Fluid Mechanics and its Applications
Professor Vijay Gupta
Indian Institute of Technology, Delhi
Lecture 6

Welcome back. Today we will discuss the description of flows.

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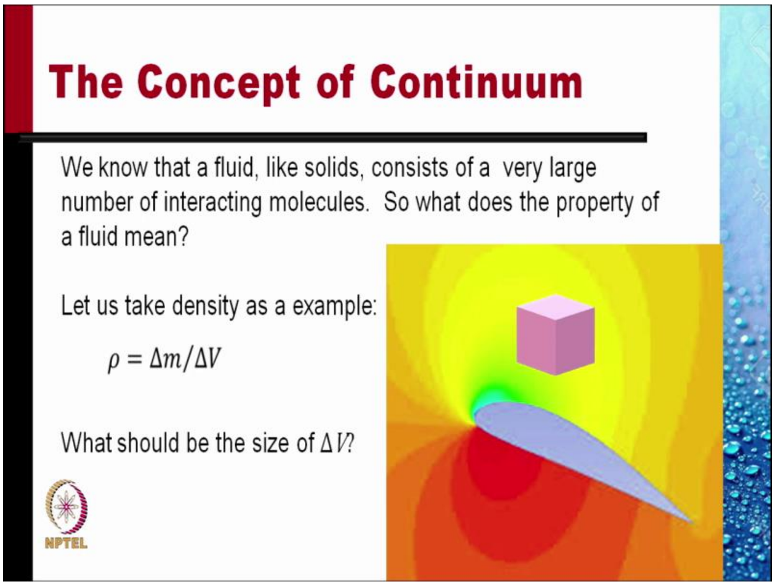

We know that a fluid like any solid consists of a very large number of interacting molecules. So, when we talk of the property of fluid, what does the property of fluid mean? Let us take example: density. To define density, we take a small volume ΔV, measure the mass of the matter contained within that volume, and the density is defined as ρ equal to $\Delta m/\Delta V$. What should the size of? Suppose, we have ΔV with dimensions approximately equal to the size of the body. The density of the matter around the body varies from point to point, from location to location.

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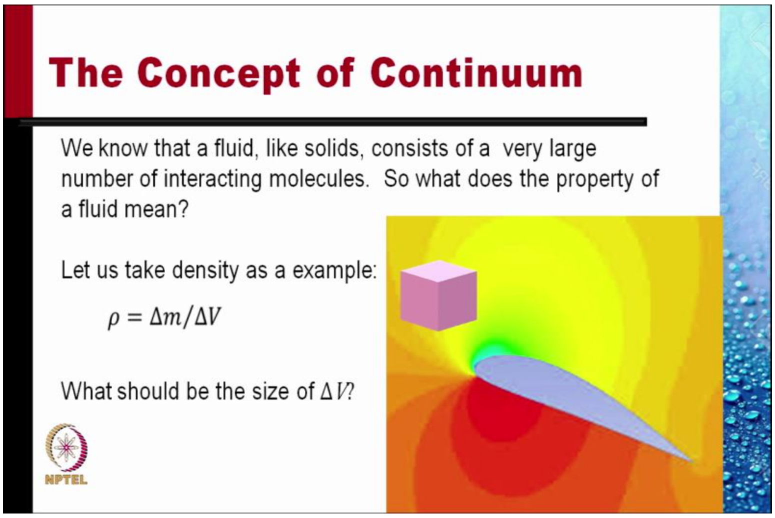

As we move this volume, the amount of matter contained within the volume would change. If the size of this volume is very large, this would smother the variations of the density over the flow field.

