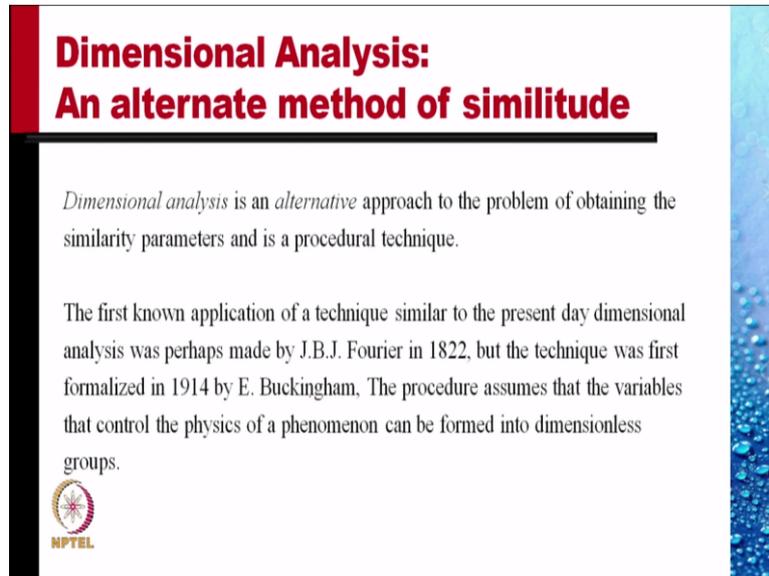


Fluid Mechanics and its Applications
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Lecture 17A
Dimensional Analysis

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**Dimensional Analysis:
An alternate method of similitude**

Dimensional analysis is an *alternative* approach to the problem of obtaining the similarity parameters and is a procedural technique.

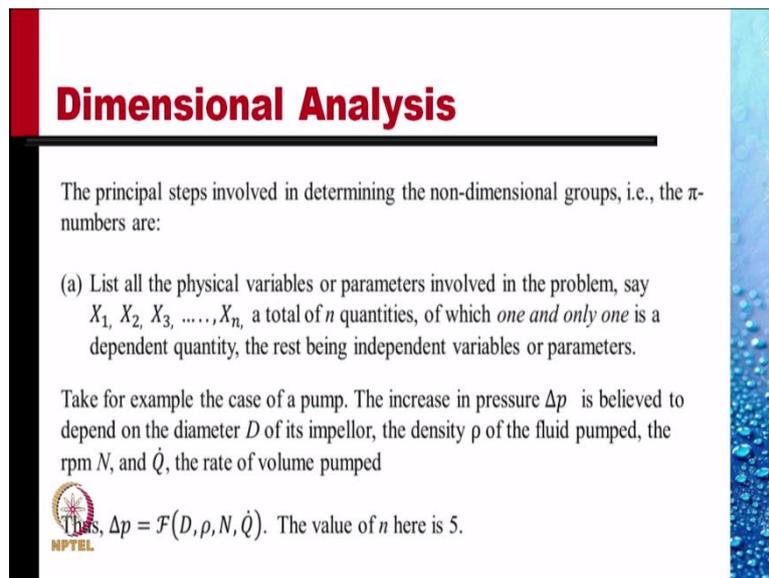
The first known application of a technique similar to the present day dimensional analysis was perhaps made by J.B.J. Fourier in 1822, but the technique was first formalized in 1914 by E. Buckingham. The procedure assumes that the variables that control the physics of a phenomenon can be formed into dimensionless groups.



We next introduce an alternative method of similitude which is known as dimensional analysis. Most books in fluid mechanics deal only with dimensional analysis as a method of similitude. We believe that this is a method which is less powerful and is more difficult to use. That is why we kept it for the end.

The first known application of a technique similar to the present-day dimensional analysis was made by J.B.J Fourier of the Fourier analysis fame in 1822. But the technique was first formalized in 1914 by E. Buckingham. The procedure assumes that the variables that control the physics of a phenomenon can be formed into dimensionless groups.

