Fluid Mechanics and Its Applications
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Lecture 11
Engineering Energy Equation

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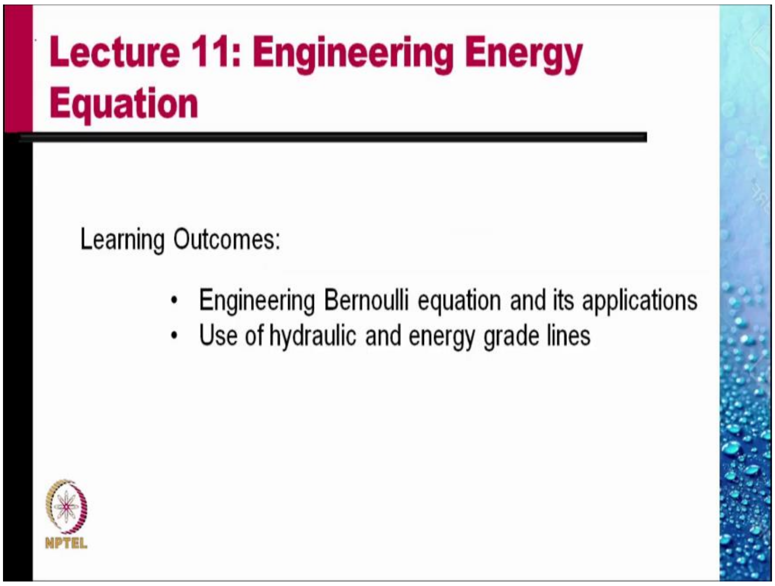

Welcome back. In this lecture, we will introduce the engineering energy equation and its applications.

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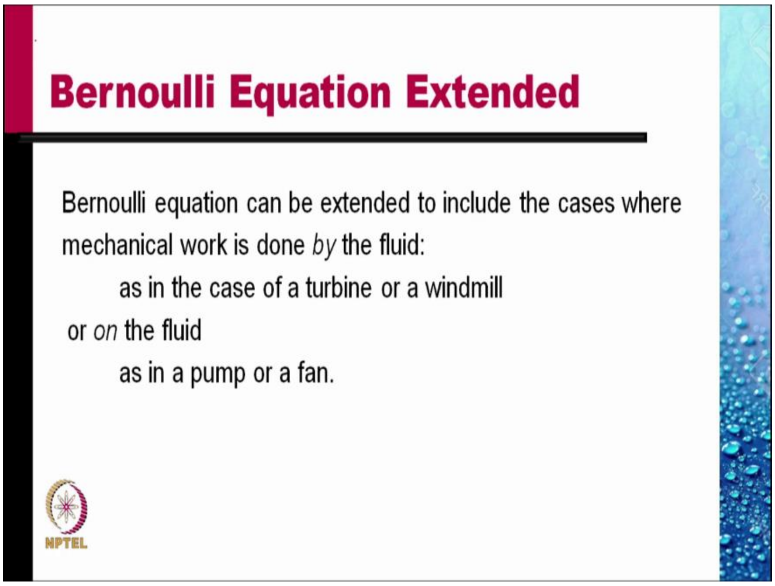

Energy equation can be extended to include cases where mechanical work is done by the fluid, as in the case of a turbine or a windmill, or it is done on the fluid as in a pump or a fan.

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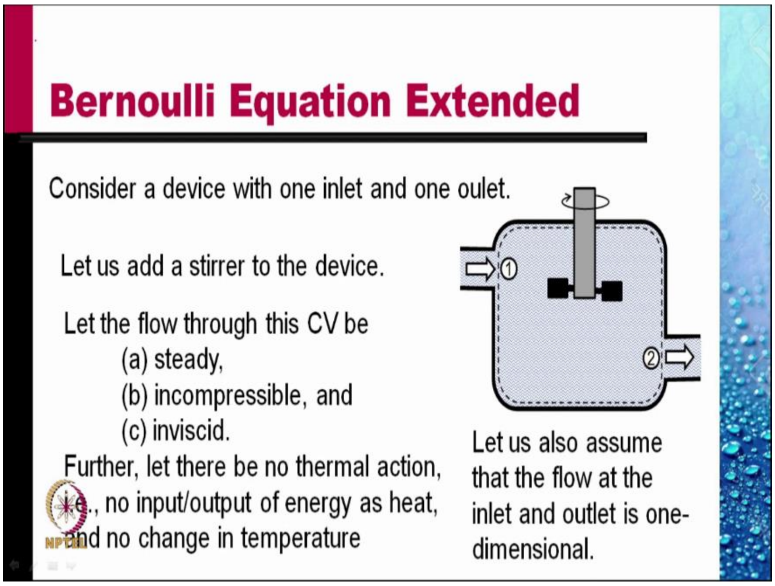

Consider a device with one inlet and one outlet. Introduce a stirrer in this device. It is a device which when rotated will do work on the fluid. Let the flow through this control volume be steady, incompressible and inviscid. Further, let there be no thermal action, that is, there is no input or output of energy as heat, and therefore, no change in temperature. Let us also assume that the flow at the inlet and outlet is one-dimensional, that is, we have uniform velocities at 1 and 2.

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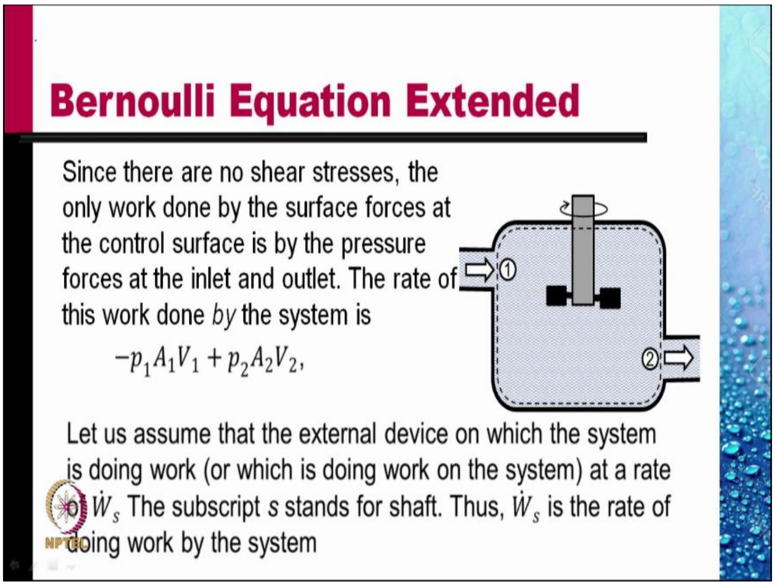

Since there are no shear stresses, the only work done by the surface forces at the control surface is by the pressure forces at the inlet and outlet, and as we showed in the last lecture, the rate of this work done by the system is $-p_1A_1V_1 + p_2A_2V_2$. $p_1A_1V_1$is the work done on the system while pushing the fluid in at the inlet, and therefore, it is negative. $p_2 A_2 V_2$ is work done by the system in pushing the material out at the exit, and that is why it is positive.

Let us assume that the external device on which the system is doing work, or which is doing work on the system at a rate of $\dot{W}_s$. The subscript s stands for shaft because this usually is a work through a rotating shaft. Thus, $\dot{W}_s$ is the rate of doing work by the system. In this case of this stirrer, $\dot{W}_s$ would be negative.

