Fluid Mechanics and Its Applications
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Lecture 9
Energy Equation

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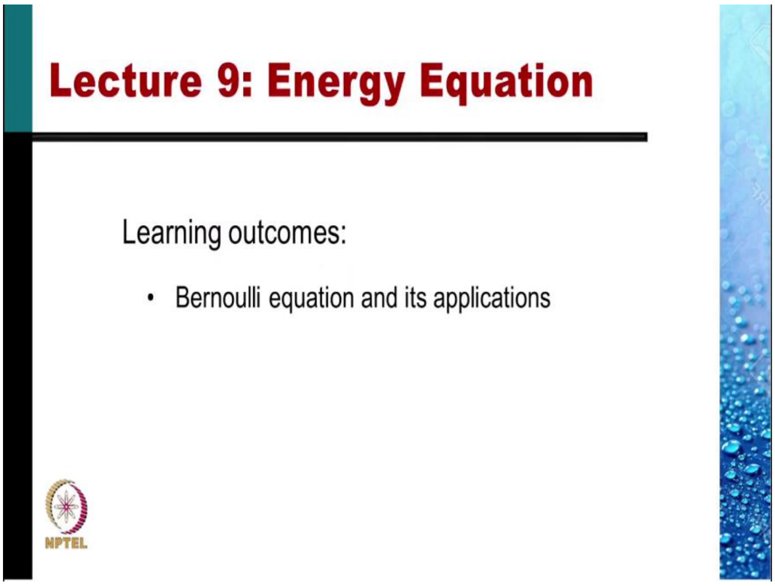

Welcome back. In this lecture, we will cover the Bernoulli equation and its applications.

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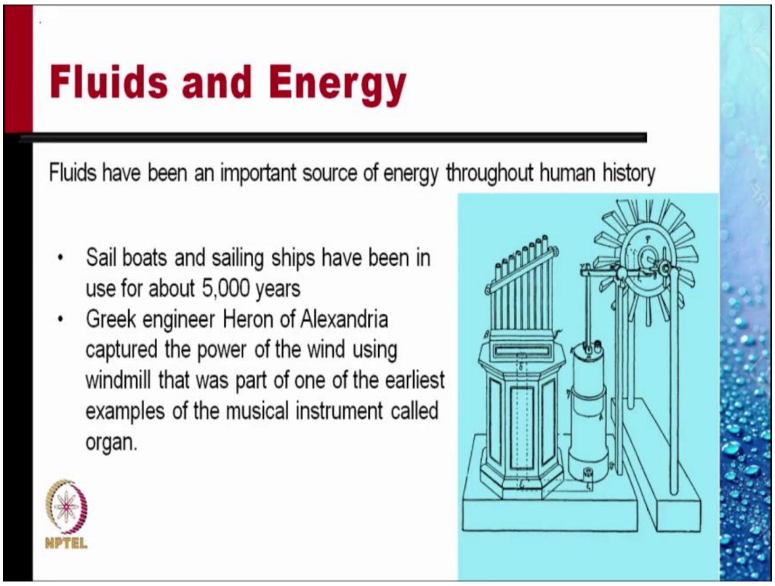

Fluids have been an important source of energy throughout human history. Sail boats and sailing ships have been around for more than 5,000 years. A Greek engineer, Heron of Alexandria, had captured the power of wind using windmill that was part of one of the earliest examples of the musical instrument called organ. In this, this windmill will rotate and would pump air into this organ, so the organ would sound.

Wind-driven prayer wheels: a prayer wheel is a cylindrical wheel on a spindle made of metal or wood. Traditionally the mantra ‘Om mani padme hum’ is written in Newari script of Nepal on the outside of the wheel. At the core of the cylinder is a life tree with mantras written on or wrapped around it. According to the Tibetan-Buddhist tradition, spinning such a wheel will have much the same meritorious effect as reciting the prayer.

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MHI Vestas V164, 9.5 MW

Rotor dia: 164 m

Water Power

Spiti Valley, HP

Water wheels

12th century, Belgium

Windmills used to drive the flour mills were quite common in Dutch and English countryside in twelfth century. Modern day windmills are quite big. For example, this MHI Vestas has a rotor diameter of 164 m. Compare it to the height of Qutub Minar, which is only 71 m. There is a self-contained power generator. The control box at the hub of the windmill is a passenger-bus sized container which contains among other things a generator. It contains a yaw controlling device that orients the windmill such that it faces the wind. It also contains device that adjusts the pitch of the blades, so that the optimum power can be generated. This windmill which is among the largest in the world produces 9.5 MW at its peak power.

Prayer wheels run by flowing water are quite common in northern India. This is a picture from Spiti Valley in Himachal where a prayer wheel is run by a stream of water. Water wheels have been used in Europe for the purpose of milling corn, for using for running hammers, etcetera.

