Fluid Mechanics and Its Applications
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Lecture 6 A
Rates of Change with Time

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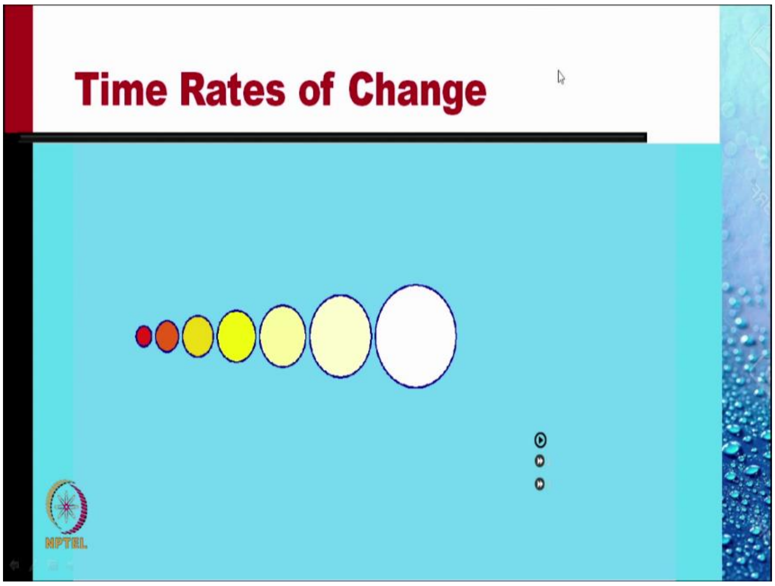

Let us now discuss the time rates of change experienced in fluid mechanics. For this, we created this image where we have a row of particles of different sizes. Suppose, the size of the particle is expressed as a variable η. The spatial derivative $\partial\eta/\partial x$which is positive, constant, the value of η, the size is increasing to the right. Let us suppose all these particles are moving with a velocity V.

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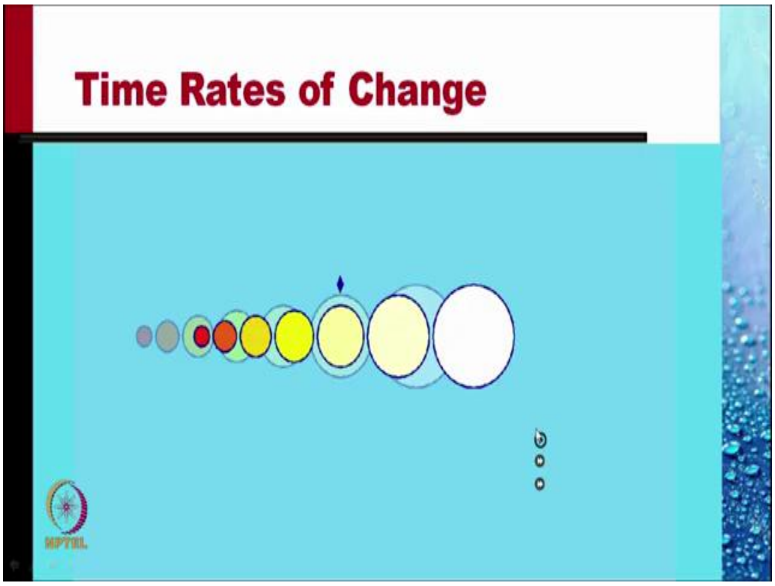

