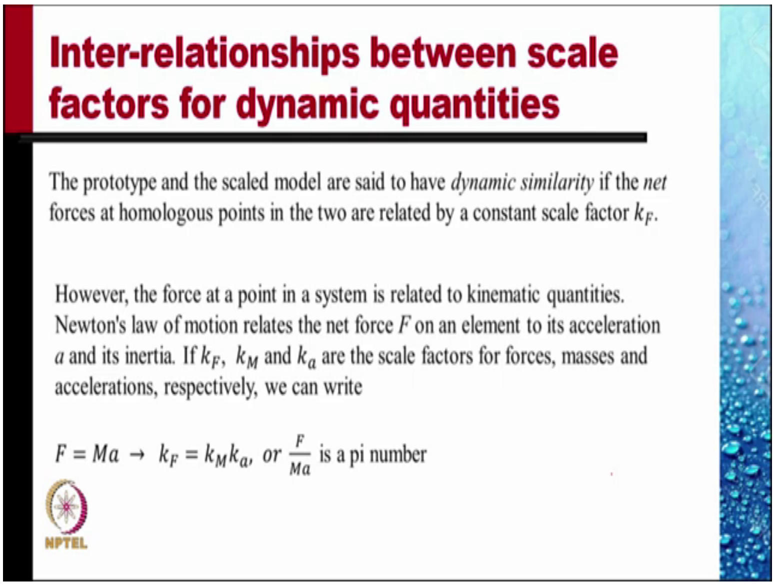
Fluid Mechanics and its Applications
Professor Vijay Gupta
Indian Institute of Technology, Delhi
Lecture – 16A
Similitude through Scale Factors (continued)

(Refer Slide Time: 00:19)

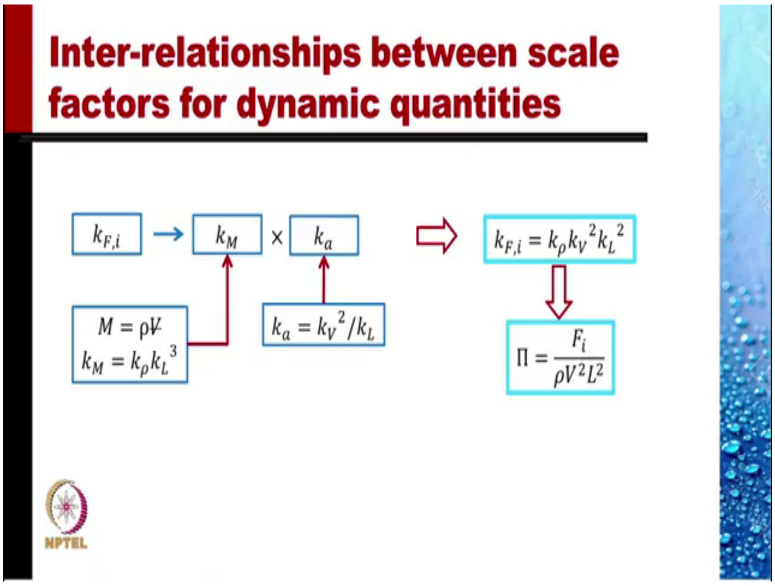


Now, let us move to dynamical quantities.

The prototype and the scaled model are said to have dynamic similarity if the net forces at the homologous points in the two are related by a constant scale factor k_F.

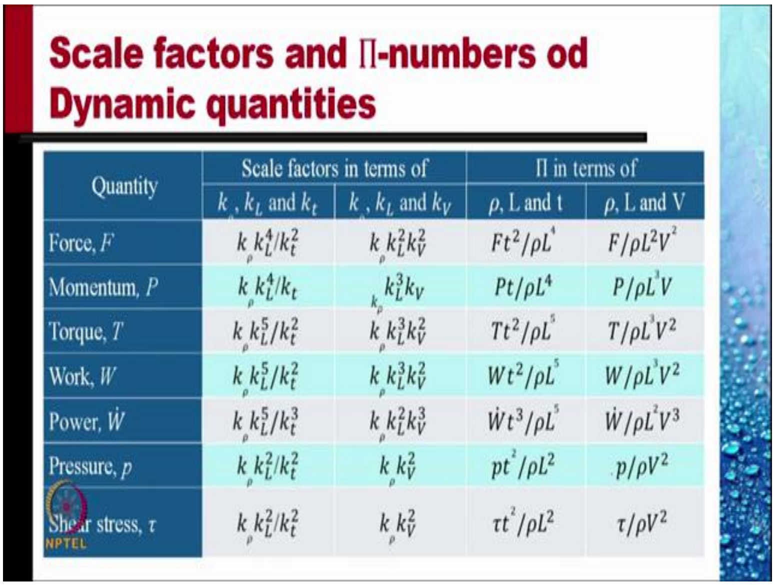
However, the force at a point in a system is related to kinematic quantities. Newton's law of motion relates a net force F on the element to its acceleration a and inertia. If k_F, k_M and k_a are the scale factors for forces, masses and accelerations, respectively, we can write $F = Ma$ which gives us k_F is equal to $k_M k_a$, or converting into pi number, that F/Ma is a pi number.

