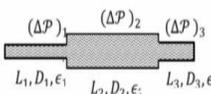


Fluid Mechanics & its Applications
Professor Vijay Gupta
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Sharda University
Lecture – 20B
Piping Networks and Non-Circular Pipes

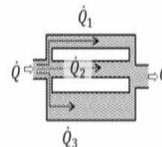
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Piping networks

Multi-pipe flow networks are common, particularly in water-supply and sewage networks. These are solved by recognizing and exploiting their similarity with electrical networks. Here too we have pipes in series and in parallel



$\dot{Q}_1 = \dot{Q}_2 = \dot{Q}_3 = \dots = \dot{Q}$
 $\Delta P = (\Delta P)_1 + (\Delta P)_2 + (\Delta P)_3 + \dots$



$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3 + \dots$
 $\text{and } (\Delta P)_1 = (\Delta P)_2 = (\Delta P)_3 = \dots = \Delta P$

NPTEL

Multiple pipe flow networks are common, particularly in water supply and in sewage networks. These problems are solved by recognizing and exploiting their similarity with the electrical networks. Here too, we have pipes in series and in parallel, as we have resistance in series and parallel in electrical networks. For example, we say these three pipes are in series.

The first pipe of length L_1 and the diameter D_1 , roughness ϵ_1 , it has a pressure drop of ΔP_1 . Similarly, a second pipe and the third pipe. Just like in the electrical networks, we have here for this series network of pipes, the volume flow rate through one should be same as volume flow rate to the second. It should be equal to volume flow rate through third, and so on. There is a common volume flow rate, just like there is a common current in three wires connected in series. But, the total pressure drop is the sum of the three pressure changes, $\Delta P = (\Delta P)_1 + (\Delta P)_2 + (\Delta P)_3 + \dots$, just like the total voltage drop is the voltage drop across the constituent resistances in a series network.

Similarly, a parallel network. Here the total volume \dot{Q} flowing through the system is equal to $\dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3 + \dots$, just like currents in a parallel electrical circuit. But, the pressure drop

through each one is the same, $(\Delta\mathcal{P})_1 = (\Delta\mathcal{P})_2 = (\Delta\mathcal{P})_3 = \dots = \Delta\mathcal{P}$. The voltage drop is same in similar electrical networks.

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Piping networks

Analysis of piping networks no matter how complex are driven by two simple rules:

1. Flow rate balance: Total volume flow rate into a junction of pipes is equal to the total volume flow rate away from the junction, and
2. Pressure drop: The pressure drop between two junctions is the same along every path between the junctions

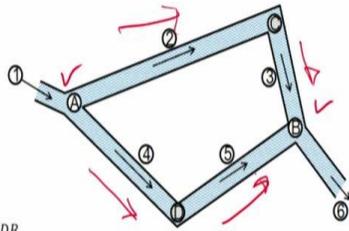


Piping networks

$\dot{Q}_1 = \dot{Q}_2 + \dot{Q}_4$ at node *A*,
 $\dot{Q}_4 = \dot{Q}_5$ at node *D*,
 $\dot{Q}_2 = \dot{Q}_3$ at node *C* and
 $\dot{Q}_6 = \dot{Q}_3 + \dot{Q}_5$ at node *B*

$(\Delta\mathcal{P})_{AC} + (\Delta\mathcal{P})_{CB} = (\Delta\mathcal{P})_{AD} + (\Delta\mathcal{P})_{DB}$





Analysis of piping networks, no matter how complex are driven by two simple rules. The flow rate balance: total volume flow rate into a junction of pipe is equal to the total flow rate away from the junction, because the volume of the flow cannot be created at a junction. Then, the pressure drop between two junctions is the same along every path between the junctions and the paths, the paths being parallel to one another. For example, consider a network in which between junctions A and B, there are two paths: through pipes 2 and 3, and through pipes 4 and 5. So, the

above two rules translate into writing, that at junction A, the node A, $\dot{Q}_1 = \dot{Q}_2 + \dot{Q}_4$. The flow coming in through pipe 1 is the flow going through pipe 2, plus the flow going through pipe 4. The net flow at A should be 0. Similarly, at node D, $\dot{Q}_4 = \dot{Q}_5$. Similarly node C and node B. And then, the second condition that the pressure drop between the two junctions A and B is the same following either path 2 and 3, or 4 and 5. So, $(\Delta\mathcal{P})_{AC} + (\Delta\mathcal{P})_{CB} = (\Delta\mathcal{P})_{AD} + (\Delta\mathcal{P})_{DB}$. We can write pressure drops in terms of the pipe lengths, diameters and the friction factors for each one of them. Depending upon the complexity of the problem, this could be a handful. Various methods have been developed to solve these equations conveniently, and there are computer calculators available for doing these pipe networks.

