Fluid Mechanics and Its Applications
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Lecture 10B
Mis-applications of Bernoulli Equation

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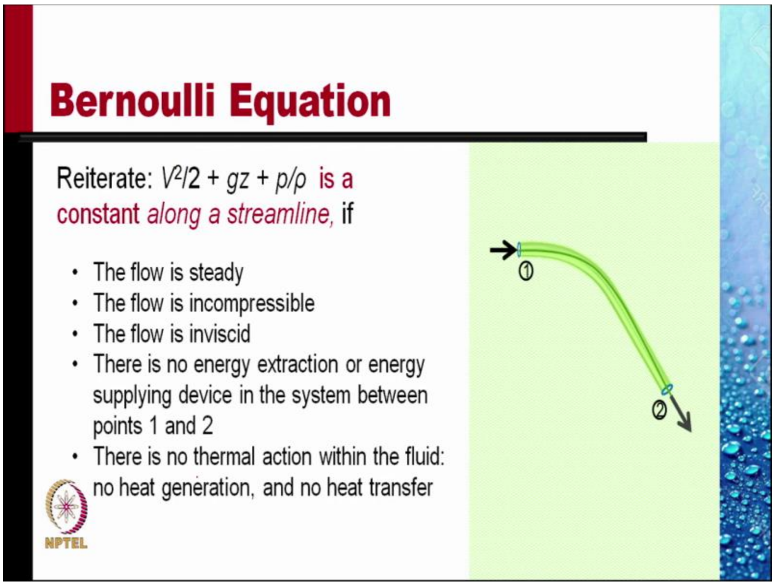

We reiterate that the Bernoulli equation is $\frac{V^2}{2} + gz + \frac{p}{\rho}$ is constant along a streamline. If, and this is a long list of conditions, the flow is steady, the flow is incompressible, the flow is inviscid, there is no energy extraction or energy supplying device in the system between points 1 and 2, and there is no thermal action within the fluid, no heat generation and no heat transfer.

The last two conditions are easy to see, and usually no mistakes are made in this. The flow is inviscid. This is the condition because of which many errors creep in. But most of the errors are there because of the conditions in the red. That is, the two points must be along the same streamline, and that the flow steady.

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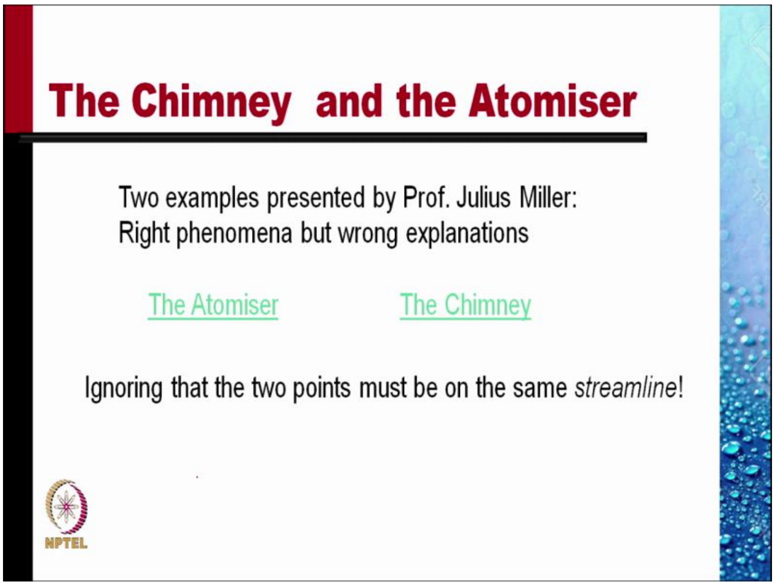

Professor Julius Miller is a famous education specialist who prepares a lot of videos to teach fluid mechanics. And in his pictures, there are a couple of examples that are given where the phenomena is right but the explanations following the Bernoulli equation are wrong. The two examples are shown here. One is the atomiser. And the other is the chimney.

