Fluid Mechanics and Its Applications
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Lecture 10
Further Applications of Bernoulli Equation

Welcome back. In today's lecture, we will discuss further applications of Bernoulli equation and some wrong applications of Bernoulli equations.

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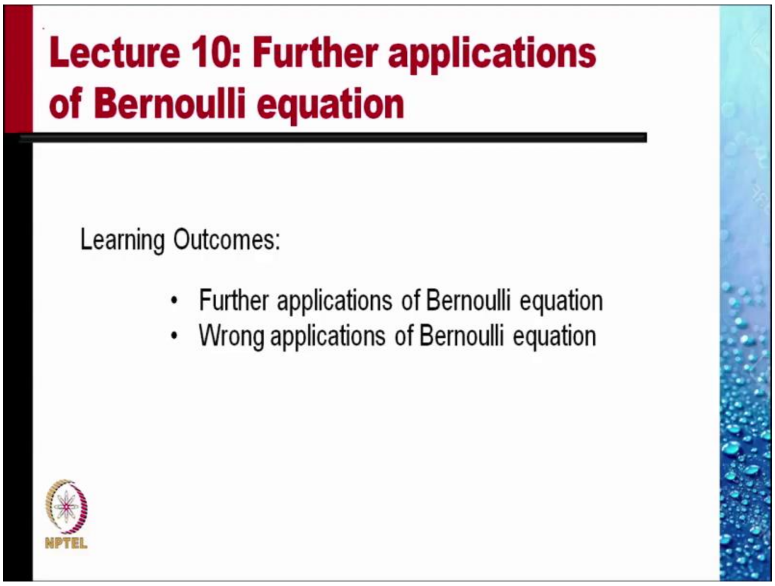

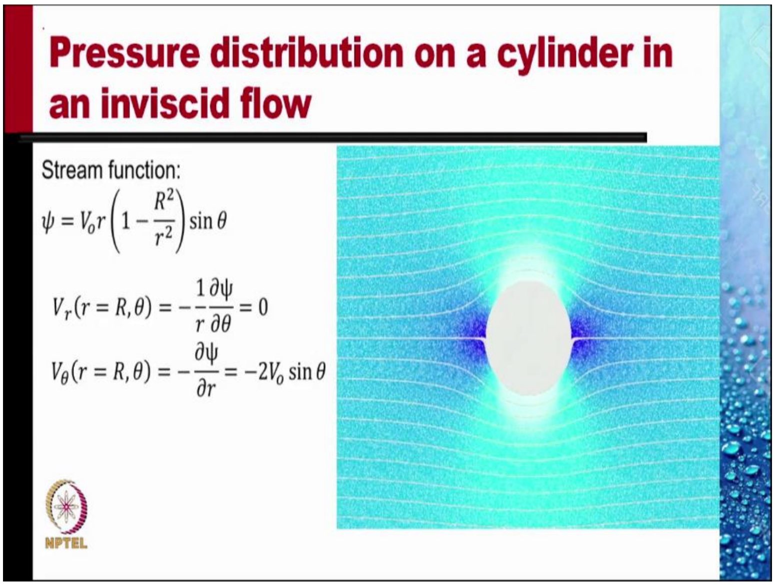

Let us start with pressure distribution on a cylinder in an inviscid flow. If we can neglect viscosity, then the flow about a circular cylinder is given by this stream function $\psi = V_o r\left(1-\frac{R^2}{r^2}\right)\sin\theta$, where capital R is the radius of the cylinder and V_o is the velocity of the fluid as it approaches the cylinder.

As we discussed, we can find out the velocity component of this flow by taking the derivatives of the steam function ψ. The r component of velocity is given by $V_r = -\frac{1}{r}\frac{\partial \psi}{\partial \theta}$, and this on $r = R$, that is, on the surface of the cylinder is 0. The velocity component theta, V_θ on the surface of the cylinder that is on $r = R$ and any θ is $-\frac{\partial \psi}{\partial r}$ at $r = R$. That turns out to be $-2V_0 \sin\theta$.

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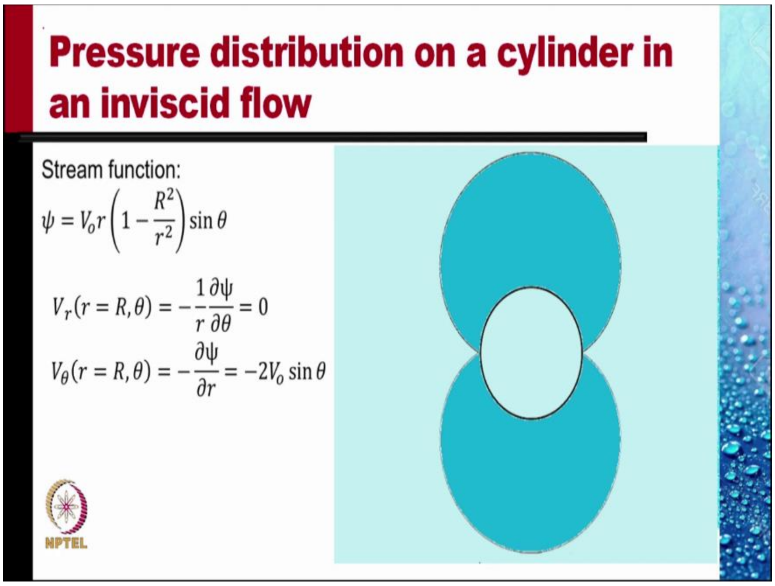

We plotted the velocity variations as a function of theta. On this, the nose of the cylinder and the tail of the cylinder are stagnation points, where the velocity is 0. The maximum velocity is at the shoulder of the cylinder. To find the pressure on the surface of the cylinder, we need to apply Bernoulli equation.

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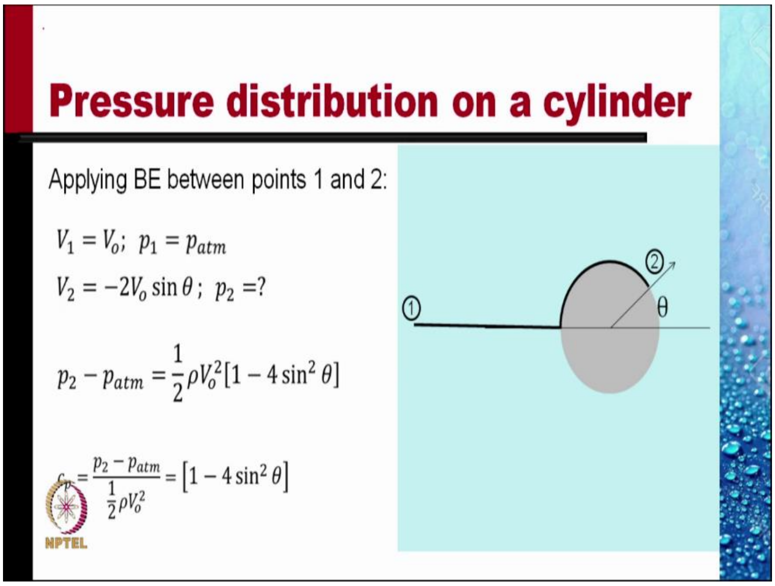

With this definition of angle theta, let us consider a streamline from a point 1 far stream of the cylinder and point 2 on the surface of the cylinder at an angle theta. Bernoulli equation requires that the two points should be on the same streamline. So, applying the Bernoulli equation between points 1 and 2, we know the velocity at point 1 is V1 is equal to V naught, and the pressure there can be taken as the undisturbed pressure p atmospheric. At point 2 the velocity we have determined earlier is minus twice V naught sine theta.

We have to determine the value of the pressure at point 2, the p2. And if we apply the Bernoulli equation, that is, the kinetic energy plus the potential energy plus the pressure term is the same at point 1 and point 2, we get this result: that p2 minus p atmospheric, that is, the gauge pressure at point 2 is one-half rho V naught square times 1 minus 4 sine square theta. We define a pressure coefficient Cp as this gauge pressure divided by 1 half rho V naught squared, and this comes out to 1 minus 4 sine squared theta.

