Fluid Mechanics and its Applications

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Lecture 7

Graphical Description of Flows and Fluid Flow Analysis

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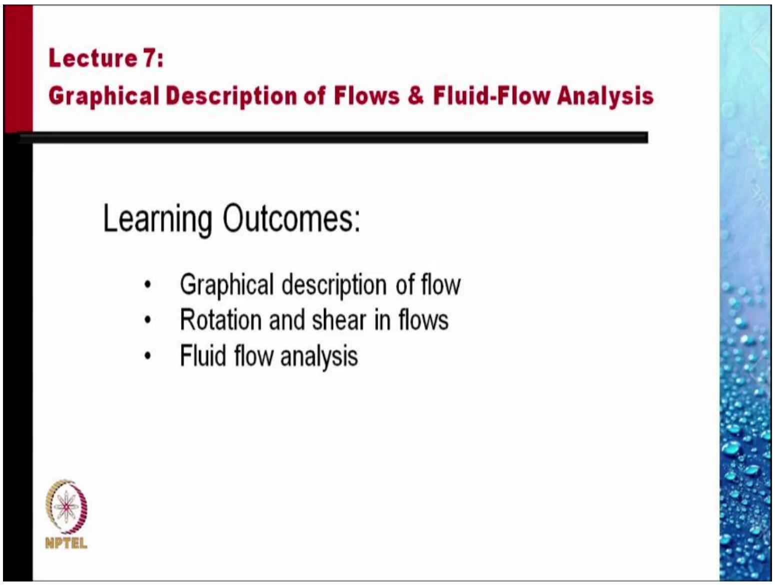

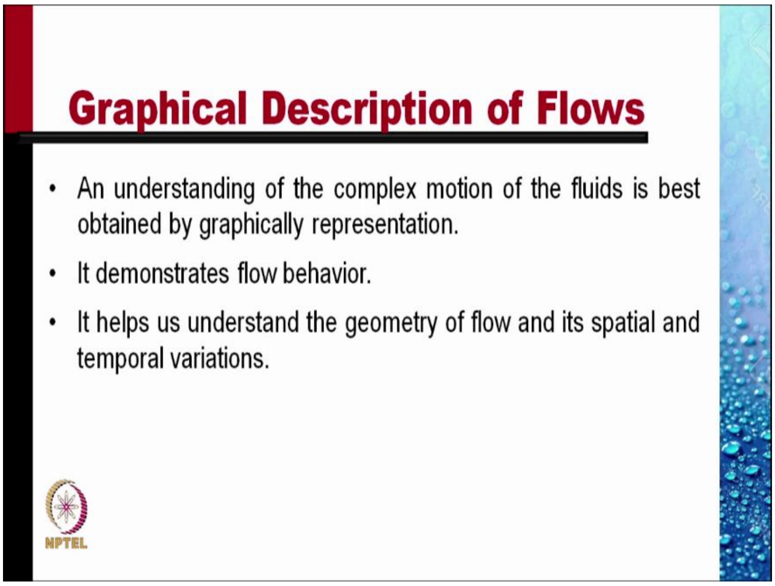

Welcome back. We continue today with the description of fluid flows and we will cover in this the graphical description and the fluid-flow analysis. Graphical description of flows is important because our understanding of the complex motionof the fluid is best obtained through it. It demonstrates flow behaviour, and it helps us understand the geometry of flow and its spatial and temporal variations.

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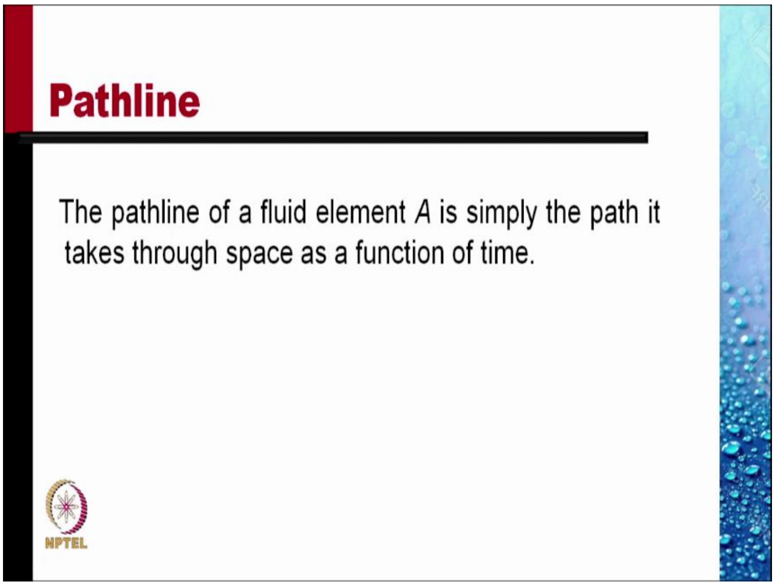

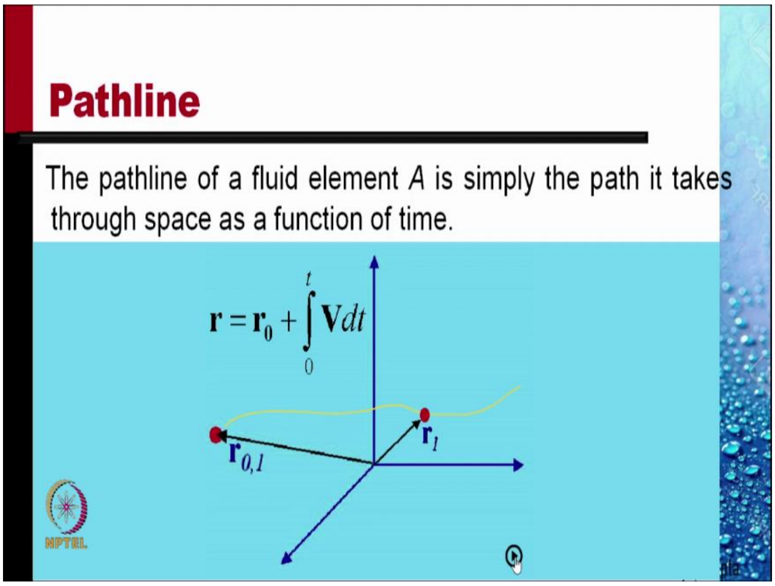

There are number of concepts in graphical description, one of the easiest one is the path line. The path line of fluid element A is simply the path it takes through space as a function of time. Consider a fluid element starting at r_o at time t is equal to zero. As time proceeds, this particle moves. At any time t its location r is given by r is equal to r_o, the original position at time $t = 0$ plus $\int_0^t \mathbf{V} dt$. We have a number of particles, so we have number of paths one for each particle.