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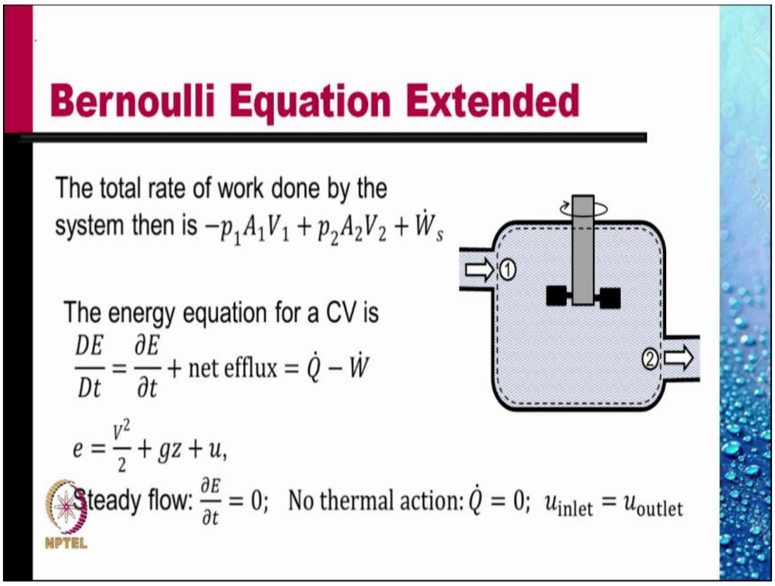

So, the total work done by the system then is $-p_1 A_1 V_1 + p_2 A_2 V_2 + \dot{W}_s$. The energy equation for the control volume that we obtained earlier was DE/Dt, the material rate of change of the energy contained in the system is equal to $\partial E/\partial t$, the rate of accumulation of energy within the control volume, plus the net efflux. And this should be, by first law of thermodynamics, $\dot{Q}$ -$\dot{W}_s$.

The capital E, the total energy contained within the system, can be found out from using the specific energy e, that is, the energy per unit mass, which consists of $\frac{V^2}{2} + gz + u$, the internal energy. The steady flow requirement states that $\frac{\partial E}{\partial t} = 0$, no accumulation. There is no thermal action so, $\dot{Q} = 0$, and also u at inlet is equal to u at outlet, since there is no change in temperature.

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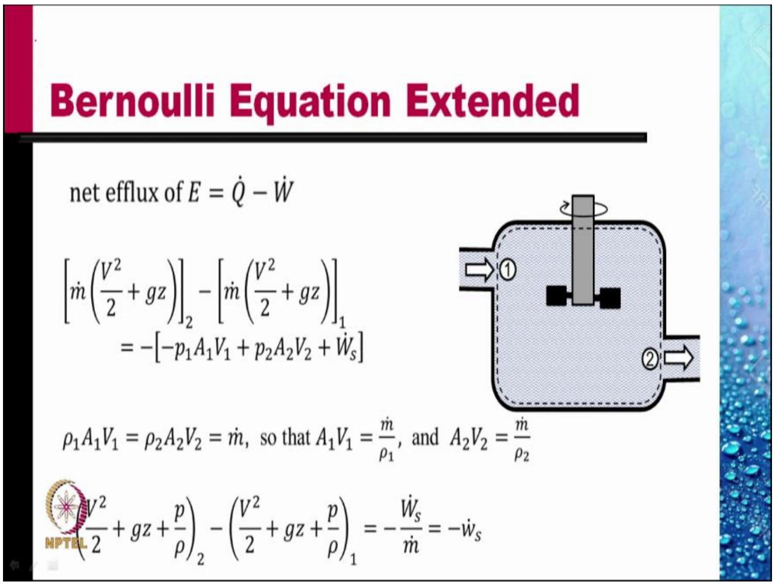

So, net efflux of E must equal to $\dot{Q} - \dot{W}_s$. Net efflux E is $\dot{m}$, the rate of mass throughput into $\left(\frac{V^2}{2} + gz\right)$, the specific energy per unit mass at the outlet, minus $\dot{m}$, the rate of mass throughput, into $\left(\frac{V^2}{2} + gz\right)$ at the inlet. This is the net efflux of energy, and that should be equal to minus the net work done by the system, and that we have shown as $-p_1A_1V_1 + p_2A_2V_2 + \dot{W}_s$.

Now, $\rho_1A_1V_1$ is equal to $\rho_2A_2V_2$ for a steady system, and that is equal to the rate of mass throughput. And so, that $A_1V_1$is $\dot{m}/\rho_1$, and A_2V_2 is $\dot{m}/\rho_2$. Using this, we can divide the energy equation by $\dot{m}$ throughout, and we get this relation by rearranging: $\left(\frac{V^2}{2} + gz + \frac{p}{\rho}\right)$ at the outlet, minus the same quantity at the inlet, is equal to minus $\dot{W}_s/\dot{m}$, which is minus $\dot{w}_s$, lowercase $\dot{w}_s$ is the rate of doing work, shaft work by the system per unit mass of fluid throughput.

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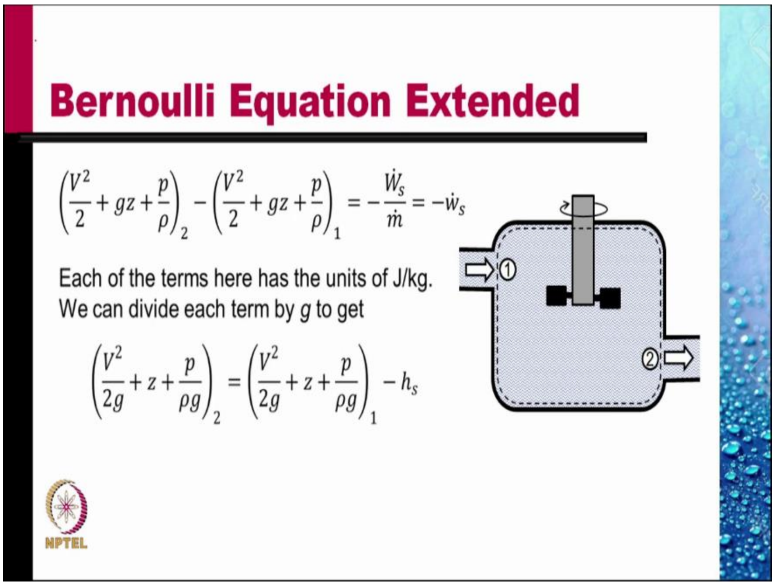

The same equation is rewritten. Each of the term here has a unit of J/kg. We can divide each terms by g to get an alternate form: $\left(\frac{V^2}{2g}+z+\frac{p}{\rho g}\right)$ at the exit. These are the sum of the various heads at the exit, $\frac{V^2}{2g}$ is the kinetic energy head, z is the potential, and $\frac{p}{\rho g}$ is what is termed as the pressure head at the exit, is equal to the same heads at the inlet minus h_s, the work done by the system expressed as per unit weight throughput. Each term has a unit of length in this case.

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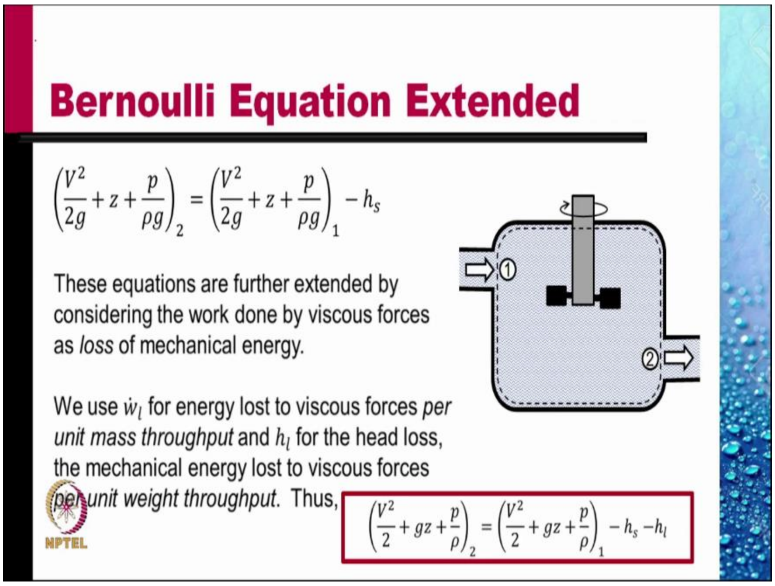

These equations are further extended by considering the work done by viscous forces as *loss* of mechanical energy. If there is any work done by viscous forces within the control volume, we will consider, consider that as a loss of mechanical energy. We use $\dot{w}_l$ for the energy loss to the viscous forces per unit mass throughput and h_l for the head loss for mechanical energy to viscous forces per unit weight throughput.