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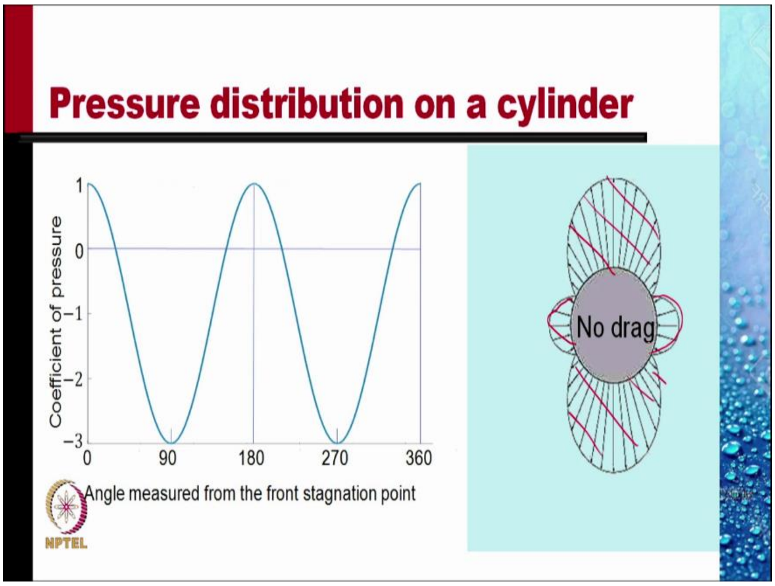

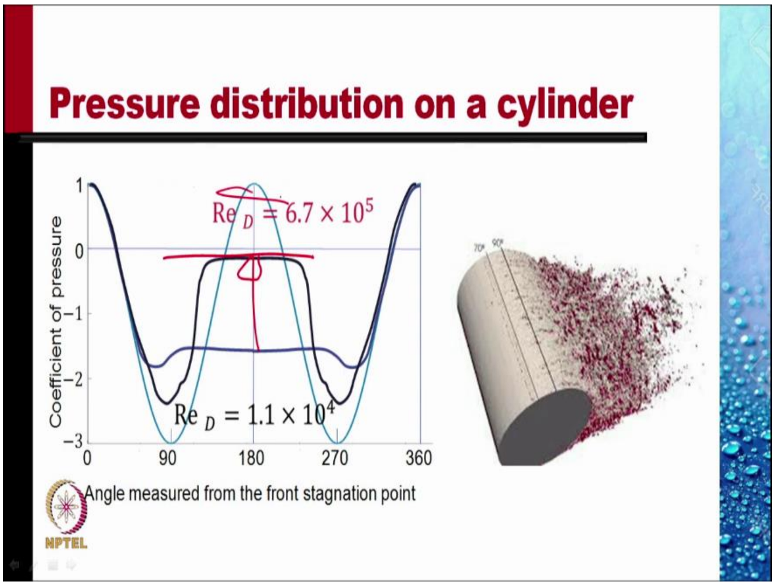

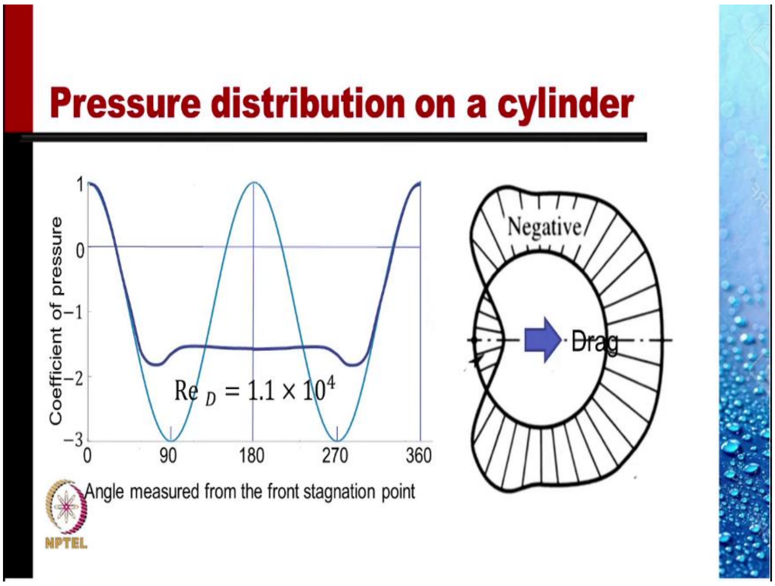

And if you plot this, we get this value. The pressure coefficient is positive to start with at the nose of the cylinder. It decreases continuously till we reach the shoulder, that is, theta is equal to 90 degrees where the pressure coefficient is minus 3, the pressure is negative. The gauge pressure is negative, below atmospheric. The pressure rises back again to 1 at the tail of the cylinder. And this is for the top half, and the same pattern repeats in the bottom half of the cylinder.

If you plot the pressure distribution in a polar plot, this is the pressure distribution that we get. This area is the negative gauge pressure, and this region here and here are the positive gauge pressure regions. Clearly, the pressure pattern is symmetrical for and aft, and top and bottom. So, there is no net force on this cylinder: no drag. We will later on study a theorem which states that in irrotational flows, the drag force on any 2-dimensional body is 0. This is the actual flow behind the cylinder.

We have assumed that the flow on a cylinder is irrotational, and when the flow is irrotational, the flow does not separate from the body, but in any actual flow there is separation. And if the Reynolds number is the order 10 to power 4 or less, then the separation takes place very early, and this pressure distribution that we obtained is like this at Reynolds number is equal to 1.1 into 10 is to power 4 about 11,000.

You notice that the pressure in the front of the cylinder, both these areas are the front of the cylinder, are very close to irrotational flow distribution, inviscid flow distribution, but the flow separates in this region, and that is reflected here. You notice that in an irrotational flow, the flow starts at the high pressure goes down at 90 degrees, it is at the lowest point and then recovers as it travels backward. At the back of this the pressure is the same as in the front.

But in actual flow, the pressure does not recover. The pressure remains in the negative region. The pressure distribution is now something like this. There is a positive pressure at the nose, but then negative pressure at the back. And this results in a rather large drag on the cylinder. The picture is quite different when the Reynolds number is increased.

For a higher Reynolds number, the boundary layer separation is delayed and the wake is narrower as you see there. The region of disturbed flow behind the cylinder is much narrower. The pressure distribution is more like this. There is more pressure recovery at the back, but still it has not recovered to the level at the nose. But there is much better pressure recovered. This is at Reynolds number is equal to 6.7 into 10 raised to power 5, about 670,000.

We will later learn that this is where the boundary layer on the cylinder has become turbulent. Turbulent boundary layer is more energetic. So it travels till further down the cylinder before it separates. And so the pressure profile is more closer to irrotational-flow pressure profile for most of the region except near the very tail. And because of this, the drag on the cylinder is much lower.

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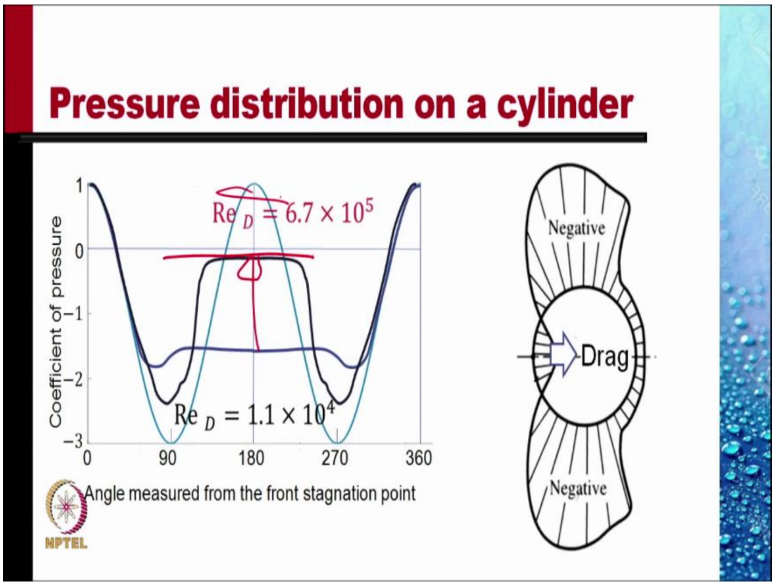

The pressure profile looks like this. The pressure is still negative at the tail, but the magnitude is much less than what it was when there was laminar separation at Reynolds number of 110,000. There is still drag, but the drag is much lower.