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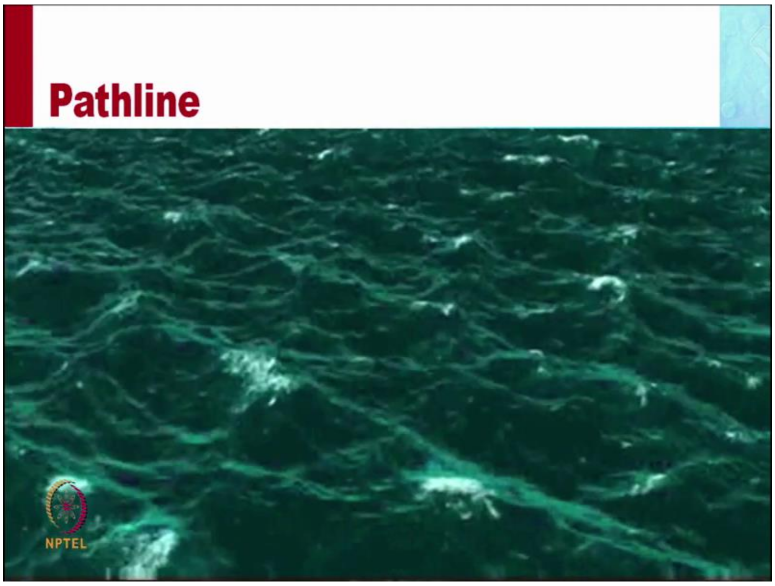

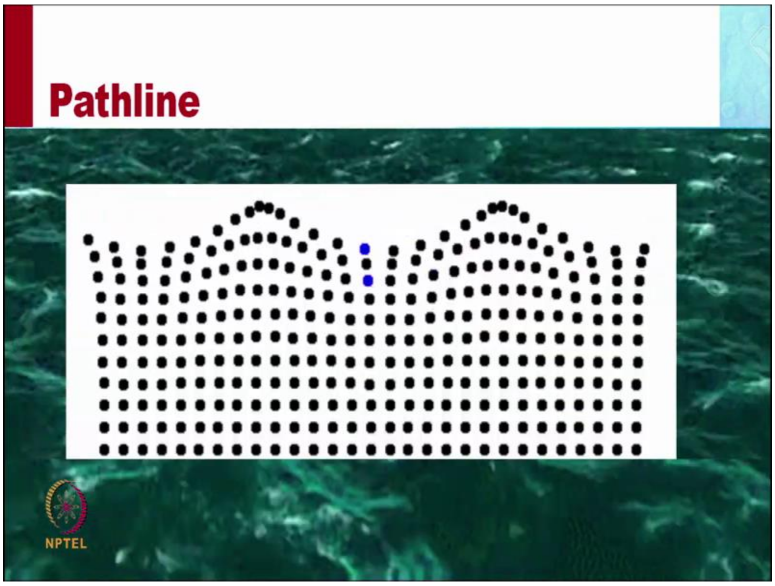

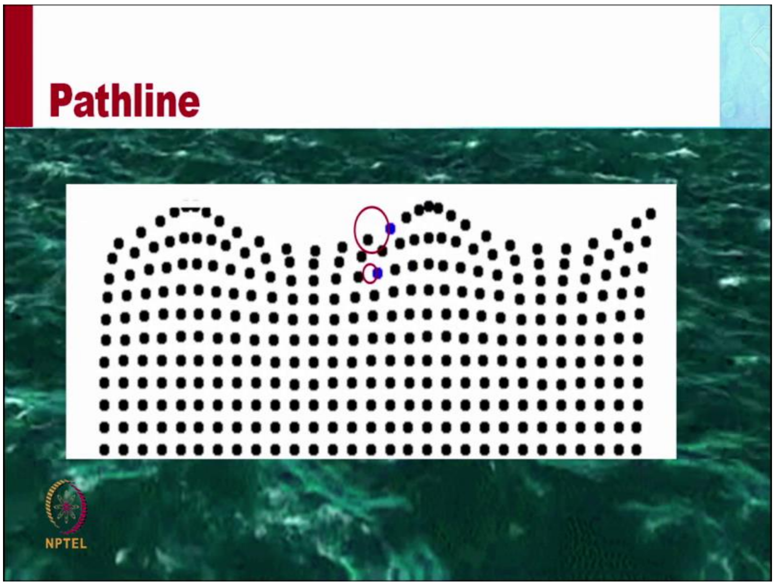

In this picture, we are showing the waves in ocean. How are the particles moving? If we look at this, the particles are moving like this. On the top, you can see the wave motion. But we have marked here a few particles with black dots and we can see their motion is strange. Let us look at two particles. And, their path is along two circles. So, the particles in this are moving purely in circular motion. So, paths of the particle are circular even though the waves appear to be traveling to the right.

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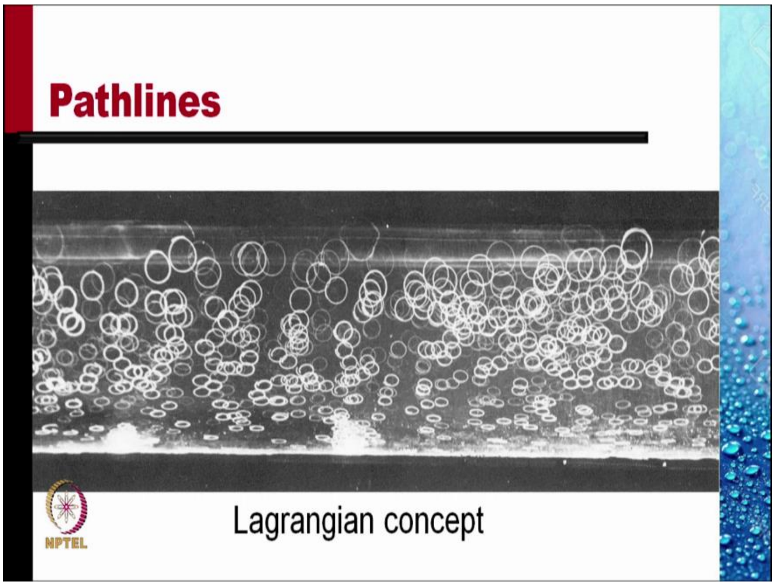

There is a time exposure picture of the same motion here. A number of particles illuminated by a white light are captured moving in circles. The particles on the top are moving through circles of larger radii, and particles near the bottom are moving through circles of smaller radii. This is, of course, the Lagrangian concept as discussed in the last chapter. Lagrangian is a material description where we follow the particles and move along with them.