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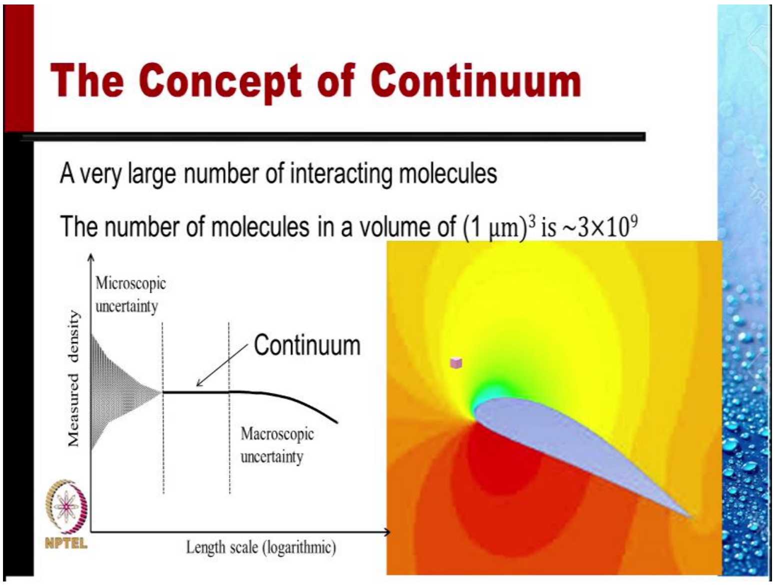

So, if we decrease the length scale of this volume, then the density measured by this volume, that is, the mass containing the volume divided by the volume, would change with the size. The narrower that we go, the more we are going to see the very density variation. After a certain limit, if we decrease the size further the density defined as $\Delta m/\Delta V$ would not change very much. There are very large number of interacting molecules in any gas, the number of molecules in a volume of 1μm cube, is of the order 3×10^9 for gases at ordinary temperatures and pressure.

This is a very large number of molecules. And so, the density defined by this formula would be approximately constant when the size of this volume is less than about 1 μm in length. If we decrease the size further, it is possible that we go into the limit where the size is so small that the number of molecules within this volume varies significantly. At some instant, they may be 20 molecules, at some other instant they be 25 molecules, at some other instance they be 15 molecules. So that, we have a microscopic uncertainty as to the density.

The region, or the size, or the length scale of this volume when the density does not change with the size appreciably is be region that can be termed as a continuum.

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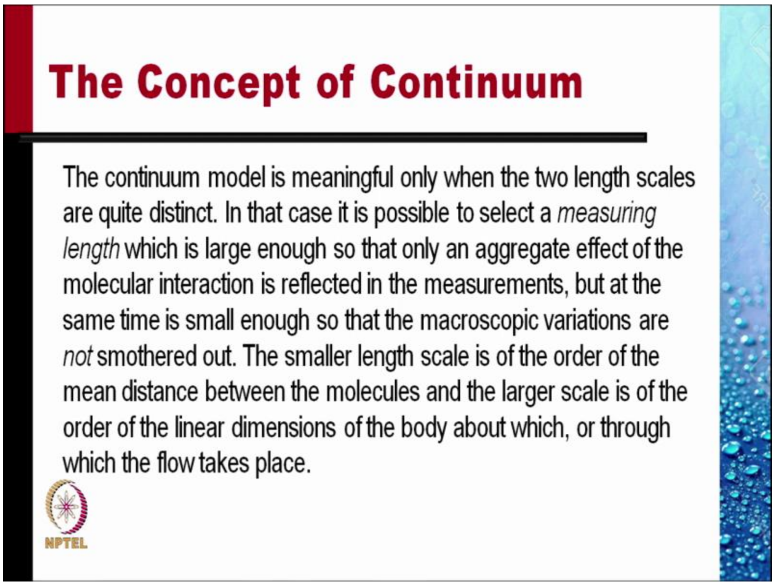

The continuum model is meaningful only when the two length scales are quite distinct. What two lengths? One, the length that smothers the variations, the length that is of the size of the body. And the other, the length which is of the size of the mean-free path of the molecules. So that, the number of molecules contained within the volume can change with time, or can change appreciably with time. In that case, when the continuum model is meaningful, it is possible to select a measuring length, which is large enough so that only an aggregate effect of the molecule interaction is reflected in the measurement. But at the same time, it is small enough so that the macroscopic variations are not smothered out.

The smaller length scale is of the order of the mean distance between the molecules. And the larger scale is of the order of the linear dimension of the body about which, or through which, the flow takes place.

