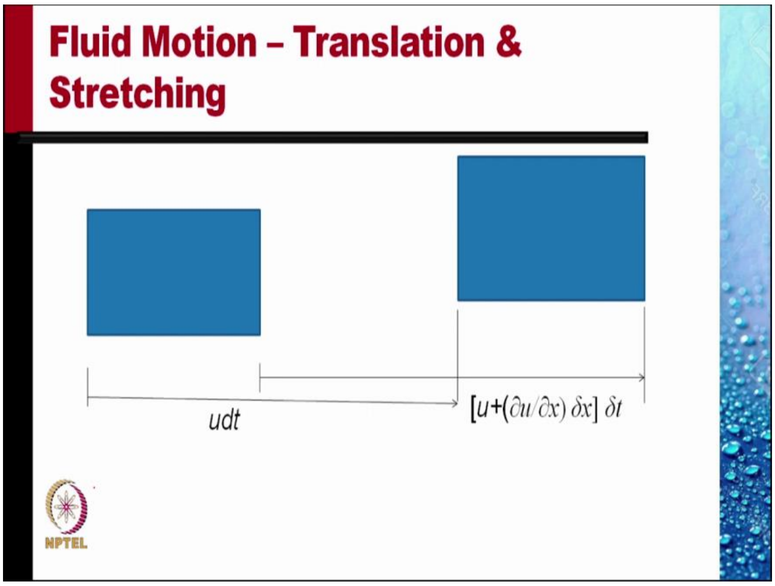
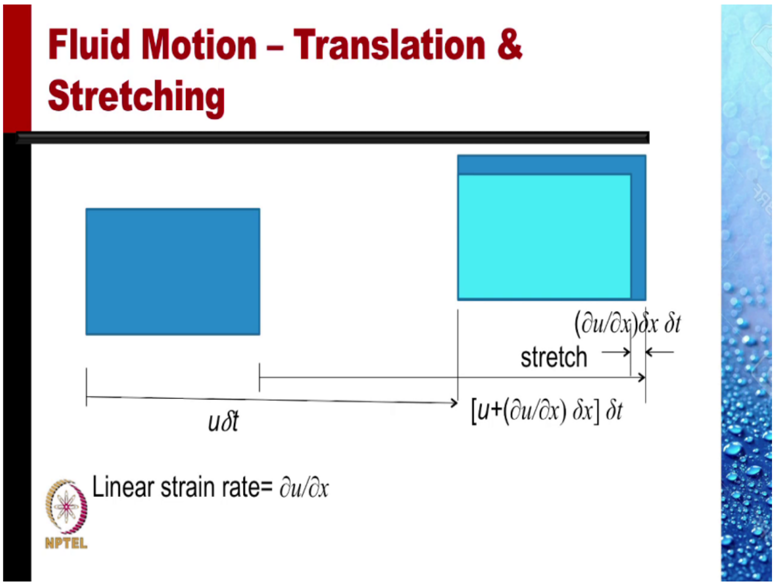
Fluid Mechanics and its Applications
Professor Vijay Gupta
Indian Institute of Technology, Delhi
Lecture 7A
Rotation and Distortion

(Refer Slide Time: 00:28)





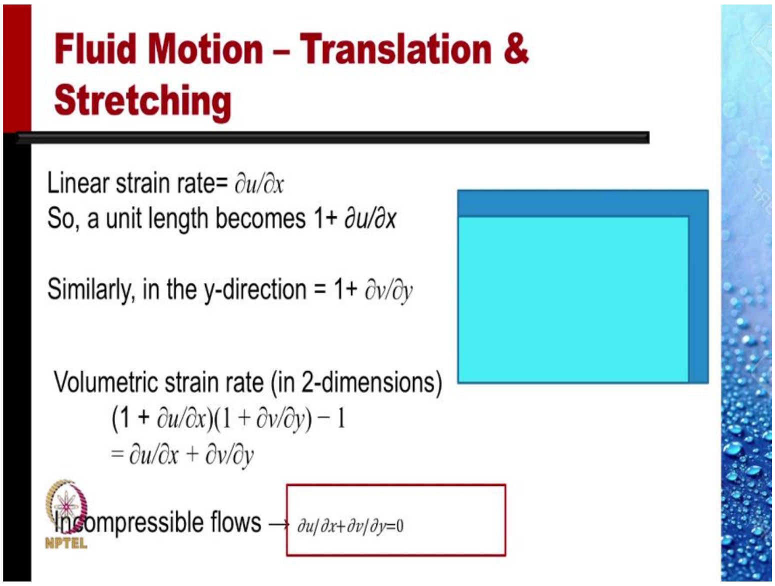
Let us next consider the rotation and distortion of fluid elements when it flows. Let a rectangular fluid element on the left inside a moving fluid move after a time δt to the location shown on the right. As you see, it has changed its shape. Let us assume it is not rotating or deforming.