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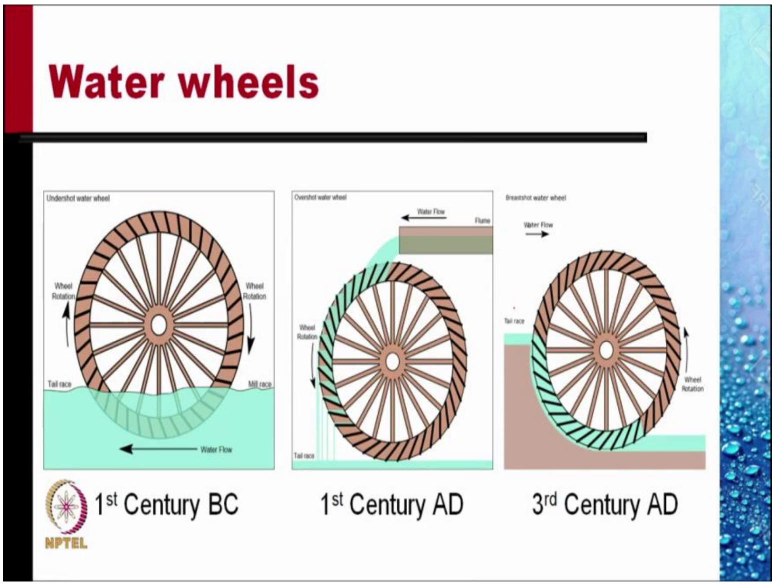

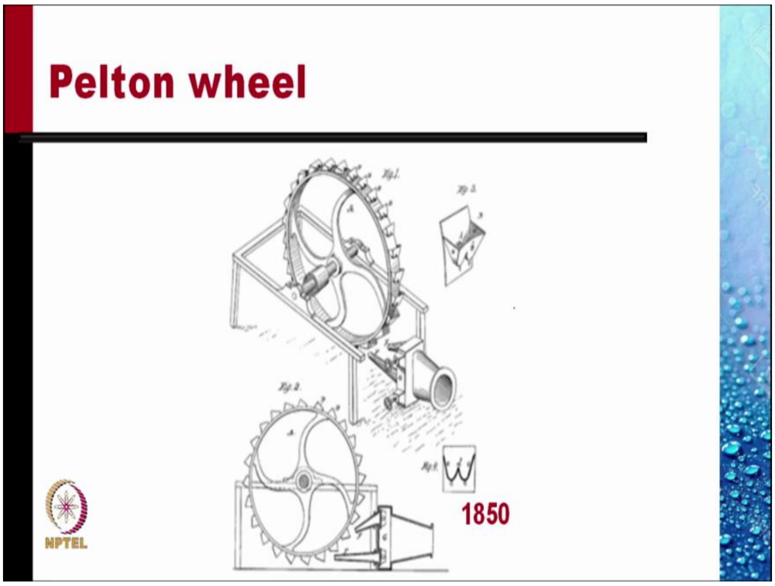

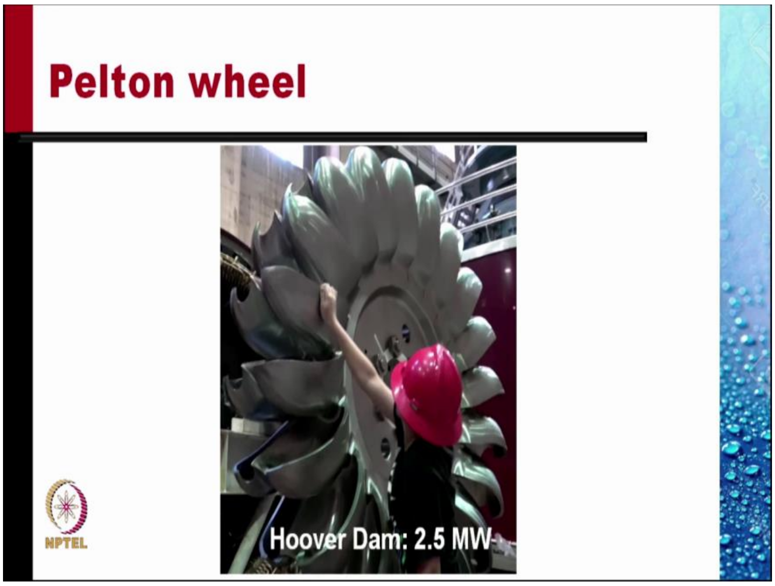

The various kinds of water wheels that date back to first century before Christ, Over the years the design of these wheels have been changing ever since. The modern hydroelectric power projects have water wheels at the core of them. One of the common designs is the Pelton wheel, which was patented in USA in 1850. This is the picture of the pelton wheel used in Hoover dam in California. It uses buckets for generating power. When a jet of water plays on these buckets; these buckets rotate. One such wheel can produce up to 2.5 MW of power.

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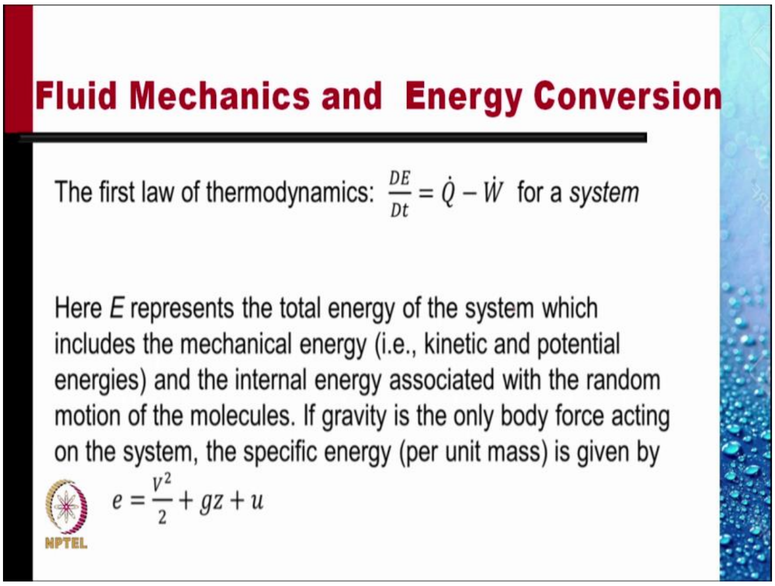

