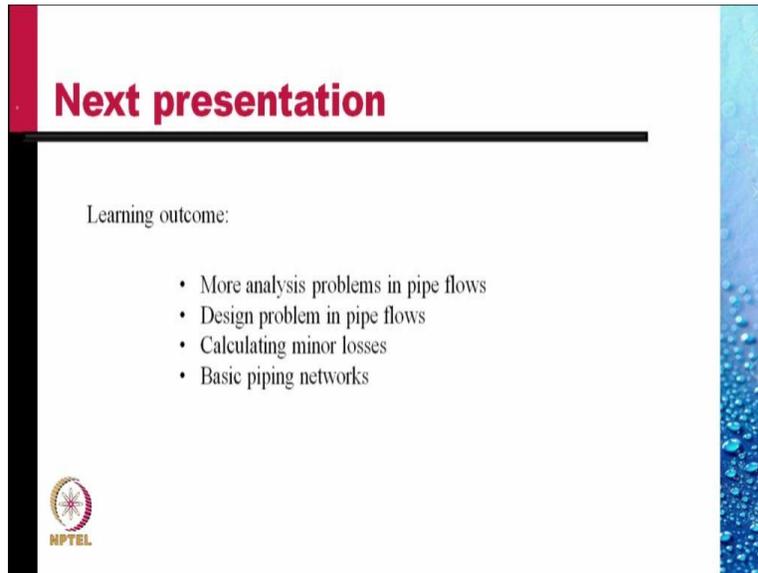


Fluid Mechanics & its Applications
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Lecture 20

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Next presentation

Learning outcome:

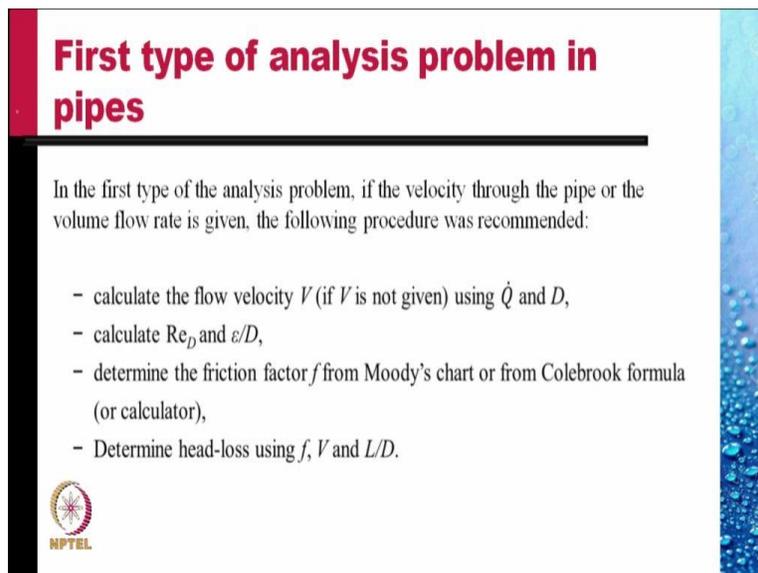
- More analysis problems in pipe flows
- Design problem in pipe flows
- Calculating minor losses
- Basic piping networks

 NPTEL

Welcome back.

In this presentation, we will do more problems of the analysis in pipe flows. We will also do a few problems on the design in pipe flows, and then we will cover some basic piping networks.

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First type of analysis problem in pipes

In the first type of the analysis problem, if the velocity through the pipe or the volume flow rate is given, the following procedure was recommended:

- calculate the flow velocity V (if V is not given) using \dot{Q} and D ,
- calculate Re_D and ϵ/D ,
- determine the friction factor f from Moody's chart or from Colebrook formula (or calculator),
- Determine head-loss using f , V and L/D .

 NPTEL

In the first type of analysis problems, if the velocity through the pipe or the volume flow rate through it is given the following procedure to calculate the pressure difference or the pressure loss through the pipe is recommended.

Calculate the flow velocity, if the velocity is not given, using the volume flow rate and the diameter of the pipe.

Next, calculate the Reynolds number and the roughness factor. Then determine the friction factor from the Moody's chart or from a Colebrook calculator. Once we know the value of f , we can determine the head loss using f , V and L/D .

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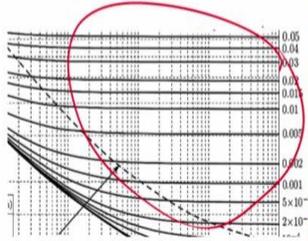
Second type of analysis problem

In the second type of the analysis problem, the pressure drop in a length L of the pipe is given, and we need to determine the volume flow rate.

We need both Re_D and ϵ/D for determination of f , but Re_D is not known.

Recall, that the fully-developed turbulent flow part of the Moody chart looks like this. The value of f is largely independent of Re and depends only on the value of $\frac{\epsilon}{D}$.