(Refer Slide Time: 01:26)



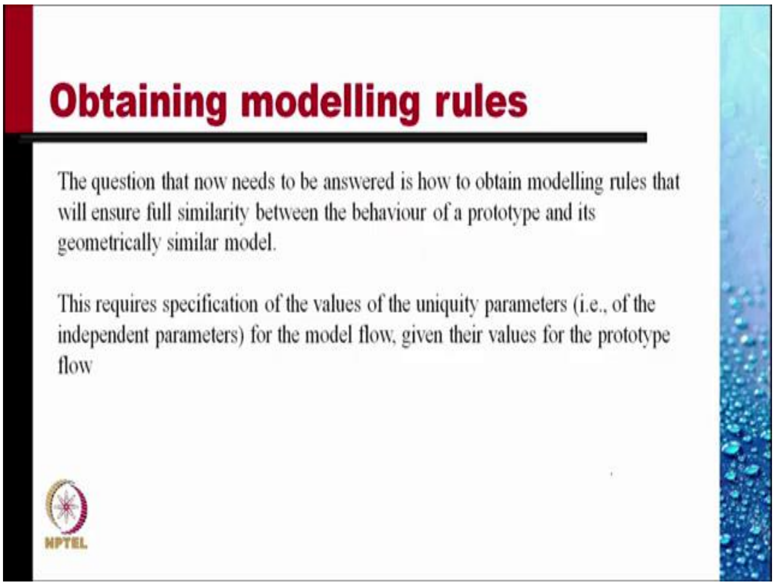
Now, we can manipulate them like this, $k_{F,i}$ gives you $k_M \times k_a$. But k_M is related to the density times the volume, mass is ρ times the volume, so k_M is like $k_\rho k_L^3$. Similarly, the acceleration is $k_V{}^2/k_L$. How? Recall from Euler acceleration formula, the convective acceleration was $V\frac{dV}{ds}$. V scaled by k_V, $\frac{dV}{ds}$ is scaled by k_V/k_L k V. So, that k_a becomes $k_V{}^2/k_L$. So, that $k_{F,i}$ is equal to $k_\rho k_V{}^2 k_L{}^2$, and from this we obtained a pi number. So, pi is equal to $\frac{F_i}{\rho V^2 L^2}$.

(Refer Slide Time: 03:07)



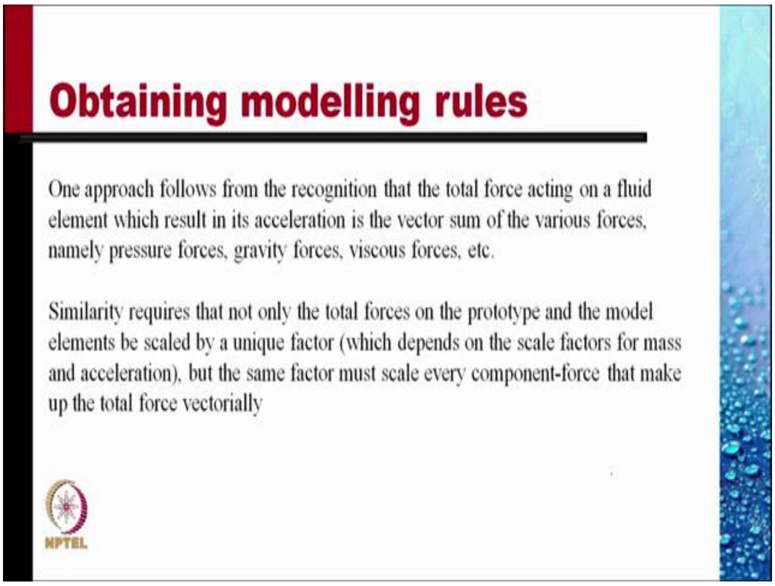
Here we have listed various dynamic quantities, like force, momentum, torque, work, power, pressure and shear stress. We have first written the scale factors in terms of k_ρ, k_L and k_t, and then in terms of k_ρ, k_L and k_V. And then from these scale factors relation, we have developed the pi numbers.