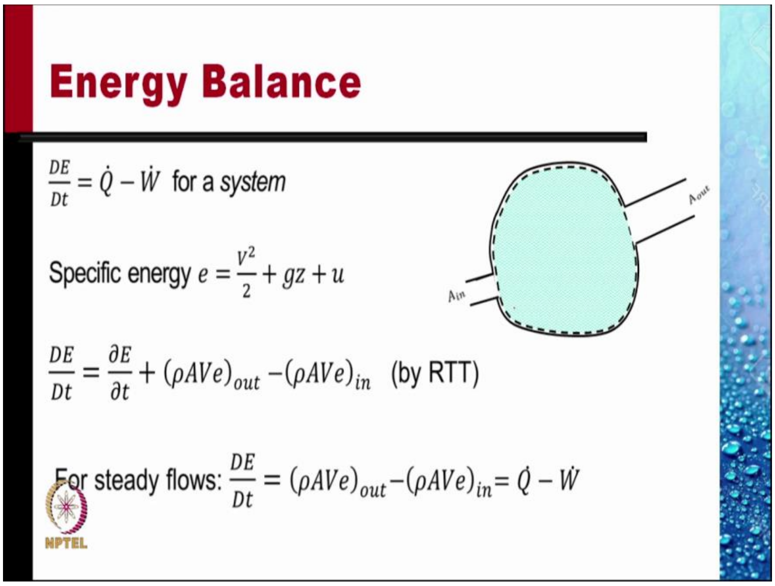

In both wind and hydraulic wheels, the mechanical energy of the flow is extracted as work done and so, the first law of thermodynamics $\frac{DE}{Dt} = \dot{Q} - \dot{W}$ for a system is applicable. Here E represents the total energy of the system which includes the mechanical energy, that is kinetic and potential energies together, and the internal energy associated with the random motion of the molecules.

If gravity is the only body force acting on the system, this specific energy which is the total energy per unit mass is given by $e = \frac{V^2}{2} + gz + u$. So for a system the capital E is the total energy which is the integral of this e over the whole system, and $\dot{Q}$ represents the rate at which the heat is transferred to the system and $\dot{W}$ is the rate at which the work is done by the system. $\dot{Q}$ is positive when it is transferred to the system. $\dot{W}$ is positive when it is done by the system.

Let us consider a control volume, a device with one inlet and one outlet area. The total energy content of this is this specific energy times the mass integrated. The rate of change of total energy contained within this control volume is given by the Reynolds Transport theorem as the local rate of change of energy contained within the system, that is, the rate at which the energy accumulates within the system, plus the rate at which the energy leaves the control volume minus the rate at which the energy enters.

If ρAV is the mass rate at which the mass enters the system, then ρAV times e evaluated at inlet is the energy that enters the system. Similarly, ρAVe at outlet is the energy rate at which the energy leaves the system. For steady flows, the rate of accumulation is 0, and so DE/Dt, the material rate of change of energy associated with the control volume is nothing but the net outflux of energy from the system, and which should by first law of thermodynamics be $\dot{Q} - \dot{W}$.

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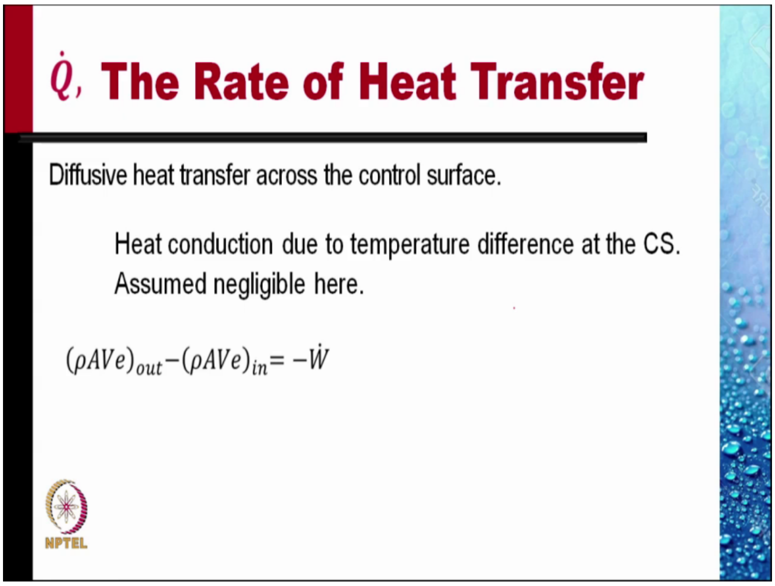