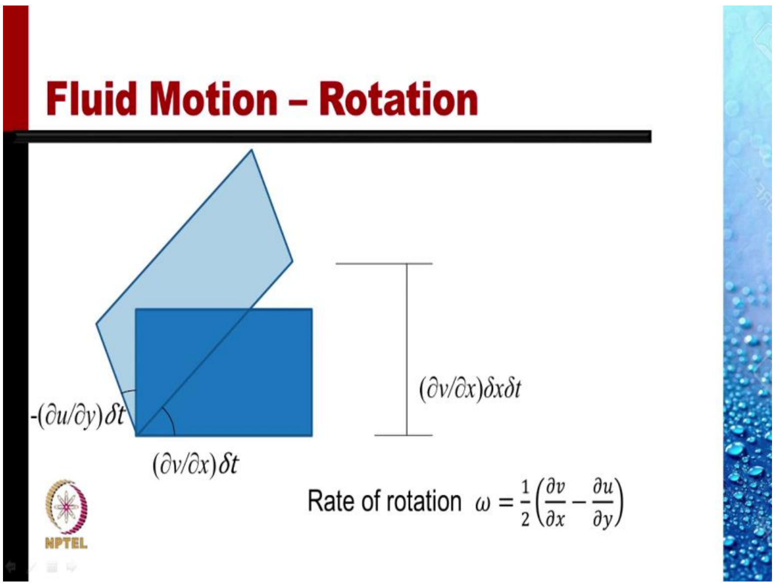
Now if we consider the left most lower edge of the fluid element, in time δt it would have moved at this $u\delta t$, and the right most element on this line would have moved a distance

$\left(u + \frac{\partial u}{\partial x}\delta x\right)\delta t$. The velocity of the point on the right side is going to be little different from the velocity at the left end and the velocity at the right hand is found by Taylor's expansion.

Clearly, this would mean that the horizontal line at the bottom of the fluid element is now stretched. This is the original size of the fluid element, and this stretch is nothing but the difference of the two motions, and that would be $\frac{\partial u}{\partial x}\delta x\delta t$. This is the stretch introduced. So what is the strain introduced? Divided by δx; And what is the rate of strain? Divide again by δt. So $\frac{\partial u}{\partial x}$ is a rate of strain in the x direction, the linear strain rate in the x direction.

(Refer Slide Time: 02:50)





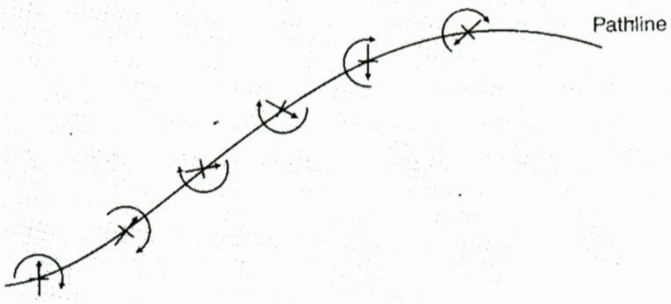
Similarly, we can find in other directions. In the y direction it will $\frac{\partial v}{\partial y}$, and so the volumetric strain rate in two dimensions would be the current volume which is $\left(1+\frac{\partial u}{\partial x}\right)\left(1+\frac{\partial v}{\partial y}\right)$ minus the original volume 1, and so this is $\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$. This is the volumetric strain rate. Now if the fluid is incompressible, then $\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$ should be 0. This is the famous continuity equation for incompressible flows that we will discuss in the next lecture.

Let us further take the rotation of this fluid element. Let this be a fluid element of size δx and δy at time t, and after time δt let the fluid element occupy this position. This element would translate also. We have brought it back so that the left bottom point is coinciding, so we removed the translation in x and y direction, we are worried only about rotation. The bottom right most point has moved up relative to its original position and that is because of the velocity gradient, because the difference of v velocity in the x direction, so this height in time δt would be $\frac{\partial v}{\partial x}\delta x\delta t$, and therefore the angle through which the horizontal line rotates is $\frac{\partial v}{\partial x}\delta t$ in time δt.

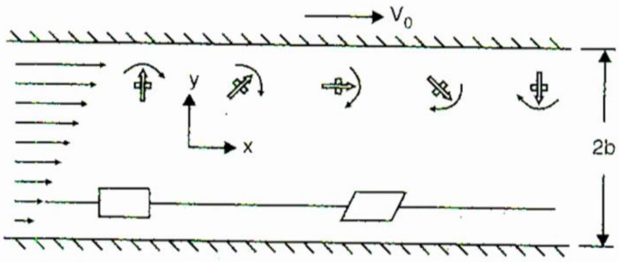
Similarly, the vertical line rotates counterclockwise by an amount minus $-\frac{\partial u}{\partial y}\delta t$ in time δt. The average rotation of the element is obtained by taking the average of these two rotations, and the rate of rotation of the element ω_z is $\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$. Those of you are familiar with vector calculus would recognize that $\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$ is nothing but curl of velocity, $\nabla\times\mathbf{V}$. So the rate of rotation ω_z is $\frac{1}{2}\nabla\times\mathbf{V}$. We will be using this fact later.