(Refer Slide Time: 03:45)



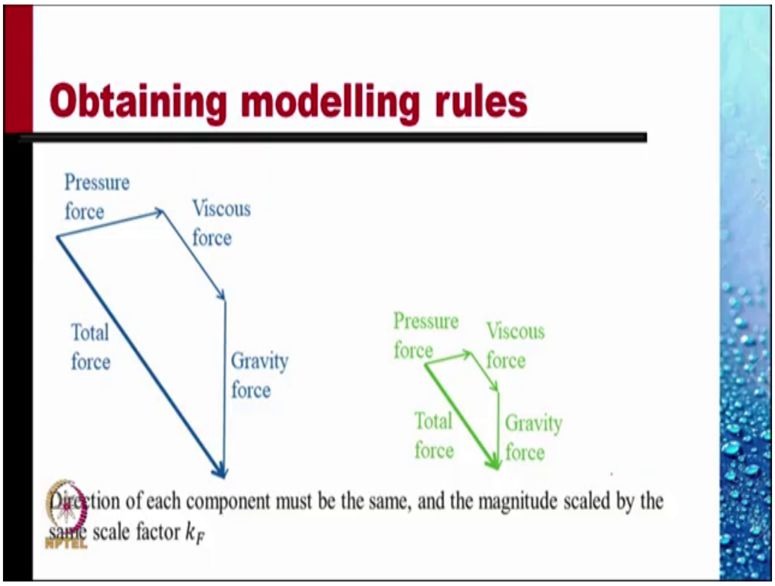
The question that now needs to be answered is, how the modelling rules are obtained? The modelling rules that will ensure full similarity between the behaviour of the prototype and its geometrically similar model. This requires specification of the values of all the uniquity parameters, that is, of all the independent parameters for the model flow, given their values for the prototype flow.

(Refer Slide Time: 04:20)



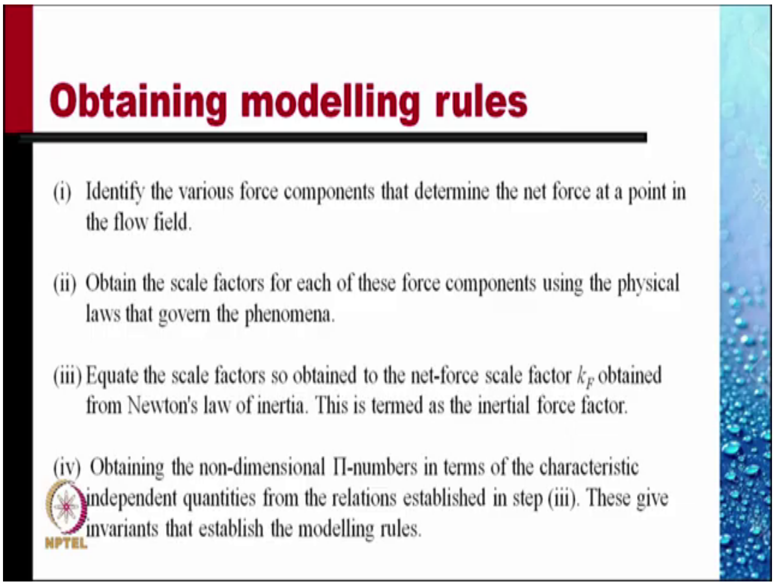
One approach follows from the recognition that the total force acting on a fluid element which results in its acceleration is the vector sum of the various forces, such as pressure force, gravity force, viscous force, etcetera. Similarity requires that not only the total forces on the prototype and the model be scaled by a unique scale factor, which depends on the scale factors for mass and acceleration, but the same scale factor must scale every component force that make up the total force vectorially.