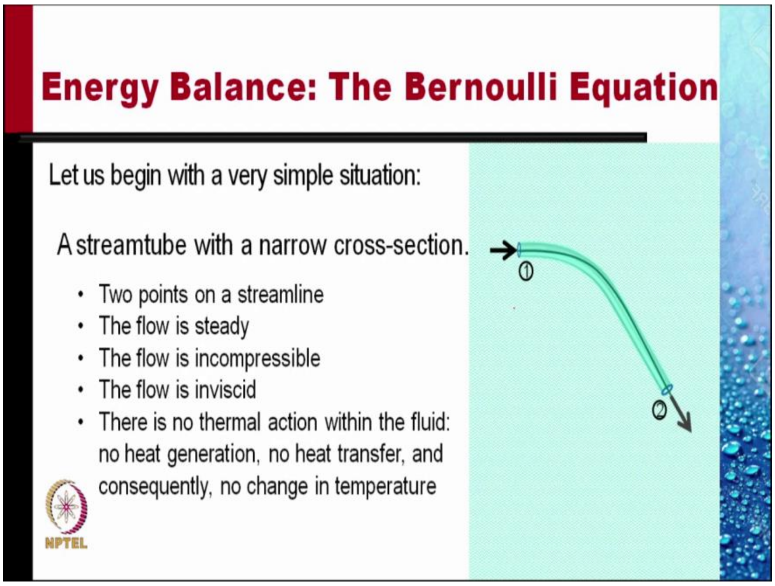

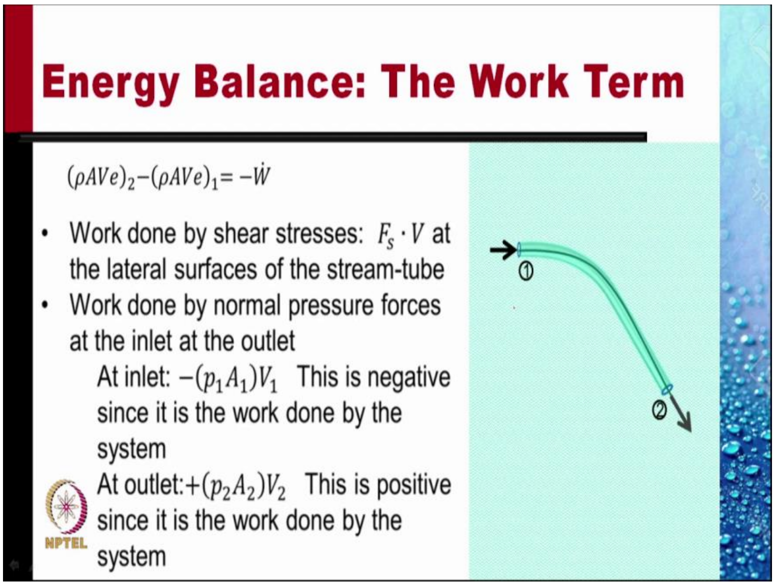

$\dot{Q},$ the rate of heat transfer is the diffusive heat transfer across the control surface. And this is because of heat conduction which is due to the temperature differences at the control surface, between the matter inside the control surface and the matter outside the control surface. In this presentation, we will assume this is negligible. So, DE/Dt is nothing but $-\dot{W}$, the rate at which the work is done by the system. Net efflux of energy out of the system is nothing but $-\dot{W}$.

Let us begin with a very simple situation. Let us consider a streamline within the flow. The flow is taking place along the streamlines. Let us mark two points, one and two, on a steamline. Let us take a control volume. And what we take as the control volume? We assume a small streamtube with a narrow cross section running along the streamline. Since this is a streamtube, no matter is crossing the surface, so if we take this as a control volume, the matter is entering only at the cross section 1 and leaving at cross section at 2.

Let us make the following assumptions. Oviously, points 1 and 2 are on the same streamline. We assume the flow to be steady. We assume the flow to be incompressible, that is, we assume the density to be constant. We also assume the flow to be inviscid, no shear stresses, no viscous action.

Also let us assume there is no thermal action within the fluid, that is, no heat generation within the fluid, no heat transfer, and consequently, no change in temperature of the fluid. If there is no change in temperature, the value of u, the internal energy, remains unchanged between point 1 and point 2. So now the first law of thermodynamics reduces to: the net efflux of energy across points 1 and 2 is equal to minus of the net work done by the system.

Let us now consider the work term. Work done by shear stresses. First of all, there are no shear stresses. We assume the flow to be inviscid. So work done by normal forces which are pressure at the inlet and outlet, can be determined. Thus, at the inlet the force of pressure is $p_1 A_1$ and the velocity of the fluid is V_1. So, the rate of doing work is $p_1 A_1 V_1$. Is this the work done by the system or on the system? Clearly, this is the work done on the system, because this pressure is pushing the fluid into the control volume. So this is negative, that is why we put a negative sign in front. At the outlet, the pressure is p_2, the force is $p_2 A_2$ and the velocity is V_2. So, the work done by pressure forces at the outlet is $p_2 A_2 V_2$. This is positive, since it is the work done by the system on the surroundings.

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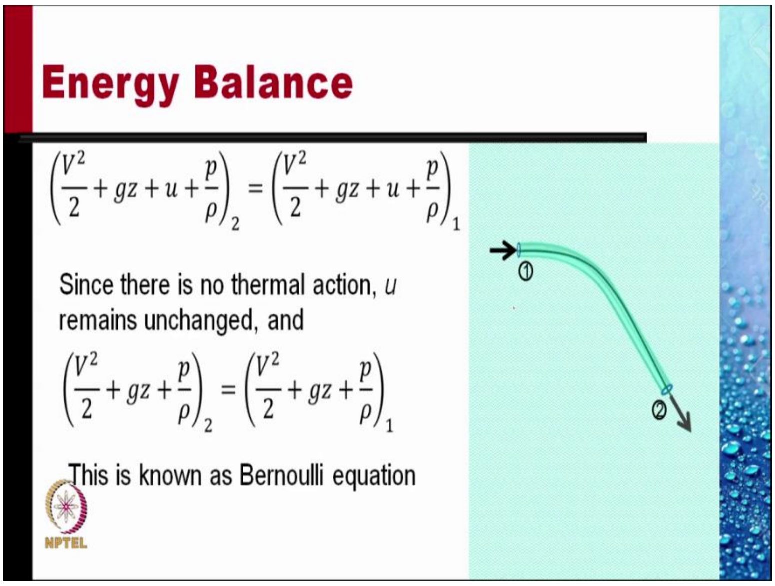

So the energy balance equation is this, in which we notice that ρAV at outlet is equal to ρAV at inlet, because of mass balance, and let us write this as $d\dot{m}$, the rate of mass throughput through the system. And if we use this in the energy equation we get $\dot{W}$, which is $p_2 A_2 V_2$ minus $p_1 A_1 V_1$, and it can be written as $\frac{p_2 d\dot{m}}{\rho_2} - \frac{p_1 d\dot{m}}{\rho_1}$. This is $\dot{W}$, the net work done by the system.

