Fluid Mechanics and its Applications
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Lecture 5A
Stability of Floating Bodies

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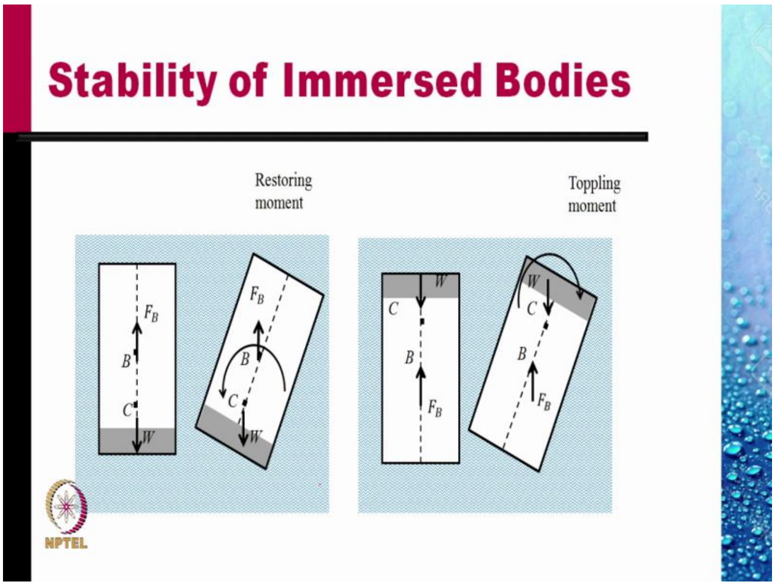

Let us next consider the stability of floating bodies. Let us consider a fully-immersed body and it is floating. So, the buoyancy force must be equal to the weight of the force. First, let us consider a geometry or an orientation in which the heavy part of this rectangular body is downwards. So, the center of gravity, C would be below the center of the body as shown.

And from this, the weight W would be acting, the buoyancy force F_B can be considered to a center buoyancy B, which is located at the center of the volume. In this case, the center of the rectangle. In equilibrium position, the two forces are equal and there is no moment on the body because the weight force and the buoyancy force act in the same line.

But if this body is disturbed slightly, so that it is tilting to the right, it is easy to see that the weight and the buoyancy force would now produce a couple, and this couple would have a restoring moment because this moment is counterclockwise, which will tend to upright the body. Consider the other case where the body is floating with a heavy end up. The center of gravity is now above the mid point. And if the body now tilts to the right as shown, the weight and the buoyancy form a clockwise couple, which will further upset the equilibrium and the body would topple over.

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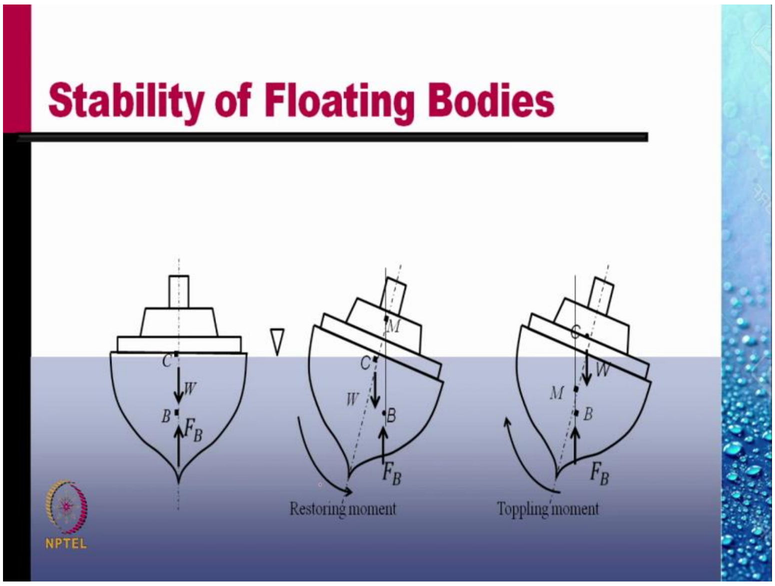

Let us consider this ship which is floating in equilibrium in an ocean. There is a weight that acts at the center of gravity C, and the buoyant force which acts as the center of buoyancy B. The center of buoyancy is at the centroid of the displaced water, that is, below the waterline. The volume of the water, the volume of the ship below the waterline. If the ship tilts to the right, then the volume of the water towards the right is more. And the volume of the water displaced is less towards the left, and so, the center of buoyancy shifts to the right. Now, we see that the buoyant force F_B acting at the center of buoyancy and the weight W acting at the centroid for a couple which is counterclockwise, which will tend to restore the ship back to vertical. This is a stable orientation, the ship.