(Refer Slide Time: 05:13)



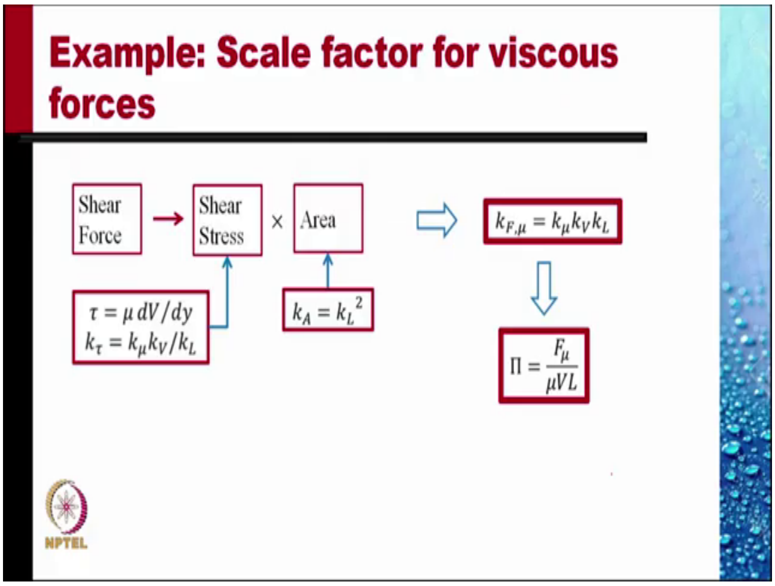
Thus, if the total force in the prototype is the sum of these three forces: pressure force, viscous force, and gravity force, the vectorial sum of these gives the total force. Then in the model, every force would be scaled by the same scale factor, their directions remaining the same. So, it is not only the pressure forces have the same scale factor, but the x component of the pressure forces also would have same scale factor, the y component of pressure forces, the direction being the same. Similarly, for all forces. This is the basis on which we construct the modelling rules.

(Refer Slide Time: 06:13)



So, the method consists of: identify the various force components that determine the net force at a point in the flow field. Obtain the scale factor for each of these force components using the physical laws that govern the phenomena. Then equate the scale factors so obtained to the net-force scale factor k_F obtained from Newton's law of inertia. This last is termed as a inertial force factor. And then, obtaining the non-dimensional pi numbers in terms of the characteristic independent quantities from the relations established in the step above. These give invariants, the Pi numbers, that establish the modelling rules.

(Refer Slide Time: 07:13)

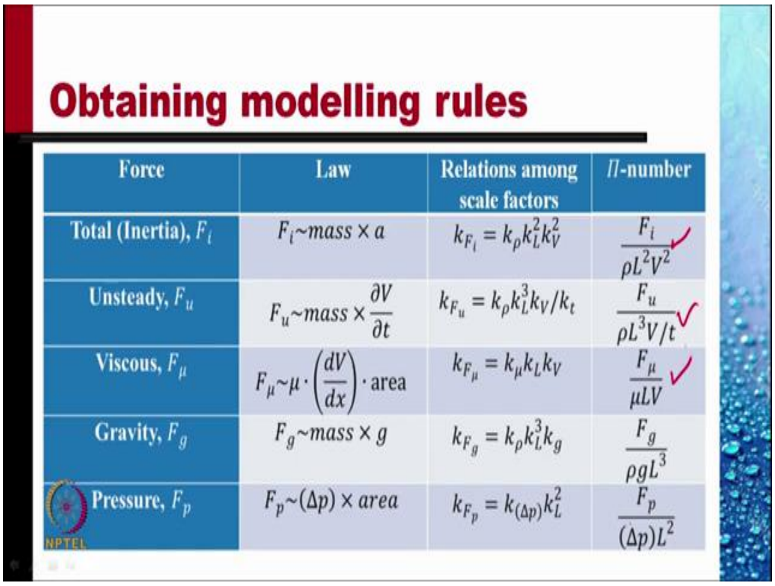


Let us do an example, let us find out the scale factor for viscous forces. Viscous forces are shear forces, which will be shear stress times the area. The shear stress τ varies like $\mu \frac{dV}{dy}$, the