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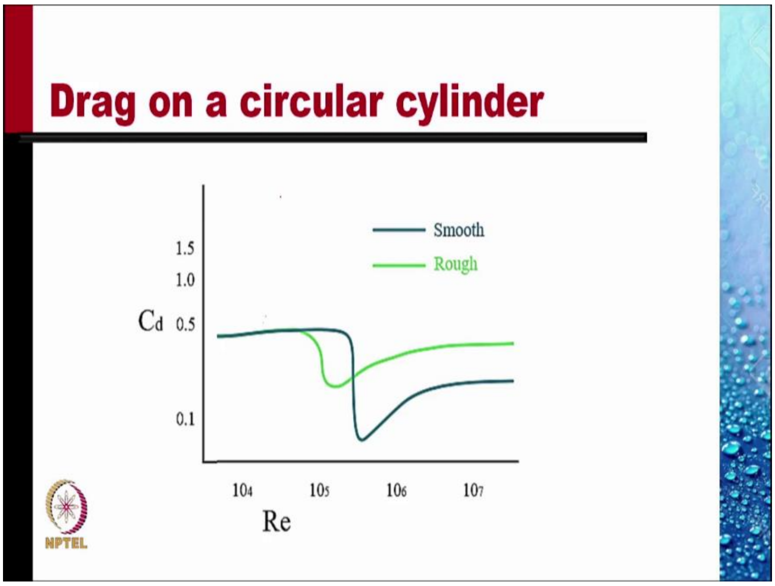

This graph shows the variation of the drag coefficient, which is defined as the drag force divided by $\frac{1}{2}\rho V_o^2 A$, where A is the frontal area of the cylinder. And we see that on the smooth cylinder, the drag coefficient is constant at about 0.44 till a Reynolds number of about 5×10^5, and then it drops suddenly to a lower value.

On a rough cylinder, the transition occurs a little earlier, because the roughness of the cylindrical surface makes the boundary layer on the cylinder turbulent at a lower Reynolds number. In fact, we will later learn that this is the reason why we make the golf balls dimpled: to reduce the drag on them.

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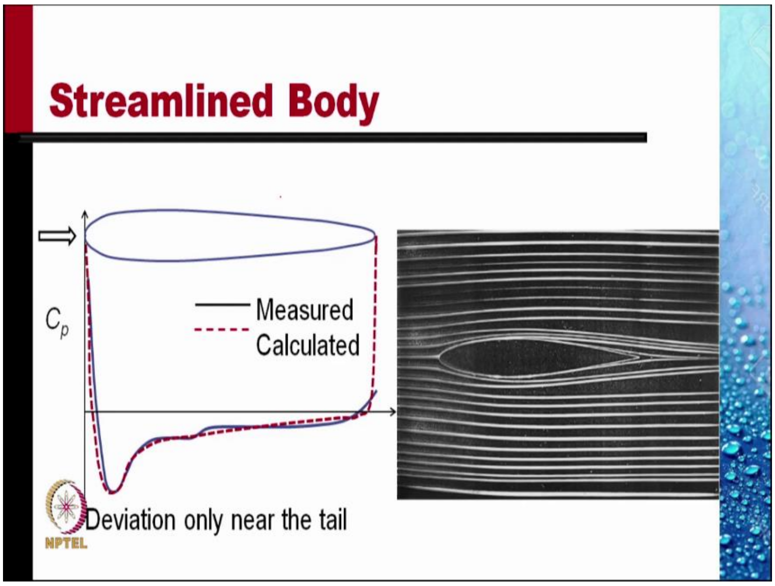

But the picture is quite different on a streamlined body. If we take an airfoil, a 2-dimensional wing, then the measured pressure coefficient and the calculated pressure coefficient under the assumption that the flow is irrotational are almost the same except at the very tail of the airfoil.

This is largely because an airfoil is a streamlined body: rounded nose and a long tapering tail such that the boundary layer separation on this is minimal, only very near the tail. And the flow around and airfoil, the real flow around the airfoil, at large Reynolds number is almost the same as the calculated flow about an airfoil with the assumption that the flow is irrotational, inviscid.

So, we can apply irrotational flow assumption to a streamline body much better than we can apply to bluff body, the bodies which are not steam-lined. But even in those bodies, the pressure distribution near the nose is the same, both in the real flow as well as in the theoretical flow with the assumption of irrotationality.

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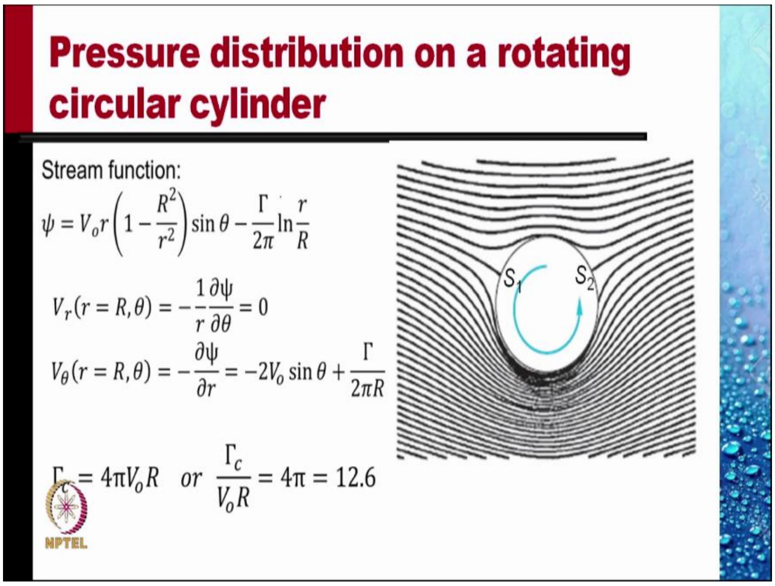

Let us next consider the case of pressure distribution on a rotating circular cylinder. The steam function for such a flow is written here. It is the same as that for a circular cylinder without rotation, plus the last term which represents circulation and causes this to behave like a rotating cylinder. $\psi = V_o r\left(1 - \frac{R^2}{r^2}\right)\sin\theta - \frac{\Gamma}{2\pi}\ln\frac{r}{R}$, where Γ for a circular cylinder is equal to the rotational velocity of the circular cylinder times $2\pi R$ times R, where R is the radius of the cylinder.

We can find out the velocity components in the same fashion as before. These are the velocity components on the surface of the cylinder at $r = R$. Of course, the radial component of velocity is 0. There is no flow penetrating or coming out of the cylindrical surface. But the theta component is $-2V_o \sin\theta + \frac{\Gamma}{2\pi R}$, where Γ is related to the rate of rotation of the circular cylinder.

If we draw this, if this is a circular cylinder, then for the flow taking place from left to right and the cylinder rotating in a counterclockwise manner, the stagnation points are now located off the major diameter. When there was no circulation the stagnation points were at the nose of the cylinder and at the tail of the cylinder.

Now, the stagnation points S1 and S2 have moved up. The flow is not symmetric on the top and bottom. The velocity at the bottom is much more than the velocity at the top. This can be seen by inspection alone, because the streamlines are much closer together at the bottom and far apart on the top. This is for a specific value of the circulation Γ. If the value of Γ increases and acquires a critical value $4\pi V_o R$ such that $\frac{\Gamma}{V_o R} = 4\pi$ which is 12.6.

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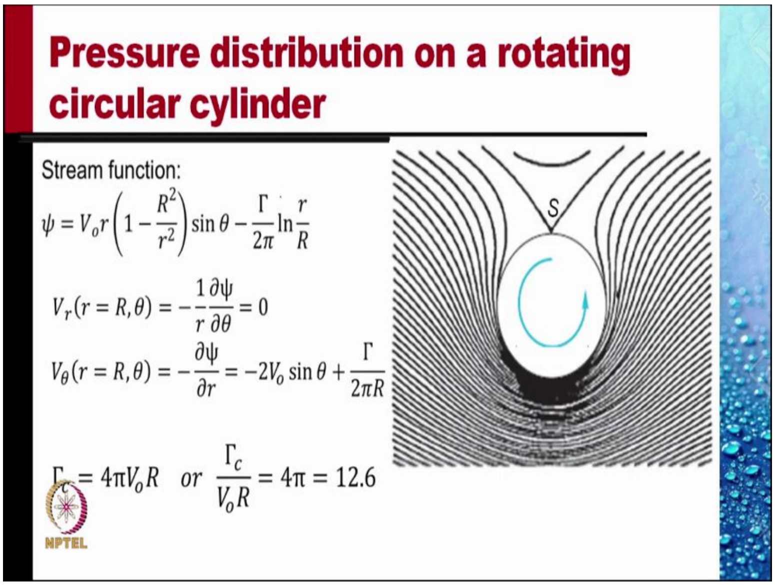