Since we know the value of $\frac{\epsilon}{D}$, we have a first estimate of f .



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Second type of analysis problem

- with this first estimate of f , estimate head losses, and using the energy equation, estimate the flow velocity,
- calculate Re_D based on this estimate of velocity, and improve the estimate of f ,
- repeat the above steps till the estimate of f converges to the required level.



In the second type of analysis problem, pressure drop in a length L of the pipe is given, and we need to determine the volume flow rate through the pipe. The pipe geometry is also given. But, since we do not know the velocity, we can not calculate the Reynolds number. And therefore, we cannot calculate f directly, because the Reynolds number is not known.

Recall, that in fully-developed turbulent flow part of the Moody chart, the value of f is almost constant for a given value of ε/D . Since we know the value of ε/D , we have a first estimate of f . With this first estimate of f , we estimate the head loss, and using the energy equation, estimate the flow velocity. We calculate the Reynolds number based on this estimate of velocity, and improve the estimate of f . We repeat the above steps till the estimate of f converges to the required level.

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Example: Cast iron pipe

Obtain the rate of flow of water carried by a cast iron pipe (of $D = 10$ cm and $L = 40$ m) when the applied head difference across it is 2 m.

The head loss is related to the velocity head and the value of friction factor f by the relation $h_l = f \frac{V^2 L}{2gD}$, so that $V = \sqrt{\frac{2gh_l D}{fL}}$, where f is a function of Re_D and ε/D .

Here, $h_l = 2$ m, $D = 0.1$ m, and $L = 40$ m.

This gives $V = \frac{0.313}{\sqrt{f}}$.

We do an example. Obtain the flow rate of water carried by a cast iron pipe of diameter 10 cm and length 40 m, when the applied head difference is 2 m. The head loss is related to the velocity head, and the value of friction factor f by the relation $h_l = f \frac{V^2 L}{2gD}$. So that we can write $V = \sqrt{\frac{2gh_l D}{fL}}$, where f is a function of Reynolds number and ε/D . Here in this relation for V , we know everything except the value of f . So, we plug in all those values, and we get a relation $V = \frac{0.313}{\sqrt{f}}$.

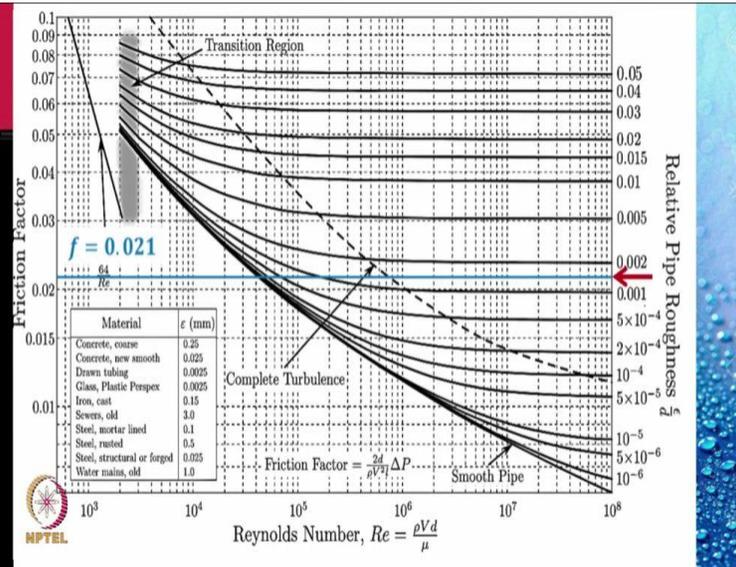
So, this should give you a hint directly. If we start with an estimate of f , we calculate a velocity V ; from that velocity V , we calculate the Reynolds number. And once I have a Reynolds number and the value of ε/D , I get the value of f from a Moody chart or from a Colebrook calculator.

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Example: Cast iron pipe

The value of ϵ for cast iron pipes is 0.15 mm so that ϵ/D is 0.15 mm/10 cm = 0.0015.

We start by assuming the flow to be fully turbulent so that f can be estimated without knowing the value of Re_D .



Example: The cast iron pipe

Thus, the value of f corresponding to fully-turbulent flow and $\varepsilon/D = 0.0015$ is 0.021.

With this value of f , we calculate V using $V = \frac{0.313}{\sqrt{f}}$, and then Re_D .

From the calculated value of Re_D and ε/D we determine the value of f from a Colebrook calculator, and use this value for a new iteration.



The value of ε for the cast iron pipe is 0.15 millimeter, so that the ε/D is 0.0015. We start by assuming the flow to be fully turbulent; so that f can be estimated without knowing the value of Reynolds number. From the chart, we enter at 0.0015, the value ε/D , the relative pipe roughness. In this we get a value of f as 0.021. We start with this value and we calculate V from $\frac{0.313}{\sqrt{f}}$, then Re_D , and do iterations.