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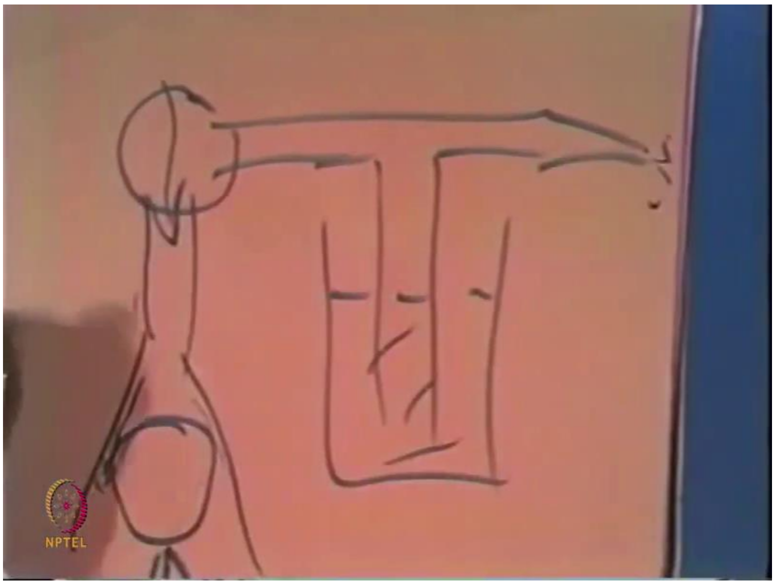

An atomiser is a tube that resides in a vessel. And here is some liquid and here is a tube across here and here is a bulb and you squeeze the bulb, air rushes across. There is a reduction in pressure. The atmosphere pushes the liquid up and then the liquid is caught in the air stream as it comes out there. I have often thought that the reason that is called an atomiser is because little atoms of the liquid come out there. Do not you like that idea, little atoms. Now here is an atomiser. And spray would come out there. Now I am going to show you an atomiser on large scale. Watch it. This is fantastic.

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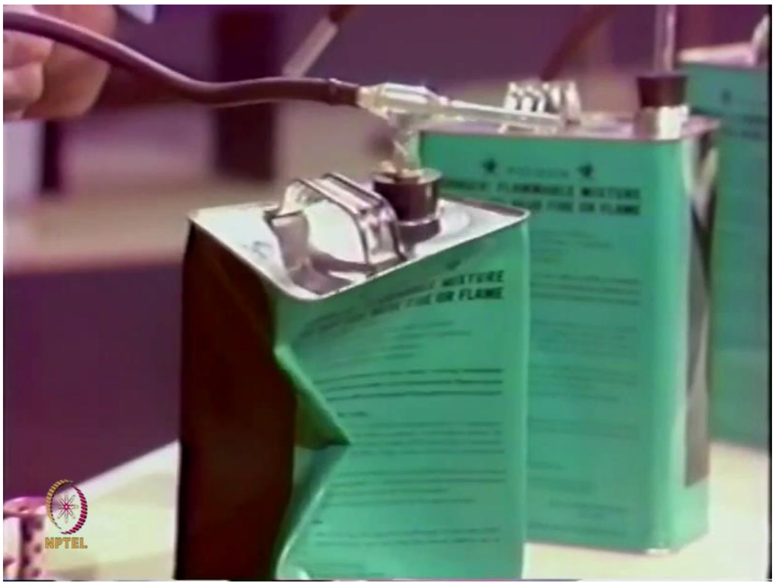

Here is a tin can. And here is a tube quite like that one residing in the atomiser. And I am going to blow a stream of air through this tube there will be a reduction of pressure in the tin can.

And now what will the atmosphere do if the can has less pressure inside. The atmosphere will collapse it because it is enormously strong. Watch it, watch.

