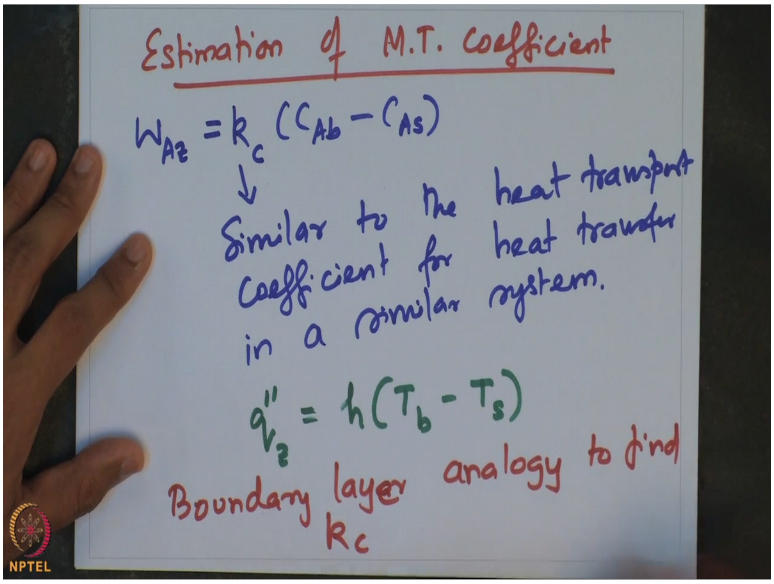
Chemical Reaction Engineering - II
Prof. Ganesh A Viswanathan
Department of Chemical Engineering
Indian Institute of Technology - Bombay

Module - 6
Lecture - 26
Estimation of Mass Transfer Coefficient

In the last lecture we defined mass transport coefficient if there is no reaction. And we looked at how to write mole balance in the thin film that may be present right at the surface of the catalyst pellet, when the bulk fluid is flowing past the catalyst pellet. We assume that the film is extremely small, extremely thin film, much smaller than the diameter of the catalyst pellet.

And therefore, we actually assumed that the film is actually planar in nature. We neglected the curvature. And in this lecture, we will look at how to find out what is the mass transport coefficient, given the hydrodynamic conditions.

(Refer Slide Time: 01:22)



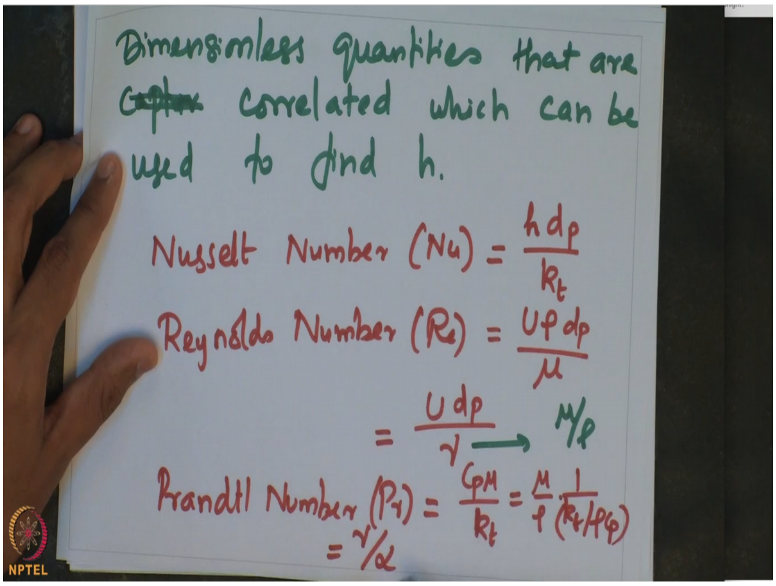
So, we particularly want to understand how to estimate this quantity called k c, which is the mass transport coefficient; which is, which basically, if the mass transport coefficient is known, we can find out what is the flux with which the species actually is diffusing or reaching the surface of the catalyst pellet. Now, this mass transport coefficient is similar to the heat transport coefficient, for heat transfer problems.

So, let us now contrast and see what is the heat transport coefficient in a heat transport problem. So, the flux of heat transport in a specific location, let us say for the same system through the film, would essentially be some heat transport coefficient, multiplied by the temperature at bulk, – the temperature of the surface.