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Flow through pipes of non-circular sections

The forces that regulate the flow through ducts, circular or non-circular, are viscous, pressure and inertia forces. We now use a technique where we relax the strict geometric similarity requirements set out earlier.

We use the fact that the viscous effects on pipe-walls are confined to a thin region very close to the wall, while the inertial and pressure forces act over the whole cross-section.

Thus, it allows us to use two different length scales, one representing the overall dimensions and the other the scale over which the velocity varies near the wall.




Let us next do flow through pipes of non-circular sections. The types of forces that regulate the flow through ducts circular or non-circular are viscous, pressure and inertial. We now use a technique, where we can relax such strict geometrical similarity requirements that has been set out earlier. We use the fact that viscous effects on pipe walls are confined to a thin region very close to the wall, particularly in turbulent flows, while the inertial and pressure forces act over the whole cross-section. That is, we are assuming that over bulk of the cross-section, the velocity gradients are quite small, and so the viscous stresses are quite small.

This allows us to use two different length scales. Up till now we have used only one scale, one length scale, the diameter of the pipe. Now, we can use two different length scales. One,

representing the overall dimension like the diameter, and the other scale over which the velocity varies near the wall. If we assume this thin layer of thickness δ , then δ would be a length scale over which the velocity varies. So, that the shear stress now can be estimated as μ times the velocity gradient, characteristic velocity gradient, which is V/δ , rather than V/L , as we have been doing previously.

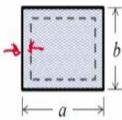
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Flow through pipes of non-circular sections

The first length scale is the so-called boundary-layer thickness δ . The velocity varies from zero at the wall to the bulk velocity over this length, so that the velocity gradients are of order V/δ .

The shear stresses are of order $\mu V/\delta$ and the shear forces are of order $(\mu V/\delta)PL$, where P is the wetted perimeter, so that PL represents the area on which the shear stresses act.

The corresponding Pi-number is $F_\mu \delta / \mu VPL$.




The first length scale is a so-called boundary-layer thickness δ . The velocity varies from 0 at the wall to the bulk velocity over this length, so that the velocity gradients are of order V/δ , this being the thickness δ for non-circular pipes. These shear stresses are of order then $\mu V/\delta$, and the shear forces are of order $\mu V/\delta$, which is the shear stress times the area over which they act. Now, the area over which they act is the perimeter of the pipe times the length of the pipe. $P \times L$ is the total area over which these shear stresses act. P is termed the wetted perimeter. So, that PL represents the area on which the shear stresses act. The corresponding Pi-number for the shear forces would be $F_\mu \delta / \mu VPL$. Earlier, we have not used this artifice, δ was taken as L , so that the Pi-number was $F_\mu / \mu VD$.

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Flow through pipes of non-circular sections

The inertia forces apply to the main flow:

$$F \sim ma, \quad m \sim \rho V, \quad V \sim AL, \quad \text{and} \quad a \sim V^2/L.$$

The resulting Pi-number is $F/\rho AV^2$, where A is the cross-sectional area of the pipe.

From these two Pi-numbers we get a Reynolds number-like Pi-number $\frac{\rho \delta V}{\mu L} \cdot \frac{A}{P}$.



The inertial forces applied to the main flow, F is like mass times acceleration, mass is like ρ times volume. Volume is now like the cross-sectional area A times the length of the pipe, and acceleration is like V^2/L . The resulting Pi-number is $F/\rho AV^2$, where A is the cross-sectional area of the pipe. And from this, we get a Reynolds number like Pi-number. The new estimate of the shear forces, a new estimate of the inertial forces gives you a Reynolds number like Pi-number which is $\frac{\rho \delta V}{\mu L} \cdot \frac{A}{P}$.

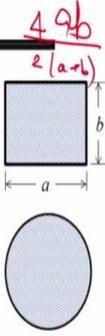
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Flow through pipes of non-circular sections

Consider two pipes of the same length carrying the same fluid at the same average speed. It stands to reason that the boundary-layer thicknesses in the two pipes would be the same and flows would be similar if $\frac{\rho \delta V}{\mu L} \cdot \frac{A}{P}$ of the two pipes are the same.

For the circular pipe $\frac{A}{P} = \frac{\frac{\pi D^2}{4}}{\pi D} = D/4$.

Thus, the diameter of a circular pipe the flow through which may be considered similar to the flow through a given non-circular pipe should be $4A/P$. This is termed as the hydraulic diameter of the pipe.