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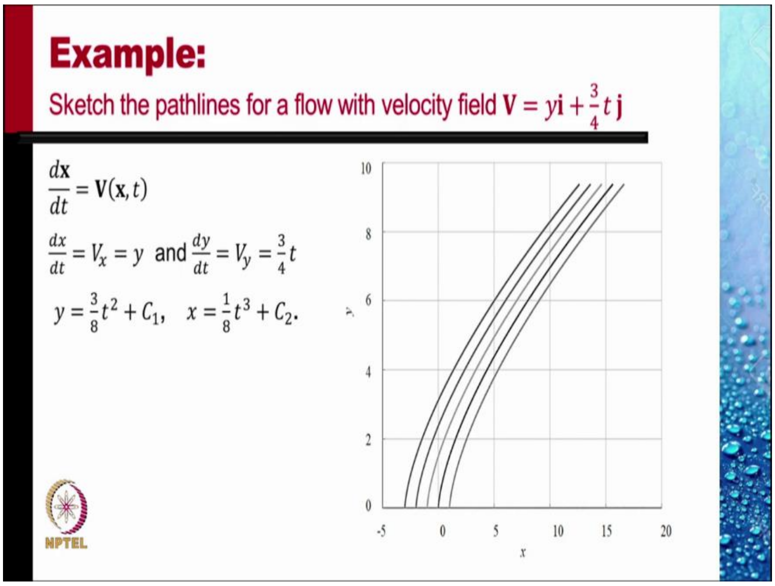

Let us do an example. Sketch the path lines for a flow with velocity field $\mathbf{V} = y\mathbf{i} + \frac{3}{4}t\mathbf{j}$. The velocity $\frac{d\mathbf{x}}{dt}$ is equal to $\mathbf{V}(x, t)$, a vector equation. The two scalar components are $\frac{dx}{dt} = V_x = y$, and $\frac{dy}{dt} = V_y = \frac{3}{4}t$, the j direction component of velocity. And the integration of this yields very easily $y = \frac{3}{8}t^2 + C_1$, and $x = \frac{1}{8}t^3 + C_2$.

We can plot this for given values C_1 and C_2. We can plot these path lines for different values of C_1 and C_2. Here we are starting with five values of C_2 from -3 to 1, and value of C_1 as 0. The paths of five particles are shown in this graph.

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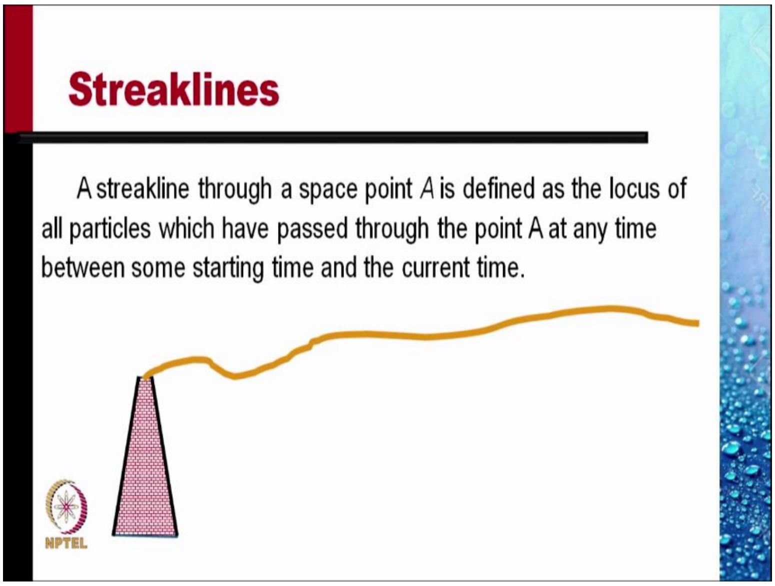