(Refer Slide Time: 06:36)

Vorticity

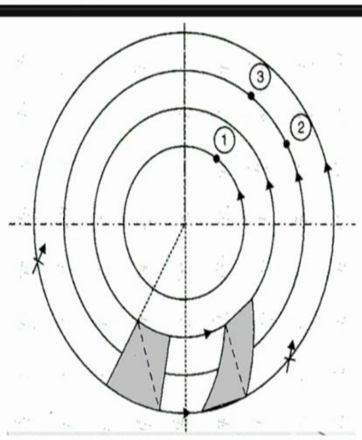


Rotational flow



Irrotational flow

$V = k/r$

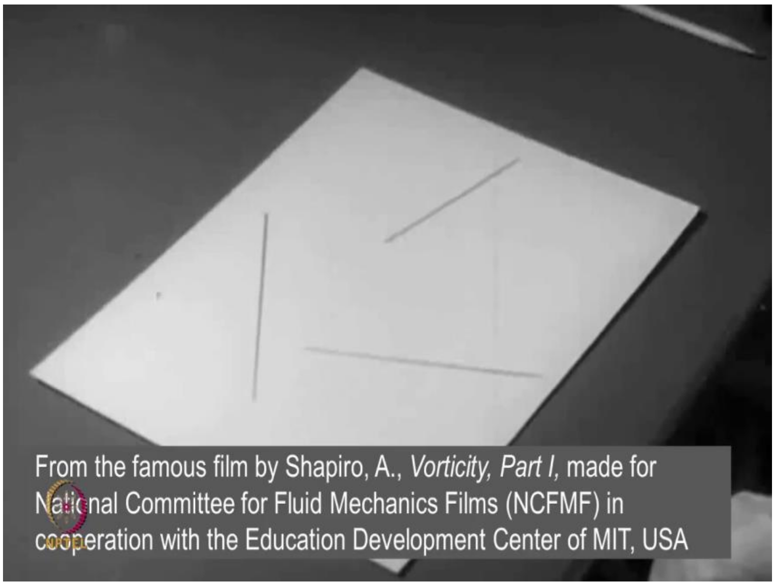


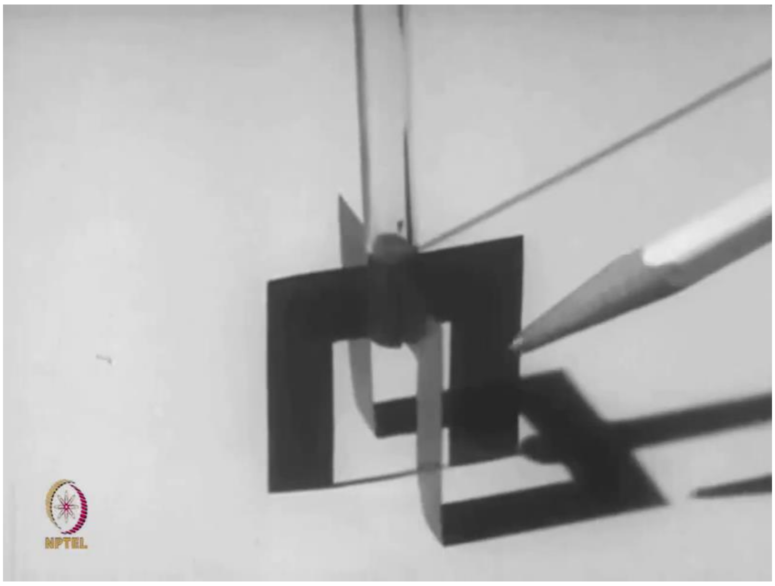
This rotation of the fluid element $\nabla \times \mathbf{V}$, which is twice the rate of rotation, is also termed as vorticity is a measure of rotation of the fluid element. Here we have shown a fluid element that rotates as it moves along its path line. The rotation is recognized by treating this fluid element, or representing this fluid element with two lines crossing one another and one of the line carries an arrowhead. And so clearly we can see the particle is rotating as it moves along its path.

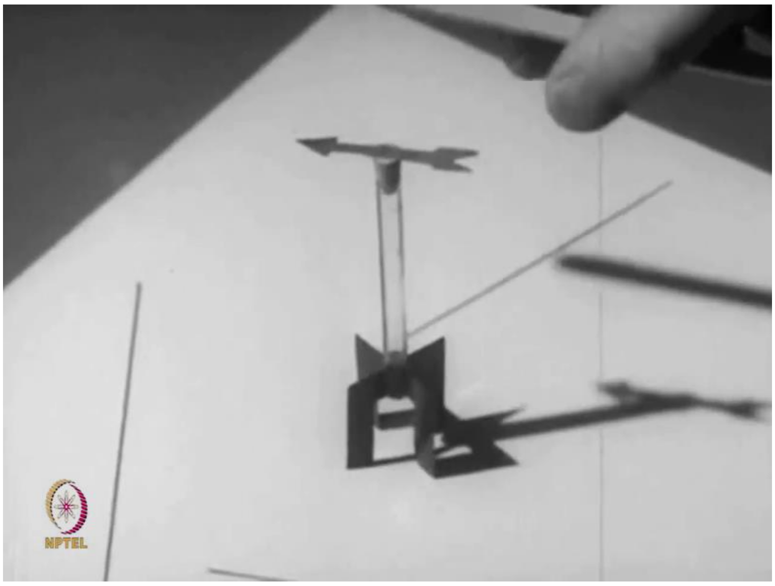
In a channel, if the upper plate is moving with the velocity V_o and the lower plate is stationary, we have seen earlier that there is a velocity profile: the velocity is large near the upper plate, and it tapers down to zero velocity at the bottom plate. A rectangular fluid element is now distorted. Also as seen by the sequence of arrows on the top, the upper end of the vertical arrow is moving faster than the lower end. So this arrow rotates clockwise. So this flow is rotational.