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Example: The cast iron pipe

The calculations are arranged in a spread-sheet (with the appropriate formulae embedded for V and Re) and we get convergence in only two iterations.

f	$V (=0.313/\text{sq.rt}(f))$	$Re (=V*0.1/0.000001)$	f from calculator
0.021	2.16	215991	0.0227
0.0227	2.08	207745	0.0228
0.0228	2.07	207289	0.0228



Thus, the flow velocity through the cast iron pipe is 2.07 m/s, and the volume flow rate is $1.63 \times 10^{-2} \text{ m}^3/\text{s}$

Colebrook calculator

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Calculates the root of Colebrook-White Equation using Simple and TRUE method.

Colebrook-White Equation mode 2.51 $\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{Rr}{3.7} + \frac{2.51}{Rn \sqrt{f}} \right)$

Rr : relative roughness
Rn : Reynolds Number

Rr 0.0015 ← ϵ/D
Rn 215991

Executes Clear Store/Read Print 14digit

Loops 5
f 0.022728784339004 ← Obtained value of f
Left 6.633028999606
Right 6.633028999606

Simple and TRUE method
(1) $X = -2 \log_{10} \left(\frac{Rr}{3.7} + \frac{2.51}{Rn X} \right)$ at mode 2.51
(2) $X_0 = 3$

So, the first value of f we start is 0.021. The value of V from the formula $V = \frac{0.313}{\sqrt{f}}$, we get the value of V to be 2.16 m/s; from which we calculate Reynolds number as 215991 from Colebrook calculator. So the value of f we get from the calculator is 0.0227. This value can also be obtained from the Moody chart. The next round we start with this value, 0.0227. And we get a value of V as 2.08, and the value of Reynolds number 207745. And the value of f from this is a slightly larger, 0.0228.

And with this value when you put in there, I get velocity as 2.07; and the Reynolds number as 207289, slightly lower, but the value of f is the same at 0.0228. Thus, the value of f has converged, and we get the flow velocity 2.07 as an estimate of the flow velocity through the cast iron pipe. The volume flow rate corresponding to this velocity is obtained by multiplying this velocity with the cross-sectional area of the pipe, and we get $1.63 \times 10^{-2} \text{ m}^3/\text{s}$.

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The design problem

The design problem is one that needs the determination of the pipe diameter to carry a given flow rate with a given head loss is rather cumbersome as the unknown diameter D occurs in both the ordinates of the Moody's chart and in the parameter ε/D .

It is useful to eliminate velocity of flow from the parameters, and express the variables of Moody's chart in terms of the flow rate \dot{Q} , D and h_L .

$$\text{Thus, } f = \frac{2h_L}{gV^2} \cdot \frac{D}{L} = \frac{\pi^2 g h_L}{8\dot{Q}^2 L} D^5,$$
$$\text{or } D = \left(f \frac{8\dot{Q}^2 L}{\pi^2 g h_L} \right)^{1/5}$$
$$Re_D = \frac{\rho V D}{\mu} = \frac{4\rho \dot{Q}}{\pi D \mu}$$



The design problem

The following iterative procedure can be used:

- since we have no idea where to start, pick *any* value as an initial guess for f . A value of 0.03 in the middle of the range of f is a good starting point.
 - estimate D from the given values of \dot{Q} , L and h_L : $D = \left(f \frac{8\dot{Q}^2 L}{\pi^2 g h_L} \right)^{1/5}$
 - estimate Re_D : $\frac{4\rho \dot{Q}}{\pi D \mu}$
 - estimate ε/D ,
 - using the estimated values of Re_D and ε/D , look-up the new estimate of f using Moody's chart,
- Iterate till the required convergence is obtained.