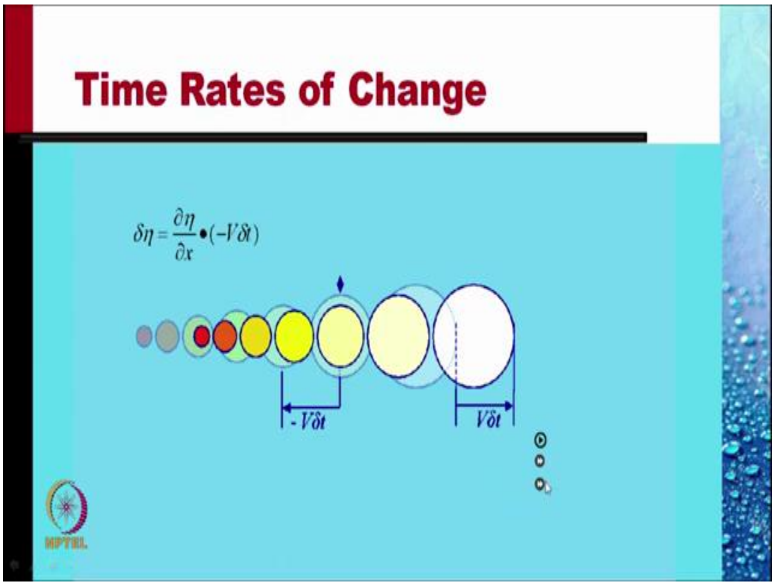

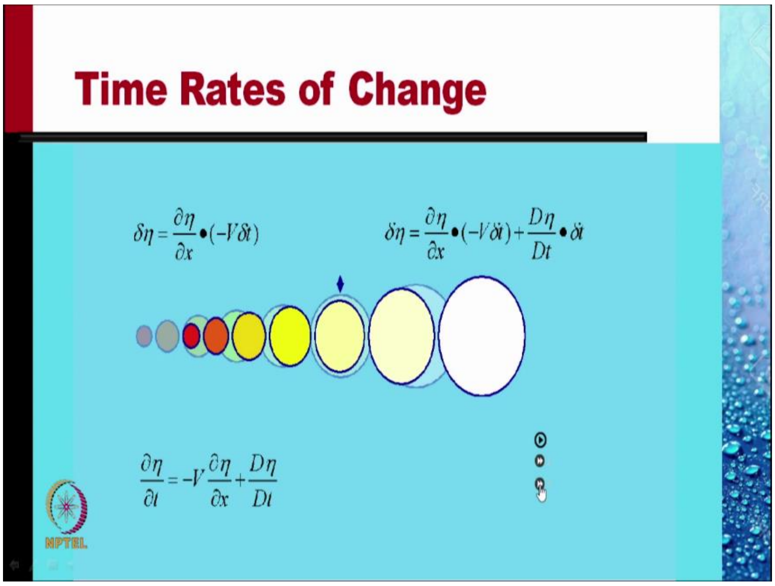

After time δt, the original row is now located at this configuration. The blue arrow is an observer observing the fluid property. If the blue observer has closed its eyes at the start of the motion and opens its eyes δt time later, then it will see this picture. It will record that the size of the particle, the property η, has decreased. What has happened? What has happened is this. Though, a particle which was V delta t distance behind the original particle at the diamond probe has occupied this location, the whole row has moved a distance $V\delta t$ to the right.

So, if the size of the particle was not changing, then originally recorded particle which was at this location x, but δt times later it was recording the size of the particle which originally was at $-V\delta t$ behind the current location. So, the change in property observed is $\delta\eta$ is equal to $\frac{\partial\eta}{\partial x}(-V\delta t)$. This change has been recorded purely because of the motion of a particle. So, this change can be called the convective change following the lead from heat transfer, where the convective heat transfer is because motion of the fluid. So, this change in property η is solely because of the motion of the fluid particles.

And of course, because of the gradient of the property η in space, because if $\frac{\partial \eta}{\partial x}$ was zero, then all particle are of the same size. No motion would show any change in the value η at the location of the probe. Now, if in addition, during this time δt, the size of each particle increases. If the size of each particle increases, then the change in η observed by this observer would be the original convective change $\frac{\partial \eta}{\partial x}(-V\delta t)$, plus the rate at which the size of the particle grows, and it denoted by $D\eta/Dt$.

$D\eta/Dt$ is the material rate of change of η with time of the actual particle: the growth of the size of each particle. We call this the material rate of change. And so, the total change in time delta t would be $(D\eta/Dt)\delta t$. So, the total change observed by this observer would be the convective change plus this material change. And the rate of change with time is given by the equation at the bottom of the slide $\frac{\partial \eta}{\partial t} = -V\frac{\partial \eta}{\partial x} + D\eta/Dt$.

$D\eta/Dt$ on the left is the observed rate of change. We call it the local rate of change, locally observed rate of change by a stationary observer. A field description, an Eulerian description. And this consists of two changes, one is a rate of change $-V\frac{\partial \eta}{\partial x}$, which is the convective rate of change, plus $D\eta/Dt$, which is material rate of change. The local rate of change is made up of two things, a convective rate of change and a material rate of change.