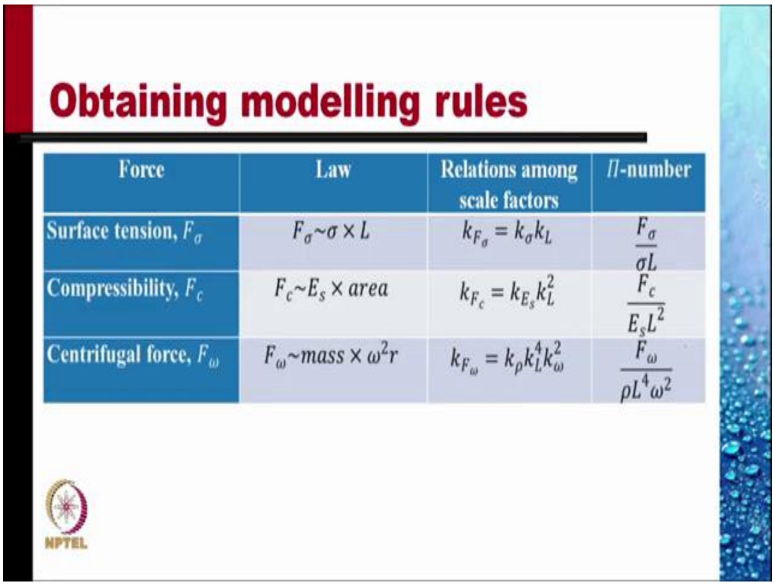
Newton's law of viscosity, so k_τ would be $k_\mu k_V/k_L$, and k_{area} we have discussed earlier is ${k_L}^2$.

So, from this we get the scale factor for the viscous forces, $k_{F,\mu} = k_\mu k_V k_L$. And from this, we obtain a pi number $\frac{F_\mu}{\mu V L}$, where the characteristic nature of the quantities is understood. Then the value in the prototype must be the same as that in the model.

(Refer Slide Time: 08:39)



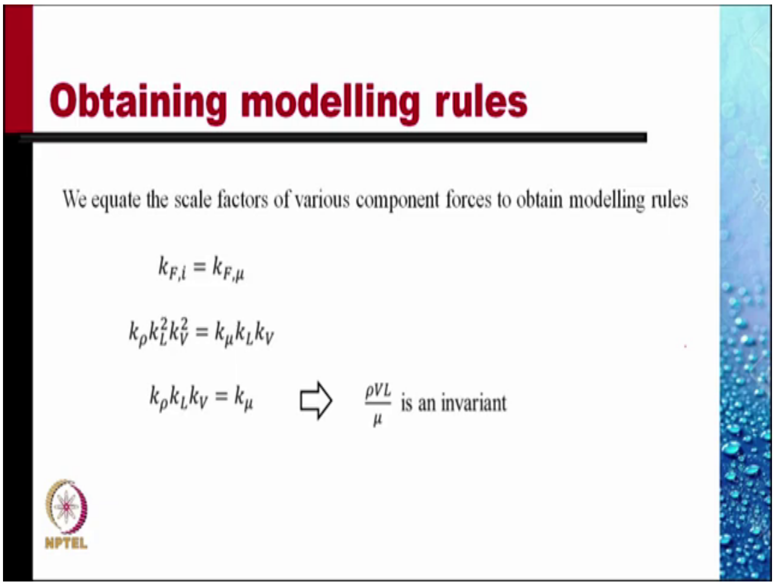
Force	Law	Relations among scale factors	Π-number
Total (Inertia), F_i	$F_i \sim mass \times a$	$k_{F_i} = k_\rho k_L^2 k_V^2$	$\frac{F_i}{\rho L^2 V^2}$
Unsteady, F_u	$F_u \sim mass \times \frac{\partial V}{\partial t}$	$k_{F_u} = k_\rho k_L^3 k_V / k_t$	$\frac{F_u}{\rho L^3 V/t}$
Viscous, F_μ	$F_\mu \sim \mu \cdot \left(\frac{dV}{dx}\right) \cdot \text{area}$	$k_{F_\mu} = k_\mu k_L k_V$	$\frac{F_\mu}{\mu L V}$
Gravity, F_g	$F_g \sim mass \times g$	$k_{F_g} = k_\rho k_L^3 k_g$	$\frac{F_g}{\rho g L^3}$
Pressure, F_p	$F_p \sim (\Delta p) \times area$	$k_{F_p} = k_{(\Delta p)} k_L^2$	$\frac{F_p}{(\Delta p) L^2}$