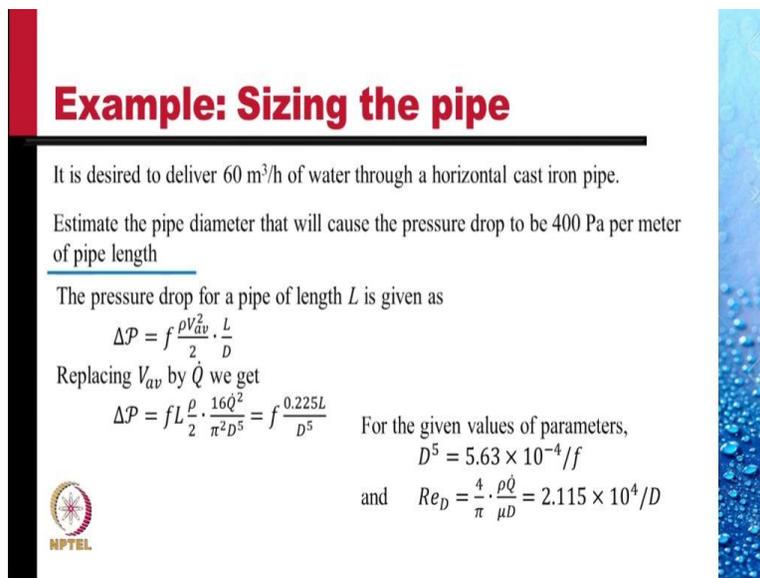
Next we do a design problem. The design problem is one that needs the determination of the pipe diameter to carry a given flow rate with the given head loss. And it is rather cumbersome as the unknown diameter D occurs in both the ordinate of the Moody chart and in the parameter ε/D . It is useful to eliminate velocity of the flow from the parameters, and express the variables of the Moody chart in terms of the flow rate \dot{Q} , D and h_L . Then, out of these \dot{Q} and h_L are known and D is the unknown. So, the friction factor which is $\frac{2h_L}{gV^2} \cdot \frac{D}{L}$ becomes $\frac{\pi^2 g h_L}{8\dot{Q}^2 L} D^5$.

And from this we can express D like this in terms of \dot{Q} and h_L . The value of f is not known here, all other quantities are known. And then the Reynolds number can be expressed as $Re_D = \frac{4\rho\dot{Q}}{\pi D\mu}$.

The following iterative procedure can be used.

Since we have no idea where to start, pick any value as an initial guess for f . A value of 0.03 in the middle of the range of f is a good starting point. Estimate D from the given value of \dot{Q} , L and h_L from the formula for the diameter that we just derived, and then estimate Reynolds number. Once we know D , we can calculate ε/D . Then using the estimated value of Reynolds number and ε/D , we can look up a new estimate of f from the Moody chart; and iterate till the required convergence is obtained.

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Example: Sizing the pipe

It is desired to deliver 60 m³/h of water through a horizontal cast iron pipe.

Estimate the pipe diameter that will cause the pressure drop to be 400 Pa per meter of pipe length

The pressure drop for a pipe of length L is given as

$$\Delta P = f \frac{\rho V_{av}^2}{2} \cdot \frac{L}{D}$$

Replacing V_{av} by \dot{Q} we get

$$\Delta P = f L \frac{\rho}{2} \cdot \frac{16\dot{Q}^2}{\pi^2 D^5} = f \frac{0.225L}{D^5}$$

For the given values of parameters,

$$D^5 = 5.63 \times 10^{-4} / f$$

and $Re_D = \frac{4}{\pi} \cdot \frac{\rho\dot{Q}}{\mu D} = 2.115 \times 10^4 / D$



We will show this process through an example. It is desired to deliver 60 m³/hr of water through a horizontal cast iron pipe. Estimate the pipe diameter that will cause the pressure to drop at 400 Pa/m of pipe length. The pressure drop for the pipe of length L is given by this formula. And replacing V_{av} by \dot{Q} , $V_{av} = \frac{\dot{Q}}{\frac{\pi}{4}D^2}$. We get ΔP in terms of \dot{Q} , D and f . And we plug in the value of the known quantities, I get $\Delta P = f \frac{0.225L}{D^5}$. For the given value the parameters, D^5 can be written as $5.63 \times 10^{-4} / f$; and the Reynolds number can be written as this.

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Example: Sizing the pipe

We start the iterations assuming the value of f as 0.03, calculate the diameter D , calculate the values of ε/D and Re_D , estimate a new value of f from the Moody's chart or from the Colebrook calculator, and repeat till the required convergence is achieved.

f	D	ε/D	Re_D	f from calculator
0.03	0.0188	0.00639	1126998	0.03280
0.03280	0.0172	0.00699	1232185	0.03376
0.03376	0.0167	0.00720	1268249	0.03408
0.03408	0.0165	0.00726	1280270	0.03417
0.03417	0.0165	0.00728	1283651	0.03417

 The required diameter converges to 16.5 mm in 5 iterations

So, now, we are ready to start iteration. We start iteration assuming the value of f as 0.03. Calculate the diameter D , calculate the value ε/D and Re_D . Estimate a new value of f from the Moody's chart or from the Colebrook calculator, and repeat till the required convergence is achieved. So, we start from f is equal to 0.03, and we get f from calculator as 0.03280.

These calculations can be done very easily on a spreadsheet like Excel with formulae embedded; so, the repeated calculation do not take much time. Then using this value in the second iteration, we get a new value as 0.03376. Value f is increasing. I plug in this value next, and I get the value as 0.03408; it is not converged yet. And now I do it two more times, I get the converged value of f as 0.03417. And then this is the converged diameter of the required pipe. That is, we need a pipe of 16.5 millimeter diameter to carry the required flow with the given pressure drop.