Fluid Mechanics and its Applications
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Lecture - 15A
Significance of some common Pi Numbers

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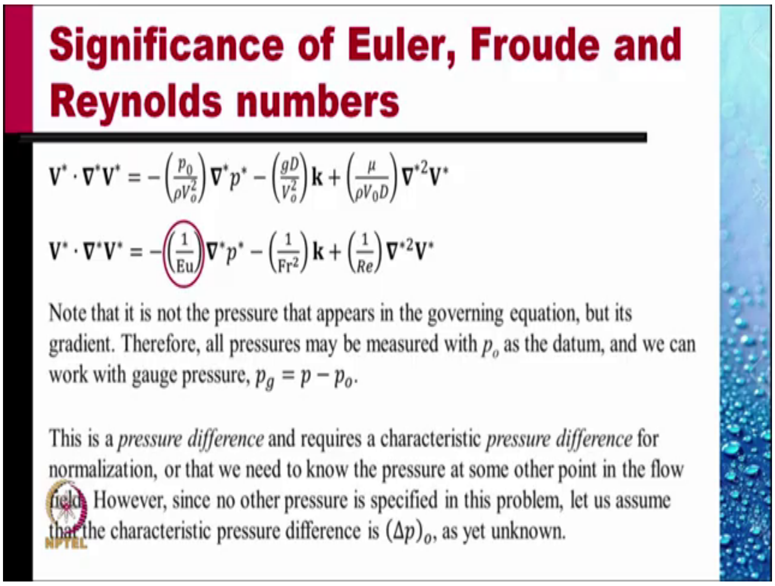

Let us next study the significance of Euler, Froude and Reynolds number introduced earlier. $\frac{p_0}{\rho V_o^2}$ is $\frac{1}{\mathrm{Eu}}$, $\left(\frac{gD}{V_o^2}\right)$ is $\frac{1}{\mathrm{Fr}^2}$ and $\frac{\mu}{\rho V_0 D}$ is $\frac{1}{Re}$. Let us first start with Euler number.

Note that it is not the pressure that appears in the governing equations but its gradient. Therefore, the pressures may be measured with p_o as the datum, and we can work with gauge pressure p_g is equal $(p - p_o)$. If we do this, the equation is unaffected, this p_g is a pressure difference, and does not require a characteristic pressure, but requires a characteristic pressure difference for normalization.

To write a characteristic pressure difference, we need pressures at two points. However, in this problem there is only one pressure p_o specified and there is no other pressure specified. So, let us assume that the characteristic pressure difference is $(\Delta p)_o$, which is as yet unknown.

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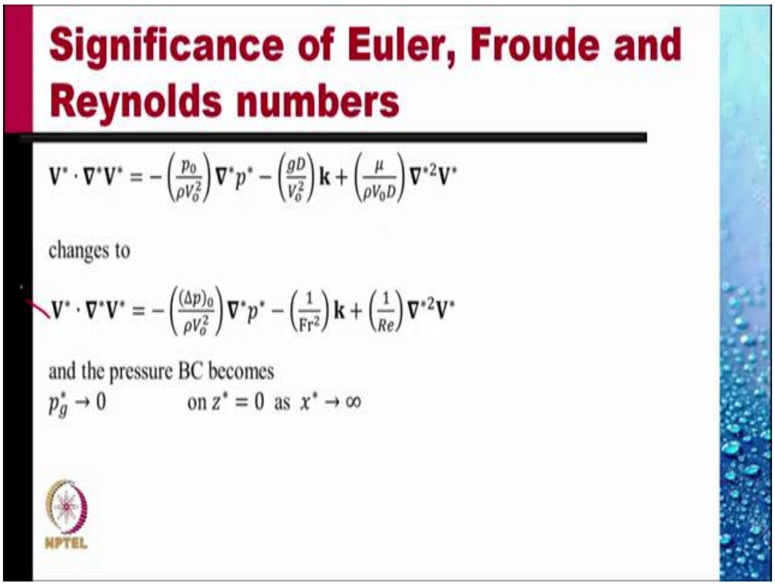

If we make the changes the equation that we started with changes into this second equation, and the pressure boundary condition becomes $p_g^* \to 0$. So, if we do this, then pressure p_o does not occur anywhere in this equation, instead a $(\Delta p)_o$has been introduced.