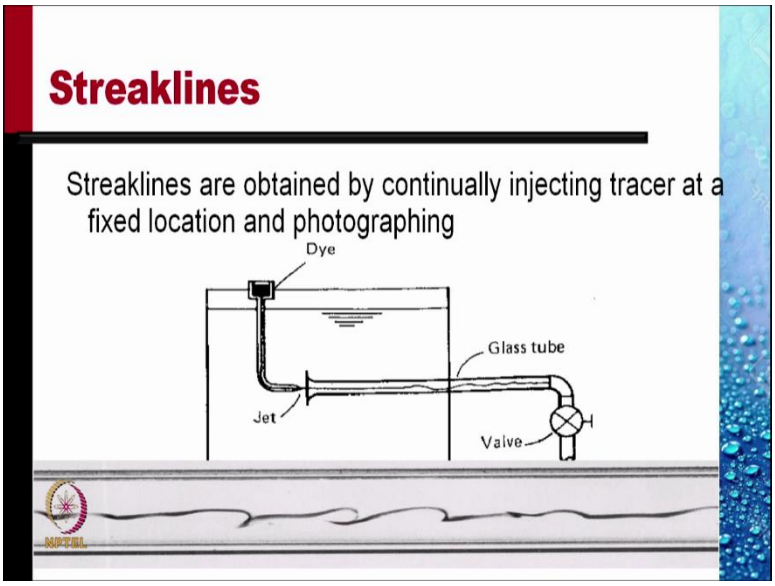

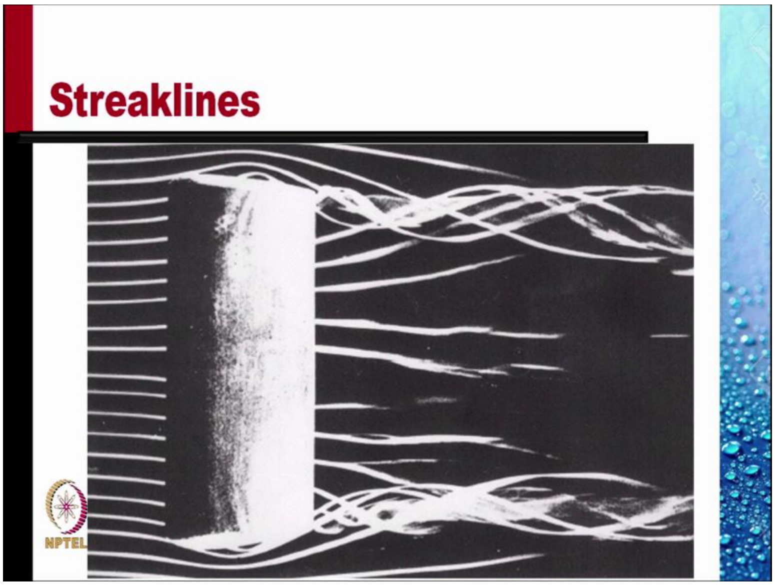

Next we talk of a streak line. A streak line through a space point A is defined as a locus of all particles which have passed through the point A at any time between some starting time and the current time. Thus, the plume of smoke coming from a chimney is a streak line, because every one of the smoke particle was at the mouth of the chimney at some time t earlier than the time at which this picture is taken. The streaklines that we saw in the Reynolds experiment are streaklines because all the particles are emitting from the same orifice of the jet.

These are the streak lines of flow past an aerofoil. This model of the aerofoil is subject to a flow from the left to the right. At number of points upstream, we inject smoke, and so each one of these lines on the left is a streakline. And we see the streaklines do tell us how the flow is behaving.

We see that the flow behind the aerofoil is kind of braided up into two braids of streaklines, one at the top end one at the bottom end. These, we will learn later on, are called the trailing vertices, and are principally responsible for creating induced drag on this aerofoil.

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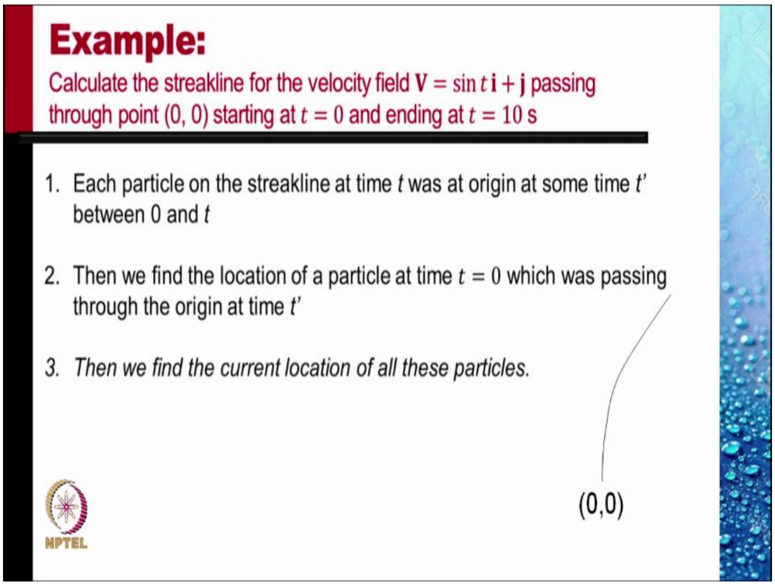

Here again is a picture where a delta wing is in a wind tunnel. The smoke is injected from the bottom and we can see how vortices are shed alternately from the two ends of the delta wing. You can see the power of visualizing the streaklines. They tell us the nature of the flow. Let us here do an example.

Calculate the streak lines for the velocity field $\mathbf{V} = \sin t\,\mathbf{i} + \mathbf{j}$. passing through the point (0, 0) starting at time $t = 0$, and ending at time $t = 0$ seconds.

This is a little complicated problem. Every particle must pass through point (0, 0). So the streakline at time t is equal to 10 seconds consists of the locations of all the particles that pass through point $(0,0)$ between time $t = 0$and $t = 10$ seconds. The particle at time t is equal to 10 seconds is still at point $(0,0)$, and the particle that passed through point $(0,0)$at time $t = 0$ is at the top end of this streakline. So the method of calculating the streakline is that we find the location of the particle at time $(0,0)$ which pass through the origin at a given time t' between 0 and t. Then we find the current location of all these points.

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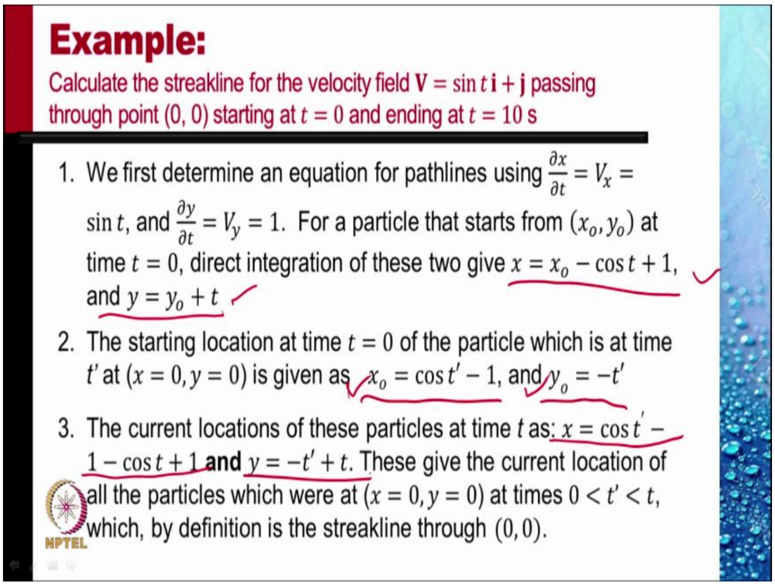