And using this in the energy equation, the specific energy e_2 at the outlet times $d\dot{m}$, the rate of mass throughput minus e_1 times $d\dot{m}$ is equal to $\frac{p_2 d\dot{m}}{\rho_2} - \frac{p_1 d\dot{m}}{\rho_1}$. We can cancel out $d\dot{m}$ throughout, and for $e = \frac{V^2}{2} + gz + u$, this gives $\frac{V^2}{2} + gz + u + \frac{p}{\rho}$ has the same value at the outlet as at the inlet.

Further, since we assumed that u does not change, there is no change in temperature of the fluid, this equation can be reduced to $\frac{V^2}{2} + gz + u + \frac{p}{\rho}$ at the outlet is the same as at the inlet. This is known as Bernoulli Equation after the scientist who discovered it. This is a very useful equation, and used extensively in fluid mechanics.

In the next couple of lectures and in this lecture we would be interested in applying this equation to various situations. It is so commonly used that many times we make wrong applications of this equation. Notice that this equation came with a lot of conditions, lot of assumptions. If any of these assumptions is not met the equation cannot be applied.

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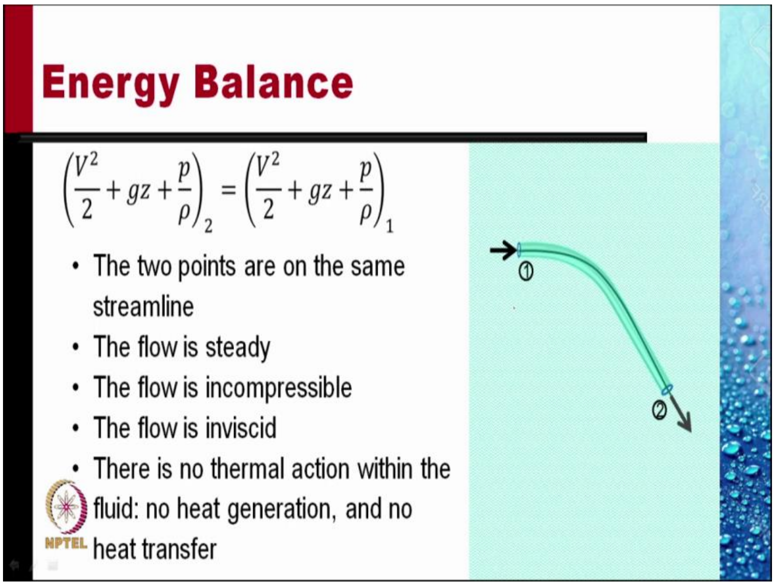

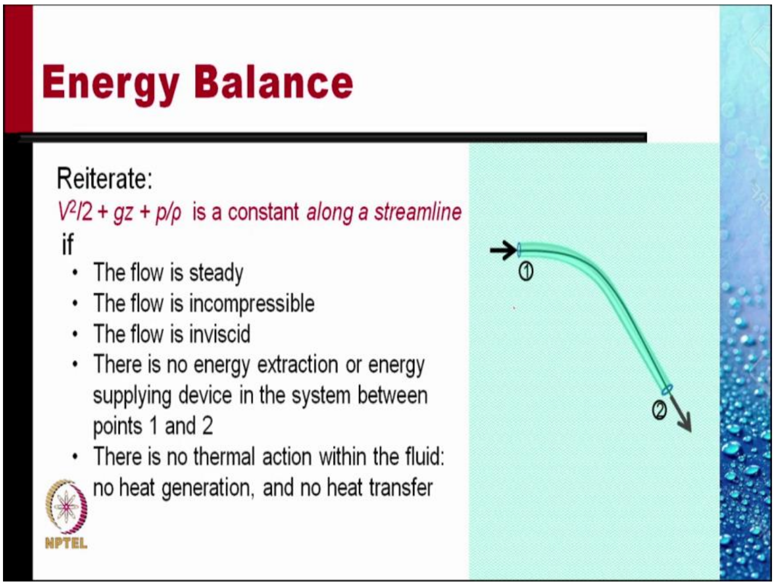

To reiterate, the energy $\frac{V^2}{2} + gz + u + \frac{p}{\rho}$ per unit mass is the same at outlet and inlet of a control volume if the two points are on the same streamline, the flow is steady, the flow is incompressible, and the flow is inviscid, in that there is no thermal action within the fluid, no heat generation and no heat transfer. Notice that we have not used the condition of incompressible flow explicitly in this derivation. But this comes in indirectly, because if u, the internal energy is not changing and the flow is inviscid, then there cannot be any change in density if there is no heat transfer.

So once again, $\frac{V^2}{2} + gz + u + \frac{p}{\rho}$ is a constant along the streamline if the flow is steady, the flow is incompressible, the flow is inviscid, there is no energy extraction and supplying device in the system between points 1 and 2. We had not stated this earlier but this goes without saying and that there is no thermal action within the fluid.

