Fluid Mechanics and Its Applications
Professor. Vijay Gupta
Indian Institute of Technology, Delhi
Lecture 9A
Applications of Bernoulli Equation

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Let us consider the phenomenon of siphon. If the tube connecting the two reservoirs is full, then the liquid with a higher level of fluid empties into the tank with a liquid at lower level. How do we analyze this?

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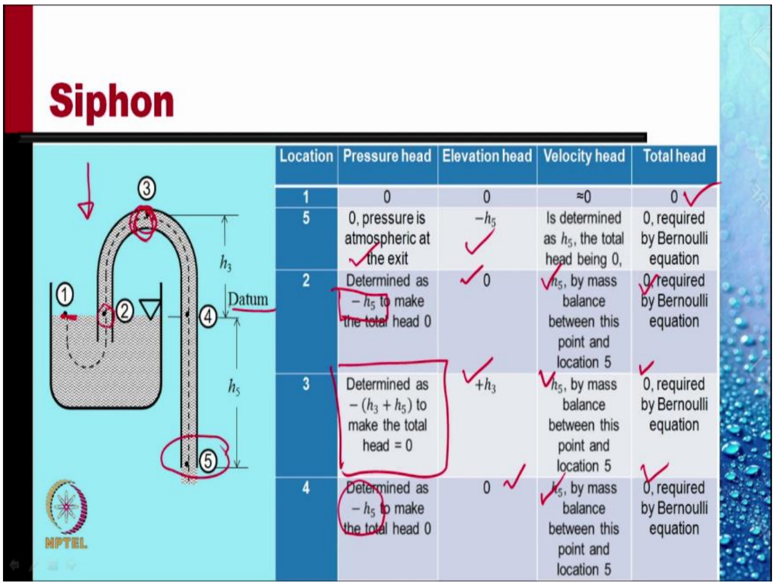

Location	Pressure head	Elevation head	Velocity head	Total head
1	0	0	≈0	0
5	0, pressure is atmospheric at the exit	$-h_5$	Is determined as h_5, the total head being 0,	0, required by Bernoulli equation
2	Determined as $-h_5$ to make the total head 0	0	h_5, by mass balance between this point and location 5	0, required by Bernoulli equation
3	Determined as $-(h_3+h_5)$ to make the total head = 0	$+h_3$	h_5, by mass balance between this point and location 5	0, required by Bernoulli equation
4	Determined as $-h_5$ to make the total head 0	0	h_5, by mass balance between this point and location 5	0, required by Bernoulli equation

To analyze this, let us consider the arrangement shown here. There is a liquid in the tank and there is a tube that runs from the tank, goes to a maximum height h_3 above the datum, and goes down to a distance h_5 below the datum. There would definitely be a streamline, something like what we have shown here: 1, 2, 3, 4, and 5. Let us apply Bernoulli equation between the various points on this streamlines.

This table organizes this determination. At point 1 on the free surface of the liquid in the tank, pressure head is clearly 0, the pressure being above the atmosphere, that is, the gauge pressure, since the pressure at 1 is atmospheric. Clearly, $p_{1,gauge}$ is 0, so pressure head is 0. The elevation is 0 because we have taken the datum at that height. The velocity head is 0 because the velocity is negligible if the tank area is large, and the total head is 0, as in this row.

Next we consider point 5. Why 5? Because we know two of the three heads at 5. We know the pressure head to be 0, because the pressure at the exit is also atmospheric. We know the elevation head, which is $-h_5$ now. And since the total head must be 0 as required by Bernoulli equation, we can determine the velocity head. And the velocity head simply would be what? h_5. Plus h_5, because the elevation head is h_5 minus h_5, the total head is 0, so the pressure head must be h_5.

So, the velocity head must be h_5. As long as point 5 is below point 1, and the siphon tube was running full, there would be a positive velocity at h_5 downwards. Then we move on to point 2. What do we know here? We do not know the pressure. This is inside the pipe. But we know the elevation head, which is 0. And we know the velocity head. Why? Because mass balance between points 2 and 5 would give you the same mass flow rate, and if the area of the tube is constant, then the velocity must be same. So the velocity head at 2 would be the same as the velocity head at 5. And velocity 5 is h_5. So velocity head at 2 is h_5.