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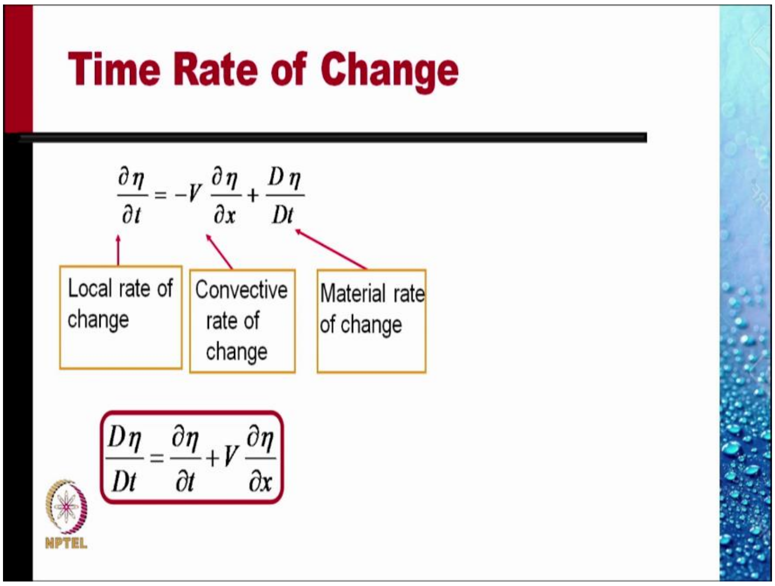

So, repeat. The local rate of change, $\frac{\partial \eta}{\partial t}$ is made up of two changes, one convective rate of change and the other the material rate of change. This equation is conventionally expressed in this form$D\eta/Dt$, the material rate of change is equal to the local rate of change, plus the convective rate of change.

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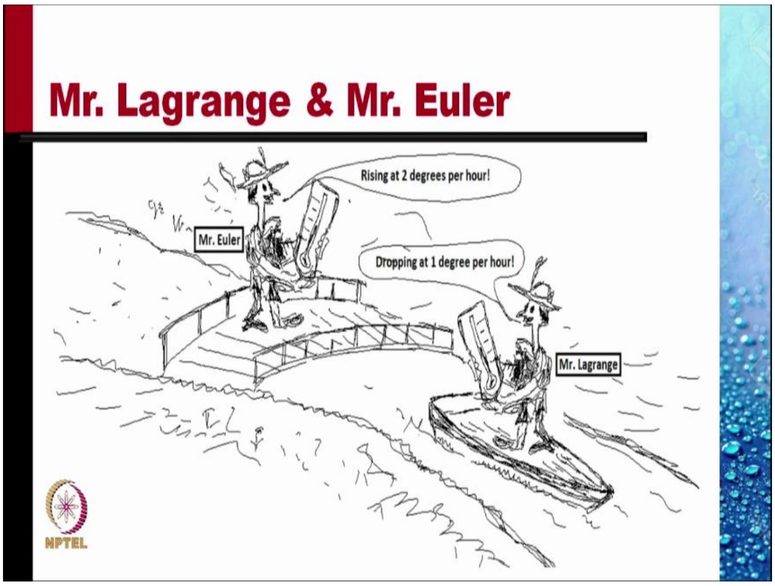

This cartoon explains this quite clearly, there are two scientists Mister Euler, who is stationary and he reports a local rate of change at 2 degrees per hour, rising at 2 degrees per hour temperature. Mister Lagrange of the material property fame is floating down the river on a boat. So, he is measuring the material rate of change and he says the temperature is dropping at one degrees per hour. What is happening? That there is a spatial gradient, there is a convective rate of change that defines this difference.

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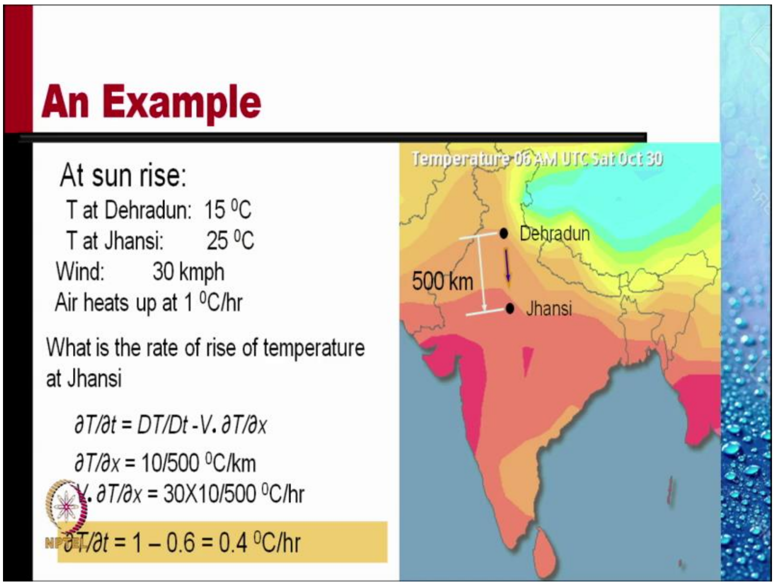