So, if I now compare the flux relationship with the concentration gradient and the heat transport flux with the temperature gradient, then I can see that the mass transport coefficient that appears here is very similar to the heat transport coefficient, where the heat transfer is actually occurring across the same boundary layer through which the mass transport is occurring. So, from the heat transport class, we know that there is a boundary layer analogy.

One can capitalise on boundary layer analogy to find the mass transport coefficient, to find the mass transport coefficient k c. Now, if we know how to find heat transport coefficient, then we, there are correlations which are present to estimate heat transport coefficient. And the correlations are very similar for mass transport coefficient because of the boundary layer analogy that is present in the boundary layer for heat and mass transport, the boundary layer analogy for heat and mass transport in the boundary layer.

(Refer Slide Time: 03:59)



So, there are dimensionless quantities that are correlated, which can be used to find the heat transport coefficient h. Now, what are these dimensionless quantities? The typical dimensionless quantities are what is called as the Nusselt number. The typical symbol used is Nu. And what is Nusselt number? It is essentially the heat transport coefficient h, multiplied by the particle diameter, divided by the conductivity of the fluid.

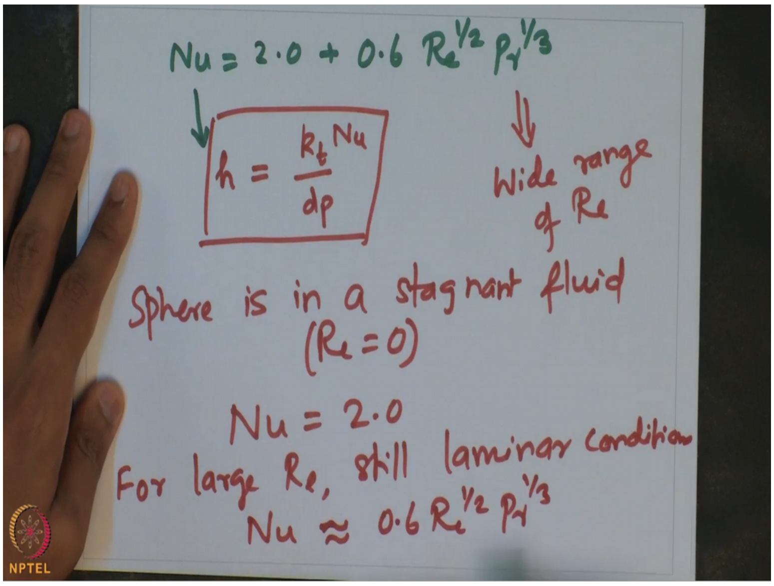
So, if we know Nusselt number for a boundary layer problem, we actually can estimate what is the heat transport coefficient for that particular problem. And then, another dimensionless number is the Reynolds number, which essentially characterises the flow of the fluid past the object, which is given by U which is the velocity and density of the fluid multiplied by the particle diameter, divided by the viscosity of the fluid.

One can rewrite this as U into d p divided by, what is called as a kinematic viscosity, which is essentially given by; this is essentially given by mu by rho. So, one can write Reynolds number in terms of U rho d p by mu or U d p nu, which is the kinematic viscosity; both these are same. And there is the third number, which is basically the Prandtl number. What is Prandtl number? Prandtl number is given by C p mu by k, where C p is the capacity of the fluid, mu is the viscosity and k t is the conductivity of the fluid.

Now, one can multiply and divide by density and we can rewrite this as mu by rho into 1 by k t by rho C p. What is k t by rho C p? k t by rho C p is essentially a thermal diffusivity. And mu by rho is the kinematic viscosity. So therefore, we can now rewrite Prandtl number as nu by alpha. So, Nusselt number is h d p by k. Reynolds number is U d p by nu and Prandtl number is essentially nu by alpha, which is the, alpha is the thermal diffusivity of the fluid.