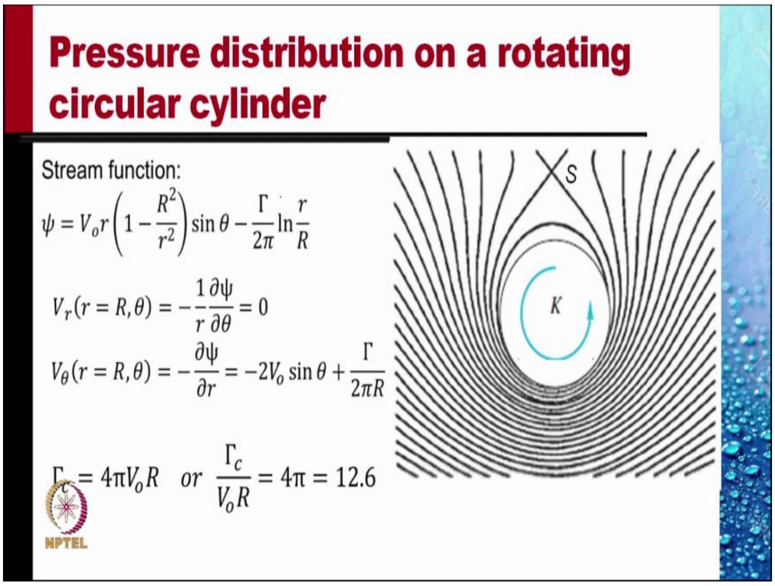

The two points S1 and S2 move till there is only one stagnation point. The asymmetry is much more for values at Γ greater than this value, the stagnation points move off the cylindrical surface as shown here. There is no stagnation point on the surface of the cylinder.

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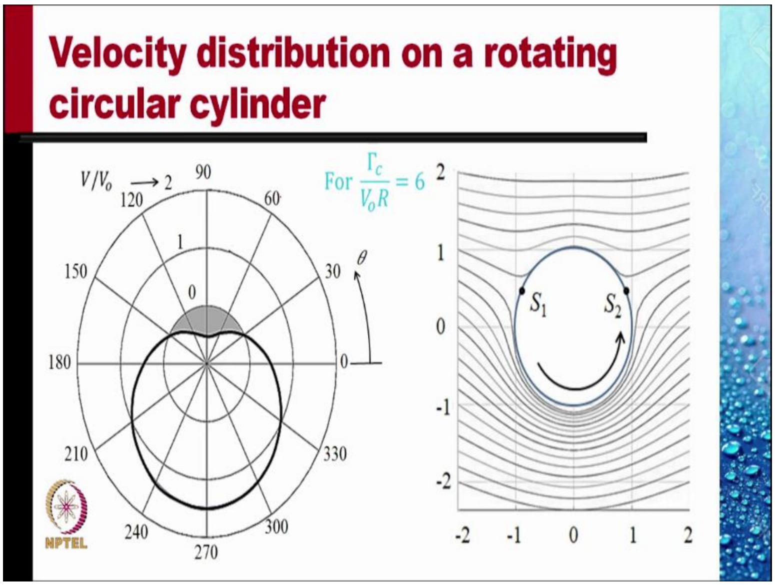

This is the polar plot of the velocity V/V_o, V_o being the free stream velocity. And this, we have plotted for $\frac{\Gamma}{V_o R} = 6$, which was less than what is critical. So there are two stagnation points S1 and S2 as shown. The stagnation points are now at a little below 30 degrees and a little above 150 degrees. The velocity V/V_o being 0 there.

The shaded area, the negative velocity, means this is the negative velocity in the context of the velocity that is defined as positive in the counterclockwise direction and negative in the clockwise direction. So between point S1 to S2, the velocity is in the clockwise direction, that it is coming out to be negative, but the value here is less than 1.

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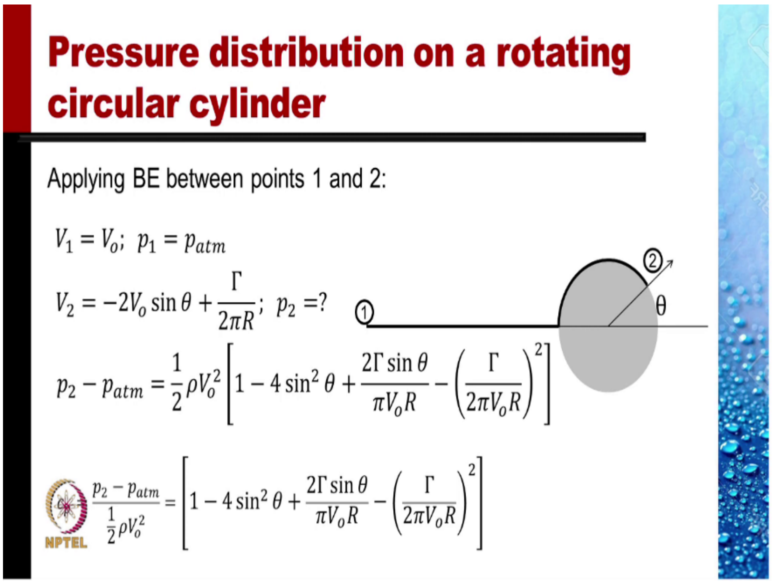

The pressure is again obtained by applying the Bernoulli equation between points 1 and point 2 on the same streamline. And as before, we can obtain the pressure coefficient as $\frac{p_2 - p_{atm}}{\frac{1}{2}\rho V_o^2}$, and that comes out as $\left[1 - 4\sin^2\theta + \frac{2\Gamma \sin\theta}{\pi V_o R} - \left(\frac{\Gamma}{2\pi V_o R}\right)^2\right]$.

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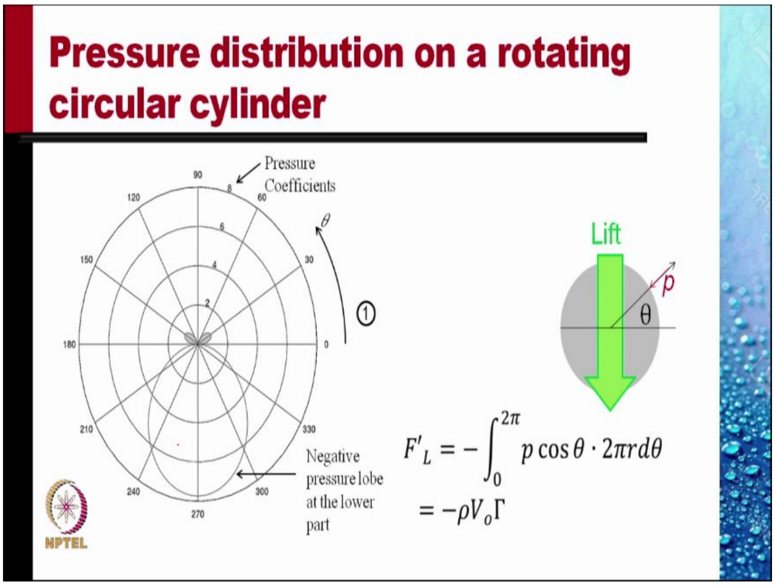

This pressure coefficient is plotted in this polar diagram. The two grey lobes here represent positive pressures, and this big white lobe at the bottom represents the negative pressure. And because of this now, there is a component of force acting on the cylinder downwards. We call it lift because it is perpendicular to the direction of flow. There the flow is from left to right horizontally, this force is perpendicular to it, so it is termed as lift.

How do you evaluate this force? Since we know the pressure at every location, we find out the pressure force at an angle theta, and take the vertical component, which is $p\cos\theta$. And the area of that element is $2\pi R d\theta$ and we integrate over 0 to 2π, and we get the force per unit length $F' = -\rho V_o \Gamma$. This is the famous Magnus rule. We will talk about this further.