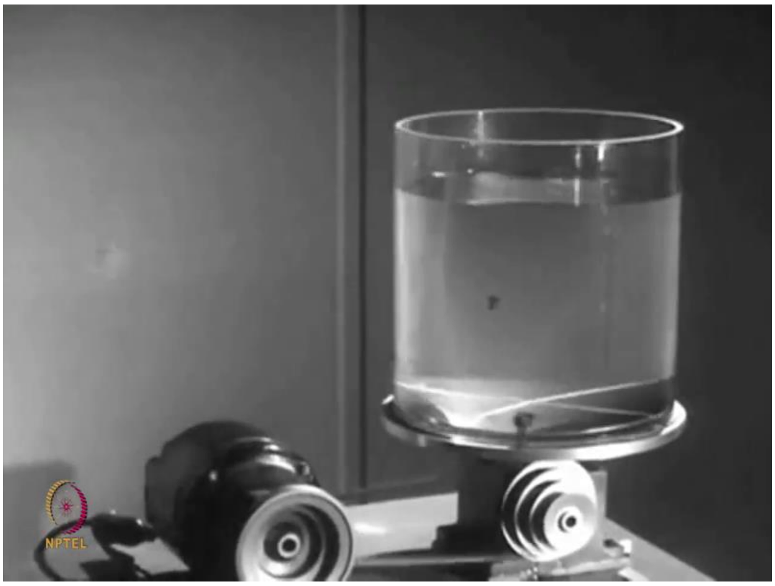
There is vorticity in the flow. The flow itself does not have to move along a circular, or a curved, streamline or a pathline. The pathlines are straight. The rotation here refers to the rotation of fluid elements about their own axes. It has nothing to do with rotating pathlines. In fact, we can show, and we will show later on, that if the velocity is like k/r, the velocity in a free vortex, or the velocity in a model hurricane or a tornado, the velocity decreases as r increases linearly, and we can show that even though the particles are moving in circular paths, each fluid element is not rotating. The two gray areas we have shown, the diagonals have the same orientation at two locations signifying that the fluid is irrotational. There is no vorticity.

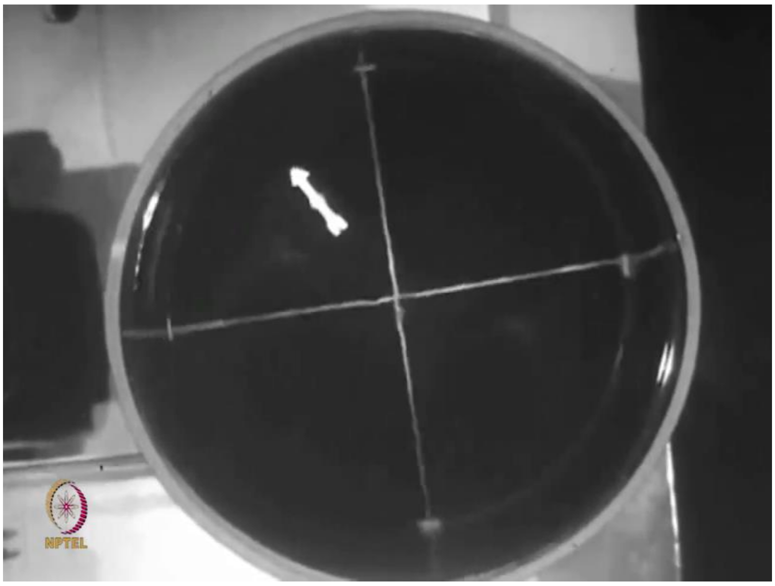
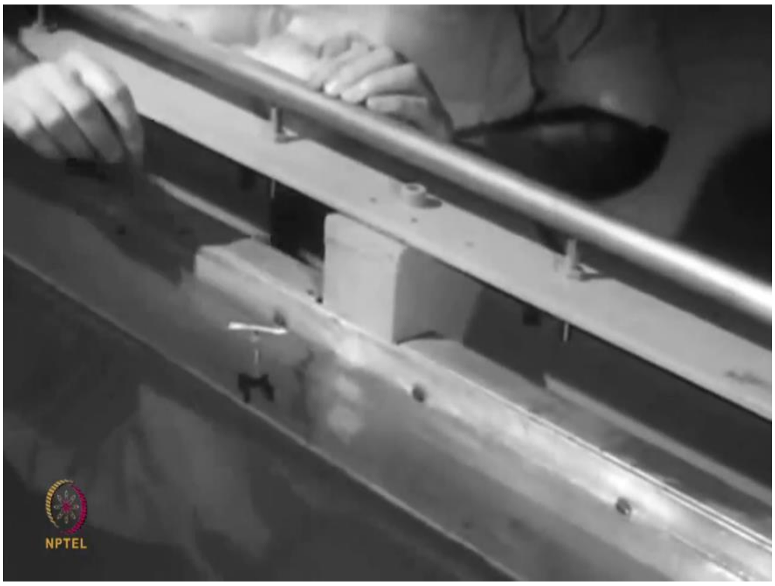
(Refer Slide Time: 09:51)