However, we can imagine a situation where if the center of gravity is too high. Then the center of buoyancy even if it shifts to the right, but the center of gravity shifts to the right more. And the two forces, the buoyant force and the weight force, form a clockwise couple which is a toppling moment, because it will topple the ship and the ship might turn turtle.

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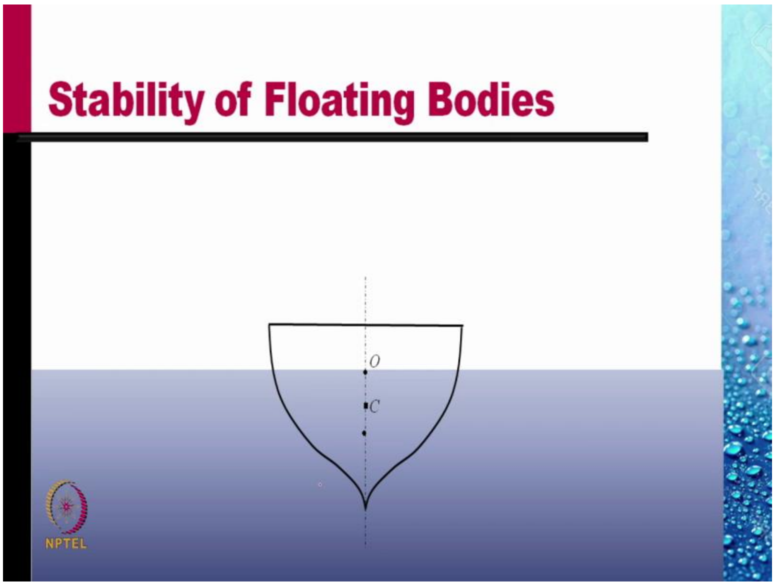

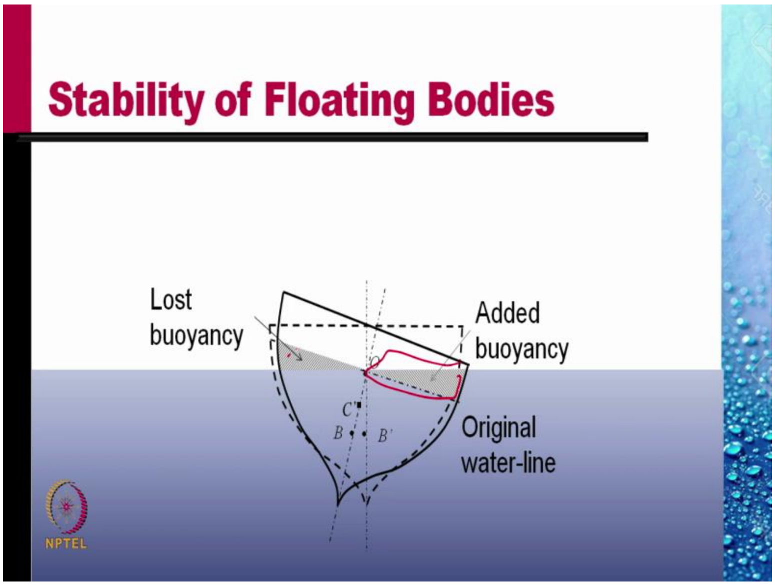

To analyze this, let us consider this simple shape of the ship where O is at the waterline, C is the centroid. Now, if the ship tilts, B is the original position of the center of buoyancy. *B'* is the new position of the center of buoyancy, which is towards the right of *B*, because there is more water on the right, which is displaced, and less on the left. So, there is added buoyancy on the right and lost buoyancy on the left. Because of this, the center of buoyancy shifts towards the right.