Let us take an example. On a given day, the temperature at Dehradun is 15 ℃ at sunrise. And at the same time, the temperature at Jhansi almost exactly south of Dehradun is 25 ℃. Jhansi is about 500 km from Dehradun. There is a 30 km/hr wind that blows due south, and the air heats up at the rate of 1 ℃/hr. What is the rate of rise of temperature at Jhansi?

To understand this, let us understand what does air heats up at 1 ℃/hr mean? Is this the local rate of change, or the material rate of change? It clearly is the material rate of change, each individual particle of air is heating up at the rate of 1 ℃/hr. There is a spatial gradient of temperature. Temperature rises from 15 ℃ to 25 ℃ over a distance of 500 km. So, $\frac{\partial T}{\partial x}$ is 25 minus 15, that is 10 ℃ divided by 500 km.

The local rate of change at Jhansi, $\frac{\partial T}{\partial t}$ is $D\eta/Dt - V\frac{\partial T}{\partial x}$, material rate plus the convective rate. $V\frac{\partial T}{\partial x}$ is equal to 30 km/hr into 10/500 ℃/hr. So, the local rate of change is 1 minus 0.6, or 0.4 ℃/hr. So, even though each particle of air is heating up at the rate of 1 ℃/hr, the local rate of change felt by the residents of Jhansi is only 0.4 ℃/hr. Because the cooler air is moving in from Dehradun.

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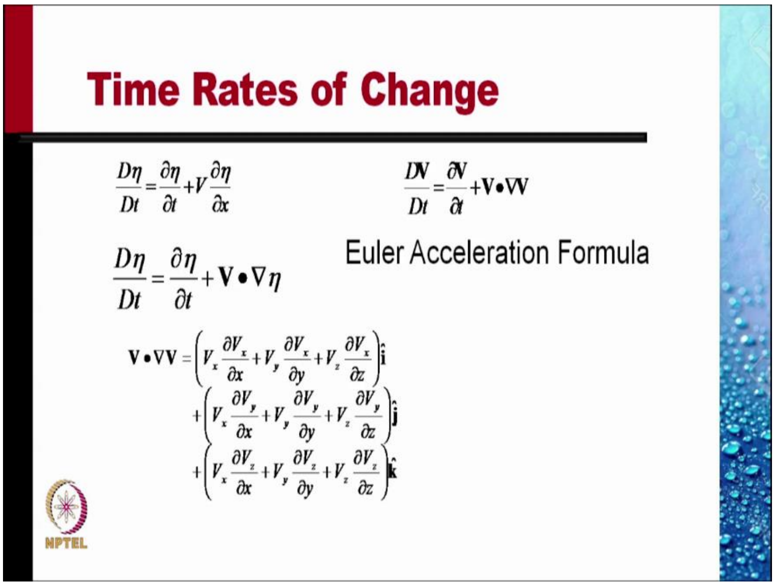

This is for the property η that we wrote this relation. For vector form, we can write this equation $\frac{D\eta}{Dt} = \frac{\partial\eta}{\partial t} + \mathbf{V}.\nabla\eta$. If η is taken as the velocity $\mathbf{V}$, then $\frac{D\mathbf{V}}{Dt}$ is acceleration. $\frac{D\mathbf{V}}{Dt} = \frac{\partial\mathbf{V}}{\partial t} + \mathbf{V}.\nabla\mathbf{V}$. This is known as Euler acceleration formula. The term on the left gives you the actual acceleration of a particle. This is a material description, a Lagrange description.

The first term on the right is the local rate of change of velocity, and $\mathbf{V}.\nabla\mathbf{V}$ is the convective rate of change of velocity, or the convective acceleration. The material acceleration, local acceleration plus the convective acceleration. Those familiar with simple vector operations would recognize that $\mathbf{V}.\nabla\mathbf{V}$ consists of 3 components i, j, and k in the 3 direction. In the i th direction, the component is $V_x\frac{\partial V_x}{\partial x} + V_y\frac{\partial V_x}{\partial y} + V_z\frac{\partial V_x}{\partial z}$. Similarly, in the y direction and the z directions.