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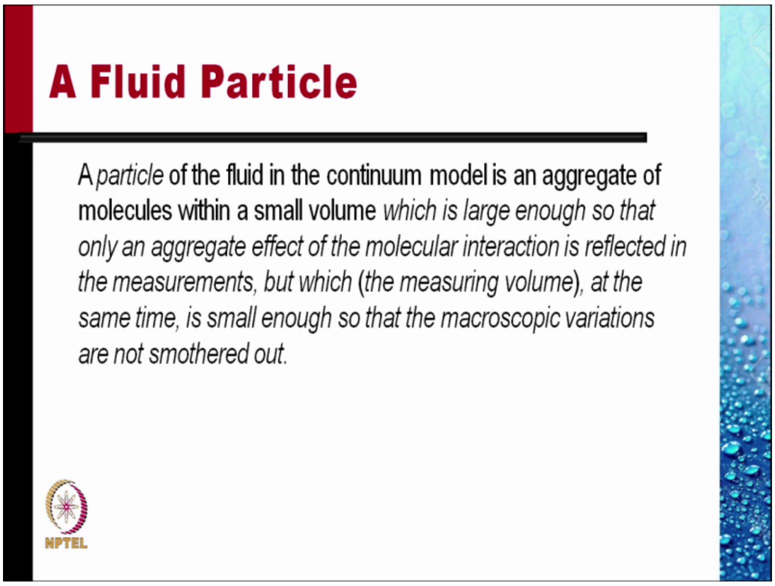

When a continuum model is possible, we can define a particle of the fluid. A particle of the fluid in the continuum model is an aggregate of the molecules, within a small volume, which is large enough so, that only the aggregate effect of the molecule interaction are reflected in the measurements, but which, that is the measuring volume, at the same time is small enough so that the macroscopic variation will not be smothered out.

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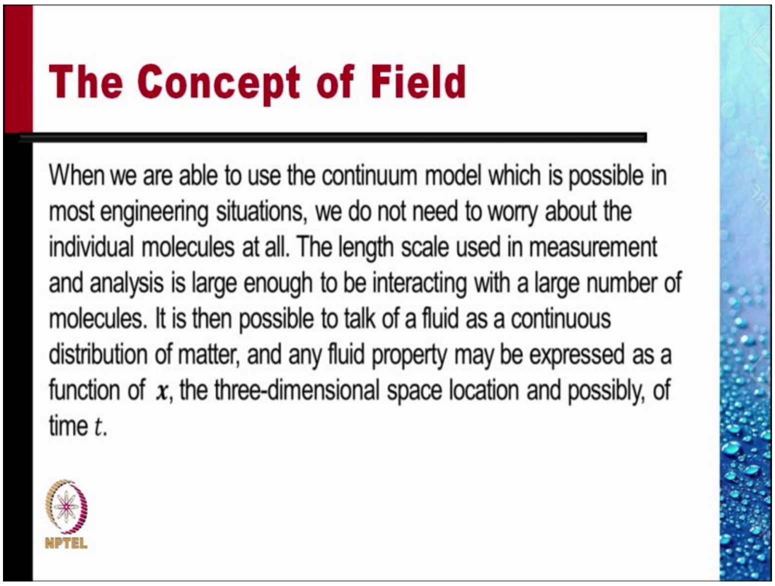

We are able to use a continuum model in most engineering situations, and when we use this, we do not need to worry about the individual molecules at all. The length scale used in the measurement and analysis are large enough to be interacting with a large number of molecules. It is then possible to talk of the fluid as a continuous distribution of matter. And any fluid property may be expressed as a function of x, the three-dimensional space location and possibly, of time t.

