Fluid Mechanics and its Applications
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Lecture 19A
Pipe Flow Problems

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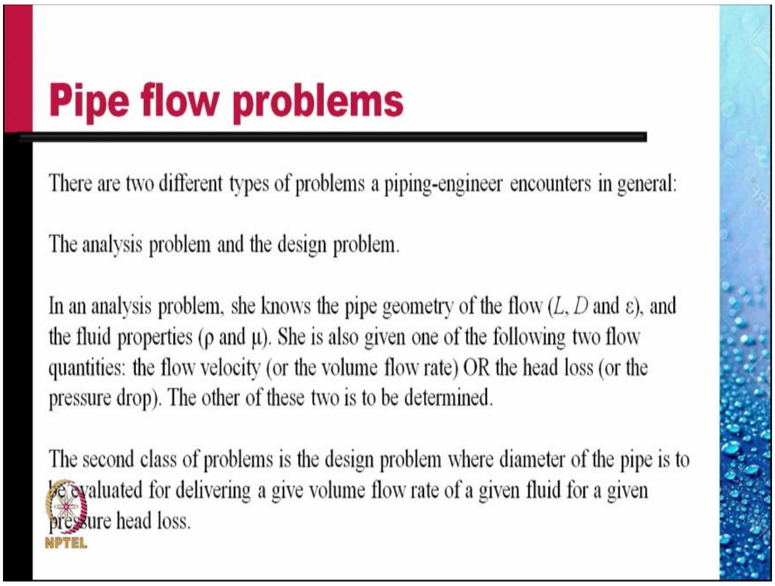

Now, there are two different types of problems that a piping engineer encounters in general. One is the analysis problem, and the other is the design problem. In an analysis problem the piping engineer knows the pipe geometry, that is the L, D and ε, and the fluid properties ρ and μ. She is also given one of the following two flow quantities, the flow velocity or the head loss. The other of these two is to be determined.

On the other hand, a design problem is one where the diameter of the pipe to carry a given flow rate for a given pressure head loss is to be determined. So, we have to determine what size of pipe we should use to carry a given flow rate with the given pressure difference. The first type of problem, the analysis problem, is easier than the second class of problems.

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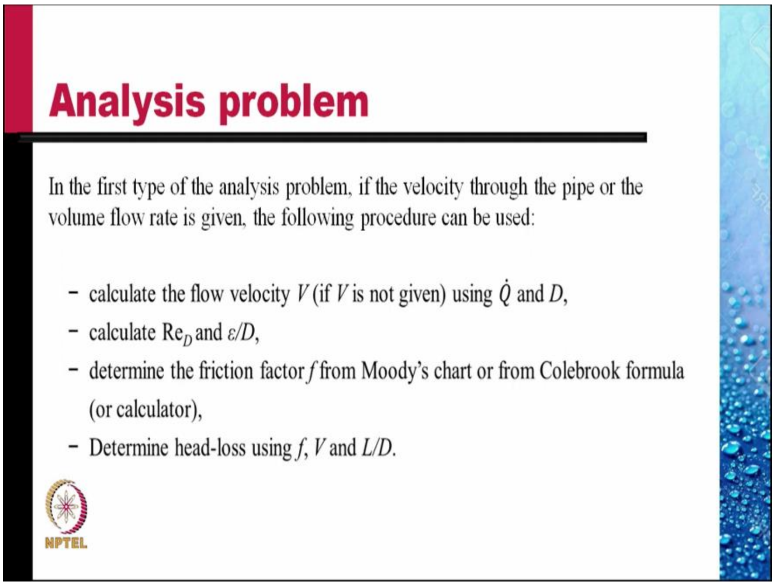

There are two kinds of analysis problems. In the first type of analysis problem, that is, when the velocity or the flow rate through the pipe is given, and we are to find out the pressure difference. The process that is used is to calculate the flow rate or rather calculate the flow velocity V, if V is not given using $\dot{Q}$ and D. Once velocity is known, calculate Reynolds number, and since the diameter is given, we can calculate ε/D. Determine the friction factor from the Moody's chart or from the Colebrook formula or calculator. And then using f, V and L/D, determine the head loss.