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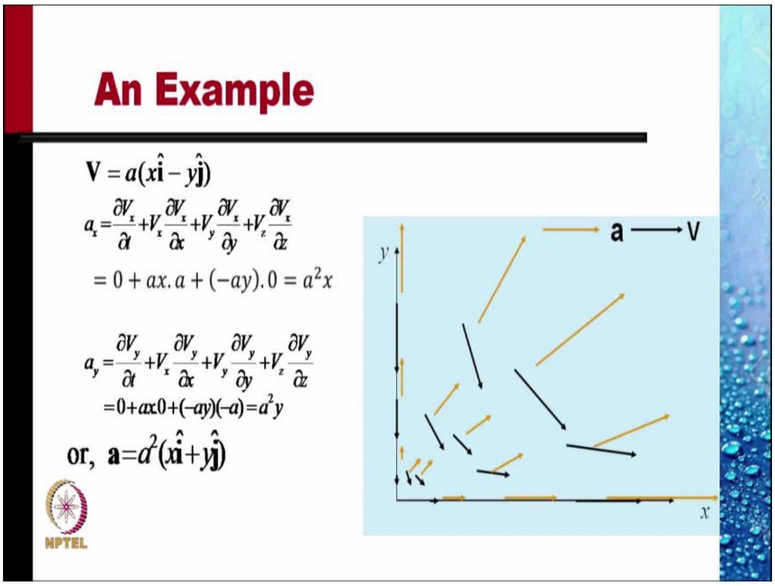

We do a simple example. Let the velocity field be $a(x\mathbf{i} - yj)$. The velocity component in the i direction, the x direction, is ax, and in the y direction is $-ay$. If you plot this velocity at a few points, we get a picture like this. Along the x-axis where y is equal to zero, the velocity is $ax\mathbf{i}$. So, the velocity is horizontal, and it is increasing as x increases. Along the y direction, where x is zero the velocity is $-ay\mathbf{j}$. So, velocity decreases towards the origin, and is directed towards the origin.

At other location, the velocity vectors are as shown. Now, let us determine the acceleration. Acceleration of particles which are at location x and y. a_x the x-component. By Euler acceleration formula wasa_x, the x component of acceleration is $\frac{\partial V_x}{\partial t}$, the local rate of change plus the convective rate of change, and if I evaluate each of this term, I get a_x as a^2x. There is no V_z component. There is no component normal to the plane of paper. a_x is this.

Similarly, a_y is evaluated, but differentiating this with respect to t, x and y, and we get a^2y. So, the acceleration vector is $a^2x\mathbf{i} + a^2y\mathbf{j}$+.This is the acceleration of the particle, which would be at the location $x\ y$, at a given time t. Note that the acceleration is independent of time. The velocity was independent of time. So, acceleration is also independent of time. If the velocity is independent of time, why is there acceleration, because acceleration is only the rate of change of velocity?

No, this is velocity field, This is Eulerian description. So, even if the velocity at a given point does not change with time, the particle at that point maybe undergoing acceleration, because when it goes/ moves to the next point where the velocity is higher, that means, it has undergone an acceleration. And if we plot this acceleration field, the acceleration field looks like this. These are the acceleration vectors at all times.

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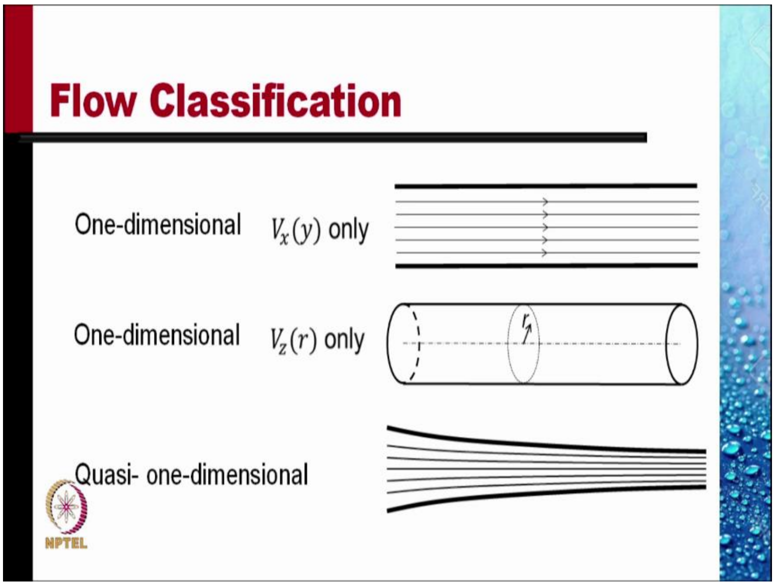