Thus, we can express the final result,: the total head at the outlet is equal to total head at the inlet minus h_s, the head of work done by the shaft minus h_l, the head loss due to viscous stresses This is known as the extended Bernoulli equation or the engineering energy equation, any of the two names.

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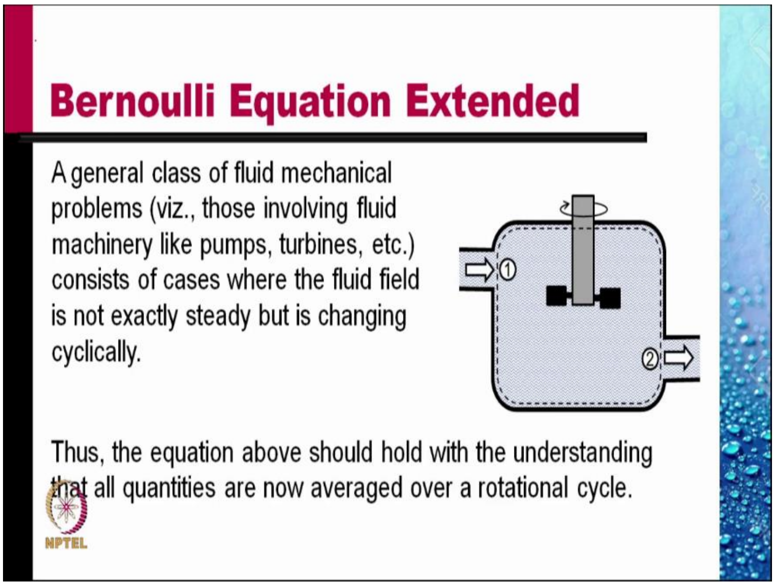

A general class of fluid mechanical problems, for example, those involving fluid machinery like pumps, turbines, etcetera, consists of cases where the fluid field is not exactly steady, but it is changing cyclically, because of a rotating shaft. Thus, the equation above should hold with the understanding that all quantities are now averaged over a rotational cycle.

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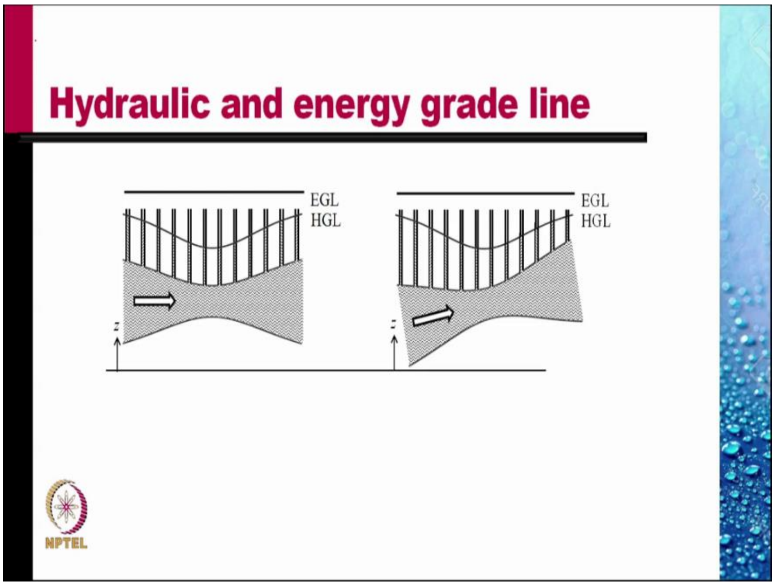

We introduce now the concept of hydraulic and energy grade lines. Consider a converging diverging channel with a liquid flowing in this. Simple application of Bernoulli equation would show that as the fluid moves down this converging diverging channel, in the converging portion, the velocity would be increasing, and so the pressure would be decreasing.

In the diverging portion, the velocity would be decreasing and the pressure would be increasing. Thus the piezometric tubes that are attached to the channel would show a level of water, first decreasing and then increasing, as shown. This represents the variation of piezometric head along the length of the channel, representing $z + \frac{p}{\rho g} z$. This is called the hydraulic grade line in civil engineering.

There is another concept, the energy grade line, which is shown horizontal, constant across this section. Because there are no losses and there is no shaft work done, so Bernoulli equation would require that the total energy, the kinetic plus potential plus the pressure, would be constant. So the variations in this case are only in the hydraulic grade line, the energy grade line is constant.

An interesting property of these grade lines is that the both the hydraulic grade line and the engineering grade line do not depend upon the inclination of the channel. If the channel were inclined at an angle as shown, as we go up the channel, the elevation head would increase, but the pressure head would decrease accordingly. So that the hydraulic grade line would have exactly the same shape as before, and so would be the energy grade line.

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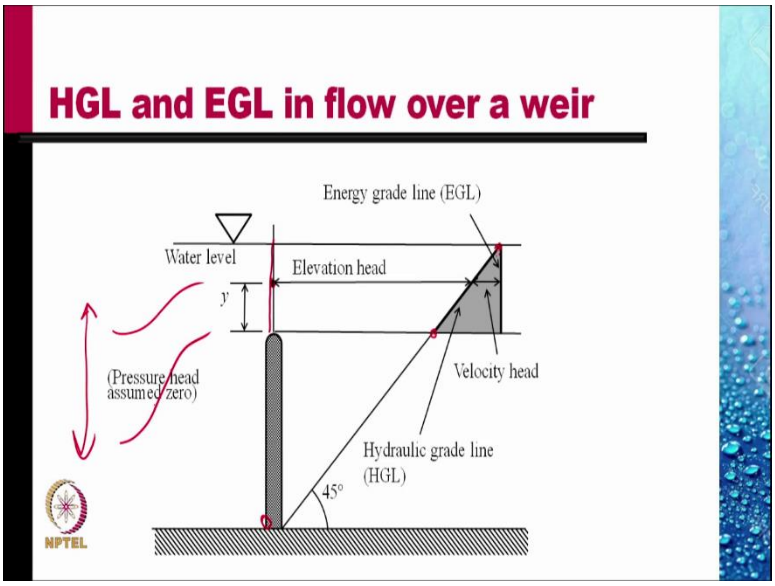

Let us do an example of plotting the hydraulic grade line and energy grade line in flow over weir. In an open channel, we have an obstruction we call a weir, and the flow is taking place above the weir. Like we did in the last lecture, this is one approximation to the flow that we can make. I draw a line at 45°. At any height *y* about the crest of the weir, the arrow here shows the elevation head. Since this line is at 45°, this horizontal line marked elevation head is exactly equal to the height of this point about the base.

So this, the pressure head all across this jet of water that emerges over the weir is atmospheric all through, and so, the pressure head is assumed 0. We talk about the gauge pressure. The hydraulic grade line then looks like this. Because pressure head is 0, so the elevation head line and the hydraulic grade line would be the same. The hydraulic head is maximum here and is minimum here. Now since the water is coming from a reservoir which is essentially at rest, the total energy along any stream line is the same.