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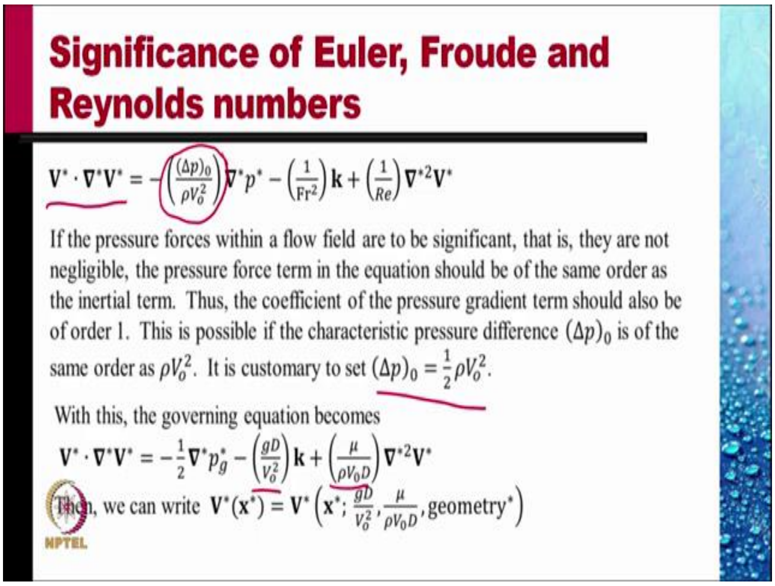

If the pressure forces within a flow field are to be significant that, is they are not negligible, the pressure force term in the equation should be of the same order as the inertial term. Thus, the coefficient of the pressure gradient term should be of order 1. This should be order 1. This is possible if the characteristic pressure difference $(\Delta p)_o$ is of order ρV_o^2.

And so, we set customarily $(\Delta p)_o = \rho V_o^2$. And with this, the equation that governs the fluid flow becomes this. Now, there are only two parameters there. The Euler number is no longer relevant. Then we can write $\mathbf{V}^*$as a function of $\boldsymbol{x}^*$, the non-dimensional space point, and the parameters $\left(\frac{gD}{V_o^2}\right)$ and $\frac{\mu}{\rho V_0 D}$, and the non-dimensionally geometry.

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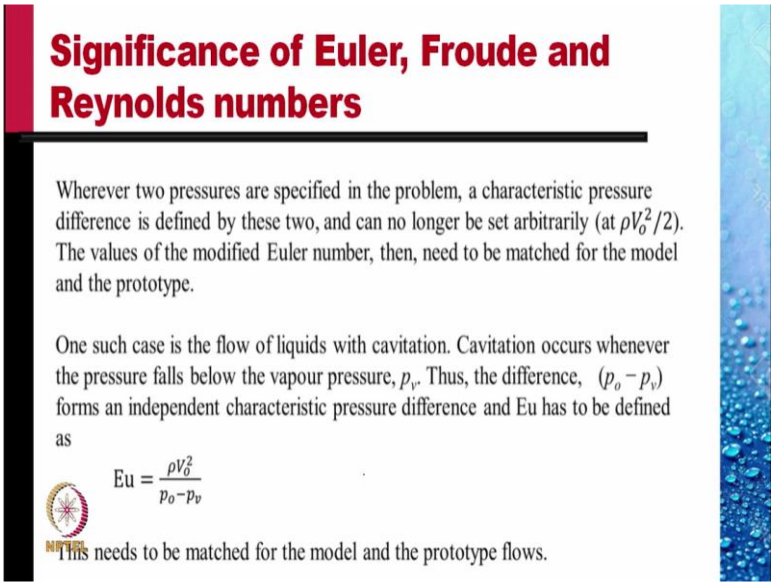