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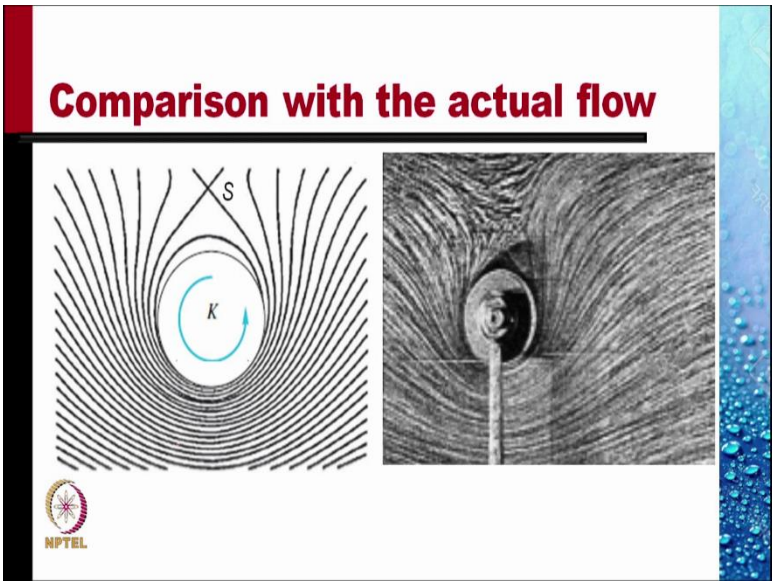

Comparison with the actual flow: Here we have shown a case where the circulation is more than the critical circulation, and you see the flow picture in the actual flow which is on the right is very similar to the flow picture of the irrotational flow calculated assuming the flow has no viscosity.

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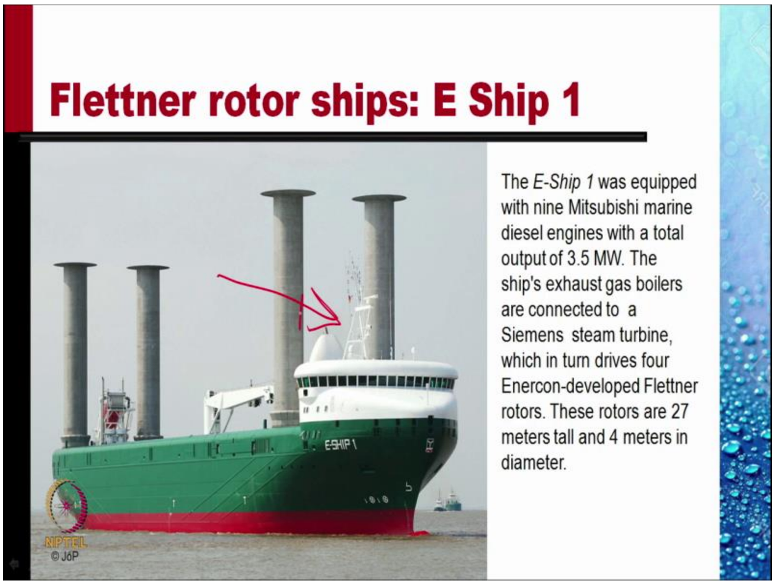

This Magnus force on a rotating cylinder has found application in what are known as Flettner rotors ships. A ship called E-Ship 1 was equipped with 9 marine diesel engines with a total output of 3.4 MW. The ship's exhaust gases from the boiler are connected to a Siemens steam turbine, which in turn drives 4 Enercon-developed Flettner rotors. These four vertical rotors.

And as these 4 verticals rotate and the ship moves forward, this results in additional thrust in the forward direction. These rotors are 27 meters tall and 4 meters in diameter.

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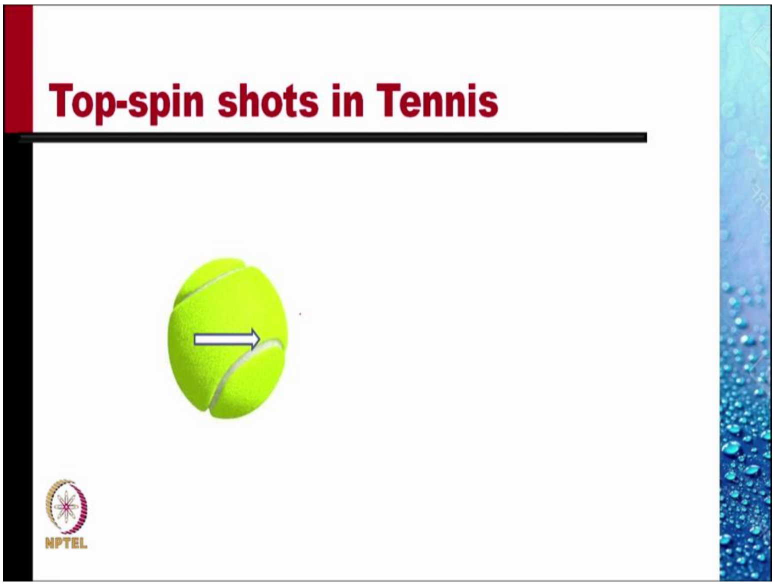

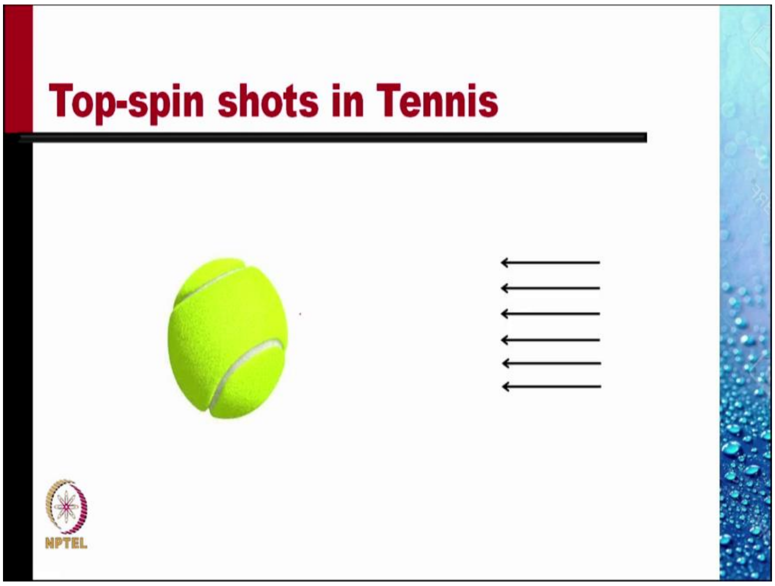

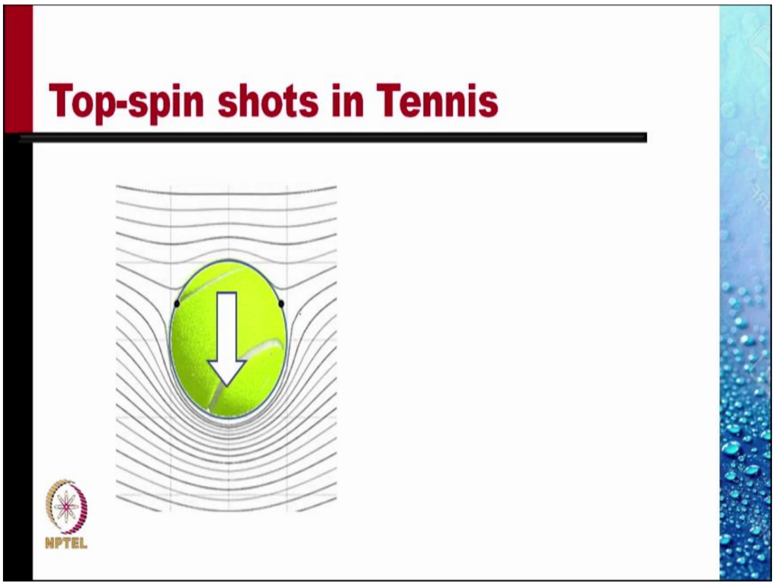

But this Magnus effect, that is the force produced on a rotating cylinder or a sphere that is normal to the direction of motion, finds wide application in sports. In sports using balls, if the balls comes straight, it does not deviate from its path, it is very easy to predict its motion. But if a ball deviates from its path, then the opponent cannot guess the trajectory the ball will follow. And that will result in making errors.

That is why we rotate a ball. A tennis ball is rotated by giving it a spin, a top spin in this case. It is almost like a rotating sphere in a flow. And this results in a force normal to the direction of motion of the ball. The ball is moving towards the right. And so, this ball is going to deviate, it will not follow a state trajectory.