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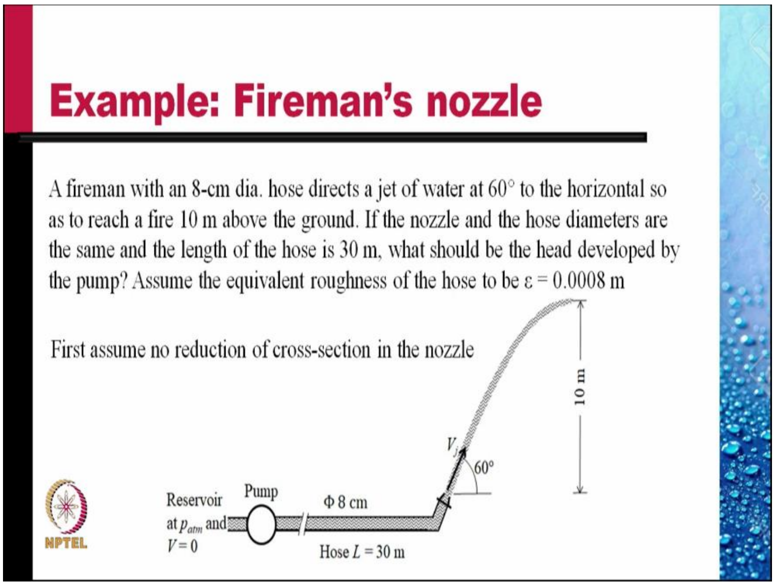

We will do an example. A fireman with an 8-cm diameter hose directs a jet of water at 60° to the horizontal so as to reach a fire 10 m above the ground level. If the nozzle and the hose diameters are the same, that is, the nozzle is not reducing the flow area, and the length of the hose is 30 m what should be the head developed by the pump? Assume the equivalent roughness of the hose to be 0.008 m that is 0.8 mm.

Now, the head required by the pump is for two purposes. One is to give the jet of water enough velocity so that it can reach a 10-m height, and the second part of the head required is to overcome the losses in the 30-m length of the hose. So, we will first determine the velocity required.

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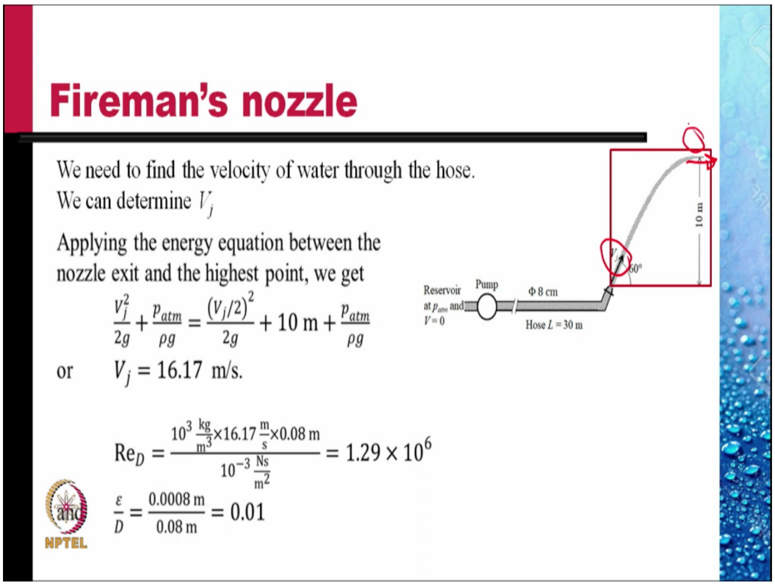

Assume no reduction in the cross section in the nozzle. We need to find the velocity of water through the hose. We can determine V_j. V_j is determined by applying the simple kinematic equations to particles of water in the jet region. If it comes out with the velocity V_j at 60° their horizontal component is $V_j \cos 60°$, and the vertical component $V_j \sin 60°$.

During the time the jet rises, the vertical component reduces because of gravitational force downwards, and at the top, the velocity component in the vertical direction is 0. The horizontal component is unaffected if we neglect the drag in air. The horizontal component to begin with $V_j \cos 60°$, that is, $V_j/2$. So, the velocity at the upper end is simply $V_j/2$, there being no vertical component. So, the velocity is V_j, the velocity here is $V_j/2$.

We can apply Bernoulli equations between these two points, the pressures at the two points are same, atmospheric, but there is a difference in elevation, and that difference in elevation is 10 m. Applying the Bernoulli equation, we get the jet velocity to be 16.17 m/s. Now, this should be the velocity through the hose. We can calculate the Reynolds number through the hose based on the diameter of the hose which is 80 mm, 8 cm, and this Reynolds number comes out to be 1.29×10^6, and given the ε/D, we can find out the roughness parameter, relative roughness parameter as 0.01.