(Refer Slide Time: 01:37)



Dimensional Analysis

The principal steps involved in determining the non-dimensional groups, i.e., the π -numbers are:

(a) List all the physical variables or parameters involved in the problem, say $X_1, X_2, X_3, \dots, X_n$, a total of n quantities, of which *one and only one* is a dependent quantity, the rest being independent variables or parameters.

Take for example the case of a pump. The increase in pressure Δp is believed to depend on the diameter D of its impellor, the density ρ of the fluid pumped, the rpm N , and \dot{Q} , the rate of volume pumped

Thus, $\Delta p = F(D, \rho, N, \dot{Q})$. The value of n here is 5.



We would give here a kind of a recipe for carrying out the dimensional analysis. The recipe is rather simple, but the results that are obtained need to be interpreted carefully. The principal steps involved in determining the non-dimensional groups, which are the pi numbers that we have introduced earlier, are a list of all the physical variables or parameters involved in the problem. Say, $X_1, X_2, X_3, \dots, X_n$, a total of n quantities, of which one and only one is a dependent quantity, a dependent variable, or a dependent parameter. Typically, a dependent parameter, the rest being independent variables or parameters.

Take for example, the case of a pump, a water pump like we discussed earlier. The increase in pressure Δp is believed to depend on the diameter D of its impeller, the density ρ of the fluid pumped, the rpm N , and \dot{Q} , the rate of volume pump. So, we can write Δp as a function of D, ρ, N and \dot{Q} . Here the value of n is 5. There are 5 parameters involved. One of them Δp we treat as the dependent parameter, and the other four are independent parameters.

(Refer Slide Time: 03:35)

Dimensional Analysis

(b) Choose a system of dimensions. For most fluid mechanics problems, a system based on mass (M), length (L), time, and (T) is an appropriate system.

(c) Write the dimensions of all the relevant variables in the chosen system. Determine the minimum number r of the dimensions required to express these variables. For most problems of fluid mechanics, this number equals 3.

Variable	M	L	T
\dot{Q} (independent)	0	3	-1
D (independent)	0	1	0
ρ (independent)	1	-3	0
N (independent)	0	0	-1
Δp (dependent)	1	-1	-3

$r = 3$



Choose a system of dimensions. For most fluid mechanics problems, a system based on mass, length and time, M, L, T , is an appropriate system. Write the dimension all variables in the chosen system. Determine the minimum number r of the dimensions required to express these variables. For most problems in fluid mechanics, this number equals 3, that is, dimensions all mass, length and time are needed.

So, for n variables, of which Δp is dependent and the other four are independent, we write the dimensions of them. For \dot{Q} the dimensions are $L^3 T^{-1}$. For diameter, obviously is L , density is ML^{-3} , for rpm it is -1 in time, and for Δp it is $ML^{-1}T^{-3}$.

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Dimensional Analysis

A fundamental theorem known as *Buckingham pi-theorem* states that

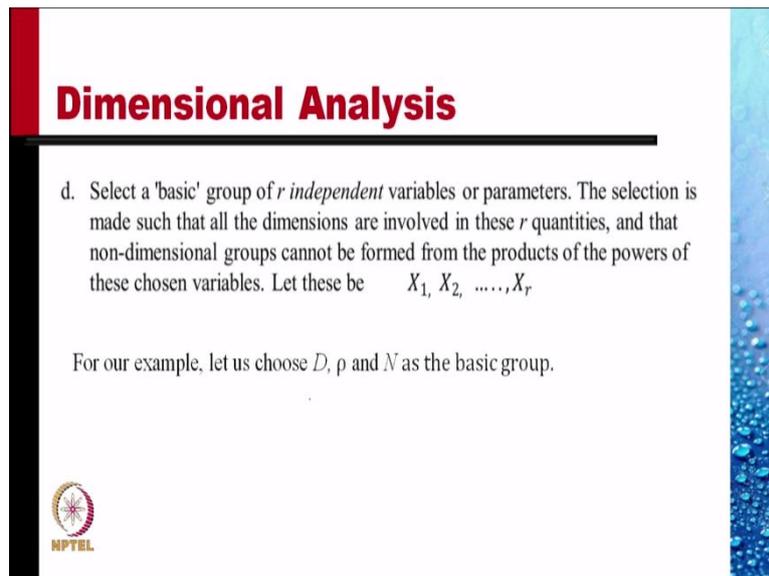
The total number of non-dimensional groups of parameters which can be formed by combining the n physical parameters and variables of a problem is equal to $(n - r)$, where r is the number of primary dimensions required to express the dimensional formulae of the n physical quantities.