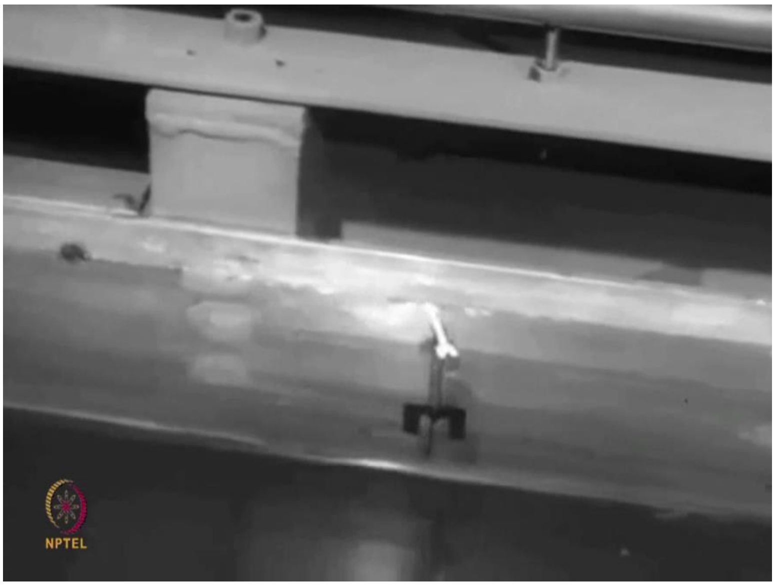








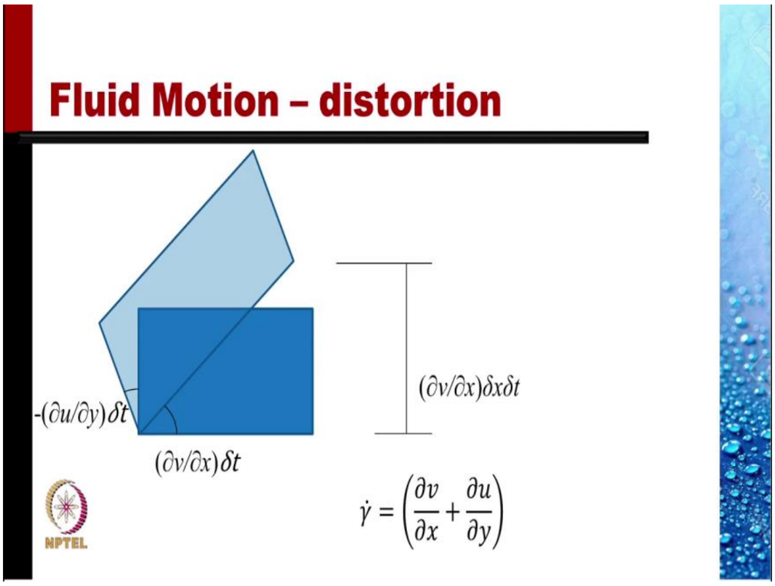

NPTEL

NPTEL

NPTEL



Video playing: This gadget is a vorticity meter, which floats in water with its axis vertical. The two veins at the bottom are driven by the angular velocities of two mutually perpendicular fluid lines so the arrow at the top turns at a rate equal to the average of these two. That is with half the vertical component of arcticity of the lump of water in which the veins are immersed. This tank has been on the turntable for a long time and viscosity has forced the water into a rigid body rotation.

The vorticity float rotates almost exactly with the speed of the crossed white lines on the bottom of the tank. Sometimes the word rotation is used as a synonym for vorticity but this does not mean that the flow has to be curved for vorticity to be present. Here for instance water flows in a straight channel. The streamlines are essentially straight and parallel to the side wall but the arrow rotates showing that vorticity is present.

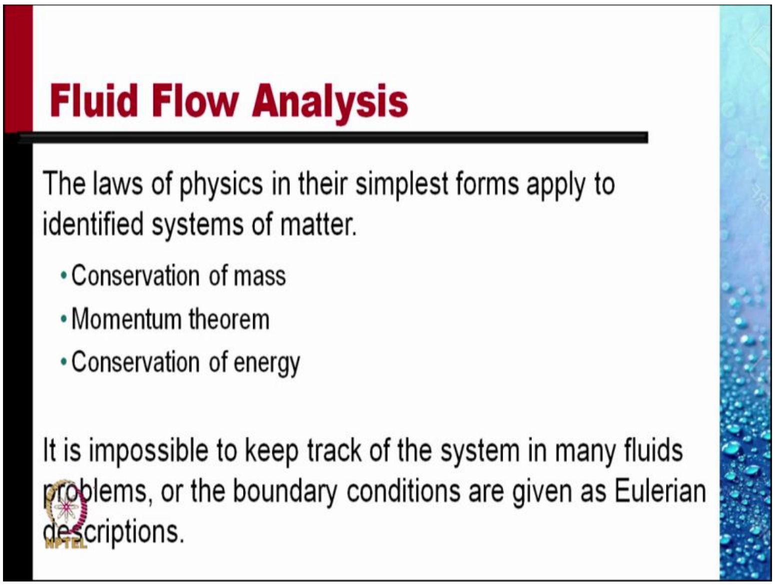
(Refer Slide Time: 11:13)