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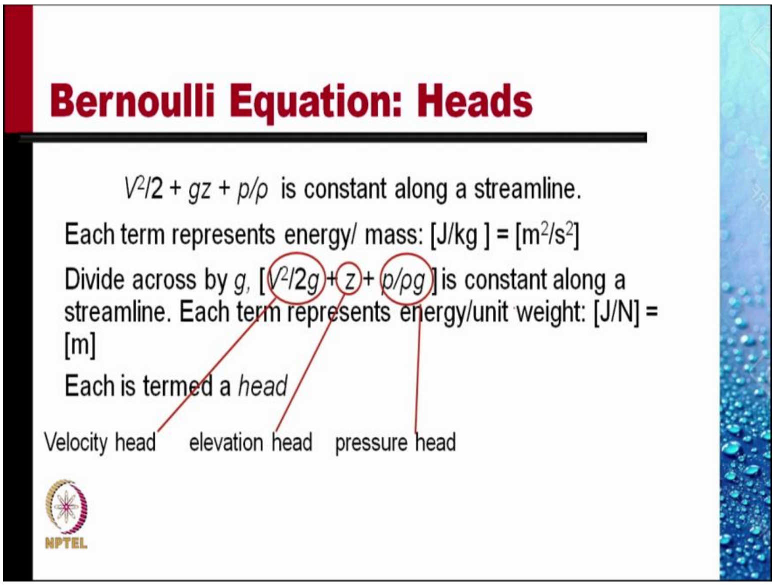

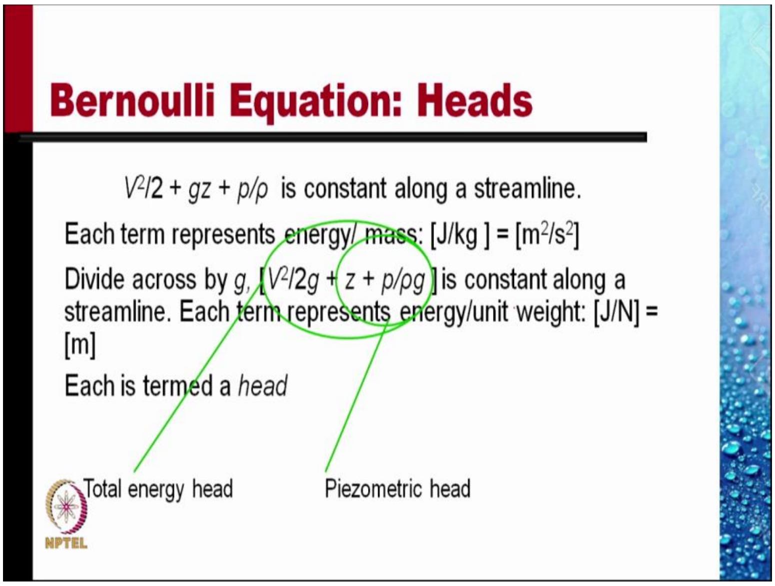

We had introduced the concept of pressure heads earlier when we were discussing fluid statics. Now in this Bernoulli equation each of the term $\frac{V^2}{2}$, gz and $\frac{p}{\rho}$ represents energy per unit mass with the units of J/kg, or m^2/s^2. If we divide across by g, the acceleration due to gravity, we get $\frac{V^2}{2g}$, z and $\frac{p}{\rho g}$. Now each term has a unit of meter, dimension of length, and each term represents energy per unit weight of the fluid.

The units are J/N, or simply meter. Each of them is termed as a head. $\frac{V^2}{2g}$ is the velocity head, z is the elevation head, and $\frac{p}{\rho g}$, as we discussed in fluid statics, is the pressure head. $z + \frac{p}{\rho g}$, that is, the elevation head plus pressure head is also given the name piezometric head, and that together with $\frac{V^2}{2g}$ is the total energy head. Using head as a unit of energy is very common in the practice of civil engineering.

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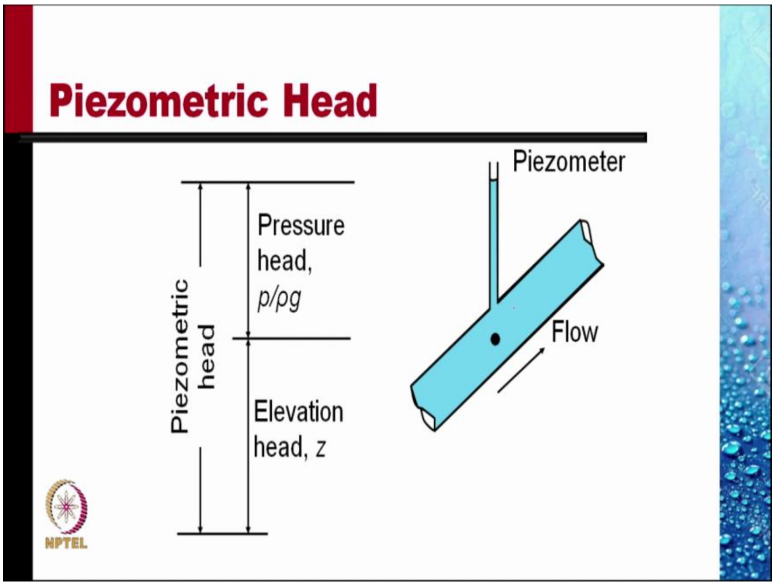

Let us talk of a piezometric head a little bit more. Suppose there is a tube through which a fluid is flowing as shown. At the point marked with the black dot, the column in the vertical tube connecting with this flow is the pressure head, is the height to which the fluid would rise. This tube is called a piezometer and if h is this height, h clearly is $\frac{p}{\rho g}$, the pressure head. The height of that point above an arbitrary datum z is the elevation head, and the sum of the two is the piezometric head.