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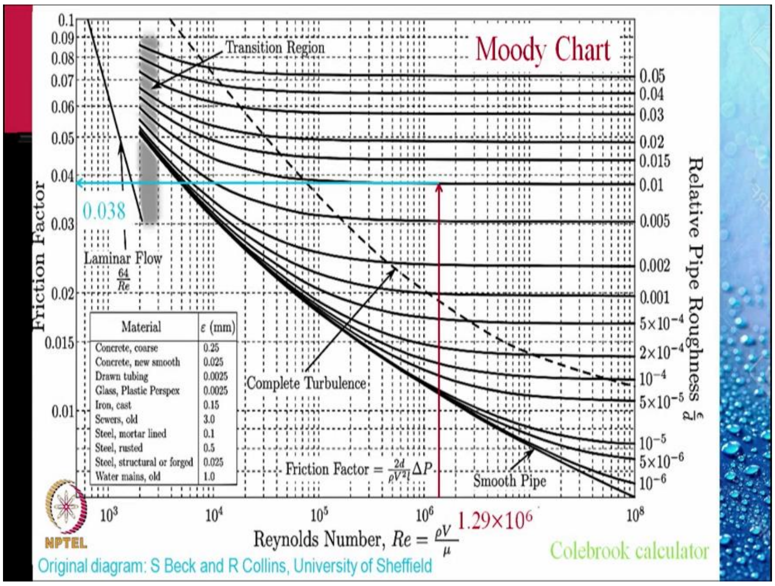

So, then we can go to Moody chart. Enter at the Reynolds number of 1.29×10^6, and go up to the relative roughness of 0.01 that we determined. And so, that is the point that we reach. And from this, we can read the value of the friction factor as 0.038 on this scale. So, the Darcy friction factor for the flow through the hose is 0.038. We could also find out the friction factor from the calculators online and we will get the same value.

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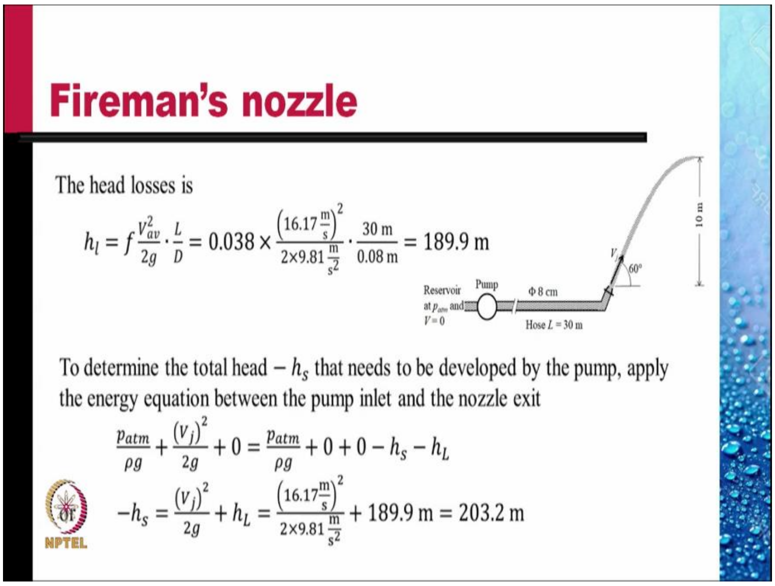

Once we know the value of f, I can calculate the head loss as $f\frac{V_{av}^2}{2g}\cdot\frac{L}{D}$, and this comes out to 189.9 m. To determine the total head, $-h_s$, that needs to be developed by the pump, negative because this is the work being done on the fluid. Apply the energy equation between the pump inlet and the nozzle exit. And we apply this equation, we find out that the head developed by the pump must be 203.2 meters.

To just pump the water 10 meters in height, we need to develop a head of 203 meters. Our calculations are not wrong, but it is too much. So, what should be done? And the solution is simple. Reduce the flow speed, because the head loss depends on the square of the velocity through the pipe. So, if we can reduce the velocity through the pipe, but the velocity at the jet must still remain the same 16.17 m/s if we have to reach a height of 10 m. So, how do you reduce the velocity through the hose while keeping the jet velocity constant:. is by using a reduction nozzle at the pipe.