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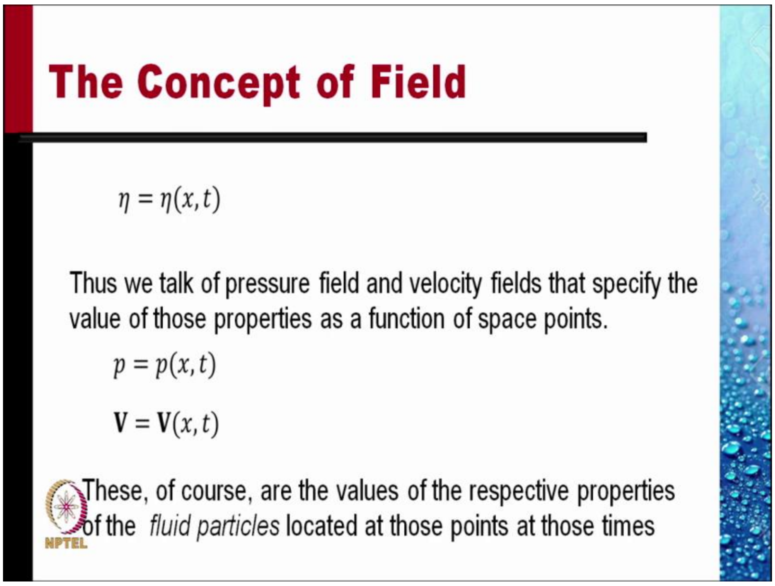

So, that any property eta can be written as function of x and t, where x is the location of that point. Thus, we talk of pressure field and the velocity field that specify the values of these properties as a function of space points and time. Velocity is a vector, so it is a vector field. Pressure is a scalar, so, it is a pressure field. These of course, are the values of the respective properties of the fluid particles located at those points at those times.

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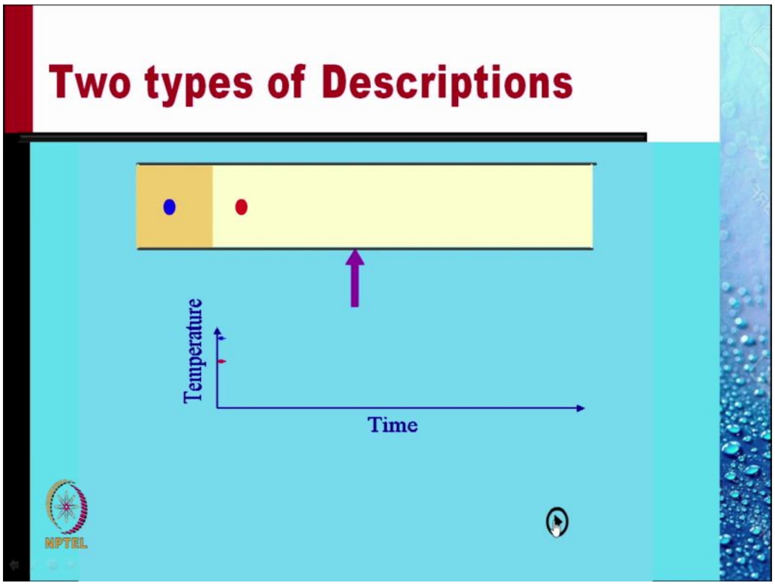

To understand the two types of descriptions that we use in fluid mechanics, let us look at this model. Here, there is a fluid, in which there are two different temperatures. The fluid with the lighter yellow colour has a lower temperature; the fluid with darker yellow colour is at a higher temperature. We have two floating temperature probes, a red probe and, a blue probe floating with the fluid. They move with the fluid. The fluid at the higher temperature, the darker coloured one, is pushing the lighter colour fluid to the right. There is an additional purple probe that is stationary, and it records the temperature.

So, let us see how these three probes see the temperature variations with time. (Video played 08:21 to 08:28)