Force	Law	Relations among scale factors	Π-number
Surface tension, F_σ	$F_\sigma \sim \sigma \times L$	$k_{F_\sigma} = k_\sigma k_L$	$\frac{F_\sigma}{\sigma L}$
Compressibility, F_c	$F_c \sim E_s \times area$	$k_{F_c} = k_{E_s} k_L^2$	$\frac{F_c}{E_s L^2}$
Centrifugal force, F_ω	$F_\omega \sim mass \times \omega^2 r$	$k_{F_\omega} = k_\rho k_L^4 k_\omega^2$	$\frac{F_\omega}{\rho L^4 \omega^2}$

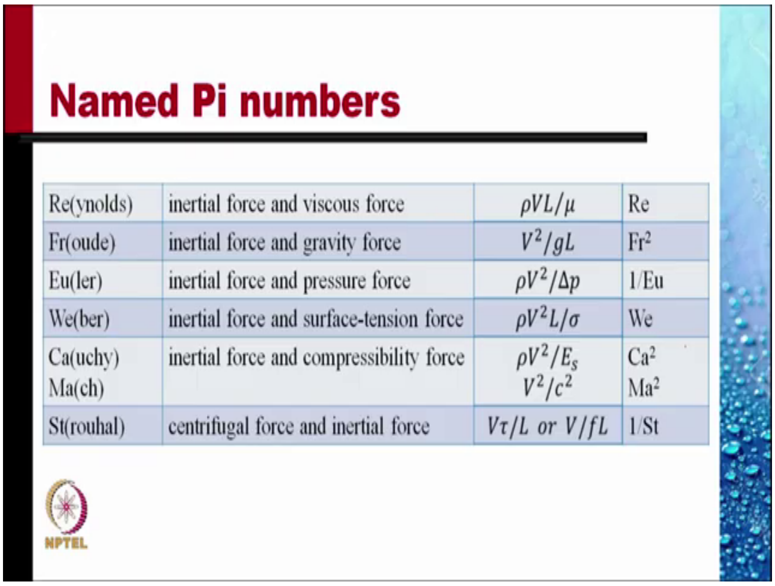
We obtained modelling rule like in this table. For the inertial force we already obtained the pi number as $\frac{F_i}{\rho L^2 V^2}$. For unsteady force F_u is $mass \times \frac{\partial V}{\partial t}$, the unsteady acceleration, and this gives you F_u. The scale factor of F_u is $k_\rho k_L^3 k_V/k_t$. And so, this gives a Pi number $\frac{F_u}{\rho L^3 V/t}$. We had just obtained the scale factor or the pi number for the viscous forces. Similarly, for gravity force, pressure force, surface tension, compressibility, and for the centrifugal force. You are advised to verify these pi numbers.

(Refer Slide Time: 09:54)



Now, we equate the scale factors for the various component forces to obtain the modelling rules. For example, when we equate the total force scale factor to viscous force scale factor, $k_{F,i}$ is $k_\rho k_L^2 k_V^2$, $k_{F,\mu}$ as shown was $k_\mu k_L k_V$. Simplifying this gives you $k_\rho k_L k_V = k_\mu$ or $\frac{k_\rho k_L k_V}{k_\mu} = 1$. And from this I get rho V L by mu as an invariant, that if the two flows are to be similar the value of this parameter $\frac{\rho V L}{\mu}$ should be identical in the model as its value in the prototype.

(Refer Slide Time: 11:08)



Re(ynolds)	inertial force and viscous force	$\rho VL/\mu$	Re
Fr(oude)	inertial force and gravity force	V^2/gL	Fr^2
Eu(ler)	inertial force and pressure force	$\rho V^2/\Delta p$	1/Eu
We(ber)	inertial force and surface-tension force	$\rho V^2 L/\sigma$	We
Ca(uchy) Ma(ch)	inertial force and compressibility force	$\rho V^2/E_s$ V^2/c^2	Ca^2 Ma^2
St(rouhal)	centrifugal force and inertial force	$V\tau/L$ or V/fL	1/St