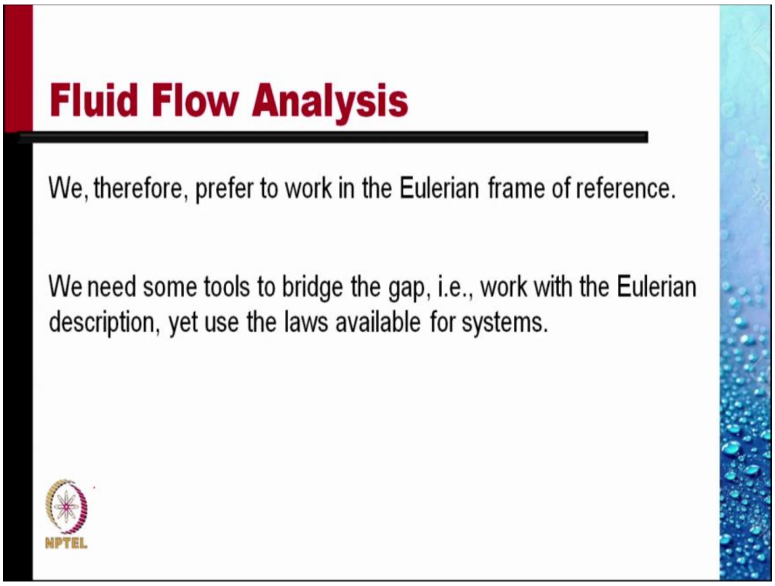


The same picture, we can invoke to explain distortion. Here the x line, the horizontal line has rotated through angle $\frac{\partial v}{\partial x}\delta t$ while a vertical line has rotated through an angle $-\frac{\partial u}{\partial y}\delta t$ as explained before. The rotation of the fluid element was the average rotation, but what is the distortion? One side is moving counterclockwise, the other side is also moving counterclockwise. So we can find the distortion. We can find the distortion by subtracting the two rotation and the rate of distortion per unit time is now given by $\frac{1}{2}\left(\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x}\right)$.

This rate of distortion is the shear straining, and this shear straining causes shear stress. We had seen earlier when we had flow through a channel in which there was only one component of velocity u which are the function of y, the rate of strain, the rate of distortion was just $\frac{\partial u}{\partial y}$. In two-dimensional flows, this would be $\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$.

(Refer Slide Time: 13:16)

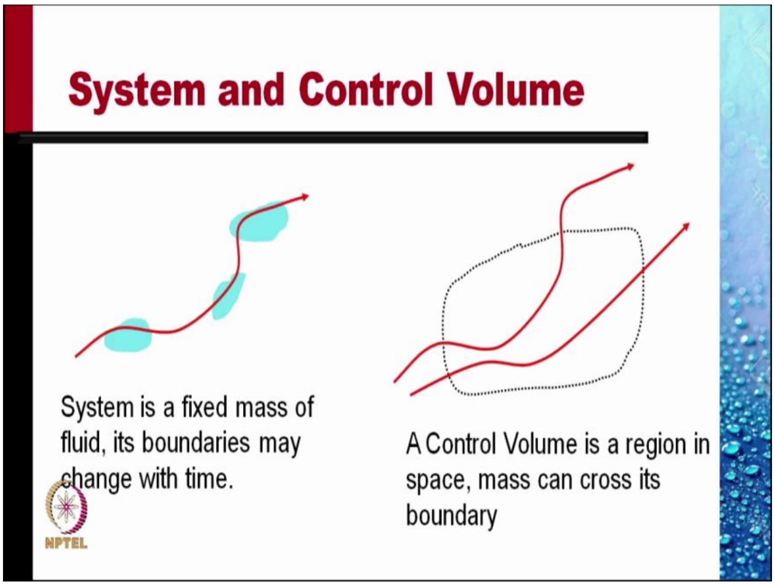


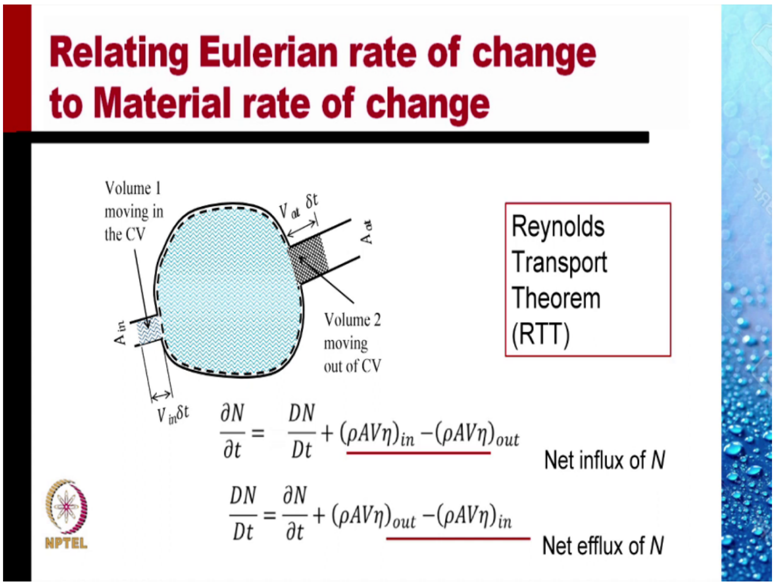


Let us now consider how do we analyze fluid flows. In solids as well as in fluids, the law of physics in the simplest forms apply to identified systems of matter. The law of conservation of mass, the momentum theorem, the law of conservation energy are applicable only to a fixed and identified quantity of mass. But in fluid, as we have said before, we like to work with control volumes, with a Eulerian approach where we do not follow the material quantity. So what do we do? It is impossible to keep track of the systems in many fluid problems, or the boundary conditions are given as Eulerian description.