So at every point along this line, the total energy is the same, and so the energy grade line would be a straight line at the top. The velocity is 0. And so, the total energy is the hydraulic head plus the velocity head at the same point. This is the energy grade line. Clearly, the shaded area represents the velocity head. The velocity head is 0 at the top and is maximum at the bottom, varying linearly.

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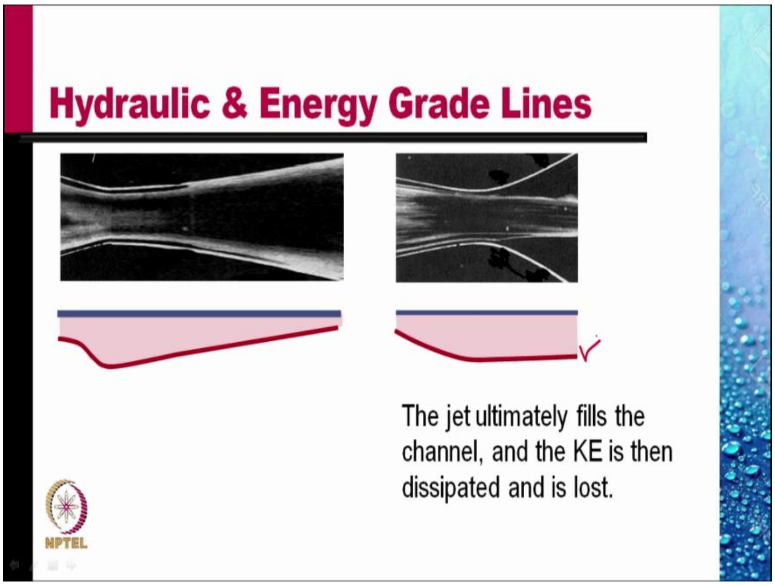

I plotted here the hydraulic and the energy grade lines in a converging diverging channel. In the first picture on the left, we have a channel in which there is a short converging portion and a very long diverging portion. The total energy shown by a blue line is constant. As the flow travels downstream the velocity increases and so the pressure decreases. And then after the throat, the area is increasing, the velocity decreases, and the pressure increases.

The red line shows the hydraulic grade line, pressure plus the elevation. The elevation is constant. So the pressure line itself is the hydraulic grade line. The shaded portion represents the velocity head: the difference between the energy grade line and the hydraulic grade line.

This picture changes drastically if the diverging portion of the channel is short. As we have discussed a couple of times earlier, now that the boundary layer forms along the channel, separates at the throat in the diverging portion, and the liquid comes out as a jet.

And since the liquid comes out as a jet, its velocity does not decrease, and so the pressure does not increase. As the red line here shows, the pressure does not recover back to its original value at the inlet. There is a loss of pressure. The jet ultimately fills the channel and the kinetic energy of this jet is then dissipated and is lost.

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The same principle is used in the design of wind tunnels. A wind tunnel is a structure to test aerodynamic phenomena, aircraft parts or similar things in a fluid flow. When the aircraft moves in a stationary atmosphere, the forces that it experiences are the same as when the aircraft is stationary and the air moves past it.

So, we use this principle to construct a wind tunnel. In this test section of the wind tunnel we mount a model of the aircraft part that we want to test, and we create a flow in this channel. The flow is created by a fan in this portion which sucks air. The inlet of this tunnel is designed such that the flow that we obtain in the test section is uniform and straight. This flow needs to slow down at the fan, and so the diverging length, termed the diffuser, is much longer than the converging length. This is to prevent the separation of the flow in the diverging portion. These wind-tunnels can be small or can be big.

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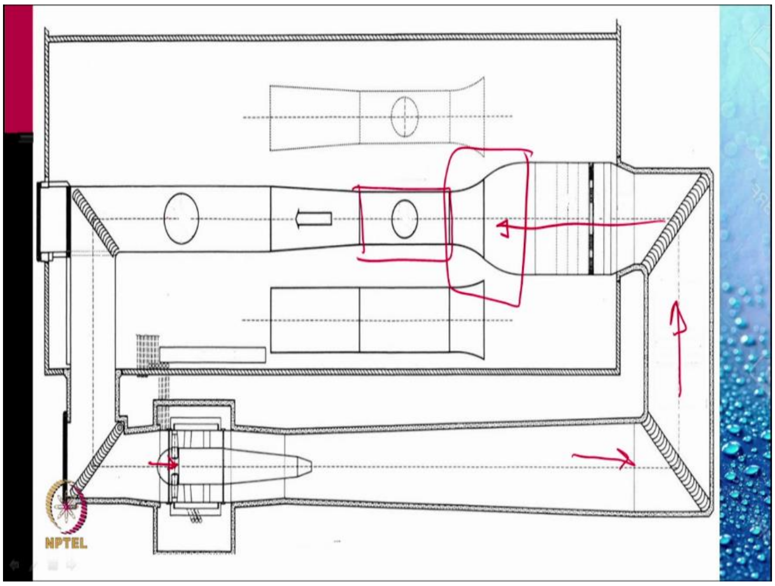

One of the very big tunnels exists in IIT Kanpur where in the National Wind Tunnel Facility we have a closed-circuit wind tunnel. The fan sucks the air and flows. The air slows down. It bends along, it goes again. The screens in this inlet section are there to make the flow smooth. The air is speeded up in this nozzle, and this is a test section, where the parts that we want to study the flow on, are mounted with proper instrumentation. The flow then slows down in increasing section and is re-circulated. The re-circulation is there essentially to save energy.

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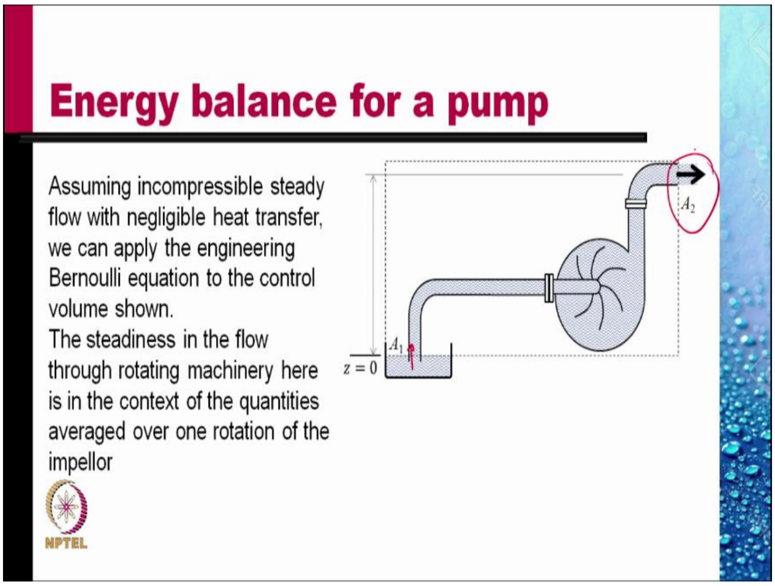

Let us apply the engineering Bernoulli equation to a pump. In this example, a pump is used to pump water through a height. The pump picks up water from this tank through this pipe, and the water is now pushed through a pipe at section A2 at a height.

Let us take a control volume shown by these broken lines. Assuming incompressible and steady flow with negligible heat transfer, we can apply the engineering Bernoulli equation to the control volume shown. The steadiness of the flow through rotating machinery here is in the context of the quantities averaged over one rotation of the impellor as discussed earlier.