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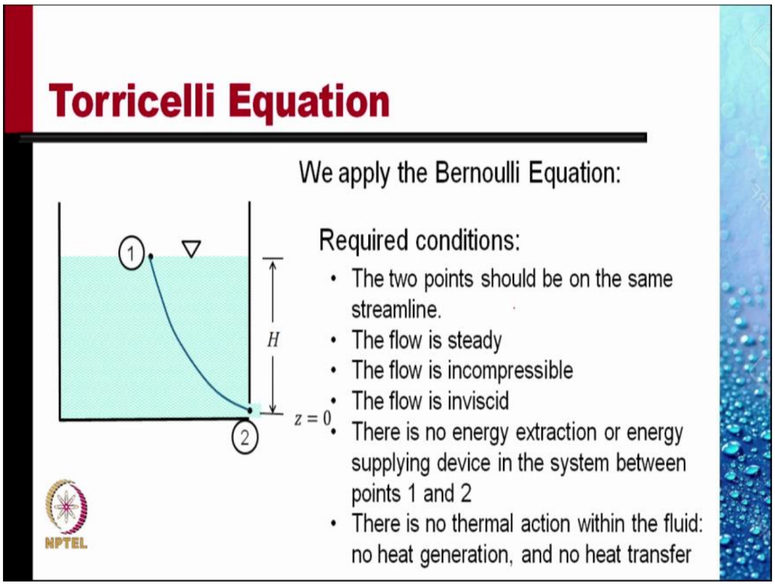

Let us now apply Bernoulli equation to a very simple situation. Consider a tank in which a liquid is present to height H. There is a free surface at H of the liquid exposed to atmospheric pressure. So the gauge pressure at the free surface is 0. There is an orifice in the tank at $z = 0$, that is at the bottom of the tank of area A. What is the rate at which the fluid issues from this orifice when they apply Bernoulli Equation? But for Bernoulli Equation we need two points along the streamline.

There must be a streamline running something like this: a point 1 on the surface to a point 2 at the orifice. It may not take exactly this path, but a similar path between a point on the surface, anywhere on the surface, and the point 2 at the orifice. Now let us apply Bernoulli equation. Let us look at the conditions required. The two points should be on the same streamline, which they are. We have chosen them to be on the same streamline.

The flow is steady. Really it is not steady. As the fluid flows out of the orifice the level at point 1 would drop, but if the orifice is much smaller than the cross-sectional area of the tank, the rate at which the level drops would be very small compared to the flow velocity. So, the flow can be taken as quasi-steady. Not truly steady, but for the purposes of applications at any given instant, it may be assumed as if the flow is steady, quasi steady.

The flow is incompressible. We are assuming a liquid, so the flow can be assumed to be incompressible. The flow is inviscid. Let us assume this to be inviscid. Any deviations would come as an error. We will discuss much later that the viscous stresses would be very small in this flow, but that is for later. There is clearly no energy extraction or energy supplying device between points 1 and 2, and there is no thermal action within the fluid, no heat generation and no heat transfer. So, the conditions required for application of Bernoulli equation seem to be satisfied.

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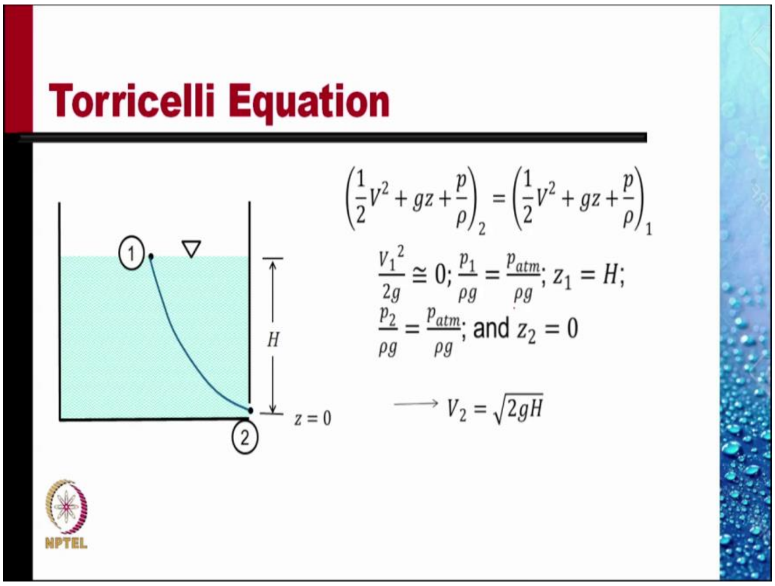