We, therefore, prefer to work in an Eulerian frame of reference. The laws are available in the Lagrangian frame of reference. So we need some tools to bridge the gap, i.e., work with the Eulerian description, yet use the laws available for systems or the Lagrangian description.

(Refer Slide Time: 14:34)





System is a fixed mass of fluid. Its boundaries may change with time. In fluids, we work with control volumes. A control volume is a region in space. Mass can and usually does cross the boundaries of the volume. This control volume could also be moving in space, could also be changing its volume, but it is a volume that we will consider. When we apply the physical laws, we need the rates of changes or material properties associated with the system. All physical laws that are available to us use the rates of change on material systems, but in fluid mechanics we deal with control volumes, and the rate of change that we know are available for controlled masses.

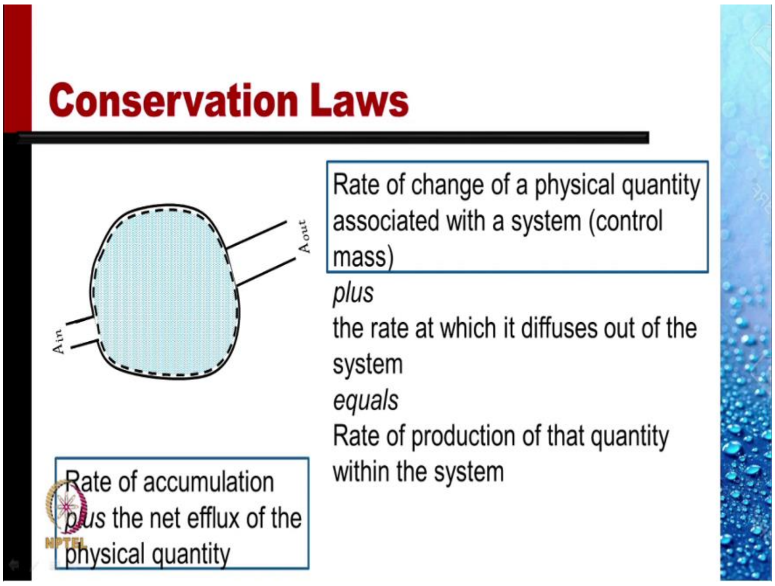
So we need a relation between Eulerian rates of change and the material rates of change. To obtain the relation, consider a controlled volume with one inlet and one outlet. In time δt, a volume of the fluid moves into the control volume, and the volume that moves in is V at the inlet times the area at the inlet times δt. The mass that moves in is ρ times the volume that moves in, and if this specific property, that is the property associated with unit mass is η, then the quantity of the property that has moved in is $\rho V_{in} A_{in} \delta t$ times η at the inlet.

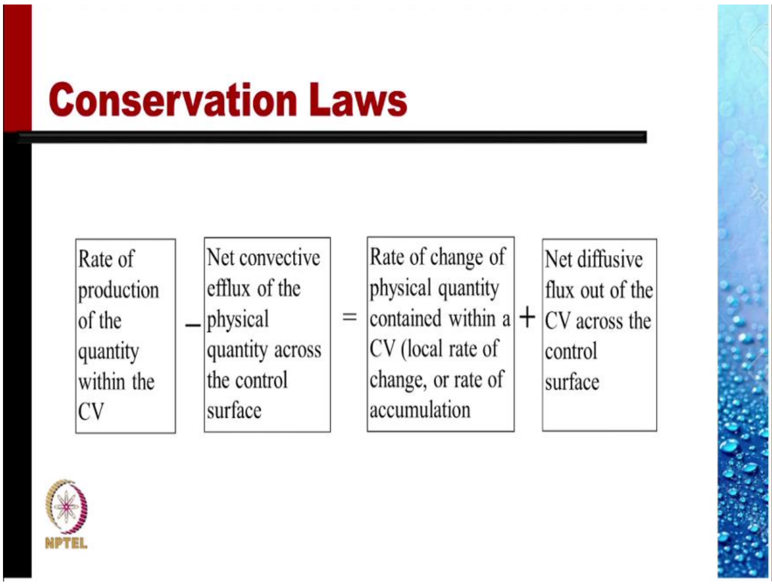
This is the quantity that moves in. Similarly a quantity would move out at the outlet in association with the fluid that moves out of the control volume. The volume moving out of the control volume would be $\rho V_{out} A_{out} \delta t$ and the property that moves in with this would be $\rho V_{out} A_{out} \eta \delta t$.

So that the change associated with the control volume would be $\partial N/\partial t$, the rate at which the property N contained within the control changes times δt is equal to DN/Dt, the material rate of change of property N associated with the volume of the control volume times δt plus the property that is moving in minus the property that is moving out. And so if we divide by δt we get this relation $\frac{\partial N}{\partial t} = \frac{DN}{Dt} + (\varrho A V \eta)_{inlet} - (\varrho A V \eta)_{outlet}$.

This is the net influx of N and we can write this in an alternate form $\frac{DN}{Dt}$ is equal to $\frac{\partial N}{\partial t}$ plus the net outflux of N. This is known as the Reynolds Transport Theorem and is a very useful theorem for analyzing the fluid flows.