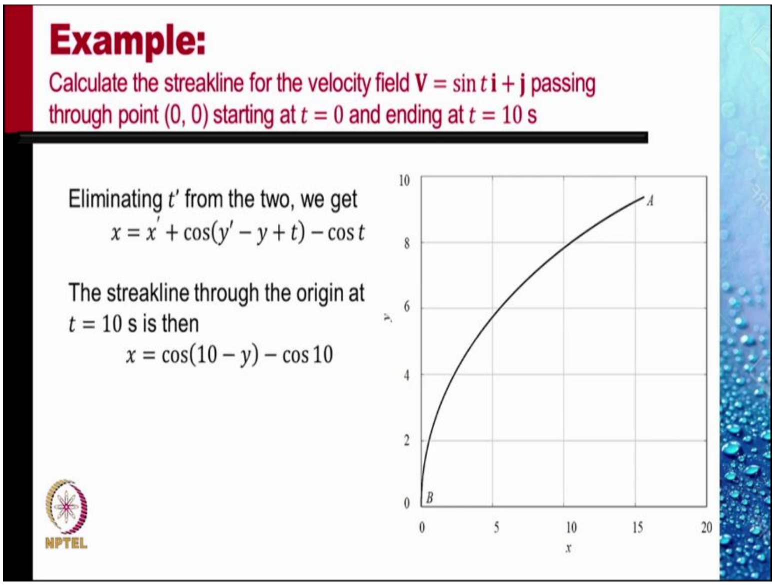

We first determine the equation for the path line using $\frac{dx}{dt} = V_x$ which was $\sin t$ and $\frac{dy}{dt} = V_y$ which was 1. The particle that starts from (x_o, y_o) at time $t = 0$, by direct integration, these give $x = x_o - \cos t + 1$, and $y = y_o + t$. The starting location at time $t = 0$ of the particle which is at time t' at $x = 0$ and $y = 0$ is given by $x_o = \cos t' - 1$, and $y_o = -t'$, because the x and y are 0 at time t equal to t prime.

So we plug in the equations in line 1, and from this we obtain $x_o = \cos t' - 1$ and $y_o = -t'$. These are the location at time $t = 0$ for the particles, which are at $x = 0$ and $y = 0$ at time t'. The current location of these particles at time t are then found by these equations where you put these values of x_o and y_o to obtain the current locations as $x = \cos t' - 1 - \cos t + 1$ and $y = -t' + t$ y.

These give the current location of all the particles which are at $x = 0$ and $y = 0$ at time between 0 and t, which by definition is the streakline through (0, 0). So we can find out the equation of steakline by eliminating t' from this. And if we do this, eleminating t' from the two, we get $x = x' + \cos(y' - y + t) - \cos t$, and this streakline originating at time t = 10 seconds is obtained by plugging in the value of t as 10 to get this expression. This is plotted as this. So the streakline for the given problem at time t is equal to 10 seconds which passes through point 0, 0 is this.

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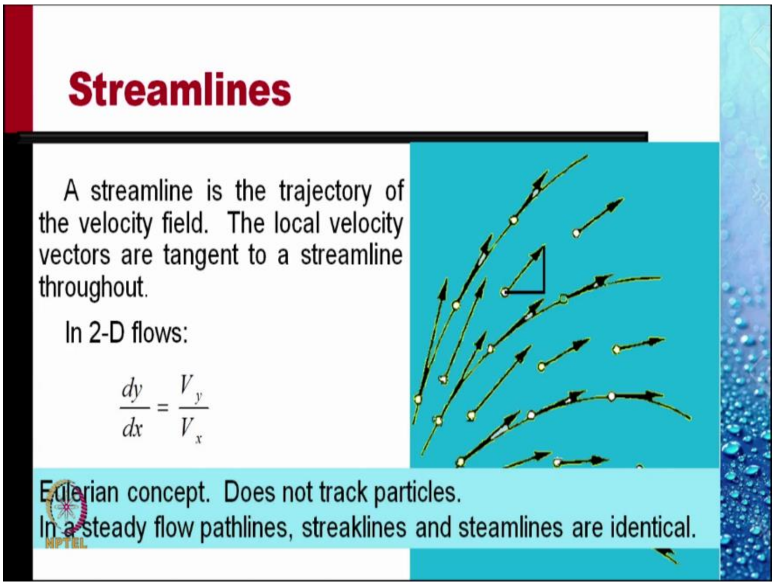

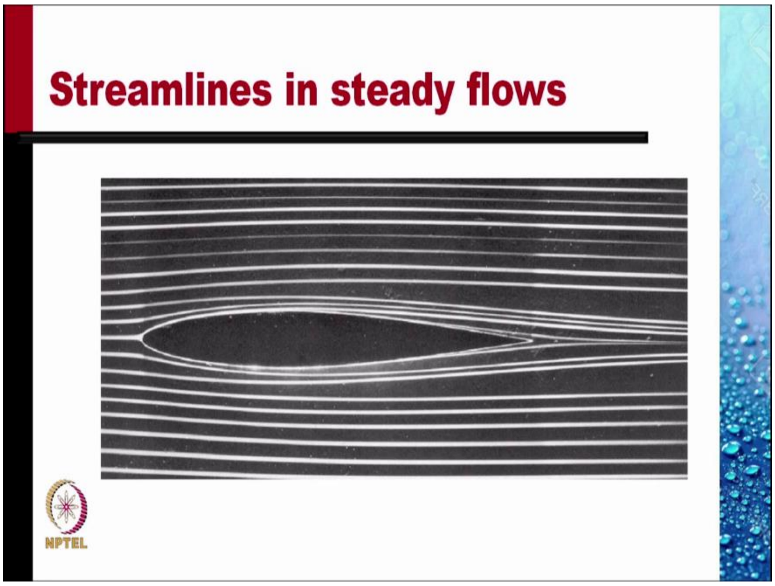

The third important concept in graphical description of the flow is a streamline. Streamline is a trajectory of the velocity field much like the trajectory of a magnetic field, or an electrical field. The local velocity vectors are tangent to a streamline throughout the flow field. In two-dimensional flow then, dy/dx, the slope of the streamline at any given point would be V_y/V_x. Streamline is an Eulerian concept. It does not track particles. In a steady flow, path lines, streak lines and stream lines are identical, but when the flow is unsteady, these could be very different.

This picture shows the streamlines in a steady flow. In fact, these are streaklines. There is a streak of smoke emitting from a rake to the left of this figure and flowing past an aerofoil. The flow is very well behaved. The flow is laminar.But these streaklines are also streamlines because the flow is steady.Bbut if the flow is unsteady, then streamlines could be very different.

The same flow can be made unsteady by keeping the camera fixed and moving the aerofoil, and if we see the velocities are tangent to these lines which are shown there.