The elevation head is 0, and since the total must be 0, this means the pressure must be $-h_5$. There would be a vacuum pressure at point 2. The pressure at point 2 would be less than atmospheric inside the pipe. Interesting.

Then we go to point 3 at the top of the siphon tube. At this point, the total head by Bernoulli equation must still be 0. The velocity head must be same h_5. All across the tube it is the same. Velocity head is h_5, the total head is 0, and the elevation is h_3 now. So calculating this pressure head is determined as $(-h_3 - h_5)$ to make the total head 0.

Let us try one more point. Point 4 at the same level as point 1, the free surface in the tank. The elevation head is 0. The velocity is same, so velocity head is h_5. The total head required is 0. So this means that the pressure head should be $-h_5$. Alright? We have been able to find pressures everywhere in this tube, the pressure is, at point 2 the pressure is sub atmospheric. As we go up the tube, the pressure decreases. The minimum pressure is at point 3, and then as we go down the other limb, the pressure increases, and increases to 0, the value at the exit of the tube at point 5.

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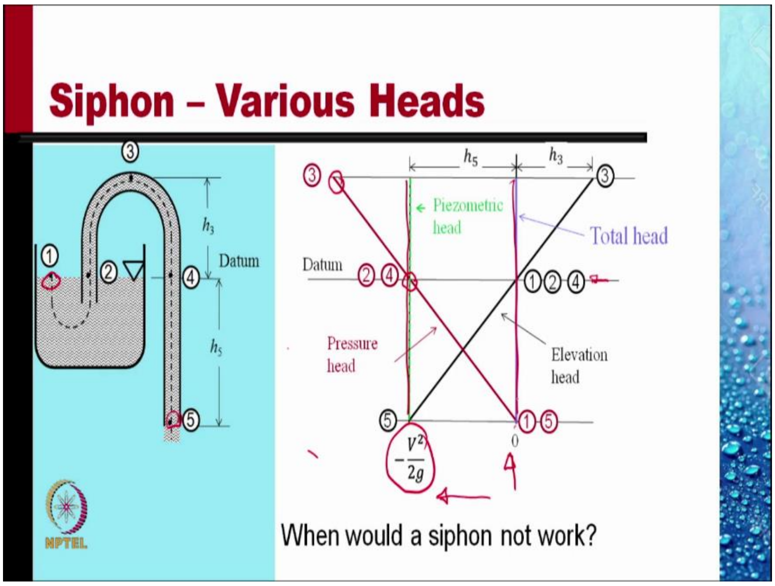

If you plot this: let us take this to be zero line. The elevation head: points 1, 2, and 4 are at the datum, so the elevation head is 0 here. Point 3 is h_3 above the datum, and point 5 is h_5 below the datum, to the left in this diagram. The pressure heads we plot: the pressure at 1 is 0, and so is the pressure at point 5. At point 1 and point 5 both, pressures are 0. Pressure at 3 is minimum, and pressure at 2 and 4 is $-V^2/2g$. This is the velocity head, $-V^2/2g$ everywhere throughout the pipe. Throughout the pipe the velocity head is $-V^2/2g$.

This is also the piezometric head. $(-V^2/2g)$ is the piezometric head. To this we add the velocity head to get the total energy head of 0. So this is the total head throughout the tube. When would this siphon not work? You see the lowest pressure is obtained at point 3. So the siphon would stop working when the pressure at point 3 is such that the water boils, that is, the cavitation occurs. And when would the cavitation occur? When the pressure becomes equal to or lower than the vapor pressure of water.

So, depending upon the temperature, the vapor pressure of water is quite low, a few kilopascals. So this sets the limit of when the siphon would work. If we neglect the small vapor pressure, then we may say that the siphon would work till at point 3 there is total vacuum, the pressure becomes absolute 0.

This means the height $(h_3 + h_5)$ is equal to the barometric head, the atmospheric pressure converted into the liquid head. If the liquid is water, then this is about 10 m. So, if the long arm of the siphon is about 10 m, the siphon would not work. So we can empty a tank only if the long arm of the siphon is less than the barometric head, which is about 10 meters for water.