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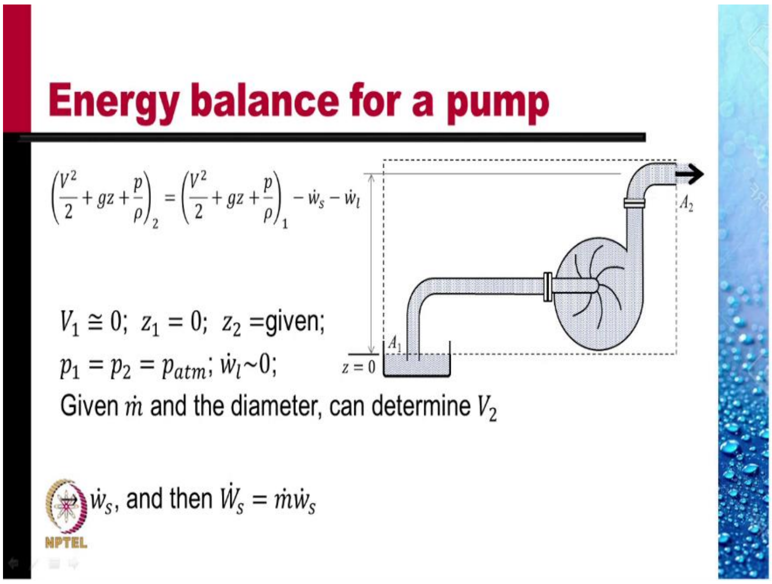

So, the applicable equation is the engineering Bernoulli equation, or the extended energy equation. Here, we know at 1 the inlet, the surface of the tank where the water is coming from, if the cross sectional area A_1 is very large compared to the area A_2, then the velocity at 1 is negligible. z_1 is taken to be 0. We define the datum there. z_2 is given. We know the height to which we are pumping water. p_1 and p_2 are both $p_{\text{atmospheric}}$. Let us neglect the losses. So, $\dot{w}_1$ is taken as 0.

Now, we are given $\dot{m}$, the mass flow rate through the pump, and know the diameter of the system. We can determine V_2, the velocity through the pipe. Let us assume the whole pipe has the same diameter. So the velocity throughout the pipe is V_2, the velocity at the exit. We are to find the power consumed by the pump in pumping $\dot{m}$ kg/s of water to the height h. $\dot{w}_s$ is what we want to calculate, and then we can find out capital $\dot{W}_s$ from $\dot{m}\dot{w}_s$. Recall that $\dot{w}_s$ is the work done per unit mass throughput, while $\dot{W}_s$ is the total work done per unit time by the system.

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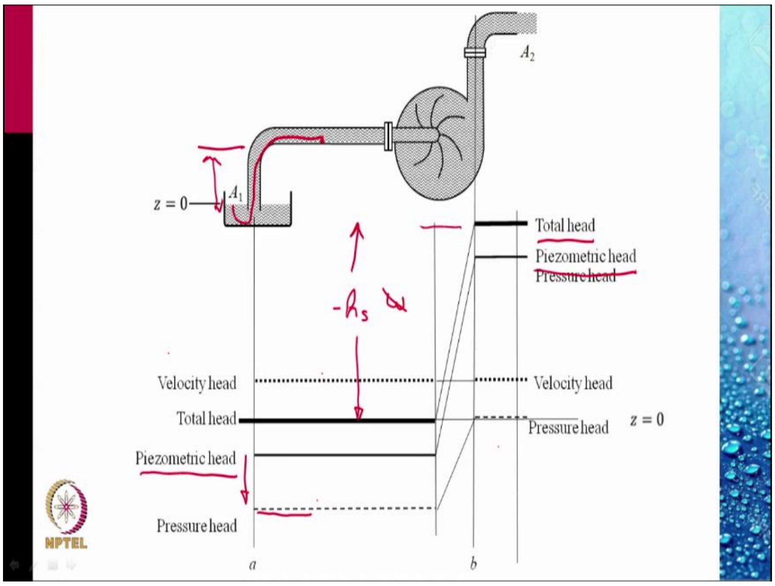

Let us plot the hydraulic and the energy grade lines, Let z is equal to 0 represents the datum. Then, the total head: total head up to the inlet is constant and is 0, because at 1, the pressure is 0, z elevation is 0, the velocity is 0. So, that the total head is 0. And since, along any stream line from here to any point within this pipe, we can apply Bernoulli equation. So, the total head remains the same at 0.

The pump increases the head. Pump does work on the system. So, h_s is negative, and that increases the head of the fluid. The total head is now in the exit pipe is at an elevated level this is h_s, the head supplied by the pump. It is actually a negative quantity, because there is work done on the system. Since, the pipe has a constant diameter, the velocity is constant throughout the pipe and so, the velocity head is a horizontal line. Clearly, the piezometric, head which is the sum of the elevation head and the pressure head is equal to the total head minus the velocity head.

So, piezometric head is a line which is below the total headline by an amount equal to the velocity head there and this line then represents the piezometric head. We have negative piezometric head through the inlet pipe, and positive piezometric head at the outlet pipe. The pressure head, pressure head and the elevation head together makes the piezometric head. So, pressure head we can obtain by subtracting the elevation from the piezometric head. This elevation we subtract from the piezometric head to get this broken line. This is the pressure head.

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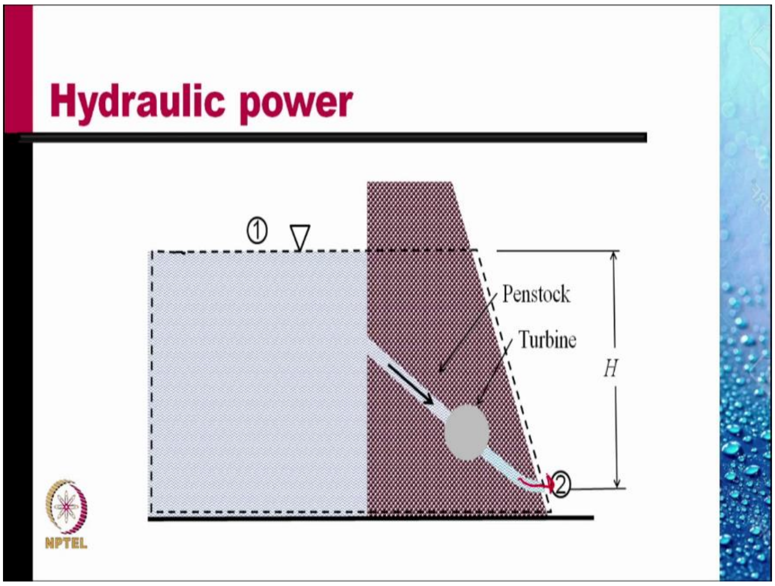

Let us take another example. It concerns a hydroelectric project where the water is stored up behind a dam. That water is directed through a tube or a pipe or a channel known as penstock to a turbine. It extracts energy from the water, and the resulting flow passes through the end of the penstock into atmosphere.

Let us draw the total energy line and the hydraulic line for this. Let us consider this as a relevant control volume, with one inch inlet at 1 where the velocity can be assumed to be 0, the pressure is 0. The elevation head is *H*. At 2 where there is a velocity, the pressure is 0, the atmospheric, and the elevation is 0. Since, we take the datum there.