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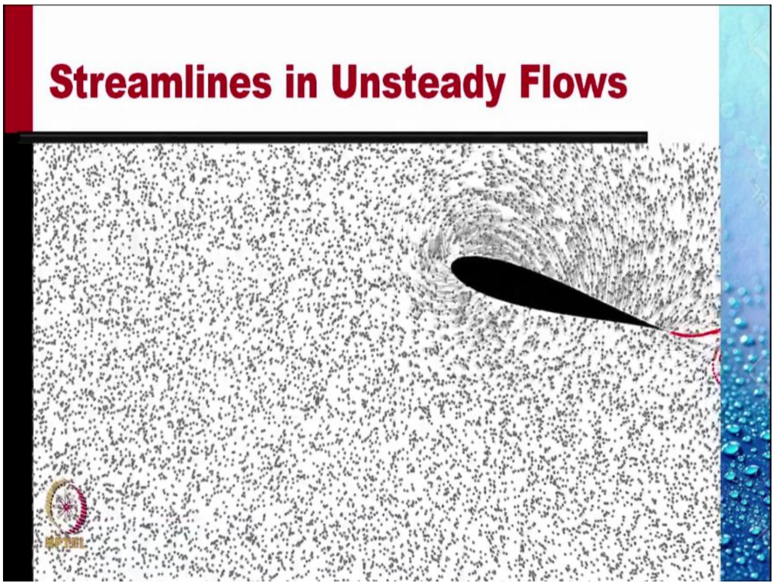

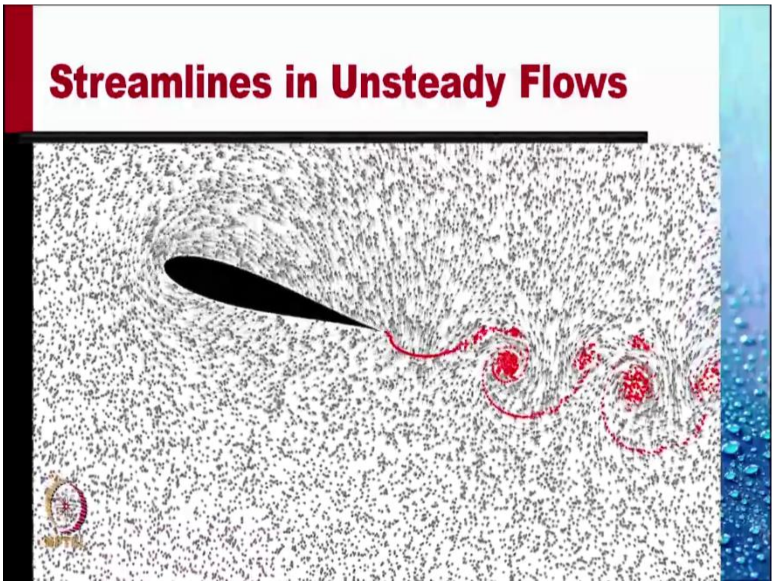

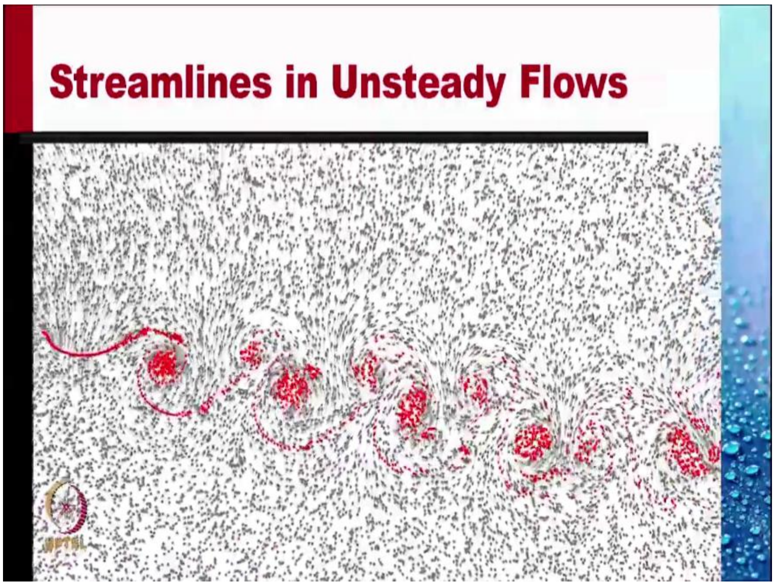

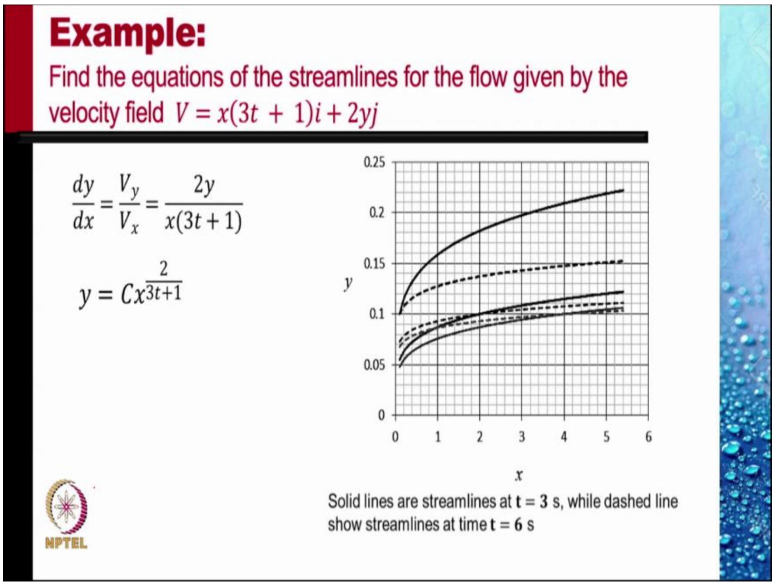

Let us do an example: Find the equation the streamlines for the flow given by the velocity field $\mathbf{V} = x(3t + 1)\mathbf{i} + 2y\mathbf{j}$. As we know the slope of a streamline in 2-dimensional flow is $\frac{dy}{dx} = \frac{V_y}{V_x} = \frac{2y}{x(3t+1)}$. And this differential equation is easily integrated to give $y = Cx^{\frac{2y}{x(3t+1)}}$.

I plotted the streamlines for two different times. You, realize that this equation between y and x changes with time. The solid lines are streamlines at time t is equal to 3 seconds while the dashed lines show the streamlines at time t equal to 6 seconds. As the time increases, the x component of velocity increases, and so the streamlines are much flatter at a larger times than they are at a smaller times.