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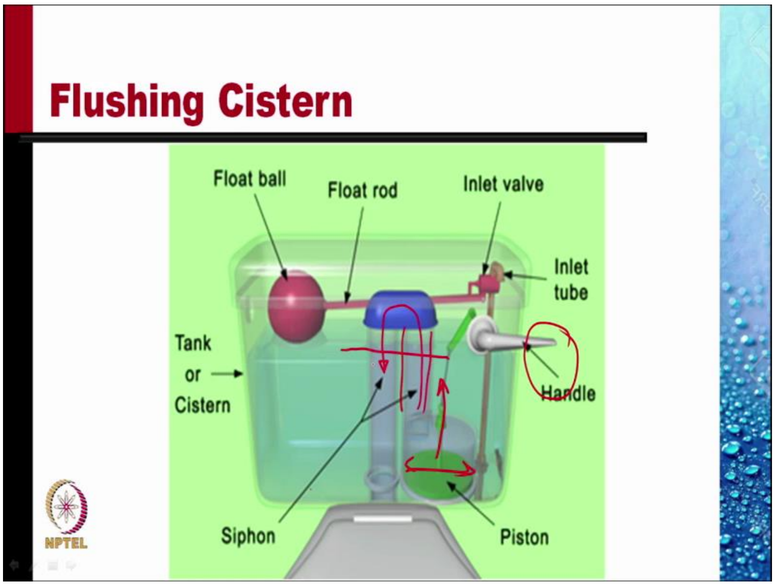

Many of the flushing systems in your toilets work on the principle of siphon. There is a cistern that holds water because of a float valve, and there is a siphon tube that runs like this. The water level is maintained at a level up to here, such that there is air on top at the siphon tube. Now as you operate the handle, this big plunger is moved up. As it moves up it displaces water inside this tube, and the water level rises and it completes the circuit, the water level rises and flows down. And once this tube flows full, it will not stop till all the water in the tank or the system is emptied, and so this results in a flush in the toilet.

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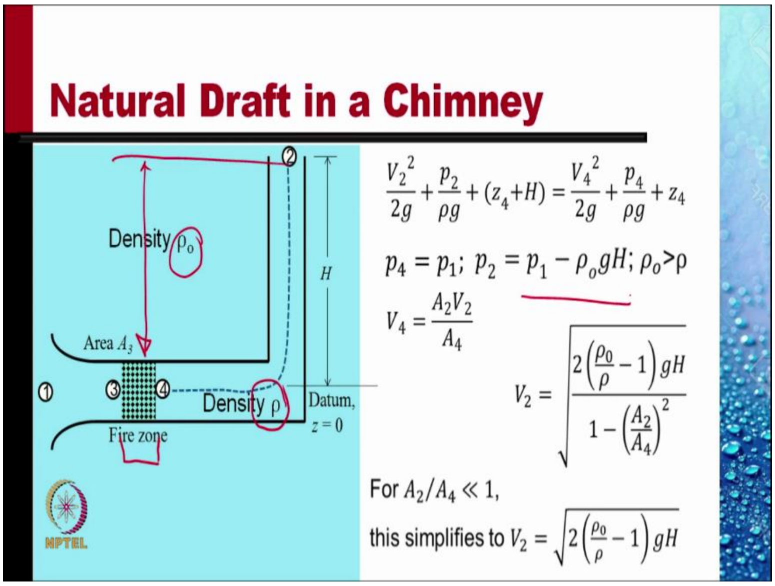

Let us consider another example, a little more complicated example of a natural draft in a chimney. Suppose we have a fireplace and a chimney to supply air. I mean natural draft does the supplying of air. How does it do it? Let us assume that the air flows across from 1 to 3 to 4 and then to 2. Point 1 is at the inlet of the chimney, point 2 is at the outlet of a chimney. We can assume the velocity of air at point 1 to be nearly 0, because of large area.

Let us assume the fire zone is a constant pressure fire zone in which the pressure does not change, so the velocity would be same. The air would be heated, and the density would be lowered. Let us apply Bernoulli equation between point 4 and point 2, point 4 just after the fire zone, and point 2 at the exit of the chimney of height *H*.

The equation of Bernoulli would be $\frac{V_2^2}{2g} + \frac{p_2}{\rho g} + (z_4 + H)$ should be equal to $\frac{V_4^2}{2g} + \frac{p_4}{\rho g} + z_4$. This z_4 is the elevation of point 4, which is 0 here. p_4 is like p_1, pressures are same. But pressure p_2, how do we obtain that? Now let us assume that the gas is heated, the pressure is same, so the density would be lower.