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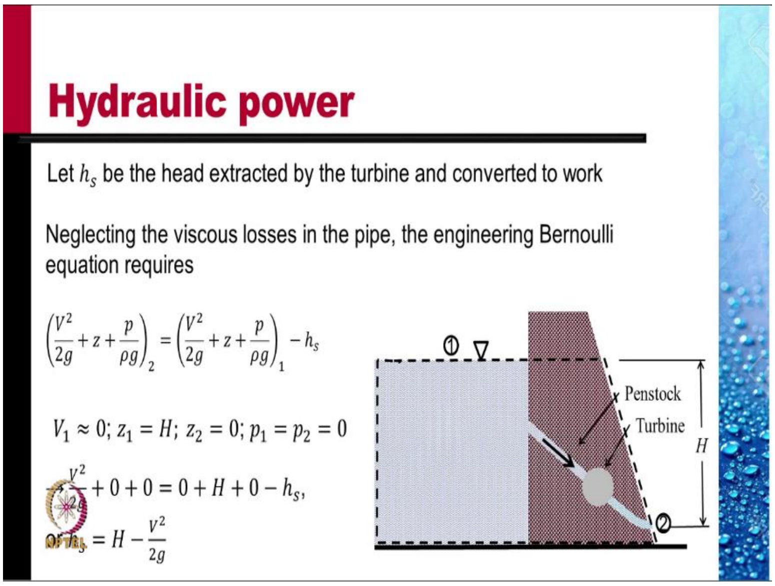

Let h_s be the head extracted by the turbine and converted to work. So neglecting viscous losses in the pipe, the engineering Bernoulli equation requires $\frac{V^2}{2g} + z + \frac{p}{\rho g}$ at the outlet 2 is equal to the total head at the inlet minus h_s, the head extracted by the turbine. This is the work done by the system. So, h_s is positive.

Let the velocity at point 2 be *V*, so the first term $\frac{V^2}{2g}$ at the exit is $\frac{V^2}{2g}$, z is 0, pressure is 0. That is equals to: velocity 0 at point 1, the elevation is *H* at point 1, the pressure is 0 at point 1 minus h_s the head extracted. So, we get head extracted is equal to *H* minus $\frac{V^2}{2g}$. This we could have seen directly. From the total head *H* available in the reservoir, we extract all of it in the turbine, except for the head of kinetic energy that the water emerges with at point 2.

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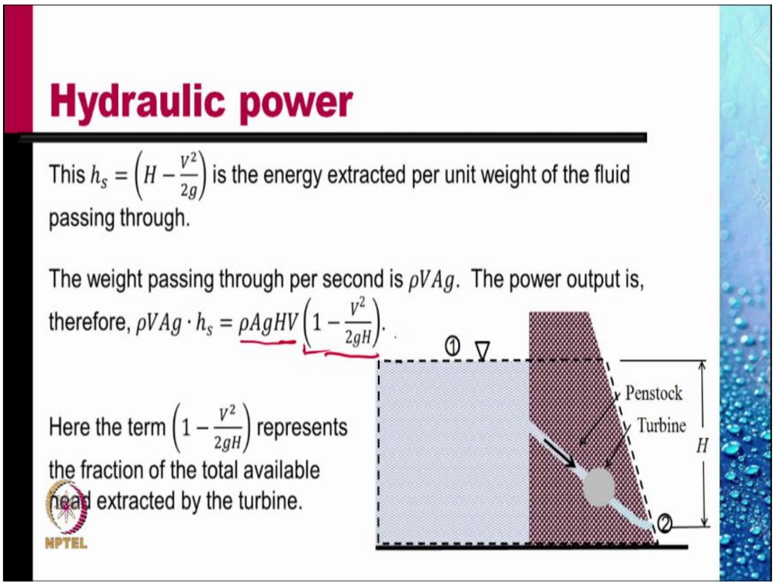

This h_s is equal to $\left(H - \frac{V^2}{2g}\right)$ where $h_s$is the energy extracted per unit weight of the fluid passing through. The weight passing through per second is ρVAg, VA is the volume passing through, ρVA gives you the mass passing through per unit time, and multiply by g will give you the weight of water passing through per unit time. So, the power output is ρVAg times h_s, which is $\rho VAgH\left(1 - \frac{V^2}{2gH}\right)$.

Now this is an interesting expression. Here, as *V* increases, this increases, but this decreases. So, one part increases and the other part decreases. Where *V* is equal to 0, clearly, the power would be 0. When *V* is equal to $\sqrt{2gH}$, that is, all the head from the reservoir is now available as the kinetic head, that is, no head is being extracted, then this term becomes 0. So, there must be a maximum somewhere in between. Here, the term $\left(1-\frac{V^2}{2gH}\right)$ represents the fraction of the total available head which is extracted by the turbine.

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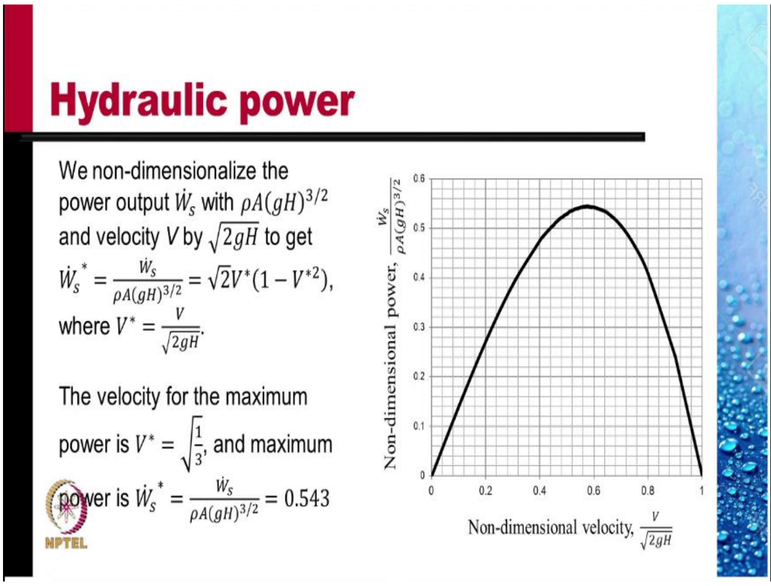

We plot here the non-dimensional velocity $V/\sqrt{2gH}$ against the non-dimensional power developed by the system. And, we notice that there is a maximum power available. We non-dimensionalize the power output $\dot{W}_s$ with $\rho A(gH)^{3/2}$. This is the total power that would be available if we extract all of this out from the system, and the non-dimensional velocity by $\sqrt{2gH}$, which would be the velocity if the total head is convert into velocity.

And so, we get a non-dimensional power $\dot{W}_s^*$ equal to $\sqrt{2}V^*(1-V^{*2})$, where V^* is V by $\sqrt{2gH}$. So, the velocity for the maximum we can determine, is $V^* = \sqrt{1/3}$ at the maximum power output, non-dimensional, is $\dot{W}_s^* = 0.543$.

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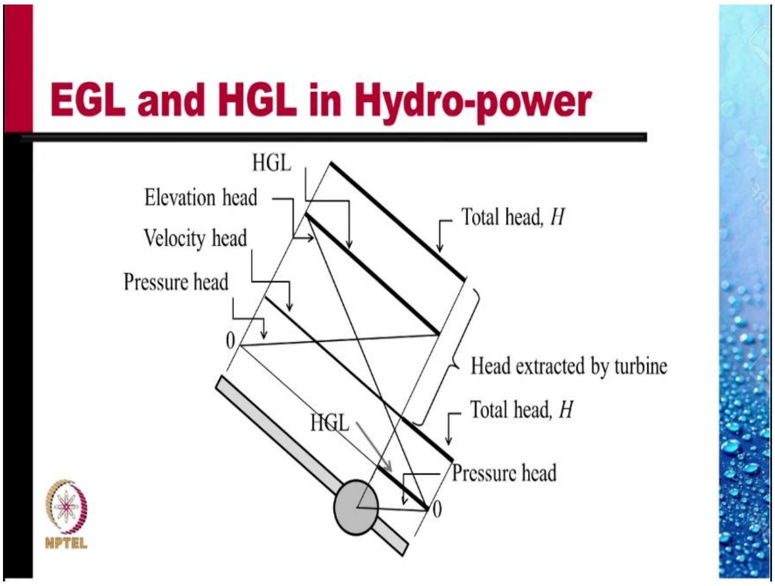

We plot the energy grade line and the hydraulic grade line for this hydro-power setup. And we do this in an inclined coordinate system. Let 0-0 line represent the datum. The elevation head. The pressure head. As we go down the pressure increases in the turbine. The pressure becomes negative after a while, and then it increases in the tail race. The hydraulic grade line is then the sum of the pressure head and the elevation head.