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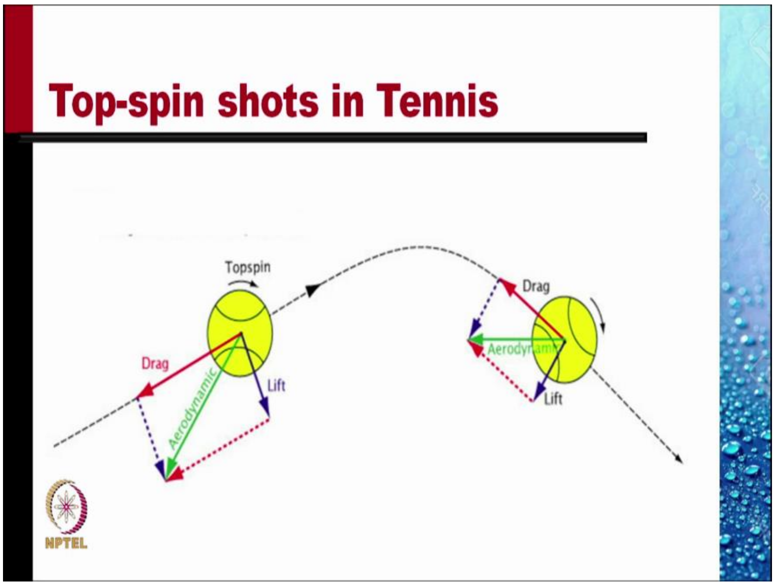

To explain this further, let us consider a tennis ball traveling toward the right. After it is hit as shown in the diagram on the left, there is a drag which is opposing the forward motion of the ball. There is a lift because of the top spin as shown, and the net aerodynamic force is the green arrow shown here. When the ball is coming down on the other side of the net, the picture is like that shown on the right. The drag force opposing the motion, the lift force acting downwards, but inclined towards the left, and the net aerodynamic force is almost horizontal.

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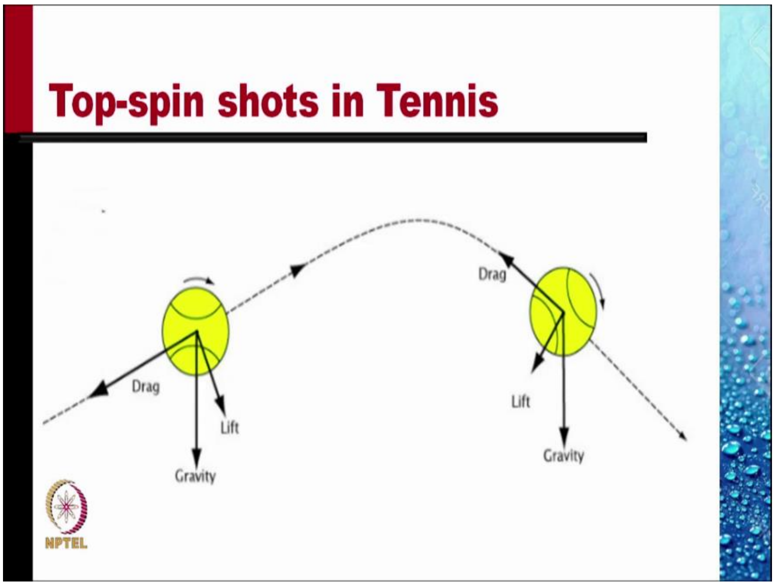

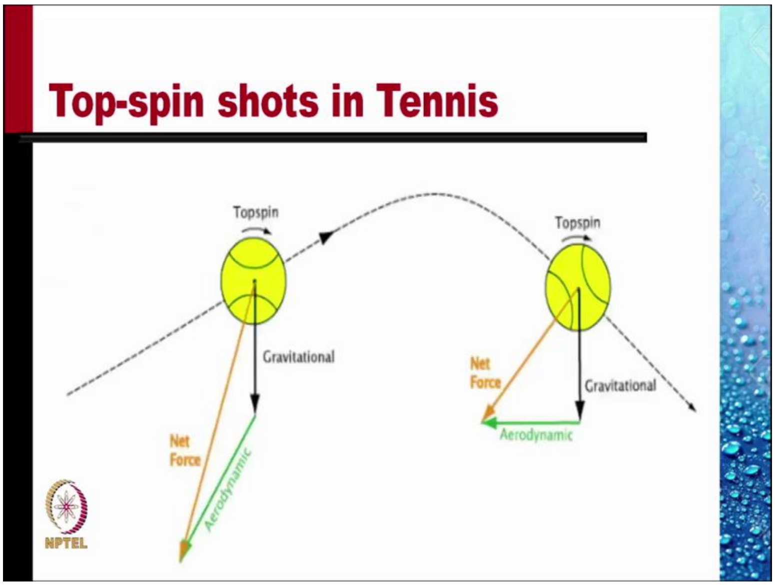

This is combined with the gravity force in each case, so that the total force on the ball, just after it is hit, is largely downwards and little backwards. When it is coming down on the other side is quite a bit backwards. This causes deviation in the path of the ball.

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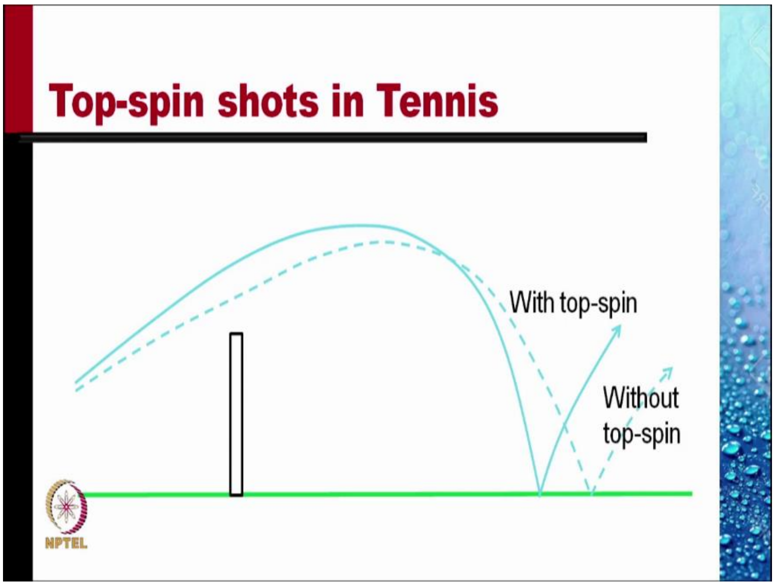

The broken line shows that the path the ball will take without the top spin. But with the top spin, the ball flattens down very quickly, and does not travel as far horizontally, because of a backward force and a downward force. So it comes down more quickly. So a shot hit with the same force is less likely to be out of court if it has top spin rather than when it does not have a top spin. This gives great advantage to the player.

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Same thing in football: the banana shot. I will show you here the animation of the curved shots created in football using the Magnus effect.

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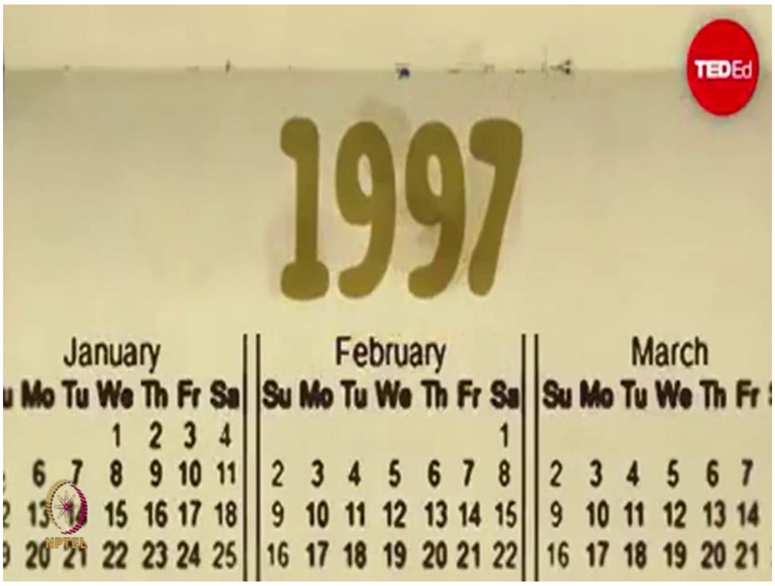

In 1997, in a game between France and Brazil, a young Brazilian player named Roberto Carlos set up for a 35 meter free kick. With no direct line to the goal, Carlos decided to attempt the seemingly impossible, his kick sent the ball flying wide of the players. But just before going out of bounds, it hooks to the left and swerved into the goal.