Let the density of gases within the chimney be ρ while the density of gases outside the air outside is ρ_0. Pressure at 2 is determined from the external atmosphere. So pressure at 2 is $p_1 - \rho_o g H$. ρ_o is the density of air in the atmosphere, going up the height *H*, the pressure will reduce by $\rho_o \rho_o g H$. ρ_o is greater than ρ. Then, V_4 is $A_2 V_2 / A_4$.

If we plug in the values in the Bernoulli equation, we get V_2, the velocity at the exit of the chimney to be this expression, and the velocity depends directly as $\sqrt{\left(\frac{\rho_0}{\rho} - 1\right)}$. It only works if $\rho_o$is more than ρ. If $\rho_o$is equal to ρ, then the temperature inside the chimney is the same as the temperature outside, so that if $\rho_o = \rho$, then this velocity would be 0.

We also notice that the velocity varies like square-root of *H*, the height of the chimney. The more the height of the chimney, more is the velocity. A typical case is when A_2 is much smaller than A_4, the area at the fireplace is much more than the area at the exit of the chimney. So A_2/A_4 is very small, and the denominator can be replaced by 1, and the velocity at the exit of the chimney is simply $\sqrt{2\left(\frac{\rho_0}{\rho} - 1\right) gH}$. Depending upon how much flow of air we want across the fireplace to keep the fire going, we can determine what should be the height of the chimney.

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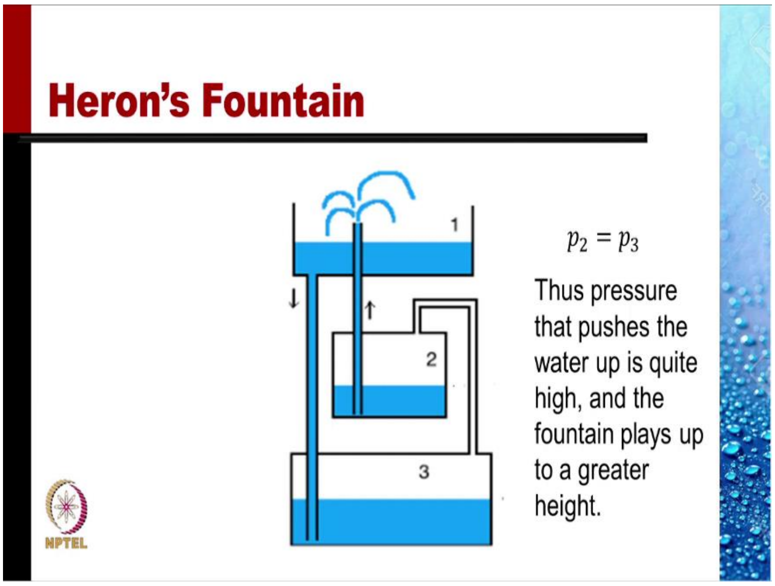

This is interesting example, a Heron's fountain in which you see the water is being thrown above the level of water, the highest level of water contained within the tank, why is this happening? From this tank the water is flowing down into tank 3. Because of this water flowing down to 3 the pressure of air within 3 is increased. The pressure of air within the 3 is communicated to air at 2, and the increased pressure at 2 pushes the water up this tube, and the water can rise above the level of water in tank 1. A simple construction of Heron’s fountain from used water bottles.

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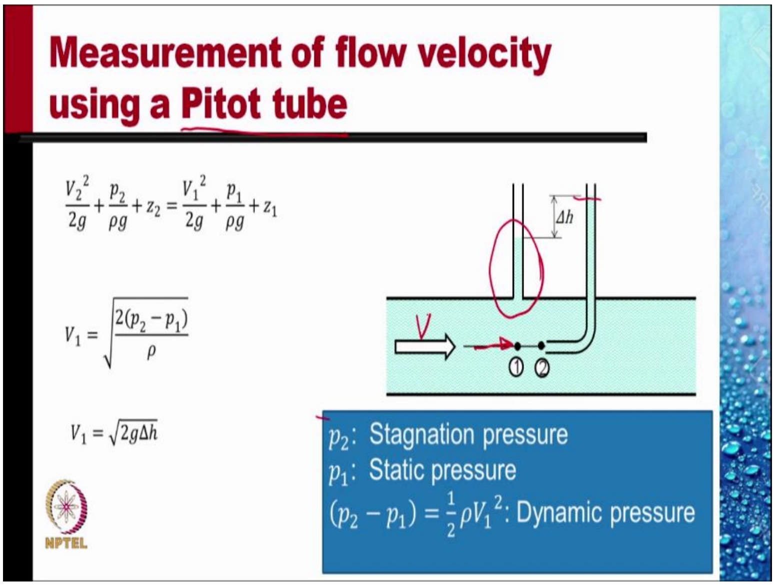