But this thing breaks down if two pressures are specified in the problem. Because if there are two specified pressure in the problem then they would determine what a characteristic pressure difference is. So we can no longer set it arbitrarily at ρV_o^2. In that case, we will use that pressure difference, the two given pressures, the difference in those pressure would be the characteristic pressure difference.

One such case is the flow of liquid with cavitation. Cavitation occurs whenever the pressure falls below the vapor pressure of the liquid. Thus, the difference $(p_o - p_v)$ forms an independent characteristic pressure difference, and the Euler number has to be defined as $\frac{\rho V_o^2}{p_o - p_v}$. And this Euler number needs to be matched in model and the prototype flows.

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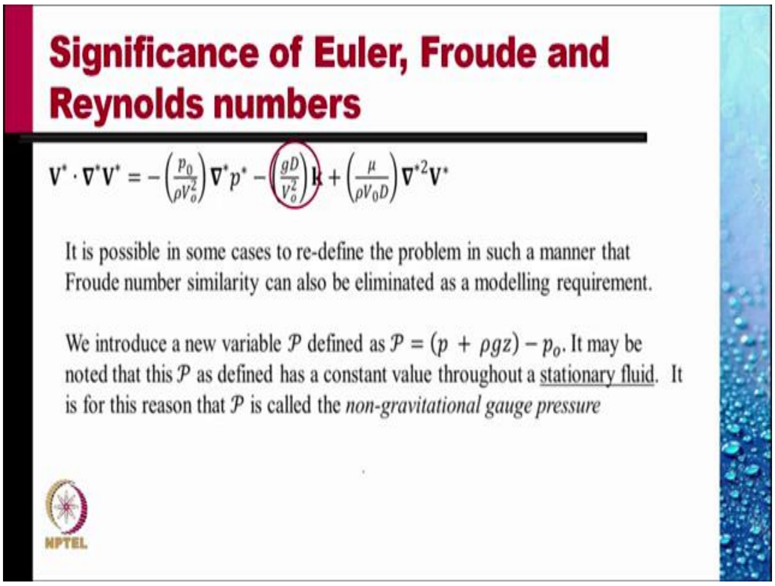

Next, we consider $\left(\frac{gD}{V_o^2}\right)$. It is possible in some cases to redefine the problem in such a manner that the Froude number similarity can also be eliminated as a modelling requirement. We introduce a new variable, modified pressure written as script $\mathcal{P}$ defined as $(p + \rho g z) - p_o$. It may be noted that this modified pressure has a constant value throughout a stationary fluid, and it is for this reason that this is called the non-gravitational gauge pressure.

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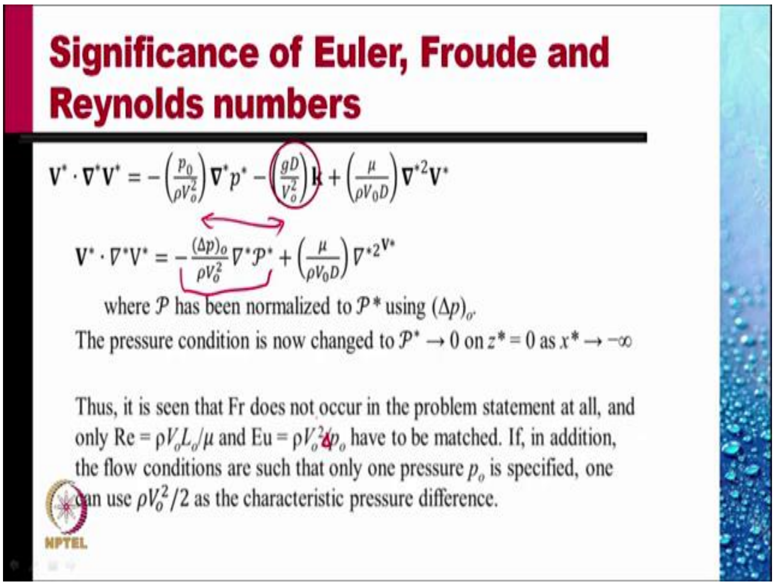