Now, how are these 3 correlated? For a spherical particle where fluid is actually flowing past a sphere, the correlation that connects these 3 dimensionless quantities is essentially given by;

(Refer Slide Time: 06:53)



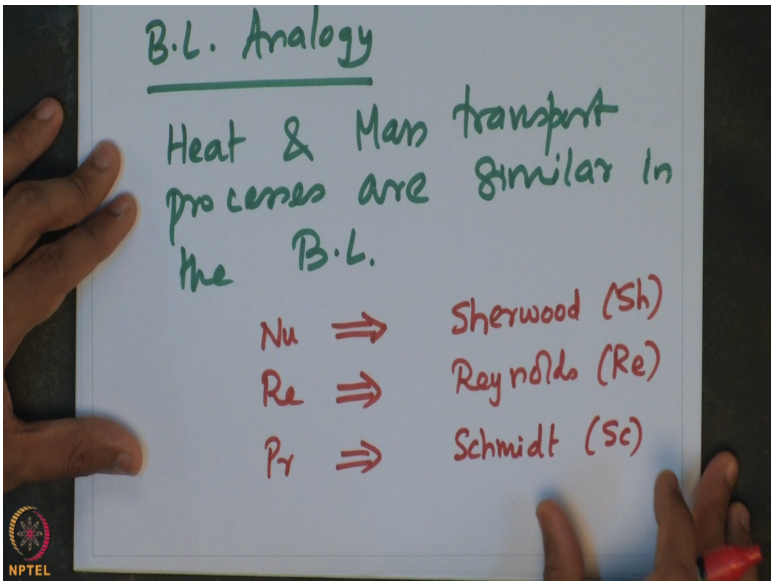
Nusselt number is = 2.0 + 0.6 into Reynolds number to the power of half into Prandtl number to the power of 1 by 3. So, if we know Reynolds number, which essentially uses, which essentially characterises the flow properties. So, if we know the flow properties, then we can actually estimate the Reynolds number. And if we know the physical properties of the fluid that is being used for flow past the sphere, we can estimate the Prandtl number.

So, once we know the Prandtl number, then from this correlation, we can actually estimate what is the Nusselt number, for that particular system where the fluid is actually flowing past a sphere. So, once we know Nusselt number; from Nusselt number we can find out that h is essentially given by the conductivity of the fluid multiplied by the Nusselt number divided by the particle diameter.

So, once we know the Nusselt number, we can actually find out what is the heat transport coefficient. Now, this correlation is actually valid for wide range of Reynolds number. If the sphere is actually in a stagnant fluid, that is, Reynolds number is = 0, then Nusselt number is essentially given by 2, where the second term is actually vanishes, because Reynolds number is 0.

And for a system where the fluid is actually flowing at a reasonable velocity but laminar, so for large R e but still laminar conditions, the Nusselt number is essentially given by approximately given by 0.6 into R e to the power of half, into Prandtl to the power of 1 by 3. This is because there is, Reynolds number is sufficiently large that this 2 is actually much smaller than the second term. So, Nusselt number can essentially be approximated as 0.6 into Reynolds number to the power of half into Prandtl number to the power 1 by 3.

(Refer Slide Time: 09:10)

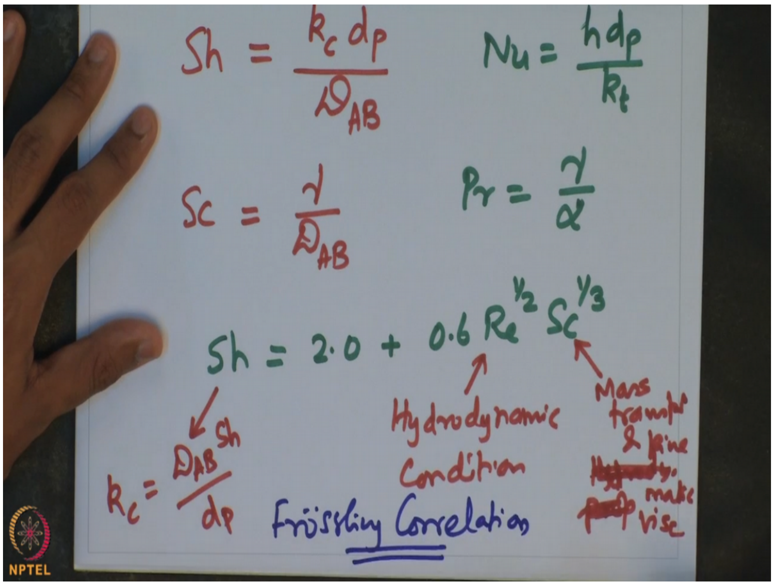


Now, we know that from boundary layer analogy, the heat and mass transport processes are similar in the boundary layer. So, which means that there must be a equivalent dimensionless quantity for each of the 3 dimensionless numbers that we have seen for heat transport. Because the processes are similar, there must be characteristically similar dimensionless quantities. So, let us look at what these quantities are.