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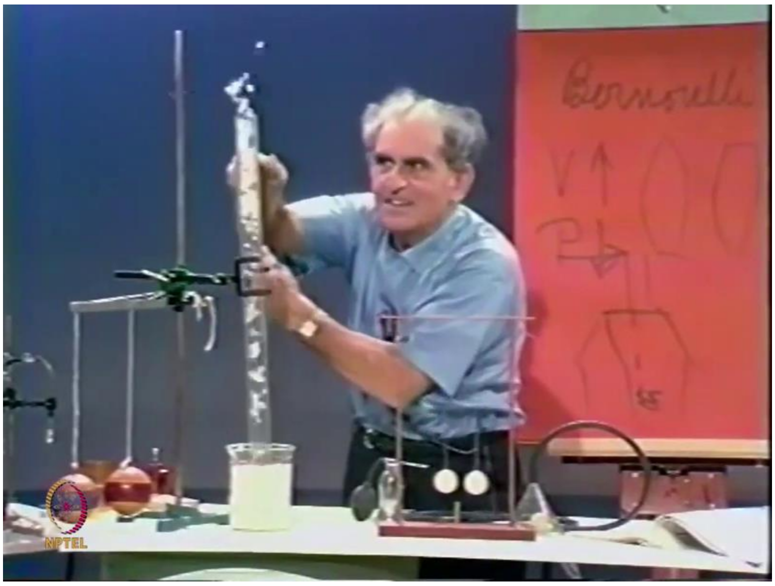

Here is my fireplace. My fireplace is a vessel with some puffed rice. Here is the chimney. I am going to blow a sharp stream of air across the top. The pressure will be reduced here in this region, the reduction in pressure will be felt in the tube, and the atmosphere will push it up, watch it. There it is. My chimney has an extraordinarily good draft watch it. There it is. I say that is a terrific.

Professor Vijay Gupta: Both these examples make the mistake that for Bernoulli equation to be true between two points: that the total energy of the two points must be same, the two points must be on the same streamline.

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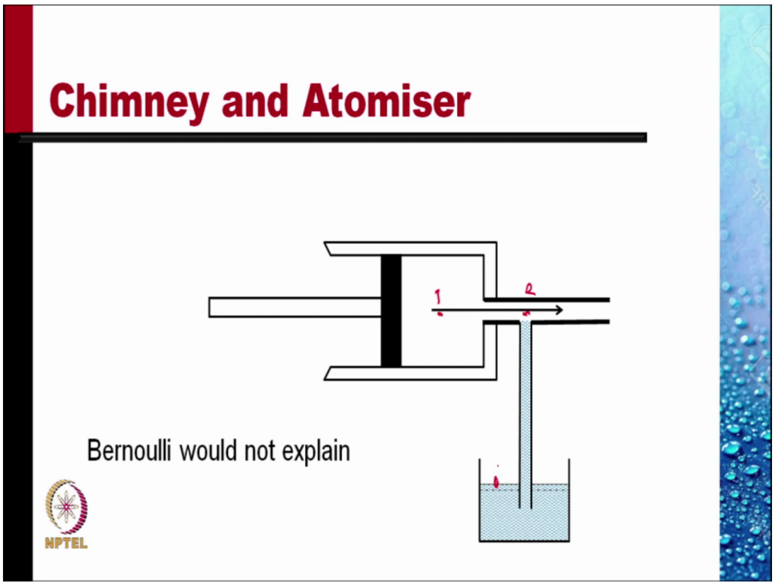

Let us simplify the picture. We have a flow in atomiser taking place like this. Professor Miller says the pressure just above the vertical limb is much less than the atmospheric pressure because of the larger velocity there. But where are the two points on which we are applying? If we say this is point 2 and this is point 1. Yes, the pressure point 2 is less, but less than 1. But, what is the pressure at 1? It is not atmospheric. So we cannot say that the pressure at 2 is less than the atmospheric pressure. And, clearly this cannot be the explanation why the liquid rises up the tube. Bernoulli would not explain.

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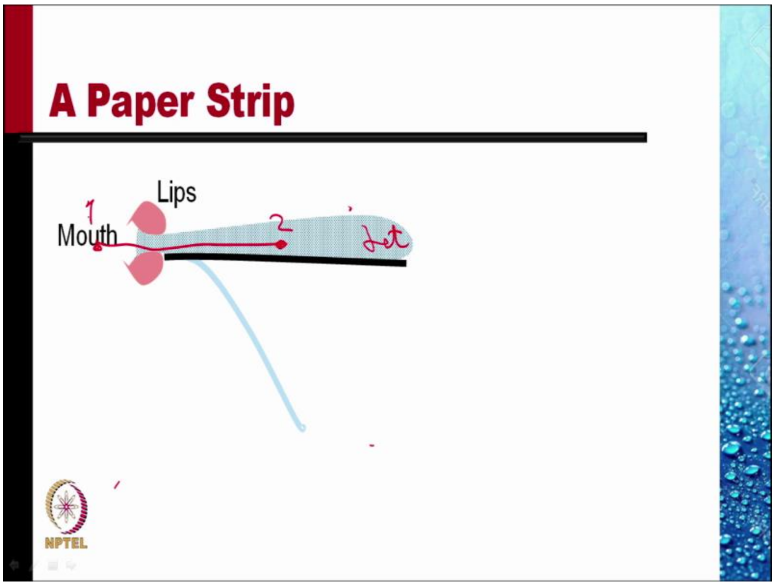