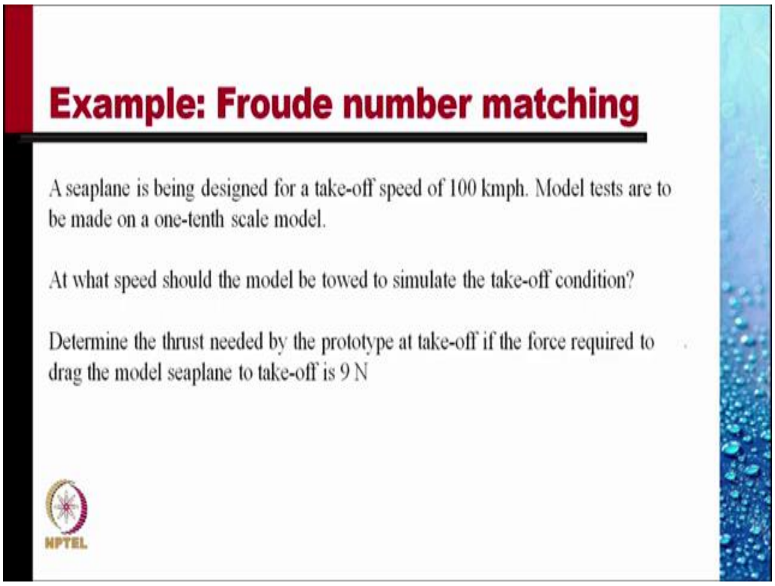
From the various components of forces, we obtain these pi numbers which are often used. There are more pi numbers which occur, but we have taken only a sample that commonly occur. These pi numbers are named in honour of the great scientists who have contributed to fluid mechanics, and they are denoted by the first two letters of the names of the scientists.

For example, the pi number that is obtained by equating the scale factors of inertial forces and viscous forces is called the Reynolds number, written as $\frac{\rho VL}{\mu}$, and is abbreviated as Re. Froude number is equating the scale factors for the inertial forces and the gravity forces, and we get V^2/gL, the square root of which is termed as the Froude number, so V^2/gL is Fr^2.

By equating the scale factors of the inertial forces and pressure forces, we get $\rho V^2/\Delta p$, which is inverse of what is called the Euler number. the Euler number is defined as $\Delta p/\rho V^2$.

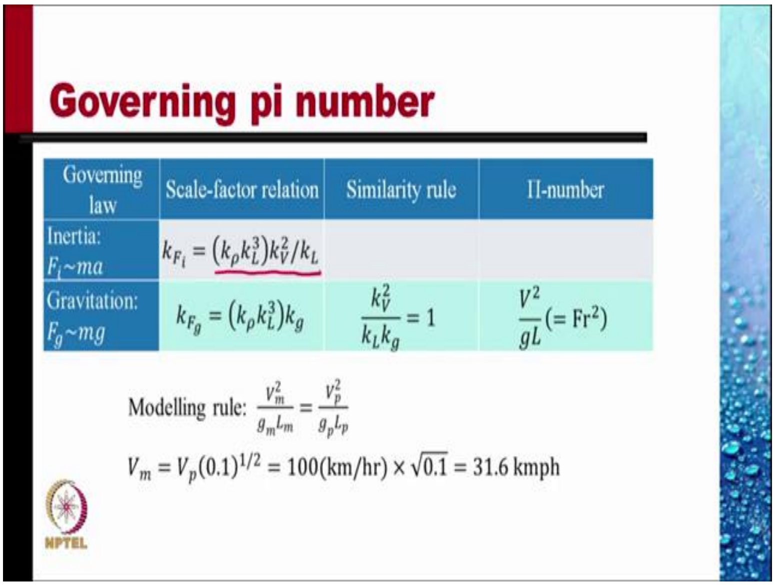
Similarly, the other numbers. Weber number relates inertial forces with the surface tension forces. Cauchy and Mach numbers relate initial forces with the compressibility forces. The Strouhal number relates inertial forces with the centrifugal forces and the unsteady forces.

(Refer Slide Time: 13:26)



Let us do an example in which we need to match Froude number. A seaplane is being designed for a take-off speed of 100 kmph. The model tests are to be made on one-tenth scale model. At what speed should the model be towed to simulate the take-off conditions? And to determine the thrust needed by the prototype at the take-off, if the force required to drag the model seaplane to take-off is 9 N. When the aircraft is taking off, it is experiencing the forces in water.