So, Nusselt number in heat transport, the corresponding number in mass transport is what is called as a Sherwood number. And Reynolds number is essentially characterising the hydrodynamic conditions. And that is going to be same as what is used for mass transport as well. In heat transport Reynolds number characterises the hydrodynamics. And the same hydrodynamic properties would also characterise mass transport.

So, it is the same dimensionless quantities that is used. And Prandtl number which essentially captures the thermal diffusivity of the fluid which is, through which the heat transport is occurring. In a similar fashion, there will be Schmidt number, which essentially characterises the diffusion of species through the fluid. So, let us look at what is Sherwood number and what is Schmidt number.

(Refer Slide Time: 10:52)



So, Sherwood number Sh is essentially given by mass transport coefficient k c multiplied by the particle diameter d p divided by the equimolar counter diffusivity D A B. Contrast this with the Nusselt number, where you have heat transport coefficient h multiplied by d p divided by the diffusivity of the species. So clearly, you can see that there is a similarity between the, these 2 numbers, where this captures heat transport coefficient.

And here you have mass transport coefficient. And here you have conductivity, which is a signature of the thermal diffusion. And here you have mass diffusivity, which is a, which is capturing the diffusivity of the species in the fluid. Let us next look at what is this Schmidt number. Schmidt number is essentially given by nu divided by D A B. And contrast that with the Prandtl number which is nu divided by alpha, which is thermal diffusivity.

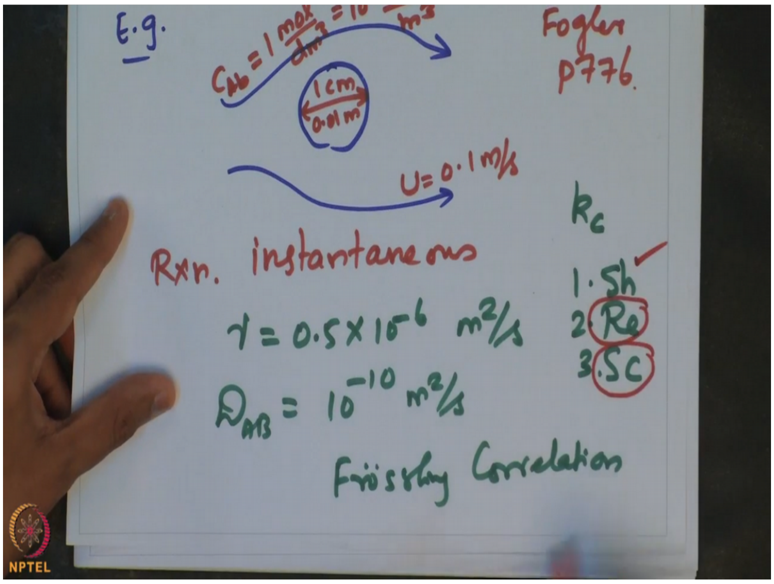
So, in place of thermal diffusivity, kinematic viscosity essentially captures the flow properties. And mu alfa is essentially capturing the thermal diffusion properties. So now we can, it is the same hydrodynamic conditions that could be used for mass transport as well. So, we essentially have to replace alpha which is the thermal diffusion, with the corresponding equimolar counter diffusivity.

Now, because of this boundary layer analogy and because of the similarity in these dimensionless numbers, we can actually see that the correlation that is used for heat transport is actually, can be directly ported to mass transport as well. And therefore, the Sherwood number can be related as 2 + 0.6 into Reynolds number to the power of half into Schmidt number to the power of 1 by 3.

So, one can actually use this correlation. If we know Reynolds number, which essentially characterises the hydrodynamic conditions and this characterises the mass transport and the kinematic viscosity, which actually has the mass diffusion and kinematic viscosity, then we can actually use these numbers to find out what is the Sherwood number. So, if we know Sherwood number, we can find out that the mass transport coefficient is essentially given by diffusivity into Sherwood number divided by the particle diameter.

So therefore, from boundary layer analogy we should be able to estimate what is the mass transport coefficient for a given hydrodynamic condition, once the properties of the system are actually known. Now, this correlation is what is classically called as Frossling correlation. And we will be using this Frossling correlation extensively in the mass transport coefficient calculations, for all these problems. Now, let us look at how these numbers look like. Let us take a specific example of mass transport without reaction.