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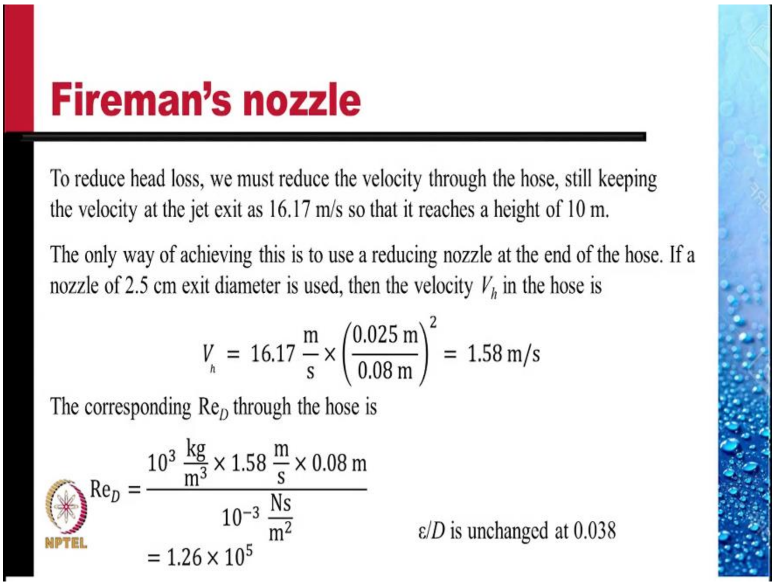

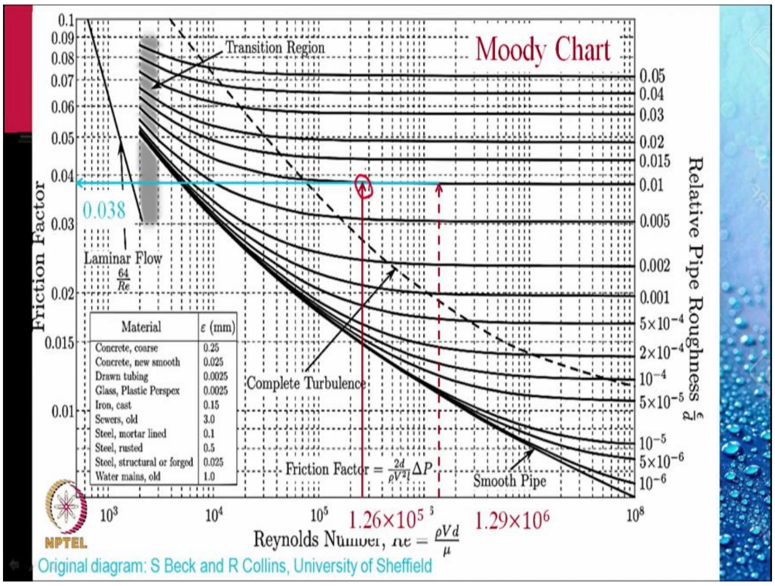

So, you must reduce the velocity through the hose still keeping the velocity at the jet exit at 16.17 m/s, so that it reaches a height of 10 m. The only way of achieving this is to use a reducing nozzle at the end of the hose. If a nozzle or 2.5-cm exit diameter is used, then the velocity V_h in the hose is 1.58 m/s.

So, while the velocity in the jet is still 16.17 m/s, the velocity within the hose is 1.58 m/s. The corresponding Reynolds number through the hose is now reduced to 1.26×10^5. ε/Dremains unchanged because we have not changed the diameter of the hose, and from the Moody chart for the new Reynolds number 1.26×10^5, the point is here and so, f is unchanged at 0.038.

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But, even with this *f*, the head loss now is a measly 1.81 m compared to about 192 m. So, the total head required now is 15.13 m, compared to 203.2 m without the use of the reducing nozzle.