There is another example that is given for Bernoulli equation. That take a slip of paper and put it against your lips like this, and blow air. And as you blow air, you would see the slip of paper would rise and become horizontal. This is the jet of air that comes out, and the paper becomes horizontal.

The explanation given is that because a larger velocity there, the pressure is less. Because the pressure is less, the paper rises. Pressure is less, but less than what? We have to follow a streamline. And on the streamline the stream of air would come from within the mouth, and the pressure there is not atmospheric. Pressure there is more than atmospheric. So there is nothing that shows the pressure at 2 should be less than atmospheric so that will create a lift on the paper. So, clearly Bernoulli equation is the wrong explanation for this phenomenon.

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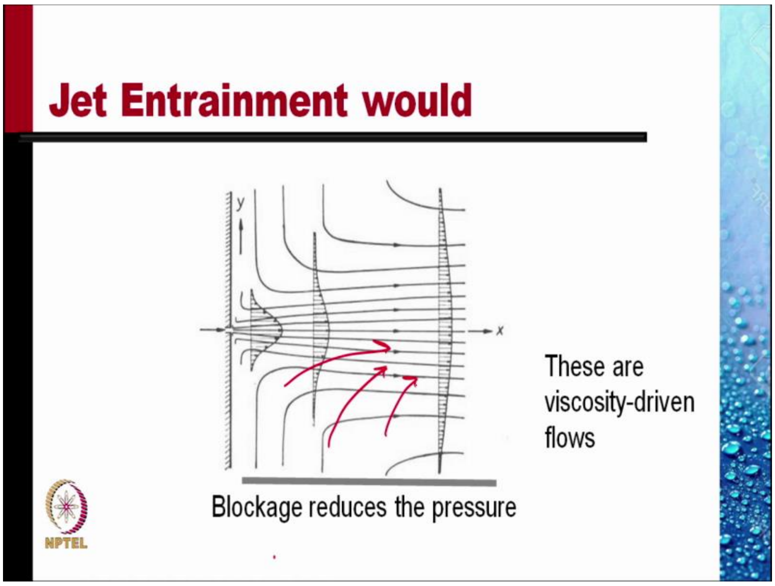

What is the actual explanation? The jet entrainment. As a fluid exits from a jet, it expands as shown. The velocity profile at three locations and the streamlines. You see the flow is being entrained in the jet from the side. It goes in there. Why does it do it? Because of viscous action. The jet coming out of the orifice drags the surrounding air into it, and the surrounding air starts moving. As the surrounding air starts moving, if we block the surrounding air, then the pressure there would reduce. If there is no blockage, the pressure throughout this region is atmospheric. There is no suction. The air is not moving upwards because of the reduction of pressure, but because of viscous action. The blockage does not let the air move in from below. And so there is a reduction in pressure because of the blockage and the viscous action combined together. So these are viscosity driven flows. Bernoulli equation would not really apply.