(Refer Slide Time: 19:10)





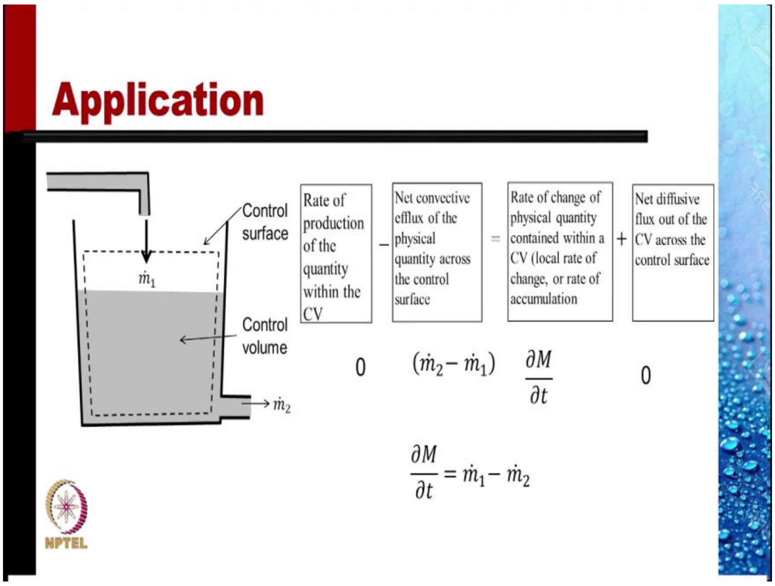
The rate of change of physical quantity associated with the system, i.e., the control mass minus the rate at which it diffuses out of the system equals the rate of production of that quantity within that system. That is how we write the conservation laws. The rate of change of physical quantity associated with the system minus the rate at which it diffuses out of the system is equal to the material rate of the change of the quantity within the system, which is also equal the rate of production of that quantity within the system.

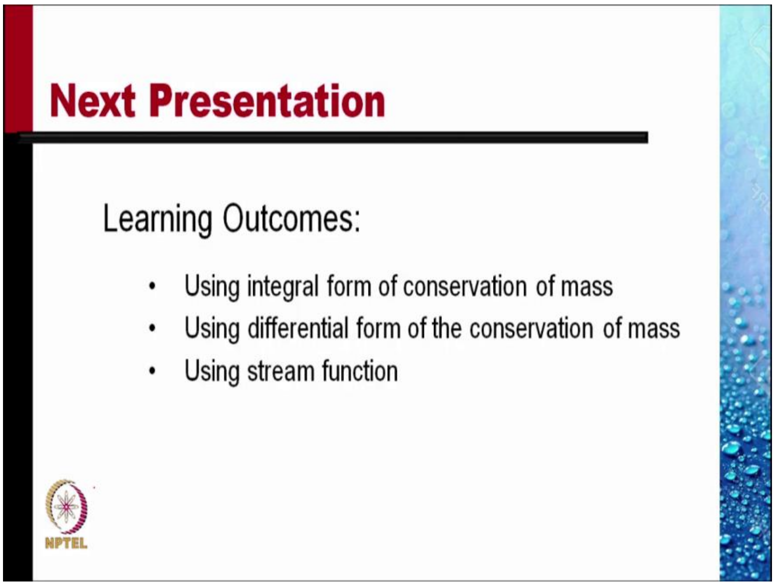
If the quantity is mass, the rate of production of that quantity is 0. If that quantity was momentum, the rate of production momentum is the net external force applied on the system. If the quantity was energy the rate of production of that quantity within the system is equal to the net rate at which the work is done on the system minus the net rate at which the heat is lost by the system.

The rate of change of physical quantity associated with the system is equal to the rate of accumulation plus the net efflux of the physical quantity. And this we can write it like this: The rate of production of the quantity within the control volume minus the net convective efflux of physical quantity across the control surface is equal to rate of change of physical quantity contained within the control volume. This is the local rate of change or the rate of accumulation plus the net diffusive flux from the control volume across the control surface.

The first efflux which is the second term on the left is the physical quantity that moves out of the control volume in association with the mass crossing the boundaries of the system, while the diffusive flux, which is the second quantity on the right, is a quantity that crosses the control surface, not in association with the mass, but because of something like temperature difference. The energy may be losing because of temperature difference, diffusive loss. Mass is not lost diffusively, momentum is not lost diffusively, but the energy is lost diffusively.

(Refer Slide Time: 22:14)





We apply this to a system in which we have a container in which the mass is coming in at the right $\dot{m}_1$, the mass is leaving at the rate $\dot{m}_2$. The control volume is defined as the volume enclosed by broken line. The broken line itself represents the control surface. The rate of production of the quantity within the control volume is obviously zero. Net convective flux of the physical quantity across the control surface convective across in association with the mass.

So $\dot{m}_1$is moving $\dot{m}_2$ is moving out, so $\dot{m}_2$ minus $\dot{m}_1$ is a net efflux. And the rate of accumulation is $\frac{\partial M}{\partial t}$. Capital M is the mass contained within the control volume, the rate at which the mass contained within the controlled volume is changing is the rate of accumulation of mass, and the diffusive flux is zero. So clearly the rate of accumulation is the rate at which the mass influxes into the system.

Thank you very much.