Let us go to classification of flow fields. We have a one-dimensional flow field called when the velocity description V_x is a function of only one space variable. In this channel made of two plates, one of the plates was moving, there is only *x* component of velocity V_x, which is a function of *y* only. So, it is called a one-dimensional velocity, one-dimensional field. Similarly, in flow through a pipe, we have seen before, that flow through a pipe at laminar regime has a velocity field which is parabolic, and it is a function only of r. V_z along the axis is a function of r. This is one dimensional.

If the channel was slowly tapering, very slowly tapering, the field is no longer one-dimensional, but if the variations with *x* direction are slow and so, it can be treated as a quasi-one-dimensional.

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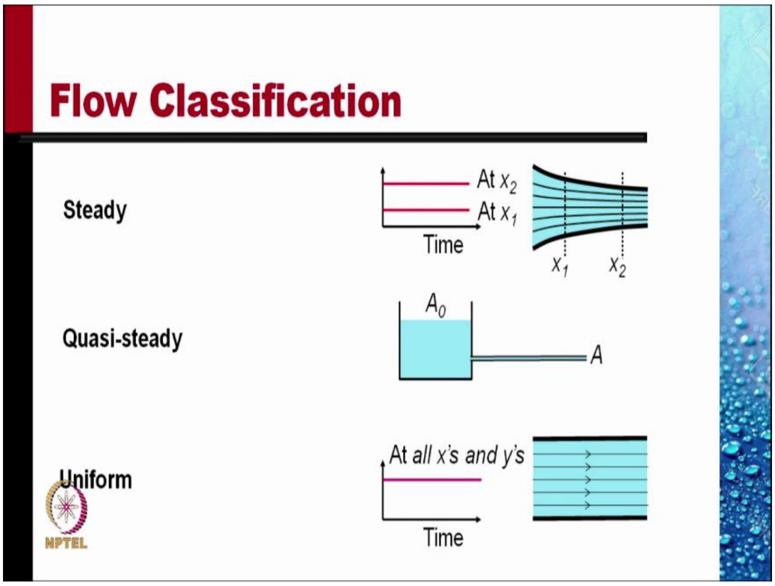

Similarly, we have a name steady flow. Steady flow is that flow field where the properties do not change with time. At location x_1 the velocity is constant with the time, and at any other location x_2 the velocity is constant at the higher value for all times. This flow is termed as steady flow. So, in steady flow the local gradient is zero. If we use the property velocity, then $\frac{\partial V}{\partial t}$ is zero. The local time rate of change of velocity is zero. But it does not mean that the acceleration is zero, as has been established earlier.

In this flow from a large reservoir to a small bore tube of long length, the level of the fluid in the reservoir would change very slowly, so that the velocity of the flow or the flow rate through the tube of area A is going to vary very slowly. And so, at a given time, this can be treated as a steady flow. Such flows, where the variations are very slow are called quasi-steady flows. We would have occasion later to amplify on this assumption. A uniform flow is a flow in which the properties are constant at all x’s and y’s, that is, everywhere all properties have the same value.

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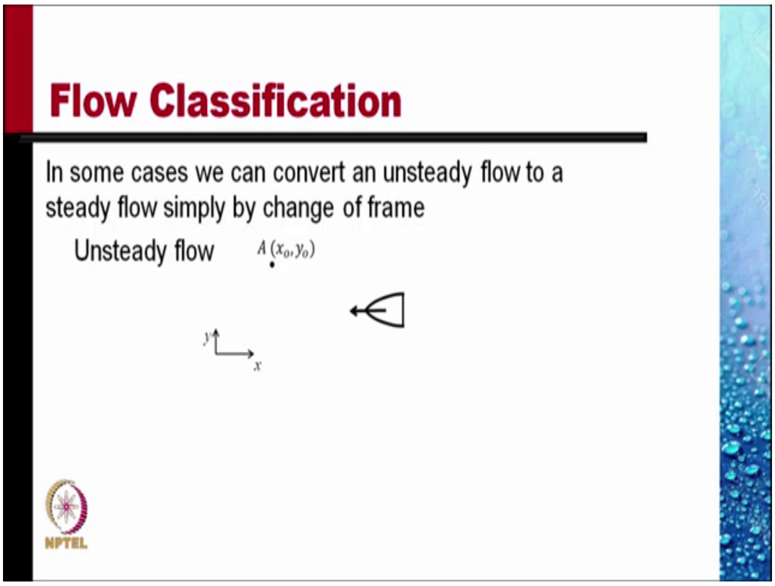