And if we introduce this, the first two terms of the governing equation combine into a single term. The pressure condition has now changed to the non-dimensional modified pressure $\mathcal{P}^* \to 0$ on $z^* = 0$ as $x^* \to -\infty$. Only Reynolds number and the modified Euler number $\rho V_o^2/(\Delta p)_o$ have to be matched.

If, in addition, the flow conditions are such that only one pressure p_o is specified, we can take $(\Delta p)_o$ is equal to ρV_o^2 as the characteristic pressure difference, and then only Reynolds number needs to be matched.

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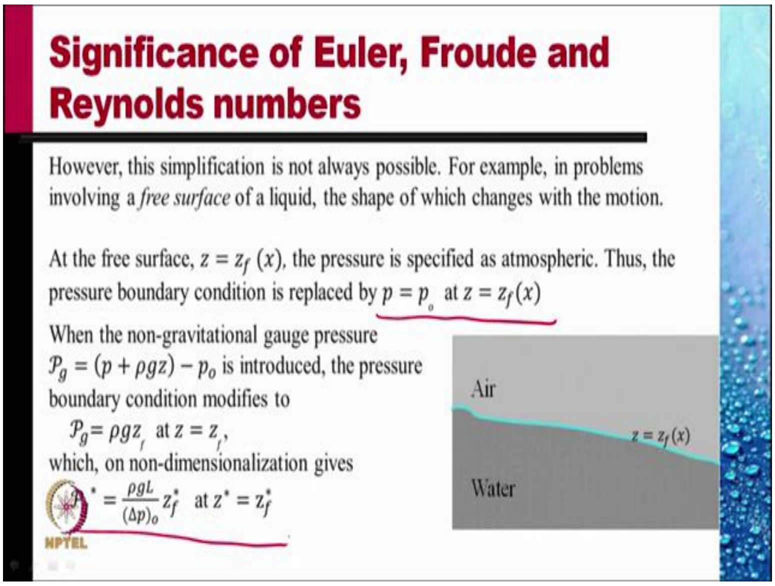

However, this simplification is not always possible. For example in problems involving a free surface of the liquid, like shown here. And this shape changes with motion. The free surface specified as $z = z_f(x)$ and the pressure at this point is p_o everywhere. So, the boundary condition is replaced by $p = p_o$ at $z = z_f(x)$.

When the non-gravitational gauge pressure $\mathcal{P}_g$ is introduced, the pressure boundary condition in this case is modified as $\mathcal{P}_g = \rho g z_f$ at $z = z_f$, which on non-dimensionalization gives this. Now, notice that g, which does not appear in the equation of motion, has now appeared in the boundary condition. Thus, gravity effects are important in problems with free surface.