The diagram illustrates two pipe cross-sections. The top one is a rectangle with width 'a' and height 'b'. Handwritten in red above it are the expressions '4ab' and '2(a+b)'. The bottom one is a circle.

Flow through pipes of non-circular sections

Thus, the pressure drop in a non-circular pipe can be predicted quite well by calculating the pressure drop in a circular pipe of diameter equal to $4A/P$



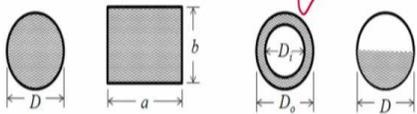
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Consider two pipes of the same length carrying the same fluid at the same average speed. It stands to reason that the boundary layer in the two pipes would be the same, and the flow would be similar, if this quantity would be the same in the two pipes. The value of A/P for a circular pipe is $\frac{\pi D^2}{4\pi D}$, it is $D/4$. So, we take $4A/P$ as the equivalent diameter of any pipe. For example, this rectangular pipe with dimension a and b , area is ab , perimeter is $2(a + b)$. So, the equivalent diameter of this pipe is four times this. This is called the hydraulic diameter, and is represented by D_h .

So, if for non-circular pipe, we calculate the hydraulic diameter, and then replace this non-circular section pipe with a circular pipe of diameter equal to the hydraulic diameter calculated, then, we should get the same pressure drop and the same flow rate. Thus, the pressure drop in a non-circular pipe can be predicted quite well by calculating the pressure drop in a circular pipe of diameter equal to $4A/P$. Using this as the diameter D , we can use Moody's chart or the Colebrook formula.

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Flow through pipes of non-circular sections



Cross-section	P	A	$D_h = 4A/P$
Circular, diameter D	πD	$\frac{\pi}{4} D^2$	D
Rectangular, $a \times b$	$2(a + b)$	ab	$\frac{2ab}{a + b}$
Annular, D_o and D_i	$\pi(D_o + D_i)$	$\frac{\pi}{4}(D_o^2 - D_i^2)$	$(D_o - D_i)$
Circular, half-full	$\frac{\pi}{2} D$	$\frac{\pi}{8} D^2$	D

In this slide we have shown the calculation of the hydraulic diameter D_h for four different shapes. First, a circular diameter pipe in which the hydraulic diameter comes out as the diameter of the pipe itself. For the rectangular pipe, it is twice ab divided by $(a + b)$ was shown in the last slide as well. For an annular pipe, the wetted parameter is $\pi(D_o + D_i)$, and the area of flow is $\frac{\pi}{4}(D_o^2 - D_i^2)$. And calculating $4A/P$, we get the hydraulic diameter as $(D_o - D_i)$. This technique can also be used for pipes which are not flowing full. In the last picture we have shown a circular pipe flowing half full, the liquid occupies only one half the area of the pipe. And then we show that for this pipe also the effective diameter is D .

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Flow through pipes of non-circular sections

Under these circumstances, the cross-sectional shape of the model may deviate from that of the prototype as long as the hydraulic radius is scaled in accordance with the length scale factor.

The concept of hydraulic diameter is widely used in hydraulics, even in open channels. It permits relaxation of geometric similarity for the cross section of the flow, *as long as the flow is turbulent and the boundary layer is laminar*.

For open channels the pressure does not change and the change in non-gravitational pressure $\mathcal{P} = p + \rho g z$ is related to change in elevation.

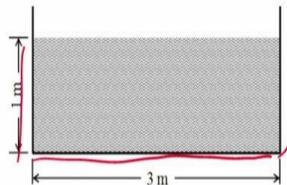


Under these circumstances, the cross-sectional shape of the model may deviate from the prototype as long as the hydraulic radius is scaled in accordance with the length scale factor. The concept of hydraulic diameter is widely used in hydraulics, even in open channels. It permits relaxation of geometric similarity from the cross-section of the flow, as long as the flow is turbulent and the boundary layer is laminar. For open channels, the pressure does not change, and the change in non-gravitational pressure $\mathcal{P} = p + \rho g z$ is related to change in elevation.

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Flow through open channels

The painted-steel channel ($\epsilon = 2.4 \text{ mm}$) is planned. Determine the slope of the channel required for a flow rate of $6 \text{ m}^3/\text{s}$ at a normal depth of 1



We first determine the hydraulic radius for the channel.

The value of wetted perimeter is $(1 \text{ m} + 3 \text{ m} + 1 \text{ m}) = 5 \text{ m}$.