Therefore, in this example we should have two, $(= 5 - 3)$ non-dimensional numbers



A fundamental theorem known as Buckingham pi-theorem states that the total number of non-dimensional groups of parameter which can be formed by combining the n physical parameters and variables of the problem, is equal to n minus r , where r is the number of primary dimensions required to express the dimensional formula of the n physical quantities. So, with 5 physical quantities, n is equal to 5 and r is equal to 3. So, we can form two independent pi numbers out of these.

(Refer Slide Time: 05:33)



Dimensional Analysis

d. Select a 'basic' group of r *independent* variables or parameters. The selection is made such that all the dimensions are involved in these r quantities, and that non-dimensional groups cannot be formed from the products of the powers of these chosen variables. Let these be X_1, X_2, \dots, X_r

For our example, let us choose D, ρ and N as the basic group.



Select a basic group of r independent variables or parameters. The selection is made such that all the dimension are involved in these r quantities, and that non-dimensional groups cannot be formed from the products of the powers of those chosen variables. Let these be X_1, X_2, \dots, X_r . In our example, let us choose D, ρ and N as the basic group. The dimensions of length are contained in D and ρ , the dimension of mass are contained in only ρ , and the dimension of time are contained only in N . All three dimensions are represented, and we cannot form a non-dimensional group out of these parameters.

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Dimensional Analysis

(e) It is now possible to form $(n - r)$ independent π 's out of the variables
 $\pi_i = X_i X_1^{a_1} X_2^{a_2} X_3^{a_3} \dots X_r^{a_r}$ for i from $r + 1$ to n .

	Variables				
	Basic group			Others	
	D	ρ	N	Δp	\dot{Q}
Dimensions	L	ML^{-3}	T^{-1}	$ML^{-1}T^{-2}$	L^3T^{-1}
Non-dimensional parameters	D^a	ρ^b	N^c		
	Values of exponent				
Π_2	-3	0	-1		\dot{Q}/ND^3
Π_1	-2	-1	-2	$\frac{\Delta p}{\rho N^2 D^2}$	

It is now possible to form n minus r independent π numbers out of these variables. π_i is X_i of the remaining variables what we have not chosen as a fundamental parameter into the first parameter to power a_1 , the second parameter raised to power a_2 , and so on. In this i varies from from $r + 1$ to n .

We organize this as in this table. We have chosen the basic D , ρ and N the other parameters are Δp which is dependent and \dot{Q} which is independent. We write the dimensions of this basic group. L is the dimension of D , ML^{-3} is the dimension of density, T^{-1} is the dimension of the rpm.

Similarly, for Δp and \dot{Q} . We form a non-dimensional parameter at D^a , ρ^b , N^c , and choose a , b and c such that they match up with the dimensions of Δp and \dot{Q} . So, we form two π numbers. The first we look at what we have formed with dependent parameters Δp .

The power a , b and c are found to be -2 , -1 , and -2 . So, that $\frac{\Delta p}{\rho N^2 D^2}$ is a π number, and the other with the independent parameter \dot{Q} and the value of exponents are 0 , -3 , and -1 , so that this comes out to \dot{Q}/ND^3 . This is for the independent parameter and this for the dependent π number.