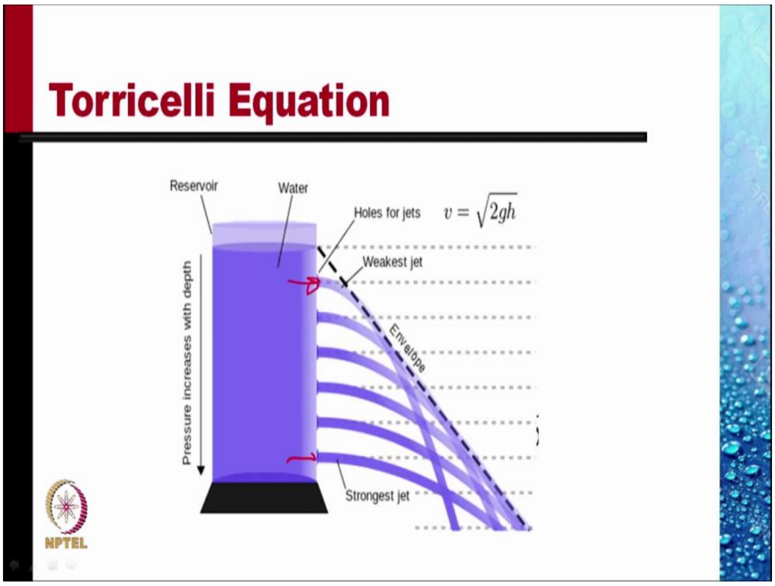

So total energy at the outlet must be the total energy at the inlet per unit mass. Let us consider the value of these: $\frac{V_1{}^2}{2g} = 0$, approximately. We have said this before. The level is decreasing at a very slow rate, a negligible rate. p_1 is p atmosphere, z_1 is H. p_2, the pressure at the orifice is again atmospheric, and z_2 is 0. We have to find out V_2. And simple application gives you V_2 is equal to $\sqrt{2gH}$, as you plug in the above values.

This was an expression obtained by Toricelli, an Italian scientist. The velocity at the outlet varies at square root of H, the level of the liquid within the tank. So what is the volume flow rate of the fluid out of this orifice? V_2 times the area of the orifice A_2. This picture shows a jar with orifices at various heights. At the topmost orifice the jet is the weakest, at the bottom-most, it is the strongest.

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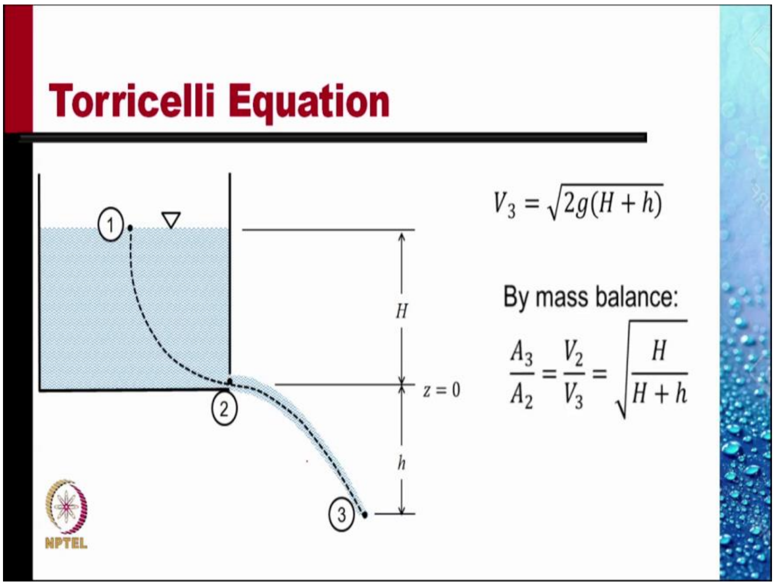

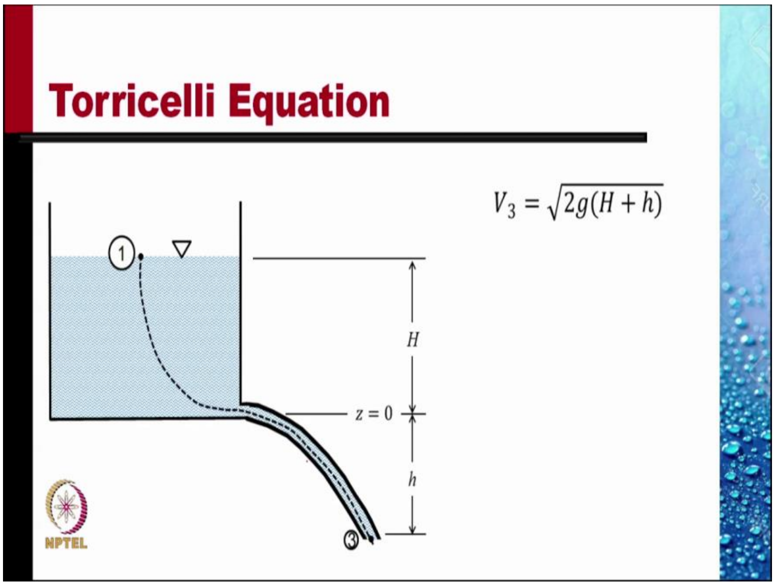

Let us now extend this a little further. Let us assume that the jet that issues out of point 2 falls down, and we can show easily that this would follow a parabolic path and falls down through a distance h, a lower case h, up to point 3. Can we find out what is the velocity at point 3, and what is the area of the jet at point 3?

Clearly, all the conditions would still be met, and we can apply Bernoulli equations between points 1 and 3 directly. And if we apply, V_3 would come out to be $\sqrt{2g(H+h)}$. The velocity would increase at the jet falls down, but the cross section of this jet wiould decrease. Why would it decrease? Because of the mass balance. The mass flow rate through section 2 must be same as mass flow rate through section 3, and since velocity at 3 is more than velocity 2, the area at 3 should be less than the area at 2.

Let us consider one other extension of this problem. Let us attach a tube to the orifice of the same area A_2 and this extends up to point 3. Now the flow takes place within the tube from 2 to 3, and so if we apply the equation directly between points 1 and 3, we will get the same velocity 3 as before, as in the last slide. $V_3 = \sqrt{2g(H+h)}$. But now the area is larger, area has not decreased.

The velocity here is V_3 and the area would be same A_2 because we use a tube. The tube is full. So, this is the way you can increase the flow rate from the tank: attach a tube and bring it down to a lower level.