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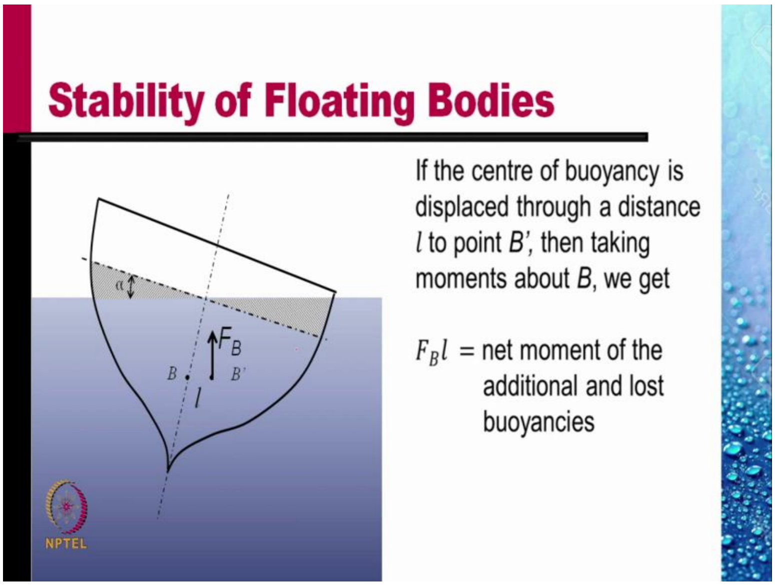

Let us take a larger view of this. Let us assume the ship tilts through angle α. *B'*is the new location of center of buoyancy. *B* is the old center of buoyancy, Let us determine *BB'*. This is *BB'* and let us denote it by *l*, the force F_B, now acts at the new center of buoyancy. If the center of buoyancy is displaced to a distance *l* to point *B'*, then taking the moment about *B*, we get F_B into *l* should be net moment of the additional and lost buoyancies, added because of the additional buoyancy and moment lost because of lost buoyancy.

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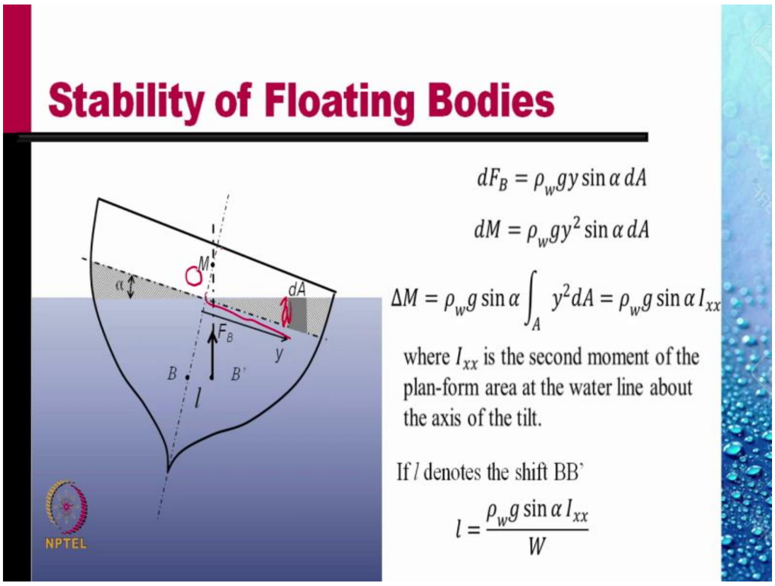

So, we consider area dA of a strip at the waterline of the ship at a distance y from the center line along the deck of the ship. The additional buoyancy dF_B because of this is $\rho_w g y \sin\alpha\, dA$ is this height. This is y. So, this is $y \sin\alpha$. $y \sin\alpha . dA$ is the volume of the grey area, the darker grey area. So, this is the additional buoyancy. And the moment it creates about the point O, this point O, is multiply this buoyancy force by y. So, dM is $\rho_w g y^2 \sin\alpha\, dA$.

So, the change in moment produced is, you integrate this over the whole area of the waterline. This additional moment is now found out to be $\rho_w g \sin\alpha\, I_{xx}$, where I_{xx} is the second moment of the plan-form area at the waterline about the axis of the tilt that is perpendicular to the screen at point O. This additional moment must be equal to *l* times the force of buoyancy, which is equal to the weight of the ship.