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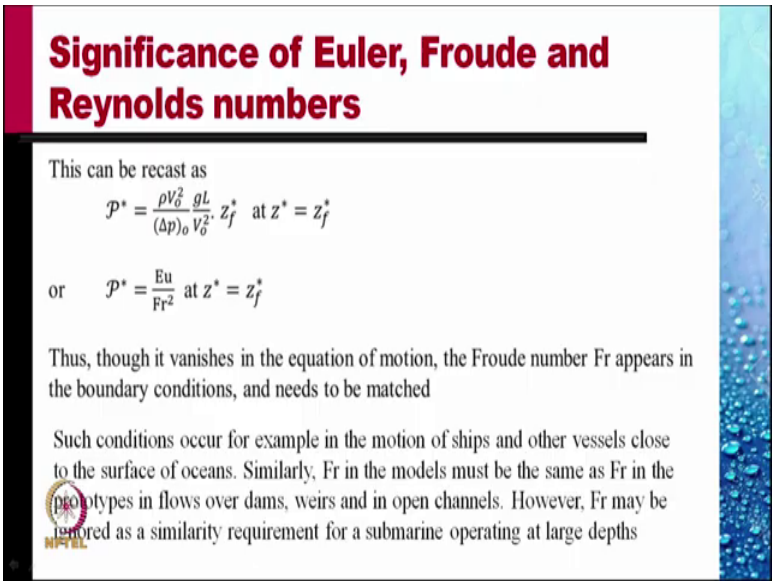

Such conditions occur, for example, in the motion of ships and other vessels close to the surface of ocean. Similarly, Froude number in the models must be same as the Froude number in the prototype in flows over dams, weirs and in open channels. However, Froude number may be ignored as a similarity requirement for submarines operating its large depths far away from the free surface.

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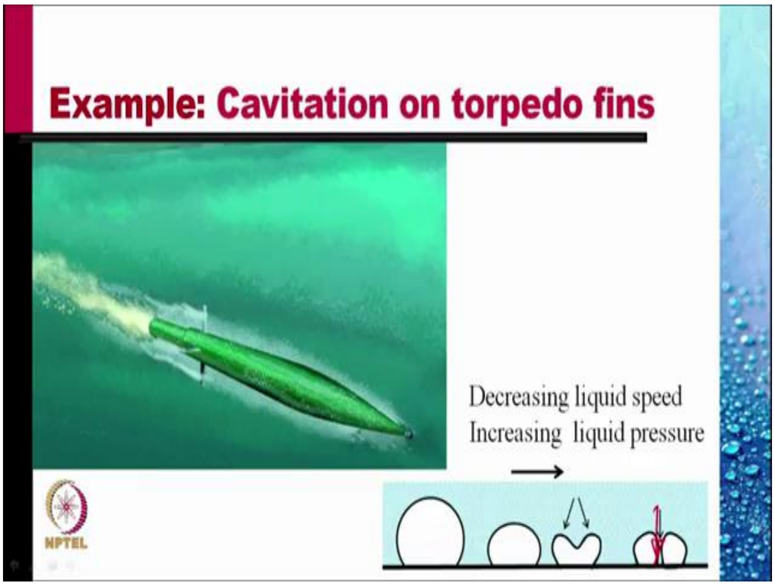

Let us do one example, cavitation on torpedo fins. You see when a torpedo is moving under sea what happens is that on fins the velocity increases and so the pressure decreases, as the pressure decreases below the vapor pressure at that temperature, then a vapor bubble is formed.

The vapor bubbles have a tendency that as the pressure increases back if the fluid slows down, they move away from the fin, the pressure increases the vapor bubbles the diameter of the vapour bubbles decreases. Then the vapour bubble splits, and suddenly a jet impinge on the surface. And this jet can cause erosion and pitting on the surfaces of fin surfaces, which will decrease the control of the vehicle, so we need to avoid cavitation.