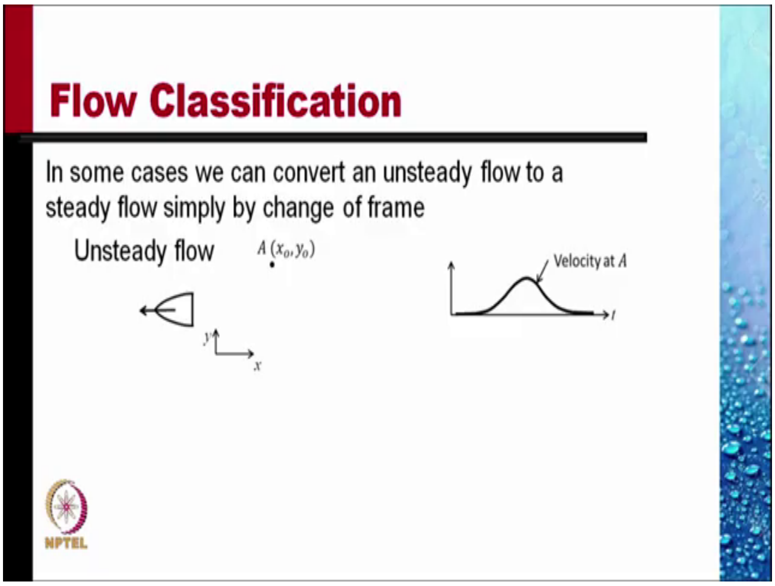

In some cases, we can convert an unsteady flow to a steady flow simply by change of frame. There is a body that is moving towards left with a constant velocity. Let us consider point A at a location $(x_o, y_o)$and record the change in velocity. Originally, the velocity there would be zero, but as the body moves towards A, the velocity increases, and as the body moves past A the velocity decrease back to zero again. So, the velocity at A would appear something like this.

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In this video, which is a famous video from 1933 recorded by the great German aerodynamicist Ludwig Prandtl, which show a body moving in a fluid and the flow is unsteady. (Video played 22:14 to 22:22) We can make these flows steady by changing the frame of reference.

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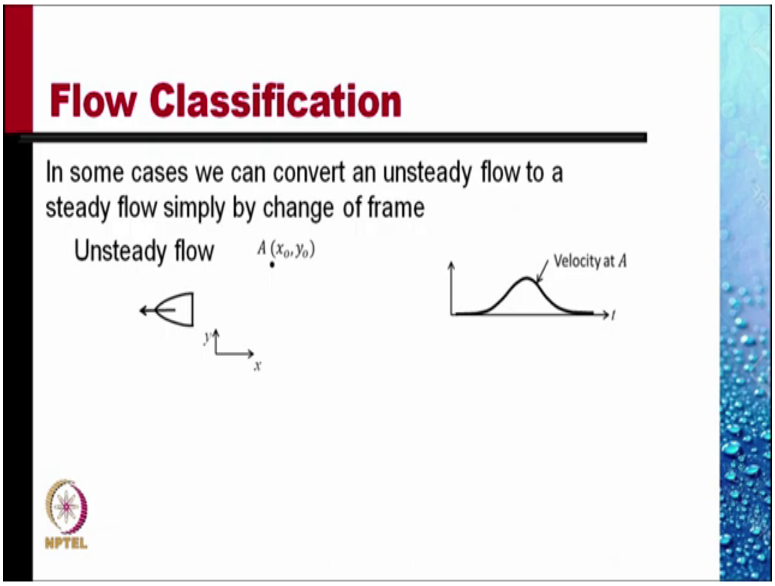