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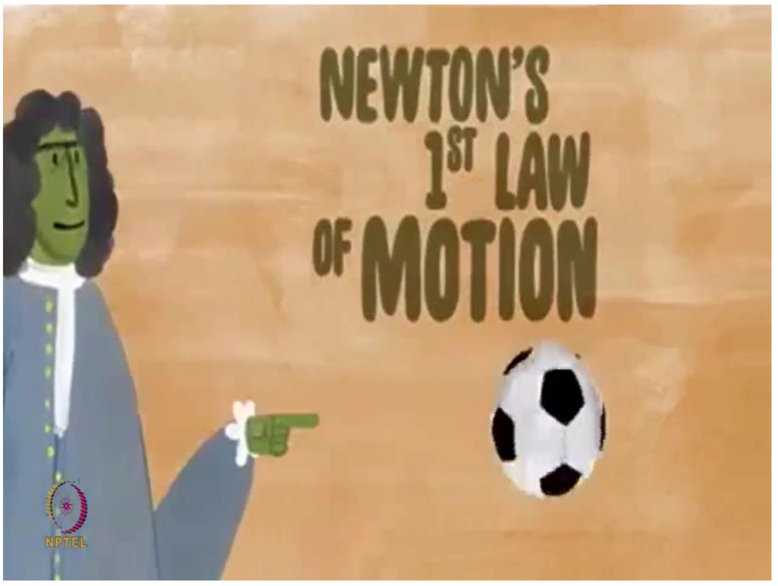

According to Newton's first law of motion, an object will move in the same direction and velocity until a force is applied on it. When Carlos kicked the ball, he gave it direction and velocity. But what force made the ball swerve and score one of the most magnificent goals in the history of the sport?

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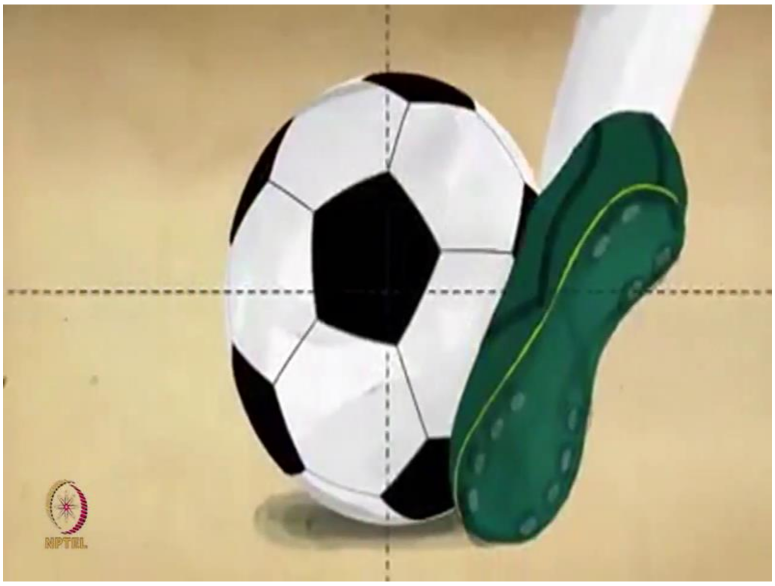

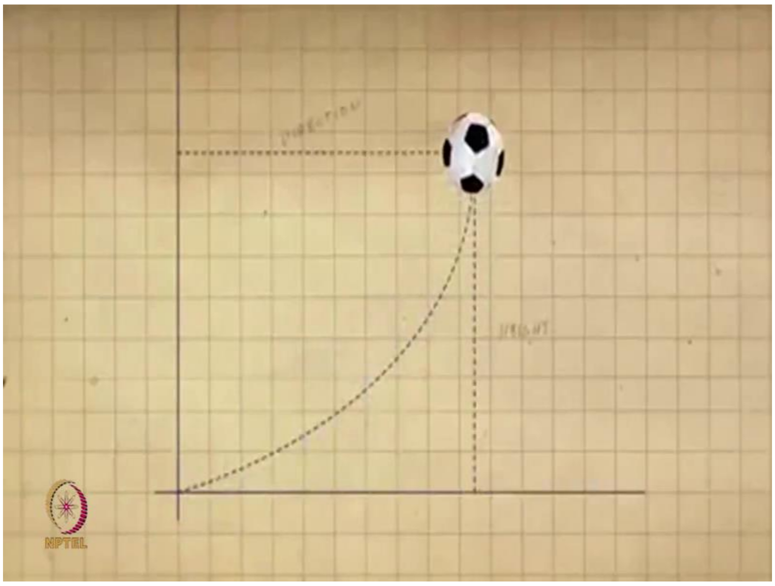

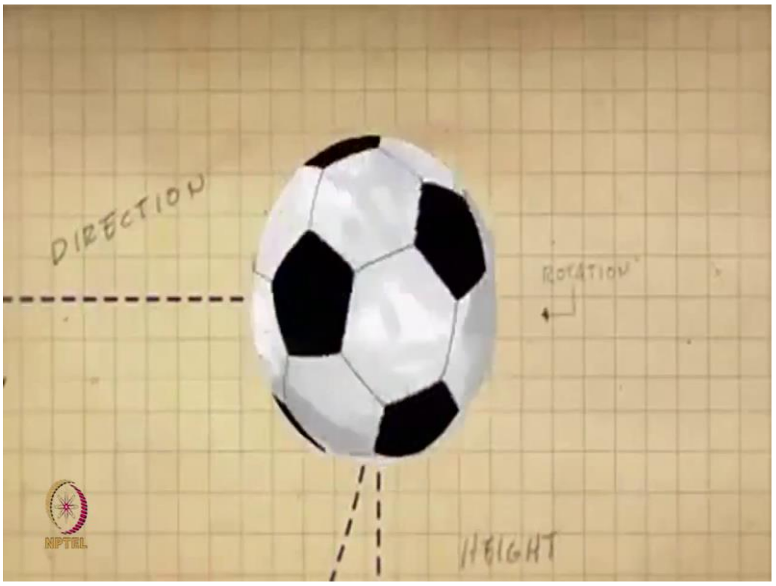

The trick was in the spin, Carlos placed his kick at the lower right corner of the ball, sending it high and to the right, but also rotating around its axis.

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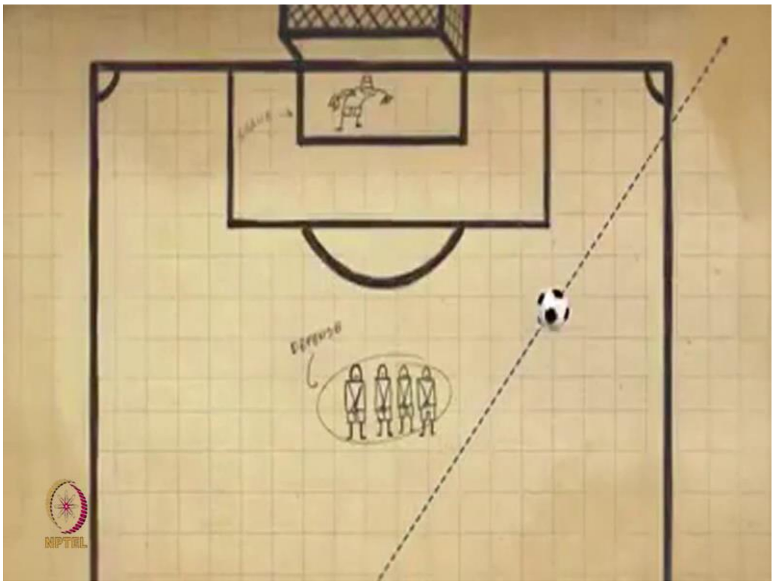