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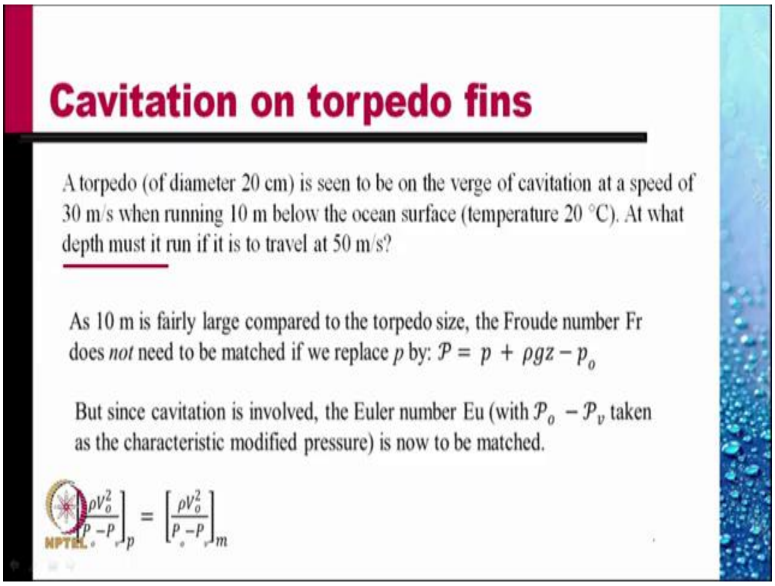

A torpedo of diameter 20 cm is seen to be on the verge of cavitation at a speed of 30 m/s when running 10 m below the ocean surface, the temperature is 20 ℃. From this temperature I can find out the vapour pressure, and it is about 3.4 kPa.

At what depth must the torpedo run if it is to travel at 50 m/s? If you want to run it faster, then the pressure difference would be larger so if cavitation does not have to occur the ambient pressure should be larger, so depth of the torpedo run must be more. As 10 m is fairly large compared to torpedo size, the Froude number does not need to be matched, and we replace p by, $\mathcal{P} = p + \rho g z - p_o$, the non-gravitational pressure difference.

But since cavitation is involved, the Euler number with with $\mathcal{P}_o - \mathcal{P}_v$ taken as the characteristic modified pressure is now to be matched. So, we have to match these two for prototype and model.

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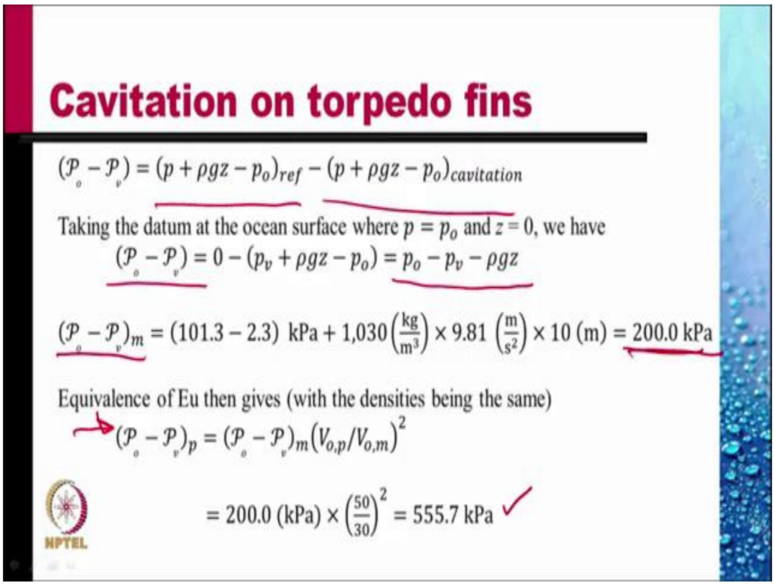

$(\mathcal{P}_o - \mathcal{P}_v)_p = p_o - (p_v + \rho g z)_p$ is like this, at reference pressure p_o and at cavitation. Taking the datum at the ocean surface $p = p_o$ and $z = 0$, we get this value is equal to $p_o - p_v - \rho g z$. So, this for the model is, we plug in the values, and we get 200 kPa.

Equivalence of Euler number then results in this expression as the modelling rule. Euler number must be the same, so the characteristic pressure difference in the prototype should be characteristic pressure difference in the model multiplied by the velocity in the prototype divided by velocity in the model whole squared. And this gives you that the characteristic pressure difference in the prototype should be 555.7 kPa as against 200 kPa in the model. And this is obtained at the depth of 45.2 m.

Thank you very much.

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