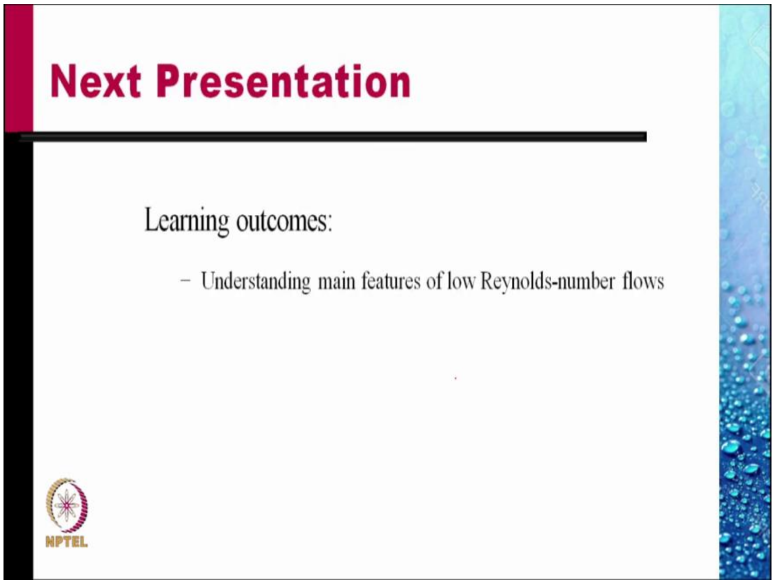

The two solutions, the outer region solution and the boundary layer solution gives rise to a velocity profile that looks like this. The velocity rises very sharply from 0 velocity to a maximum velocity some distance away from the wall. And then, in this outer region, viscosity is neglected, and we get a solution which is inviscid. So, this part represents the inviscid solution.

The two solutions merge together at the edge of the boundary layer. In fact, the boundary condition needed to solve the boundary flow says that the velocity at the edge of the boundary layer should be equal to the velocity at the wall given by the inviscid flow. This is a very interesting problem, and we will spend considerable time to this problem of boundary layer flow. The boundary layers on bluff-bodies play a central role in the dynamics of fluid at large Reynolds number, and we shall explore it in some details later.

Thank you very much.