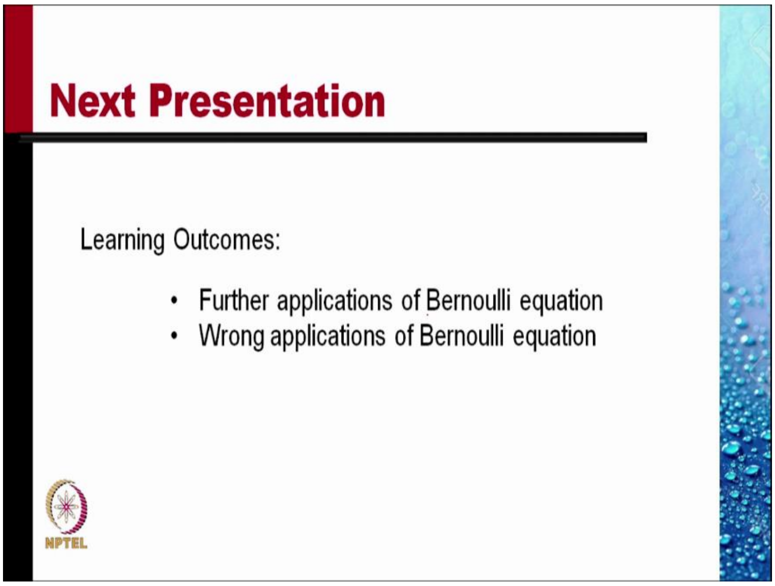

Bernoulli equation can also be used to calculate the flow velocity in a pipe. Consider a liquid flowing through this. We use a bent tube aligned with the flow. At the lower end it is aligned with the flow, and the upper end rises vertically. Because of the energy of the flow here, the water rises into this tube. This bent tube is known as Pitot tube after the French scientist who invented it. There is also a simple piezometer attached here to the wall of this tube and the water rises in this tube to the height of the piezometric height.

Now to calculate the velocity of the flow, assume a straight streamline between point 1 and 2. At point 2, which is directly at the entrance of the bent pitot tube, there is no flow taking place. So the velocity at point 2 is zero. And if we assume the velocity at point 1 is the velocity of the water flow, this velocity V_1. Then Bernoulli equation gives you $\frac{V_2^2}{2g}+\frac{p_2}{\rho g}+z_2$ is equal to the same quantities with subscript 1. Here clearly, z_1 is equal to z_2, the velocity at point 2 is 0, V_2 is 0, so V_1 is given by $\sqrt{\frac{2\Delta p}{\rho}}$, where Δp is the pressure difference.

By applying the hydrostatic rule, we can find out that $(p_2 - p_1)$ is nothing but $\rho g \Delta h$, where Δh is the difference in levels between the piezometric tube and the pitot tube, so that the velocity at 1 is given as $\sqrt{2g\Delta h}$. This Pitot tube is used for velocity measurements very extensively. Almost all aircrafts measure their velocities through use of a properly designed pitot tubes, and we will discuss more of this in later chapters.

There are special names given to these pressures. p_1 is named the static pressure; p_2 is named the stagnation pressure, because this is the pressure where the flow has come to rest. The flow taking place along the streamline 1 to 2 has come to rest at point 2, or is stagnating at point 2. So that is why p_2 is termed as the stagnation pressure. And the difference $(p_2 - p_1)$ which by Bernoulli equation is nothing but $\frac{1}{2}\rho V^2$ is termed as dynamic pressure, the pressure because of the velocity of the fluid: dynamic pressure.

Thank you very much.