The hydraulic grade line here hass a constant value. The velocity head is constant through the pipe, if we assume the diameter of the penstock and the tailrace is constant. The total head is the energy the hydraulic grade line plus the velocity head through the pipe. Head extracted by the turbine is shown here.

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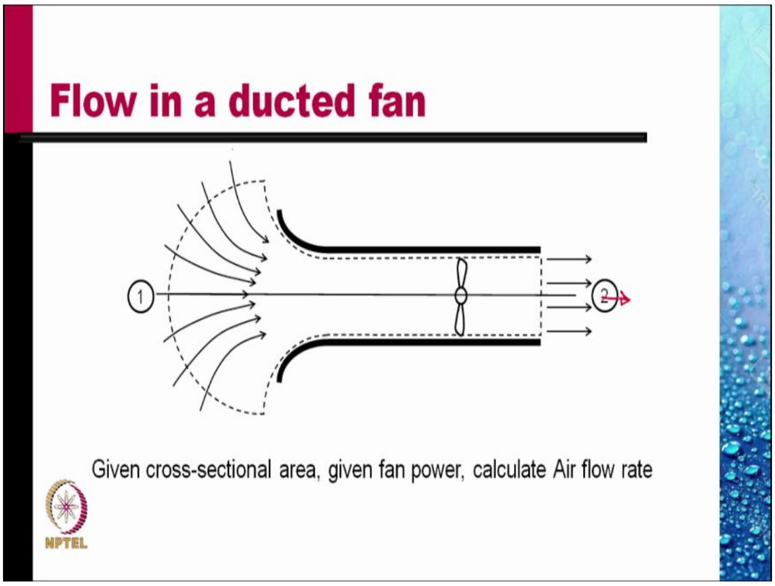

Let us do another example of flow in a ducted fan operating in a duct, with a smooth bell shaped entrance. This fan sucks in air from all around, and the air exits as a jet. Given the cross sectional area and given the fan power, calculate the air flow rate, that is the problem. So, assume this control volume. We assume a very large area at the inlet. The air is coming from all sides. The pressure is atmospheric there and the velocity is very small, because the area is very large.

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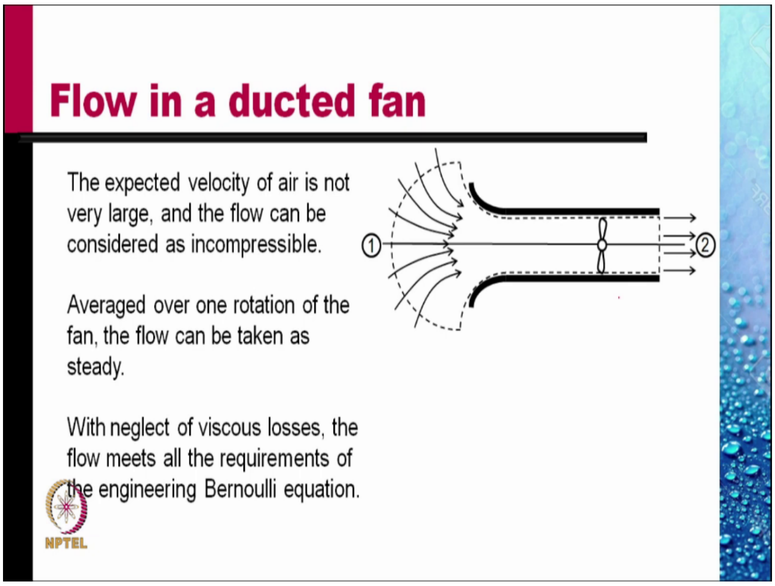

If the air velocity within the duct is not very large, the flow can be considered as incompressible, averaged over one rotation of the fan. The flow can be taken as steady. We neglect the viscous losses. The flow meets all the requirements of the engineering Bernoulli equation.

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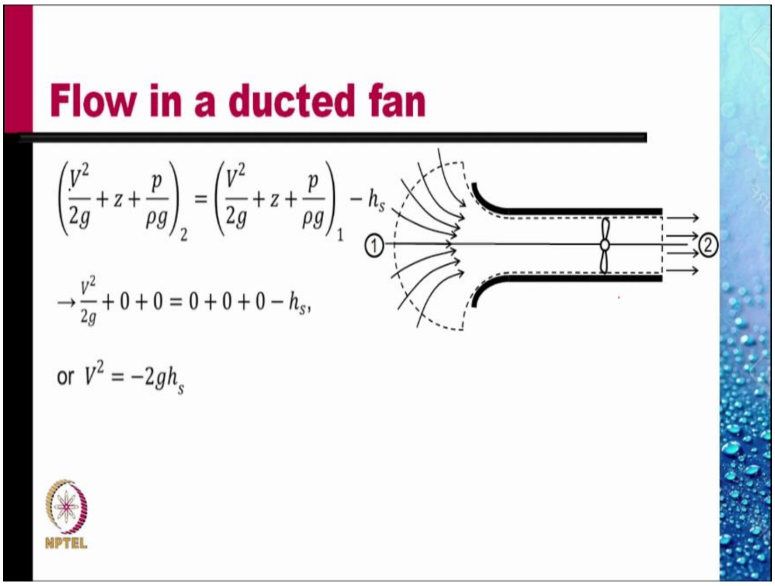

And so, we apply this equation, z at the exit is 0, p at the exit is 0. $\frac{V^2}{2g}$ at the inlet is 0, the elevation z is 0, pressure at the inlet is 0 again, minus h_s. So, this gives you $V^2 = -2gh_s$. Since the fan does work on air, h_s is negative. h_s just is the head supplied by the fan.

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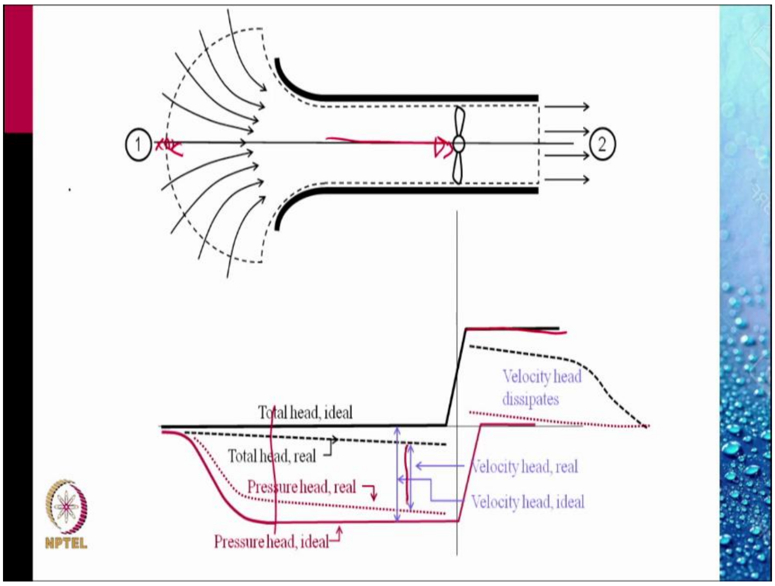

Let us plot the variations of the heads. The total head up to fan is 0, because the total head here is 0. Up till the fan there is no input of energy, and if we neglect viscous losses, the total head would remain 0 here. The fan supplies a head h_s and so, in the remaining part of the tube behind the fan the total head is h_s. The pressure head: z is same. So pressure head is same as the hydraulic head, as the piezometric head. So pressure head line is the same as a hydraulic grade line, and it is obtained by subtracting the velocity head from the total head.