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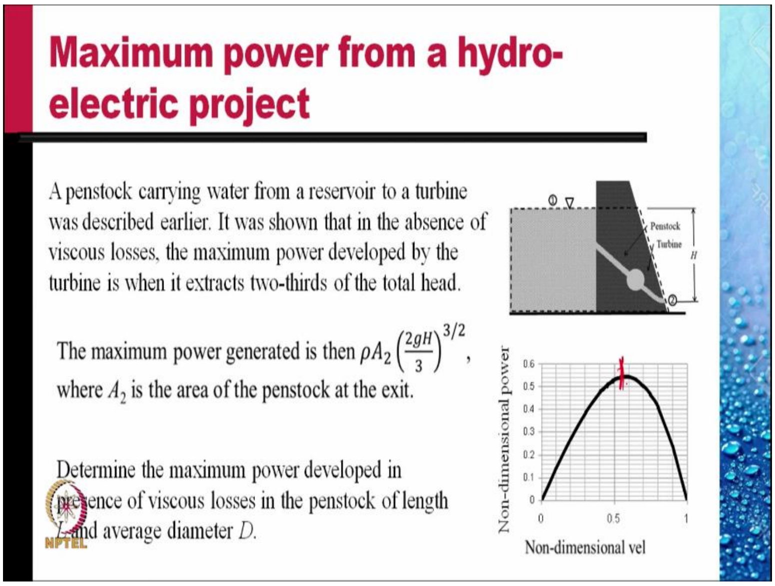

Let us do one more example. The maximum power from hydroelectric project. A penstock carrying water from a reservoir to a turbine was described earlier. It was shown that in the absence of viscous losses, the maximum power developed by the turbine is when it extracts two-thirds of the total head, so that the maximum power generated is $\rho A_2 \left(\frac{2gH}{3}\right)^{3/2}$. The curve of non-dimensional velocity versus non-dimensional power was something like this. So, we get a maximum power when two-thirds of the head was extracted, and one-third of the head was left with the velocity. Now, determine the maximum power developed in the presence of viscous losses in the penstock. Until now we have not taken the viscous losses in the penstock. Penstock is of length L and average diameter D.

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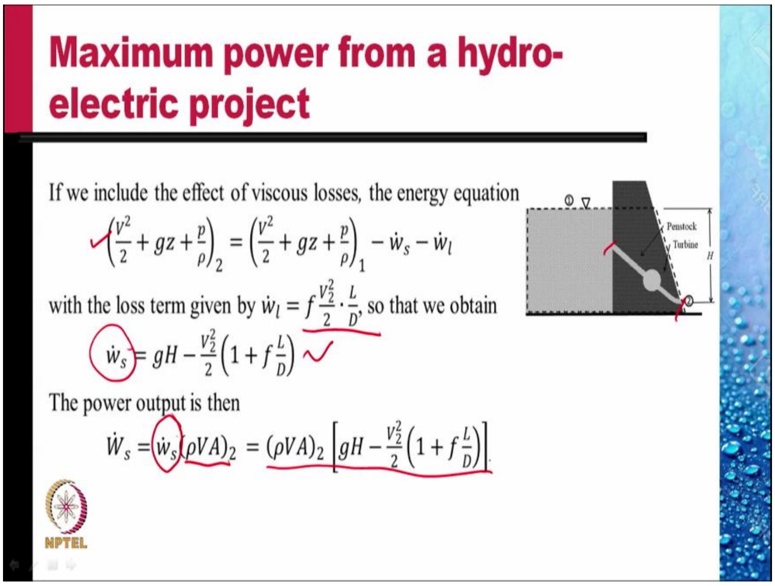

If we include the effect of viscous losses, the energy equation, which is this, and the loss is $f\frac{V_2^2}{2}\frac{L}{D}$. Velocity through the penstock from here to the exit V_2 is the same at V_2. So, we are using this as the head loss, the energy extracted per unit mass throughput is now given as this. The power output is the energy extracted per unit mass throughput times the mass throughput, mass flow rate through the penstock. And this is the expression that we get.

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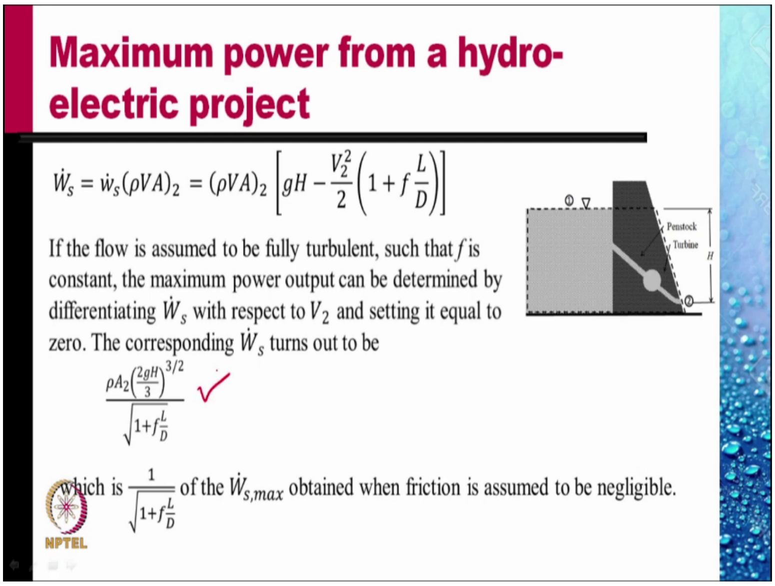