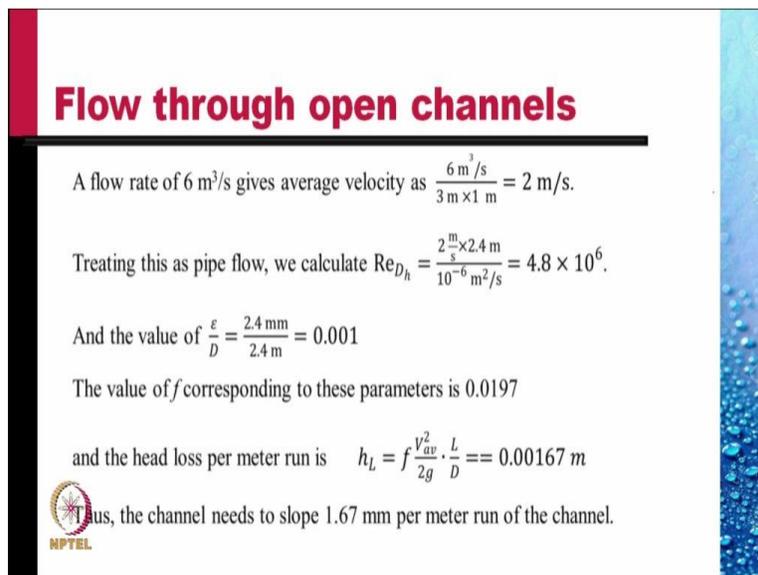
The flow area is 3 m^2 .

Therefore, $D_h = \frac{4A}{P} = 2.4 \text{ m}$



We do an example. Consider a painted steel channel, and we look up the table and find that ϵ is like 2.4 mm in such painted steel channels. Determine the slope of the channel required for a flow rate of $6 \text{ m}^3/\text{s}$ at a normal depth of 1 m as shown. So, to treat this as a pipe flow problem, we calculate first the hydraulic radius. The wetted perimeter is 1 m + 3 m plus 1 m, a total of 5 m. The flow area is $3 \text{ m} \times 1 \text{ m}$, and therefore, the hydraulic diameter is $4A/P$, that is, 2.4 m. So, this could be treated as a circular pipe of diameter 2.4 m.

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Flow through open channels

A flow rate of $6 \text{ m}^3/\text{s}$ gives average velocity as $\frac{6 \text{ m}^3/\text{s}}{3 \text{ m} \times 1 \text{ m}} = 2 \text{ m/s}$.

Treating this as pipe flow, we calculate $Re_{D_h} = \frac{2 \text{ m/s} \times 2.4 \text{ m}}{10^{-6} \text{ m}^2/\text{s}} = 4.8 \times 10^6$.

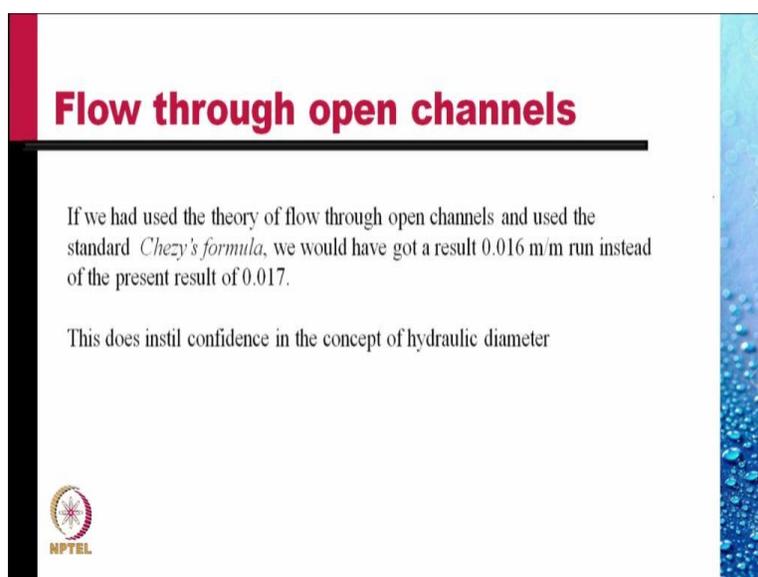
And the value of $\frac{\epsilon}{D} = \frac{2.4 \text{ mm}}{2.4 \text{ m}} = 0.001$

The value of f corresponding to these parameters is 0.0197

and the head loss per meter run is $h_L = f \frac{V_{av}^2}{2g} \cdot \frac{L}{D} = 0.00167 \text{ m}$

Thus, the channel needs to slope 1.67 mm per meter run of the channel.