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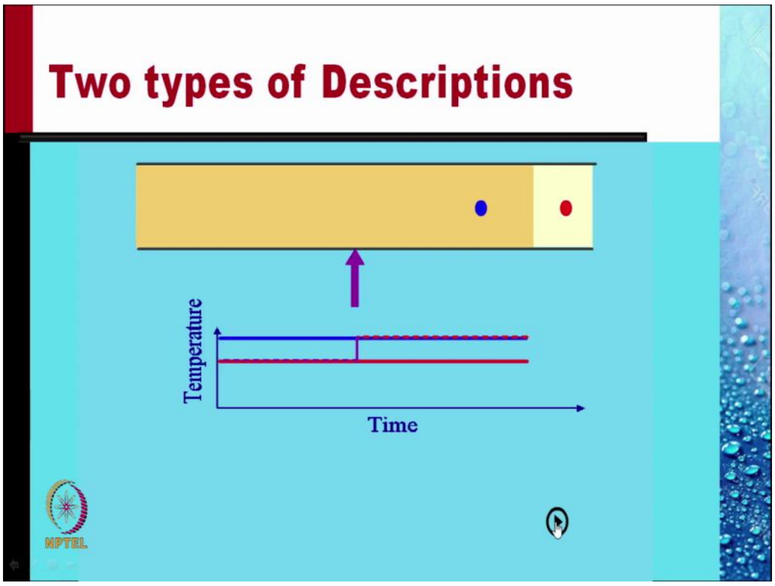

You will notice that the red probe is constantly measuring a constant temperature of the fluid with a lower temperature. The blue probe is constantly measuring the temperature of the hotter fluid, the darker yellow fluid. When there is no change of temperature recorded in the either of the two probes. But on the purple probe sitting in between, the temperature changes as the interface between the two fluids passes through that location. Let us look at it again. (Video played 09:21 to 09:29).

The yellow temperature, which is lower, and now it shifts up to the temperature of the darker yellow fluid. The red and the blue probes measure the temperature of the fluid as it flows with the fluids, but the purple probe, measure the temperature at a fixed location.

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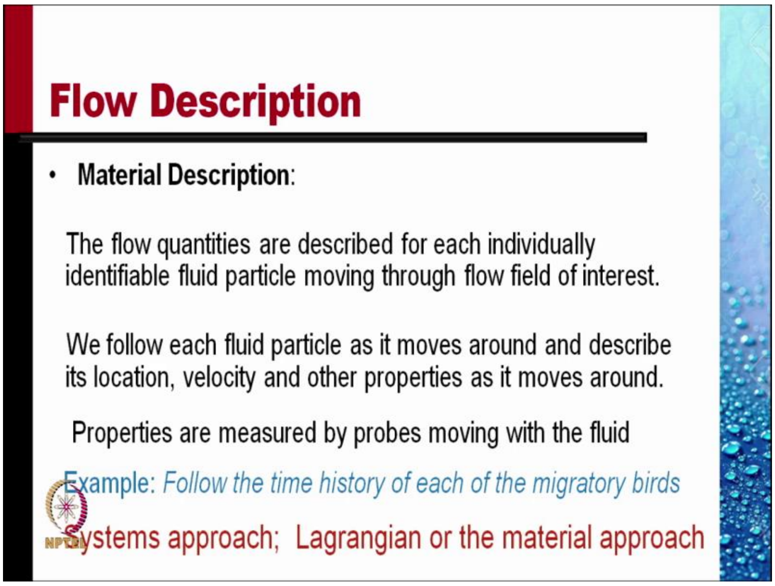

We have two kinds of flow descriptions as I said. One is what is called a material description. Here the flow quantities are described for each individually identifiable fluid particle moving through the flow field of interest. We follow each fluid particle as it moves around and describe its location, velocity, and other properties as it moves around. Properties are measured by the probes moving with the fluid. In the last example, the blue probe and the red probe for measuring the properties of material particles, because they were moving with the fluid, so, they were giving the temperature of the fluid particles in the immediate contact with the probes. And the probe was moving with the fluid. So, particles in contact with the probes were fixed. An example of this is, if we follow the time history of each of the migratory birds. This is each bird goes where to where and at what time? At what velocities are they flying? This would be the material description. When you study the motion of solid particles in solid mechanics, what you are usually doing is the material description. You follow each particle and worry about their locations, their velocities and accelerations with time. This material description is also known as a system approach or, the Lagrangian approach or, the material approach. These are equivalent names given by different writers.