The velocity starts out at 0. Here it increases and then becomes constant in this duct. So the pressure head looks like this shown here. From the total headline, we subtract the velocity head. In this portion, the velocity is increasing up to the inlet, and after that the velocity is constant, ideal velocity head. If there was viscosity present, there would be losses against the walls of the duct, and the total head would decrease constantly.

This broken line shows the decrease in total head. The pressure would also decrease constantly along this. And so the real pressure head would be this broken red line, and the real velocity head is this.

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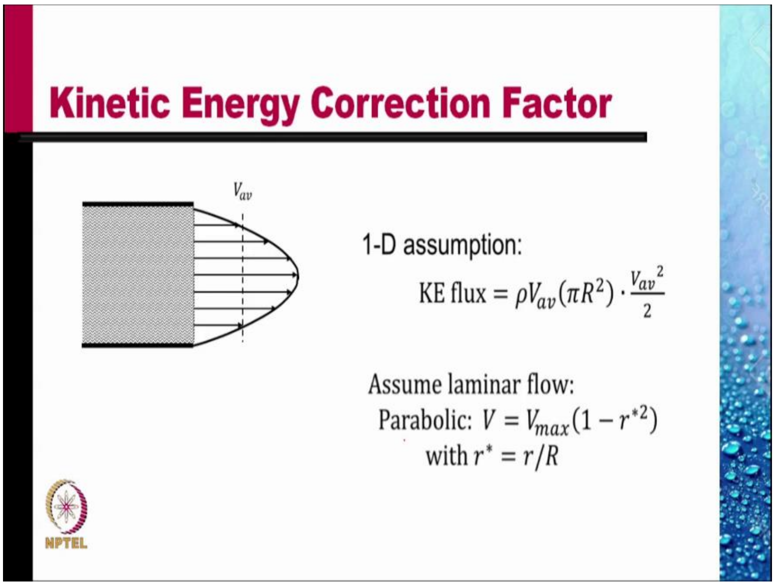

Let us do one more concept called the kinetic energy correction factor. We have so far assumed that the flow through the inlet and outlet are one-dimensional. That is, we assumed that there are no variations in the velocity across the inlet and the outlet sections. But there actually are. This can be taken care of by using a kinetic energy correction factor. So we calculate the kinetic energy flux by using the average velocity, and then we multiply it by a correction factor to find out the actual kinetic energy flux.

If we have a circular pipe of radius R, then the one-dimensional assumption would require that we assume the velocity, the average velocity all across the pipe section. So, V-average, V_{av} is the average throughout the pipe. The mass flow rate would be $\rho V_{av}\pi R^2$, and then kinetic energy per unit mass at the average velocity is taken ${V_{av}}^2/2$. So, this is the kinetic energy flux with one-dimensional assumption.

Now, Let us assume a laminar flow. In a laminar flow, the velocity profile is given as $V = V_{max}(1 - r^{*2})$, the r^* is r/R, the radius of the lead pipe. We have shown earlier that $V_{max} = 2V_{av}$.

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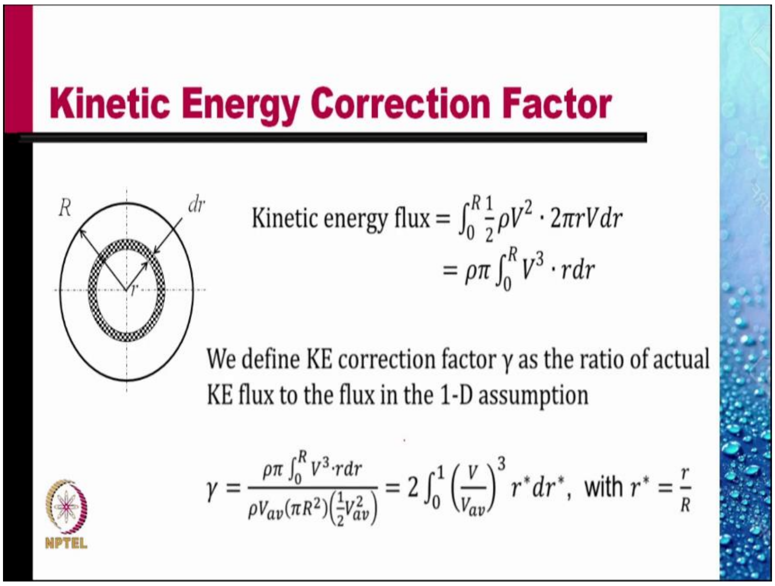

Now, the actual kinetic energy flux would be obtained by considering a ring of thickness *dr* at radius r. The kinetic energy through this is mass flow through this which $2\pi r V dr$ times the kinetic energy of , which is $\frac{1}{2}V^2$ per unit mass, and so, this is simplified into, $\rho\pi \int_0^R V^3 r dr$.

So, we define γ, the kinetic energy correction factor, as the actual kinetic energy flux shown in the numerator divided by the kinetic energy flux with one-dimensional assumption which is obtained in the last slide. And this simplifies to $2\int_0^1 \left(\frac{V}{V_{av}}\right)^3 r^* dr^*$ where r^* is r/R. Now, we can go the laminar or the turbulent route.

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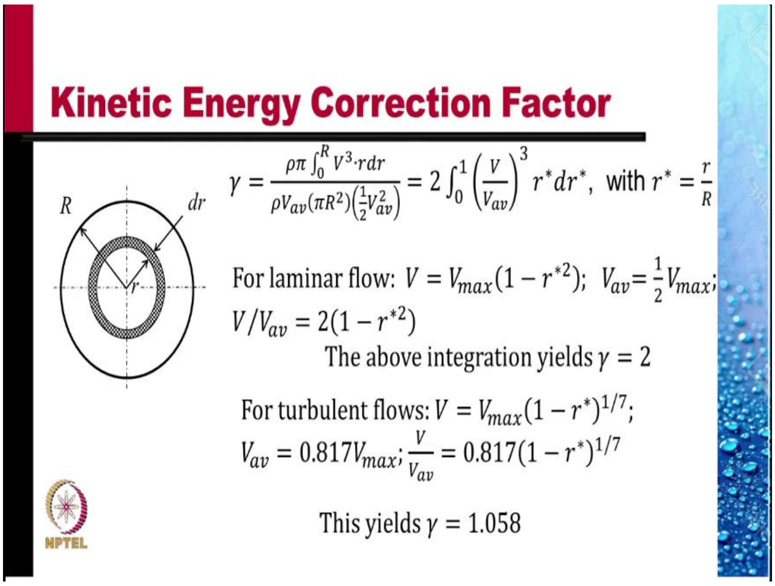

For the laminar flow, $V = V_{max}(1 - r^{*2})$. V_{av} is one half V_{max}. $\frac{V}{V_{av}} = 2(1 - r^{*2})$. The above integration yields γ, the kinetic energy correction factor, as 2.

We could use turbulent flow also, and in turbulent flow, the most commonly used velocity profile is the one seventh power law profile $V = V_{max}(1 - r^*)^{1/7}$. V_{av} turns out to be 0.817 V_{max} max and $\frac{V}{V_{av}} = 0.817(1 - r^*)^{1/7}$. This yields γ is equal to 1.058.

The kinetic energy correction factor for turbulent flow is very close to 1. So, we might as well not use the kinetic energy factor in the case of turbulent flow. As was mentioned earlier, most pipe flows of commercial interest are turbulent, and so there probably is no need of using kinetic energy correction factors. Thank you.