Flow through open channels

If we had used the theory of flow through open channels and used the standard *Chezy's formula*, we would have got a result 0.016 m/m run instead of the present result of 0.017.

This does instil confidence in the concept of hydraulic diameter



A flow rate of $6 \text{ m}^3/\text{s}$ gives an average velocity of 2 m/s flow rate divided by the cross-sectional area. Treating this as a pipe flow, we calculate the Reynolds number, which comes out to

4,800,000, and the value ε/D as 0.001. The value of f corresponding to these parameters from the Moody chart is 0.0197. Actually I used the Colebrook calculator to determine this value.

And the head loss per meter run then is given by $h_L = f \frac{V_{av}^2}{2g} \cdot \frac{L}{D}$, and if I plug in all the values, I get a head loss equal to 0.00167 m. And this head loss is obtained purely as the head loss in elevation, since the pressure is constant. It is atmospheric throughout the channel and the velocity is same. So, the channel needs to slope 1.67 millimeter per meter run of the channel.

There is a well-developed theory of open channel flows, where we do not make these approximations. And there is a standard Chezy's formula. And if we use that, we would have got a result of 0.016 meters per meter run instead of the present result of 0.017 meters per meter run.

So, this does instill confidence in the concept of hydraulic diameter. So, even though the open channel has a separate theory to deal with, we could model it as a pipe, based on the principle of hydraulic diameter, and get a result which is well within the range of accepted values. This shows the power of characteristic lengths properly used.

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Flow through open channels

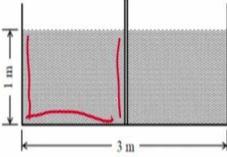
In the channel of the previous problem, we now introduce a painted-steel barrier as shown. Determine the reduction in total flow rate

In the presence of the barrier, the channel splits into two exactly similar channels.

Let us calculate the flow through one channel.

We first determine the hydraulic radius for one channel. The value of wetted perimeter is 3.5 m. The flow area is reduced to 1.5 m².

Thus, hydraulic diameter is $D_h = \frac{4 \times 1.5 \text{ m}^2}{3.5 \text{ m}} = 1.71 \text{ m}$.



We extend this a little further. And we said that in the channel of the previous problem, we now introduce a painted-steel barrier in the middle as shown, just erected a thin barrier in the middle Determine the reduction in flow rate.

Now, in the presence of the barrier, the channel splits into two exactly similar channels. Let us calculate flow through one channel. We first determine the hydraulic radius of one channel. The value of wetted parameter is now 3.5 m, 1 m + 1.5 m + 1 m. The flow area is reduced to 1.5 m × 1 m, and so, the hydraulic diameter is 1.71 m.

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Flow through open channels

Since the slope remains the same, the head loss remains the same at 1.67 mm per meter run of the channel.

The head loss is related to the velocity head and the value of friction factor f by the relation $h_l = f \frac{v^2 L}{2g D_h}$, so that $V = \sqrt{\frac{2gh_l D_h}{fL}}$, where f is a function of Re_D and ϵ/D .

Here, $h_l = 0.00167$ m, $D_h = 1.71$ m, and $L = 1$ m. This gives $V = \frac{0.237}{\sqrt{f}}$.

The value of ϵ for painted steel is 2.4 mm as before so that ϵ/D_h is 2.4 mm/1.71 m = 0.0014.



Flow through open channels

f	V	Re	f from Colebrook calculator
0.0213	1.62	2776864	0.02144
0.02144	1.62	2767783	0.02144

$V = \frac{0.237}{\sqrt{f}}$
 $Re = 1.71 * V / 10^{-6}$

Thus, the flow velocity through the channel is 1.62 m/s, and the volume flow rate is 2.43 m³/s through one half of the channel or 4.86 m³/s for the whole channel.

Thus, the barrier reduces the volume flow rate by almost 20 per cent



Since the slope remains the same, the head loss remains the same at 1.67 m per meter run of the channel. The head loss is related to the velocity head and the value of friction factor f is obtained, and then we determine the velocity value V . The value of ϵ for the painted steel is 2.4 mm, as

before, so the ε/D_h is 2.4 mm divide by 1.71 m, which is 0.0014. I go through iterations again, and determine the value of V as 1.62 m/s. The value of f has converged.

Thus, the flow velocity through the channel is 1.62 m/s, and the volume flow rate is 2.43 m³/s through one half of the channel, or 4.86 m³/s for the whole channel. Thus, the introduction a barrier reduces the volume flow rate by almost 20 percent, because of the additional shear stresses on the central barrier, which reduces the flow velocity.

Thank you.