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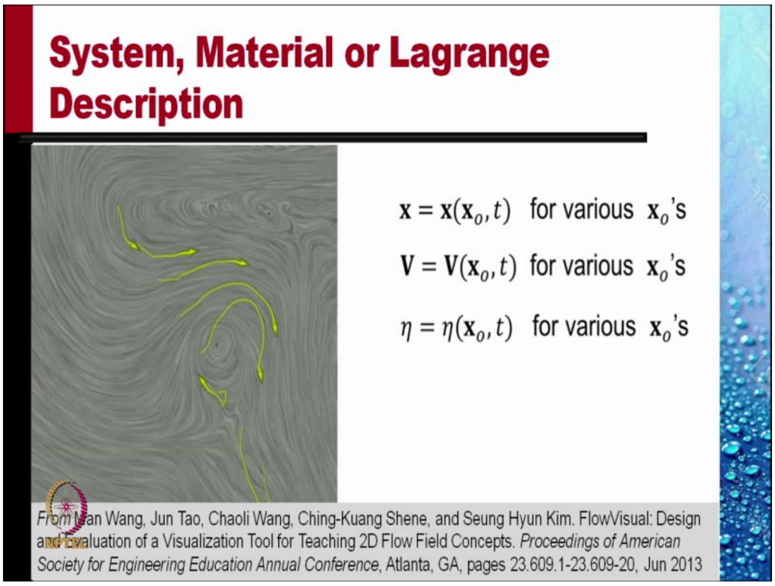

(Video played 12:06 to 13:23) In this flow, we are following each particle as it moves around. If we write the location x, the velocity V or any other property, say, pressure or density as a function of x_o, the location of the particle at time t equal to zero. This location x_o, that is the location of the particle at time t equal to zero, is used here as a label of the particle. So, each particle has the same value $x_o$at all times. It is a label, and different particles have different values of x_o's, their locations at time t equal to zero.

So, the location x at any given time, is a function of the original location at time t equal to zero, and the current time t.

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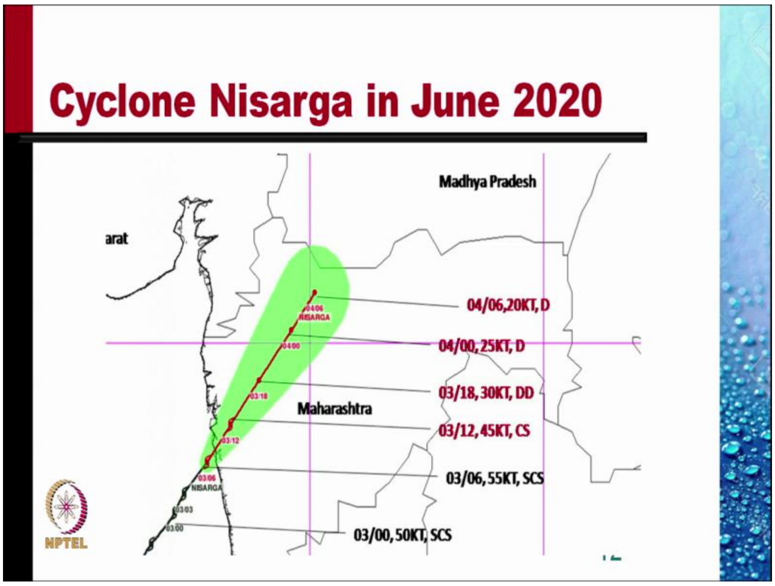

This map shows the movement of cyclone Nisarga in June 2020 on the west coast of India. It marks the location of the core of the cyclone on different times as it moves. This largely a material description, because the core of a cyclone consists essentially of the same matter.