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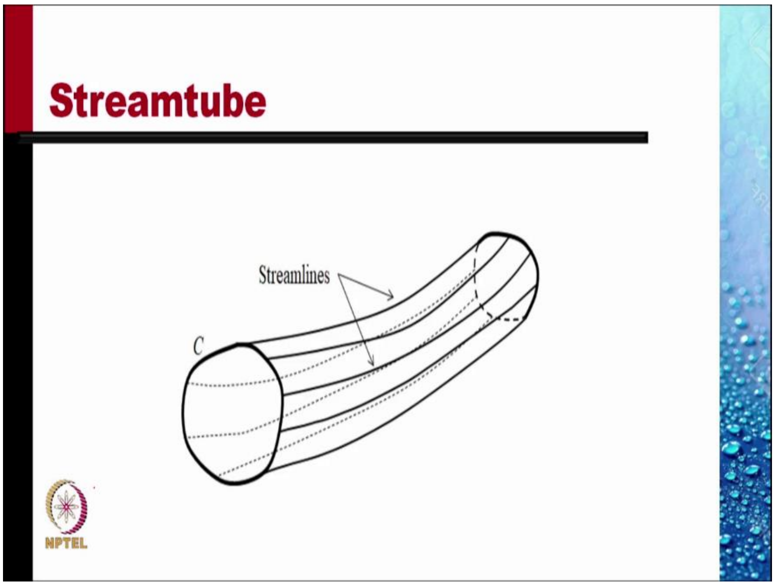

Another associated concept is a streamtube. If we take a small circuit C within the flow field, then there is a streamline passing through every point of the circuit, and the streamlines would form a surface enclosing a small thin volume with a cross section equal to the circuit C at that location. This surface, which consists of streamlines, is called a stream surface, and the volume enclosed by this closed surface is called the streamtube. Clearly, since the velocity is tangent to streamlines at all points, the velocity is tangent to stream surface at all points. This concept of steamtube, we will use in a later chapter.

As was said before, in steady flows the streamlines, streaklines and path lines are identical. This picture shows a slender body in which smoke is being emitted through various orifices arranged in a line and we get a beautiful picture of vortices. The flow is steady. So these streaklines are also streamlines as well as path lines.

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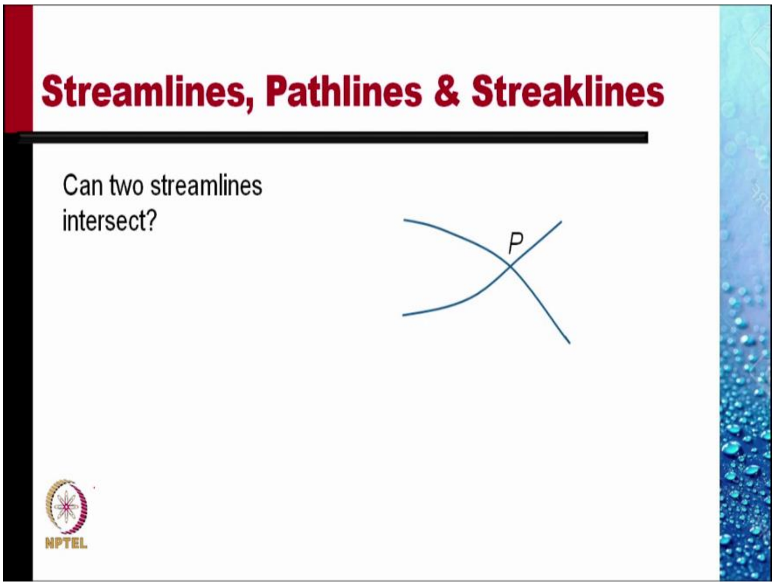

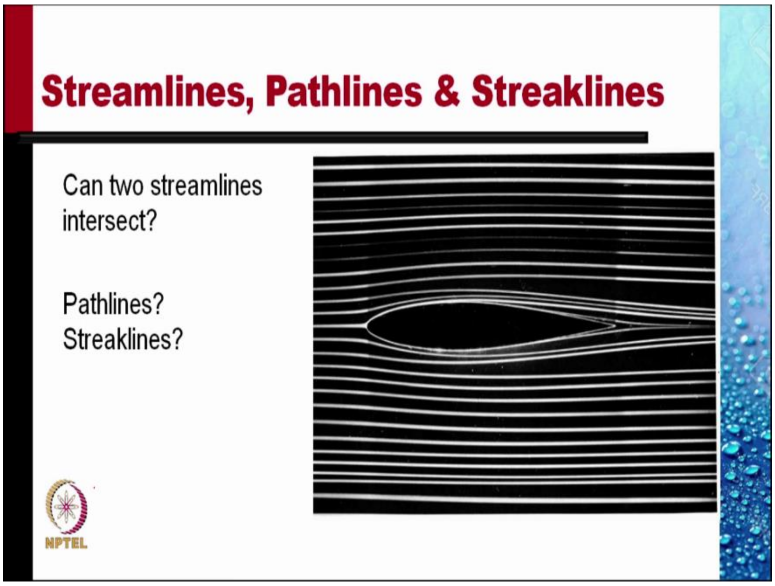

Can two streamlines intersect? Suppose they intersect at a point P then clearly the tangent to a streamline is in the direction of velocity. There are two directions of velocity at point P one along one streamline and the other along the other streamline. Clearly, that is not possible. The velocity can have only one direction. This is possible only if the velocity at point P is 0. In that case, there is no direction to a velocity, and the two streamlines intersect. Such a point is a stagnation point where the fluid is stagnant.

You see the streamline at the nose of this body is dividing into two portions, one going along the upper surface and the other along the lower surface. So this point is clearly the stagnation point, the velocity at the left tip, the upstream tip of the aerofoil is clearly a stagnation point.

Can pathlines intersect? Can streakline intersects? Yes, they can, because if the two pathways are intersecting that does not mean the two particles are at that point of intersection at the same time. They could be at different times. So pathline and streaklines intersect when the flow is unsteady. They cannot do so in a steady flow, because in steady flows they would also be streamlines and the streamlines cannot intersect.