(Refer Slide Time: 14:08)



Let us take a specific example problem, where a fluid is actually flowing past a sphere, where the fluid is actually flowing past a sphere. And let us say that the diameter of the particle is about 0.1, 0.01 metres or essentially 1 centimetre. And this problem has actually been taken from the textbook Fogler, page number 776. So, suppose let us say that the reaction is instantaneous.

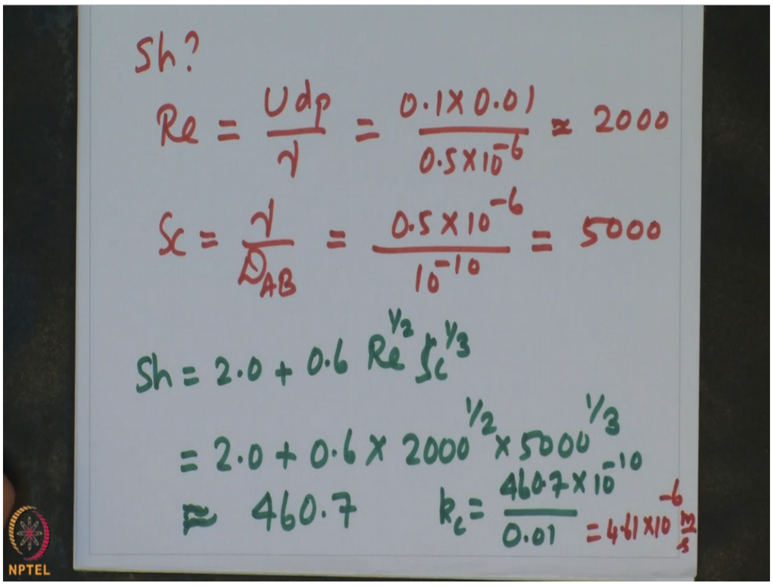
Let us assume that the reaction is actually instantaneous. And if the concentration of the species in bulk is = 1 mole per decimetre cube, which is essentially 10 power 3 moles per metre cube. And if you assume that the fluid velocity is essentially 0.1 meters per second.

And the properties of the fluid essentially are; if you say that the property of the fluids are, the kinematic viscosity is 0.5 into 10 power – 6 metre square per second.

And the equimolar counter diffusivity of the species is essentially = 10 to the power of – 10 metre square per second. So, we need to find out what is the mass transport coefficient k c. How do we find this? We know that there is a correlation relating Sherwood number which captures the mass transport coefficient and the Reynolds number and Schmidt number. So, let us use the Frossling correlation.

So, in order to use the Frossling correlation, what are the quantities we need to estimate? We need to find Sherwood number, we need to find Reynolds number and we need to find Schmidt number. So, these are the 3 quantities we need to estimate in order to use the correlation. So, let us estimate the Sherwood number first.

(Refer Slide Time: 16:10)

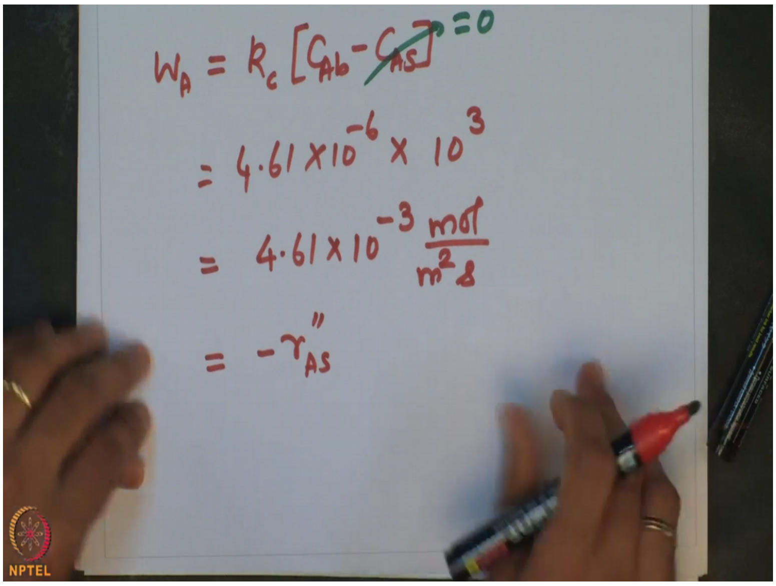


So, Sherwood number; Sorry. We need to find out what Sherwood number is. So, we need to essentially estimate what Reynolds number is and what Schmidt number is and we need to find out what is the Sherwood number. So, let us try to find out what is Reynolds number. Reynolds number is given by U times d p divided by nu. Which is = 0.1 into 0.01 divided by 0.5 into 10 power – 6, which is = 2000. Approximately = 2000.