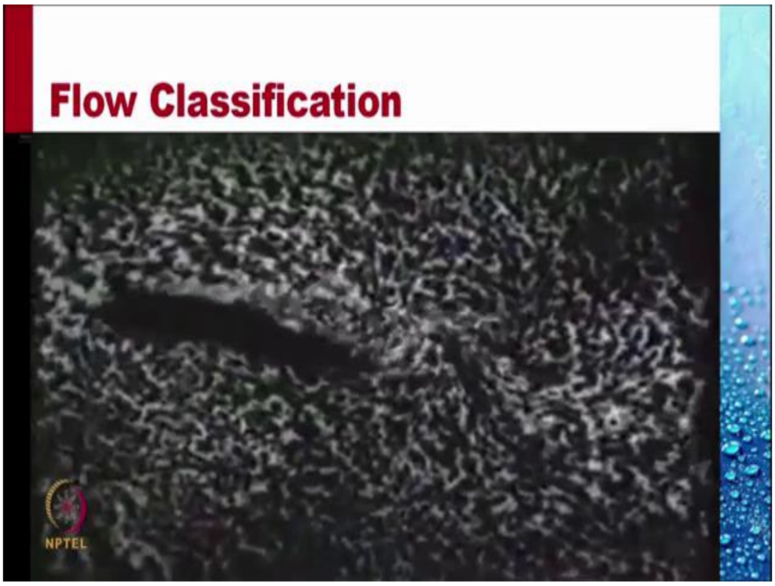

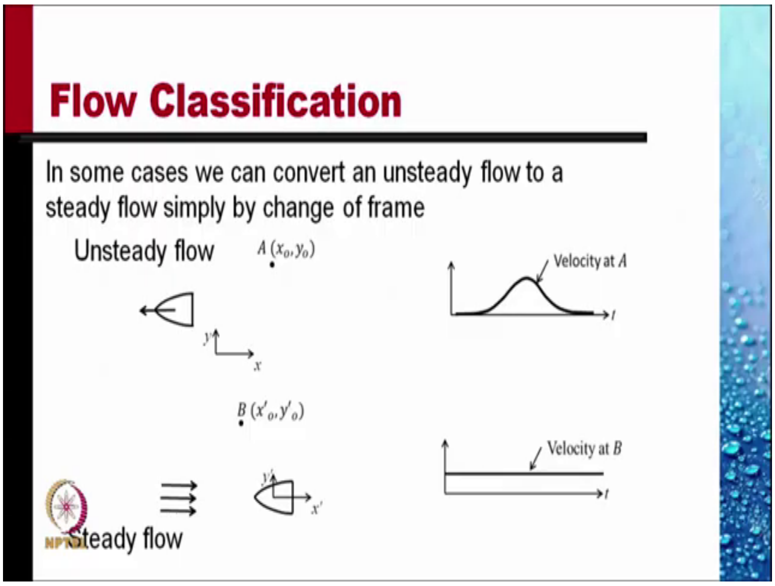

Now, we attached the frame to the body. Here the point B is in the frame (x'_o, y'_o), the body is stationary in this frame, the fluid moves past this from left to right at a velocity V. Then the velocity at point B would always be same, invariant with time. The value would be depending upon the location of B relative to the body. If it is closer to the body, if it is at nose of the body, the fluid at the nose is always at rest. But far away, the velocity is equal to V, the speed of the body. This is shown in this video again by Ludwig Prandtl (Video played 23:46 to 23:56). The flow is now steady.

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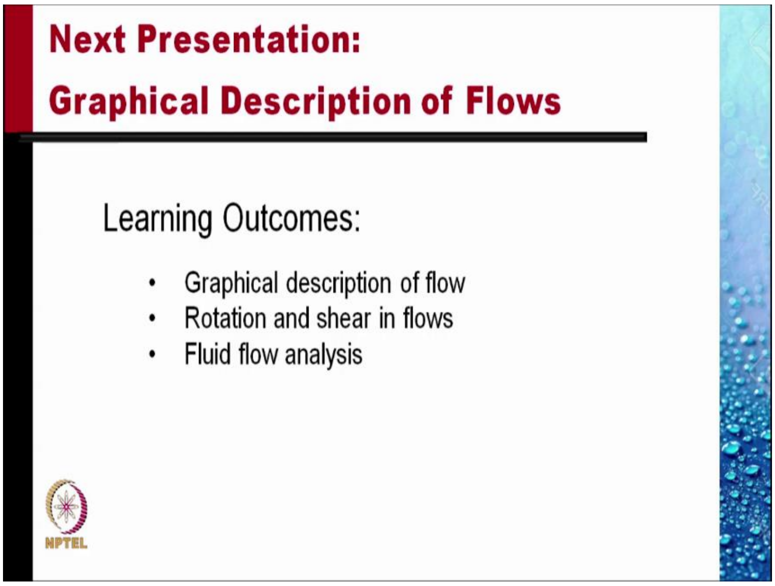

Thank you very much.