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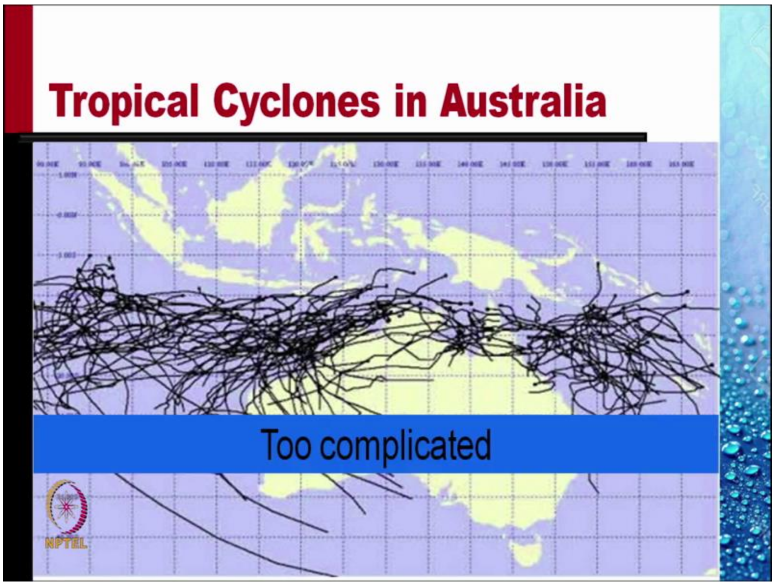

This is a little more complicated picture showing the tropical cyclones in Australia over a matter of a year. You see, describing this is getting to be too complicated. And so we like to use a different description of the material properties in most cases.

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(Video played 14:38 to 15:45) Luckily, we can do without it quite often. In this picture, we are showing a jet of water cutting a metal plate. If we are going to analyse this, we need to figure out the forces that the jet applies on the metal plate. It is a fixed location. We do not worry about each individual particle of water. We do not worry about what is happening to the particle over time. What we are worried about is: what is the force that this jet applies at a location on the solid boundary, the solid plate, where this jet cuts the plate? So, not only is a material description quite complicated, but we quite often do not need it.

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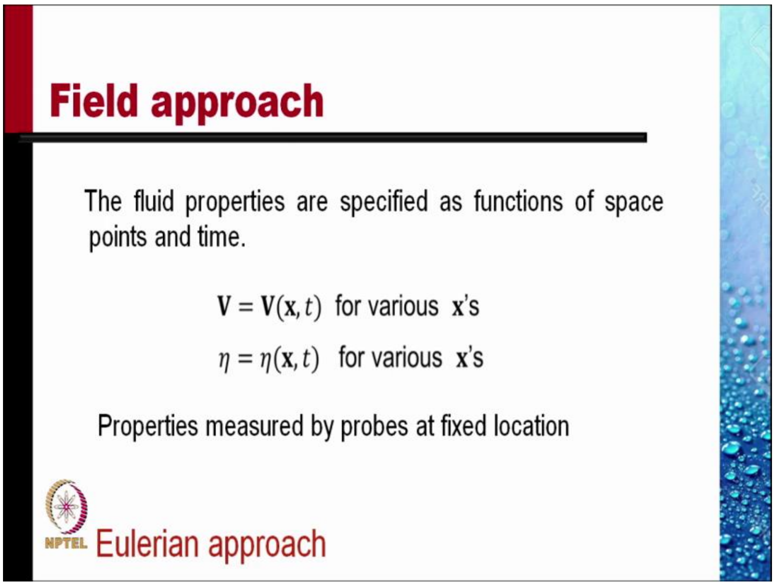

And therefore, we use an alternate approach, what we call a field approach. Here the fluid properties are specified as functions of space points and time. So, V is a function of x, the space point and time. It is the velocity of whatever particle is at that location at that time. We do not follow the particle. So, at a time $t + \Delta t$, this function would be specifying the velocity of the particle which is at the location x, and which most probably would be a different particle than the particle which was there a time t.

Similarly, η. η could be any other property, pressure, temperature, density, stresses. These properties are measured by probes at fixed locations. Because, we are expressing that as a function of space location x. This is also known as the Eulerian approach, after the scientists Euler.