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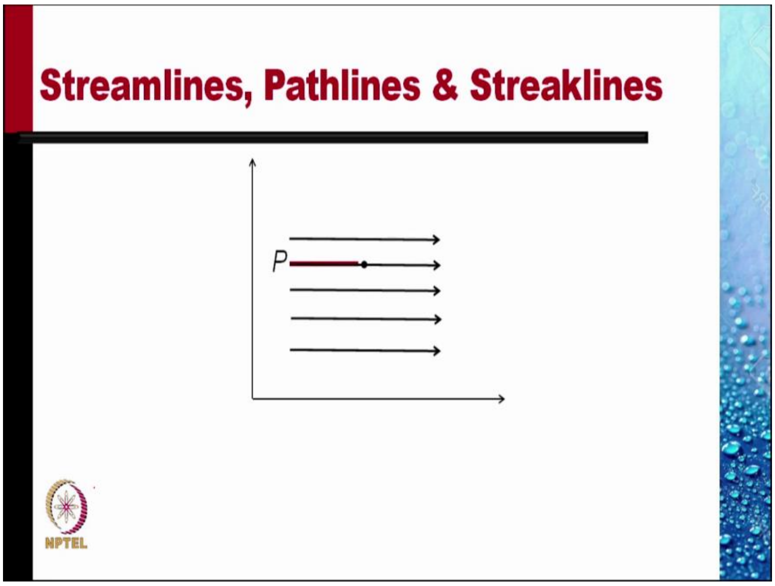

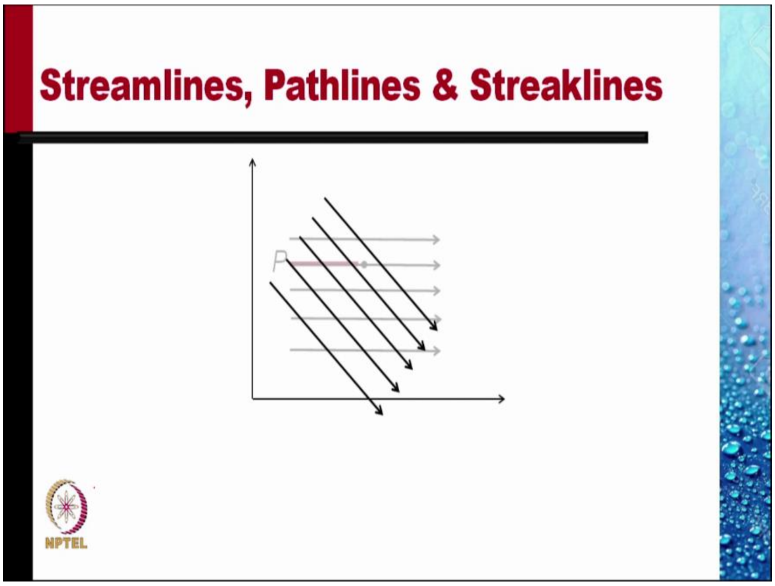

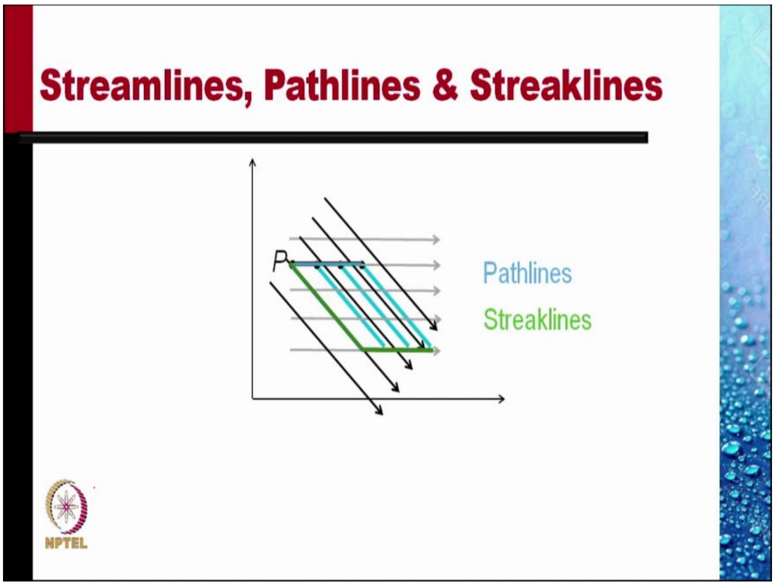

Let us explain these differences between streamlines, pathlines and streaklines once again. Let us consider a point P in a flow field where the flow is from left to right. So the streamlines would look like these arrows, black arrows from left to right. A particle at P would travel along this line. A particle that follows will also travel along this line, so this red line here would both be a streakline, as well as pathline, besides being streamlines as we started out with.

Now at time t is equal to t_o the direction of the flow changes, and now the flow is inclined at 45° in that direction. If everywhere the flow direction is same, then the current pattern of streamlines would be these dark black lines. But the particles which have passed through the point P up till time $t = t_o$ are still occupying this line and but now they start moving down in that direction, and so what is the final position of the particles, those particles which pass through point P up till time $t = t_o$ is this and the particle which pass through point P after time $t = t_o$ are along that inclined green line.

So the green line is the streakline at this point of time. The green line is the streak line at this point of time. And what is the pathline of the particle that started out first from point P? The pathline is this blue line. They start from P horizontally and then take this 45° incline line, so the green line is an inclined L and pathlines is inclined L upside down. I hope this clarifies the differences between streamlines, pathlines and streaklines.

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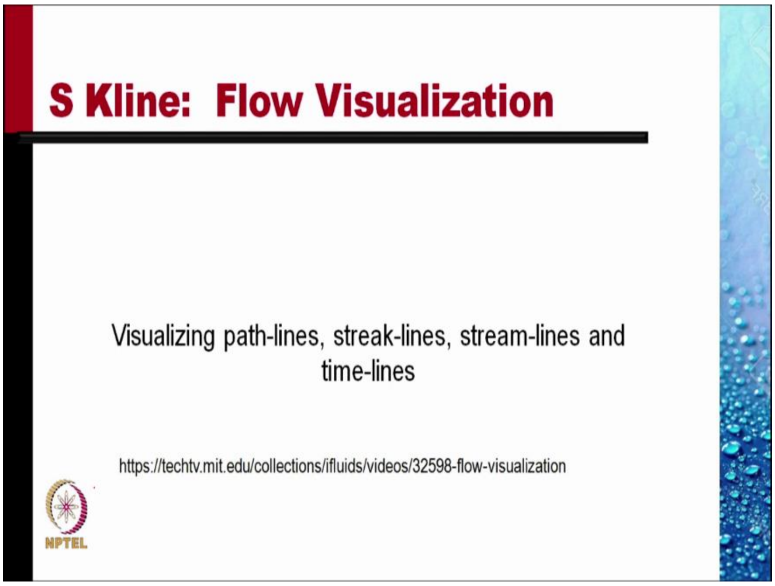

I invite you to watch this famous video by Professor S. Klein of Stanford University. This was made in 1960 and demonstrates the various flow visualization elements very beautifully. This video is available on Youtube and it is also available on a site of MIT, I have given here the link.