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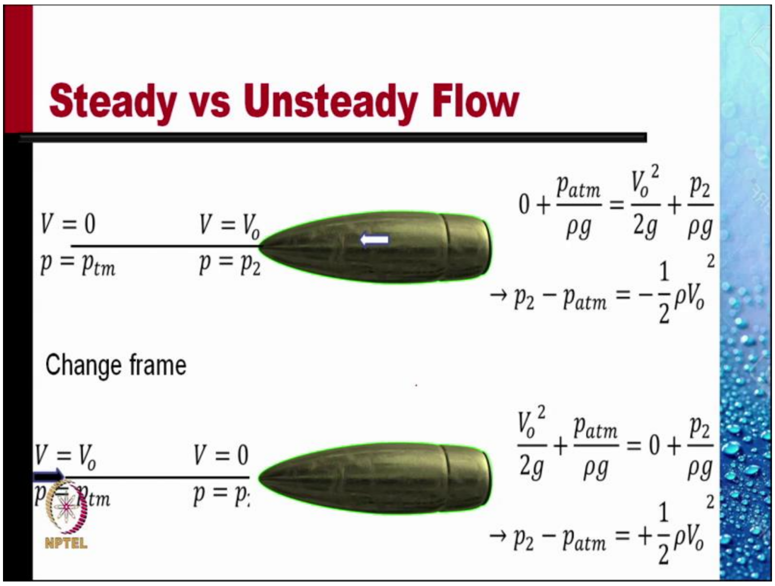

As another example, consider a projectile moving to the left through atmosphere which is stationary and pressure is p atmospheric all around. What is the pressure at the nose of this projectile? If you apply Bernoulli equation to this, at a point far upstream, the atmosphere is stationary, the velocity is 0, the pressure is $p_{\text{atmospheric}}$. At the nose, the air particle is moving with the velocity of the projectile, velocity is V_o and the pressure is to be determined. If we apply Bernoulli equation between these two points, I get $(p_2 - p_{\text{atmospheric}})$ is $-\frac{1}{2}\rho V_o^2$, that is the pressure at point 2 is below atmospheric pressure by an amount $\frac{1}{2}\rho V_o^2$. This is clearly against the observed phenomenon. The pressure at the nose of this projectile is always more than the atmospheric pressure.

So, where have we gone wrong? This projectile is moving fast, at a large speed so that the Reynolds number can be taken as very high. We are on the same streamline. The velocities are written correctly. So where have we gone wrong? Where we have gone wrong is that the flow is not steady. Bernoulli equation applies only to steady flows in the form that we have derived it. So we have to make the flow steady before we apply the Bernoulli equation. How do we make this flow steady: by changing the frame of reference.

Let us attach the frame of reference with the projectile. And if we do so, the projectile become stationary, and it is the atmosphere moving past it at the velocity V_o far away where the pressure is $p_{\text{atmospheric}}$. Then far away, the velocity is V_o towards the projectile, pressure is $p_{\text{atmospheric}}$. On the nose of the projectile the velocity is 0 and the pressure is given by Bernoulli equation now as $(p_2 - p_{\text{atmospheric}}) = \frac{1}{2}\rho V_o^2$, positive. This is the correct solutions.

Moral of the story: Bernoulli equation in the form that we derived is applicable only to steady flows. So unless we can ensure that the flow is steady, we cannot apply the Bernoulli equation.

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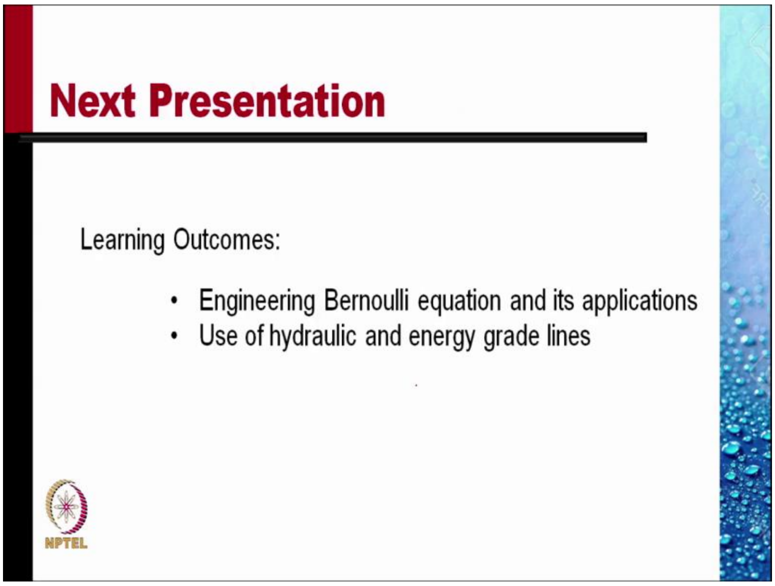

Thank you very much.