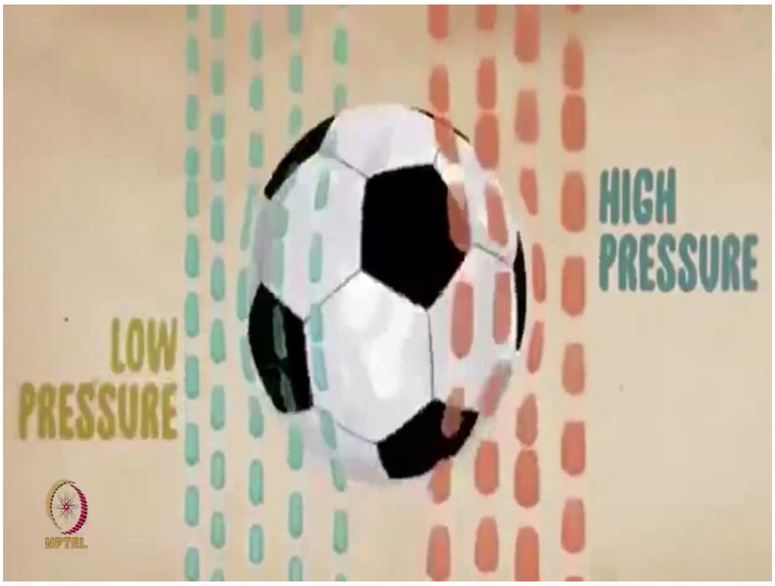
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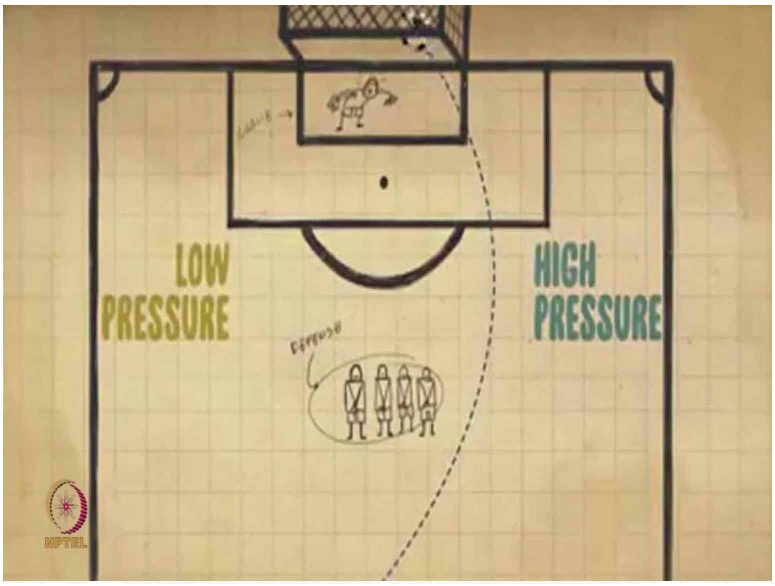
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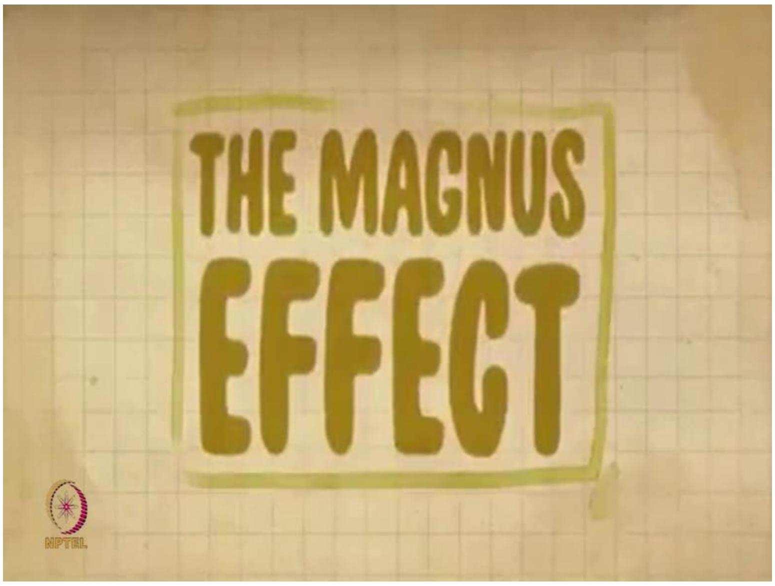
THE MAGNUS
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The ball started its flight in an apparently direct route, with air flowing on both sides and slowing it down. On one side, the air moved in the opposite direction to the ball spin causing increased pressure, while on the other side, the air have moved in the same direction as the spin creating an area of lower pressure. That difference made the ball curve towards the lower pressure zone. This phenomenon is called the Magnus effect.

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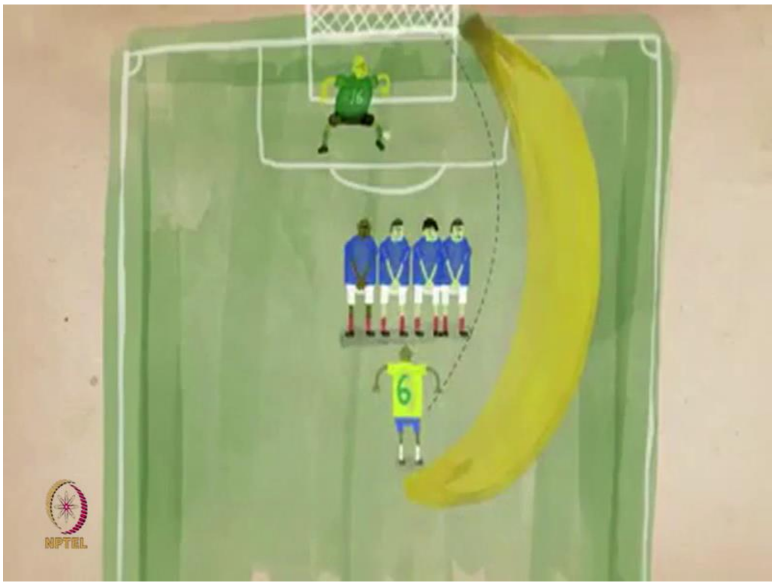

This type of kick, often referred to as a Banana Kick, is attempted regularly. And it is one of the elements that makes the beautiful game beautiful. But curving the ball with the precision needed to both bend around the wall and back into the goal is difficult: too high and it soars over the goal, too low and it hits the ground before curving too wide and it never reaches the goal.

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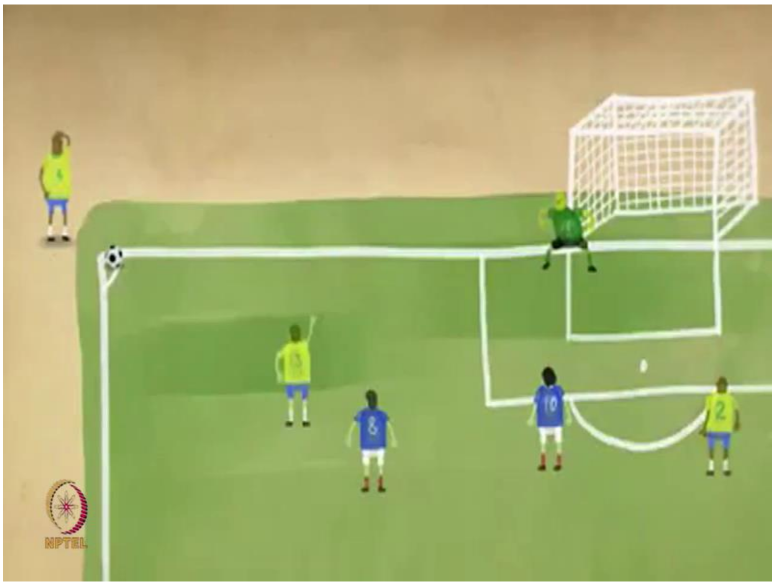

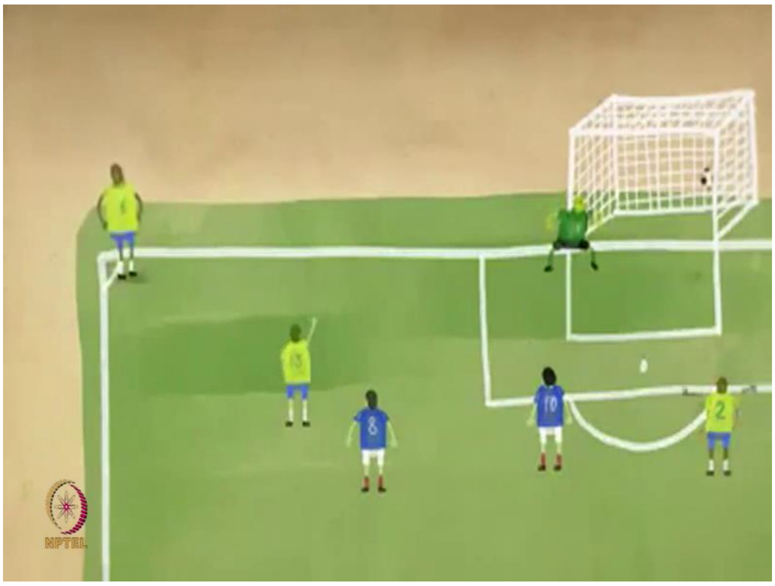

Not wide enough and the defenders intercept it, too slow and it hooks too early or not at all. Too fast and it hooks too late. The same physics make it possible to score another apparently impossible goal, an unassisted corner.