If this flow is assumed to be fully-turbulent, we can assume f to be constant, independent of the Reynolds number, and then the maximum power output can be obtained by differentiating $\dot{W}_s$with respect to V_2 and setting it equal to 0. The corresponding $\dot{W}_s$ turns out to be this, which is $\frac{1}{\sqrt{1+f\frac{L}{D}}}$ of the maximal power in the absence of friction.

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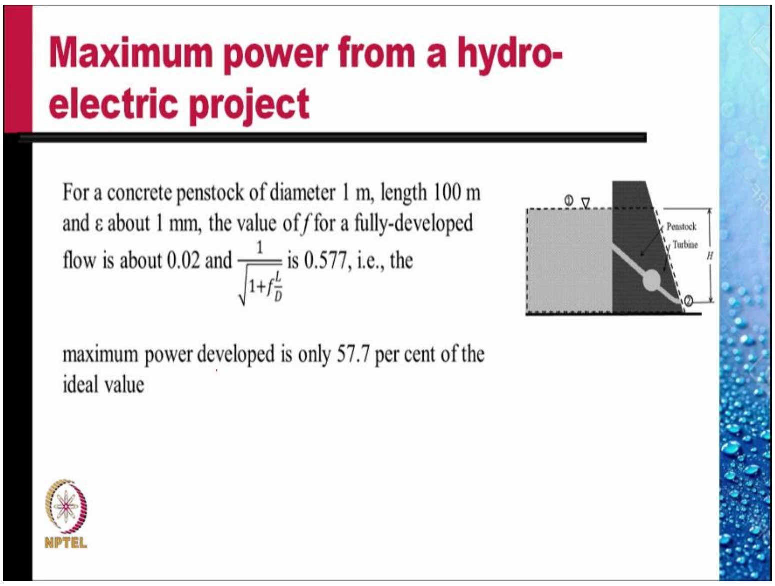

For a concrete penstock of diameter 1 m and length 100 m and ε about 1 mm for concrete, the value for the fully developed flow is f is equal to 0.02, so that $\frac{1}{\sqrt{1+f\frac{L}{D}}}$ is 0.577, that is, the maximum power developed is only 57.7 percent of the ideal power when friction was negligible. There is quite a bit of loss of power.

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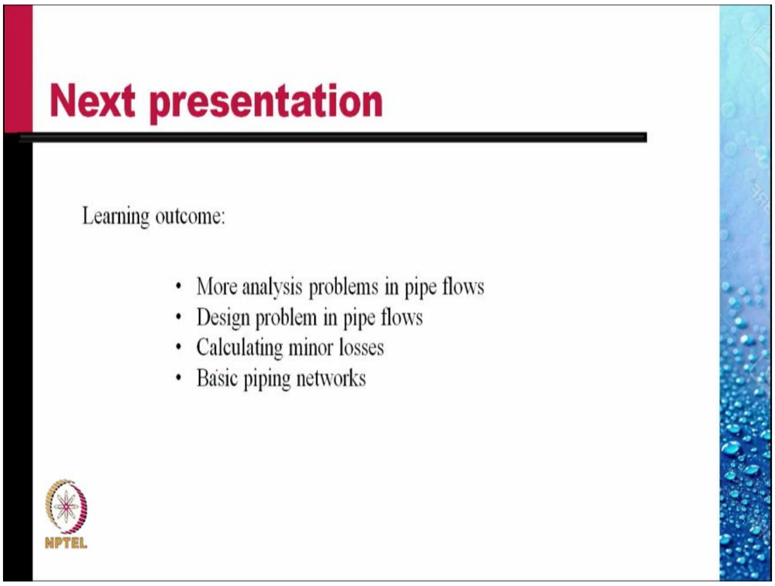

Thank you very much.