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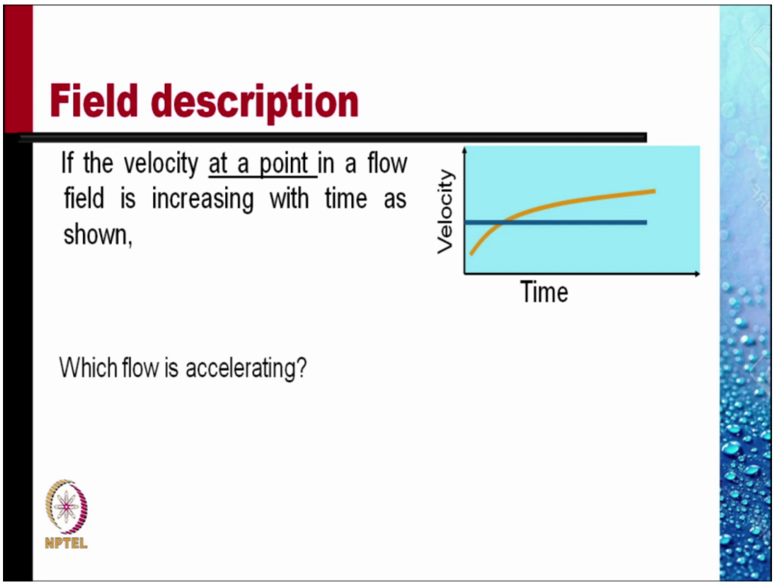

If the velocity at a point in the flow field is increasing with time as shown, velocity versus time. Is the flow accelerating? Most probably. If the velocity at a location is fixed with time, constant with time. Is the flow accelerating? It could be. It is not necessary that the fluid has a constant velocity. The fluid may still be accelerating, even though the velocity at a given location is constant with time. This needs further explanation.

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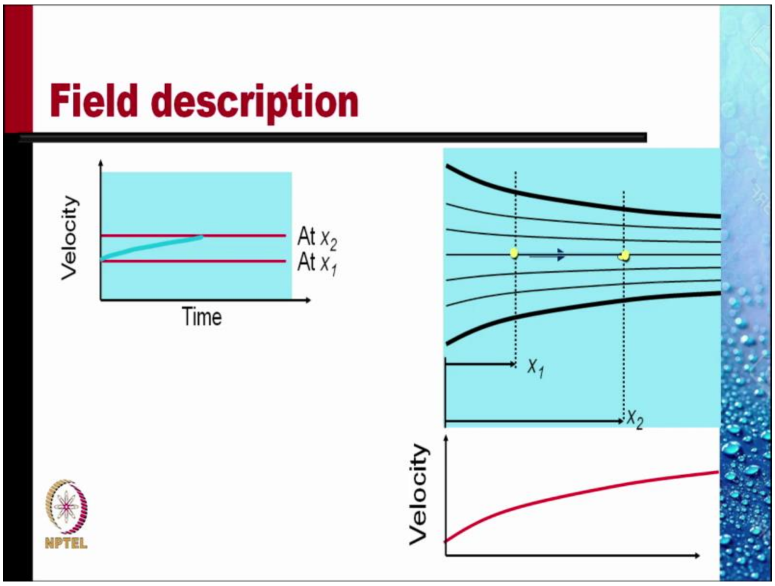

Consider a converging channel in which a fluid is flowing like this. At the left end, the area of the channel is larger than it is at the right end. And it is possible to have a flow where the mass flowing through the channel does not vary with time, and in that case, the velocity at the left end would be lower than the velocity at the right end, because the same mass now flows through a smaller area.

If I plot the velocity versus location x, then we get a curve like this, the velocity is increasing downstream. Now, I fix a probe at a location x_1. And if I measure the velocity at location x_1 with a time, then I get a constant velocity, independent of time. It does not change. But if I fix the probe at a location x_2, the velocity there is constant with time too, but this velocity is larger than the velocity at location x_1 at all times. If a particle starts at location *x*, and starts moving, then its velocity would increase.

See the picture on the left as the particle moves. It experiences an acceleration. So, though the field of velocity V is function only of x, not of time t, each particle in the flow is undergoing an acceleration, as it moves down the channel. We need to worry about this, because if the particle is experiencing acceleration, then there must be a force on that particle, which causes an acceleration: Newton's law of motion. So, if we see the velocity at a location is not changing with time, it does not mean the fluid is not accelerating.