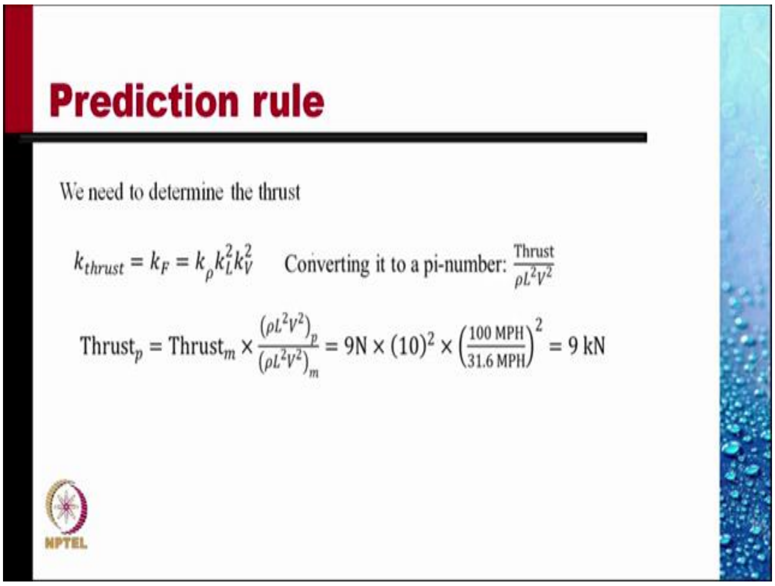
(Refer Slide Time: 14:15)



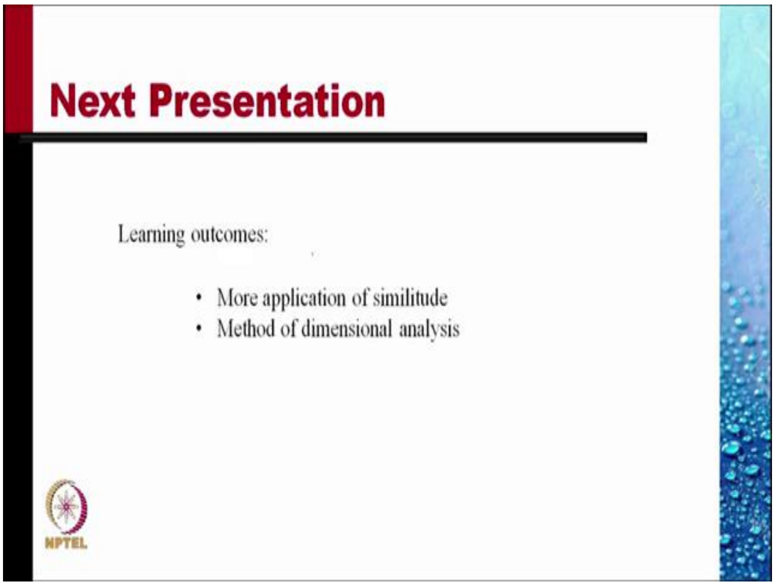
Here gravitational forces and inertial forces are important. The inertial force scale factor was determined earlier as this, which is nothing but $k_\rho k_L^2 k_V^2$. The gravitational force scale factor was the scale factor of mass which is $k_\rho k_L^3$ times k_g. And by equating these two, we get a similarity rule as $\frac{k_V^2}{k_L k_g} = 1$. And from this, we form a pi number $\frac{V^2}{gL}$, which is nothing but Fr^2.

So, the modelling rule is $\frac{V^2}{gL}$ in the model should be equal to $\frac{V^2}{gL}$ in the prototype, where the characteristic nature of these parameters are understood. So, $V_m = V_p(0.1)^{1/2} = 100(\mathrm{km/hr}) \times \sqrt{0.1} = 31.6$ kmph, instead of 100 kmph for the prototype. We need to drag the model aircraft as 31.6 kmph. So, we will need the force required to accelerate this to 31.6 kmph.

(Refer Slide Time: 16:21)



(Refer Slide Time: 17:16)



The force required, the thrust should is a force, so it should have the same scale factor: $k_\rho k_L^2 k_V^2$. Converting it to a Pi number, $\frac{\text{Thrust}}{\rho L^2 V^2}$ must have the same value in the model as in the prototype. So from this thrust for the prototype is determined as 9 kN compared to 9 N, for the thrust in the model. Thus, we have been able to use the scale factor for developing both the modelling rule as well as the prediction rule.

Thank you.