So, this is *l* which is a distance between *B* and *B'*, the shift in B, is $\rho_w g \sin\alpha\, I_{xx}/W$, the weight of the ship. If we draw a line vertically through *B*, it will intersect the centerline of the ship which passes through *B* and *O* at point *M*. This point *M* is termed as the meta center of the ship.

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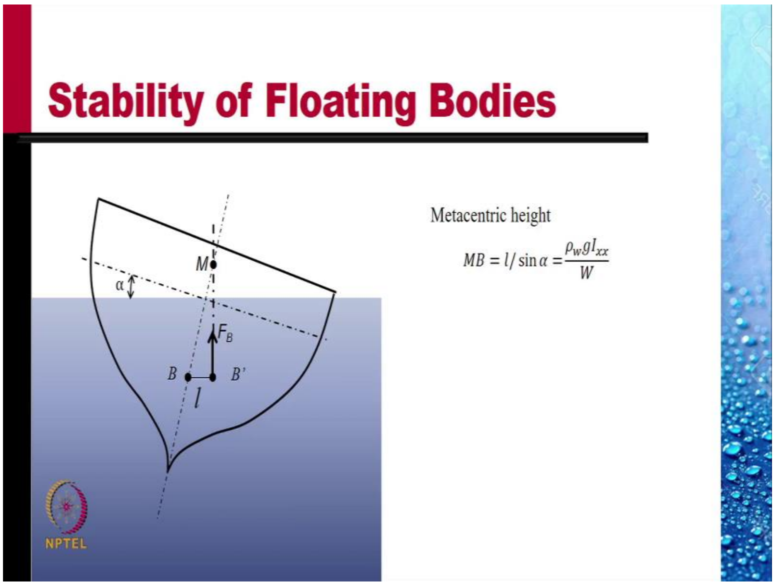

So, this point *M*, the meta center of the ship. The distance *BM* plays a very important role. It is called the metacentric height. The distance between the center of buoyancy *B* of the ship in the normal orientation and this point *M*. Clearly *BM* distance, $l/\sin\alpha$, and so, it is $\rho_w g I_{xx}/W$. This ship would be stable if the center of gravity lies within the segment BM, the lower the center of gravity, the stable, the more stable is the ship.

If the center of gravity lies above point *M*, then the ship is unstable. So, while loading a ship the captain and crew of the ship ensure that this CG of the ship is low enough, well below the meta center. The metacentric height *MB'* is the property of the ship geometry and the weight of the ship that plays a very important role in plying of the ships.

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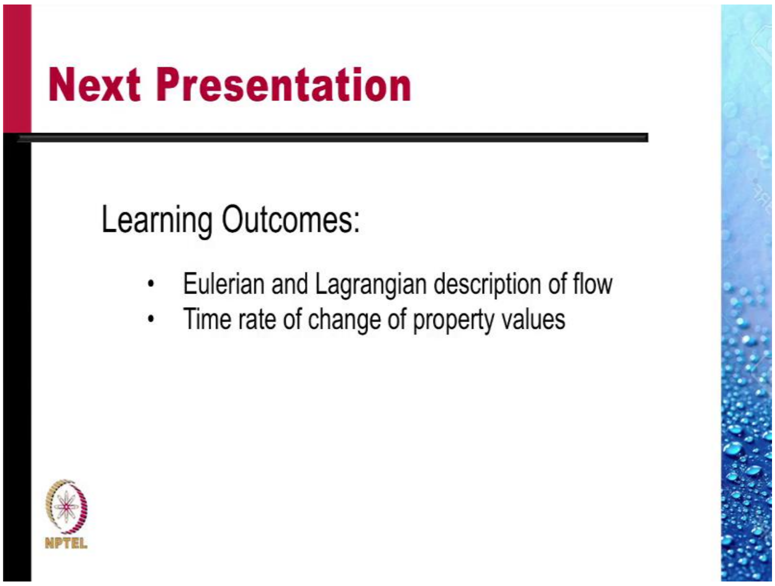

Thank you very much.