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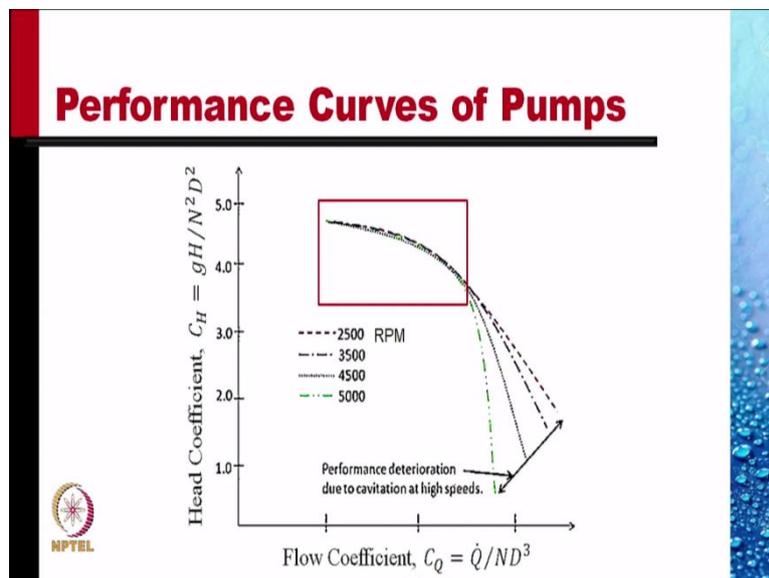
Dimensional Analysis

$\frac{\Delta p}{\rho N^2 D^2}$, the pressure developed by the pump is usually expressed in terms of the head H of the fluid, such that $\Delta p = \rho g H$. Using this our non-dimensional dependent parameter becomes $\frac{gH}{N^2 D^2}$



So, we can say $\frac{\Delta p}{\rho N^2 D^2}$, the pressure developed by pump which is usually expressed in terms of the head H of the liquid such that $\Delta p = \rho g H$. Using this our non-dimensional dependent parameter becomes $\frac{gH}{N^2 D^2}$. Dependent parameter.

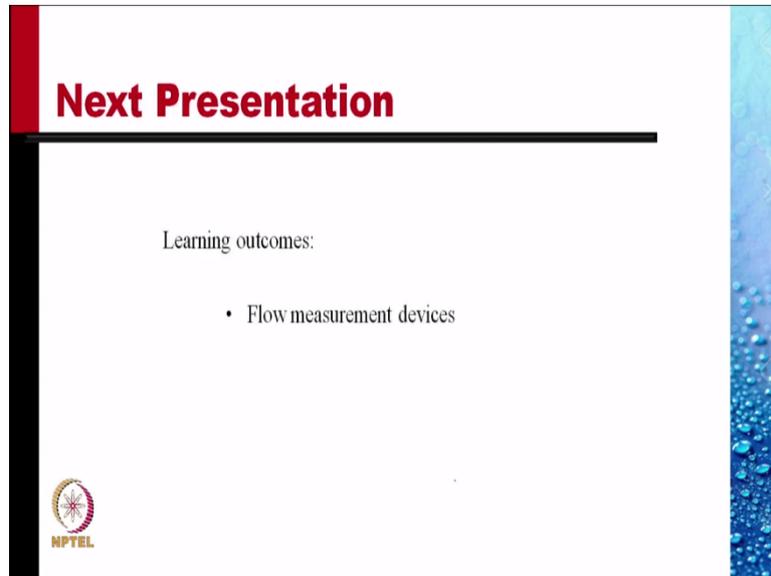
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And the independent parameter is \dot{Q} / ND^3 . So, there is only one dependent parameter and one independent parameter. So, we can get one graph. For a pump, if we plot the head coefficient C_H against the flow coefficient C_Q , we get one single curve at all rpms. The deviation at higher speeds is largely due to cavitation that takes place at higher speeds, and the higher the speeds the cavitation take place earlier. That is why there is a performance deterioration due to cavitation. These curves separate out only because a new phenomenon is introduced. As

long as cavitation is not taking place, the only relevant independent parameter was the flow coefficient \dot{Q}/ND^3 and so, the head coefficient was a simple function of this a line for all RPM's, independent of the RPM.

(Refer Slide Time: 11:23)



Thank you