And then Schmidt number is = nu by D A B, which is = 0.5 into 10 power – 6 divided by 10 to the power of – 10. Which is essentially = 5000. So, we can now plug this into the Frossling correlation. So, Sherwood number is = 2 + 0.6 into Reynolds number to the power of half, into Schmidt number to the power of 1 by 3. So, that is = 2 + 0.6 into 2000 to the power of half into 5000 to the power of 1 by 3. So, from here, we will see that Sherwood number is = 460.7.

And we can actually estimate the mass transport coefficient which is 460.7 into particle diameter is 0.01 divided by 0.01 into 10 power – 10. And that is essentially = we estimate this is = 4.61 into 10 to the power of – 6 metres per second. So, that is the mass transport coefficient for the problem for the system that is actually specified in this problem. So now, from the mass transport coefficient, we can now estimate what is the flux with which the species is actually diffusing through the boundary layer.

(Refer Slide Time: 18:20)



So, the flux essentially is given by mass transport coefficient multiplied by the concentration difference. But we assume that the reaction is actually an instantaneous reaction. So, what happens if reaction is instantaneous? As soon as the species comes to the surface of the catalyst where the active material may be present, then the reaction will instantaneously happen, which means that all the species which is available for the reaction at the surface would actually undergo complete conversion.

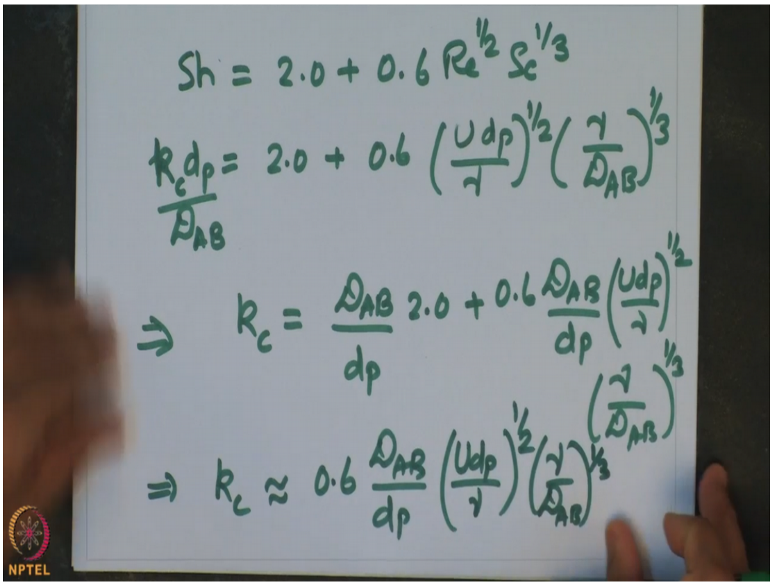
So, which essentially translates to the fact that the concentration C A S is essentially = 0. So, the surface concentration will be 0, which means that there is no species that will be present right at the surface, at any point in time. Because a reaction is actually an instantaneous reaction, where all species present are available at any moment would actually be consumed completely.

So therefore, from here we know what k c is. k c is 4.61 into 10 to the power of – 6 multiplied by, the concentration is 10 to the power of 3 moles per metre cube. And from here we can see that the flux essentially is = 4.61 into 10 power – 3 moles per metre square second. Now in fact, by definition of the rate at which the reaction happens per unit surface area, which is essentially the moles of the species that is consumed per unit surface area of the catalyst per time, is essentially = this same quantity which is here.

So, this is nothing but – r A S defined per, this is the rate that is actually, rate at which the species is consumed per unit surface area of the catalyst. So therefore, the flux with which the species diffuses to the catalyst surface is = the rate at which the species reacts, is = the rate at which the species actually is reacting per unit surface area of the catalyst. And this is for the instantaneous reaction.

So now, next, let us look at how do we improve the mass transport coefficient? So, is it possible to improve the mass transport coefficient? So, suppose there is no reaction, then how do we improve the mass transport coefficient? The Sherwood number correlation, the Frossling correlation actually gives us a an idea of how to actually improve the mass transport coefficient.

(Refer Slide Time: 20:46)



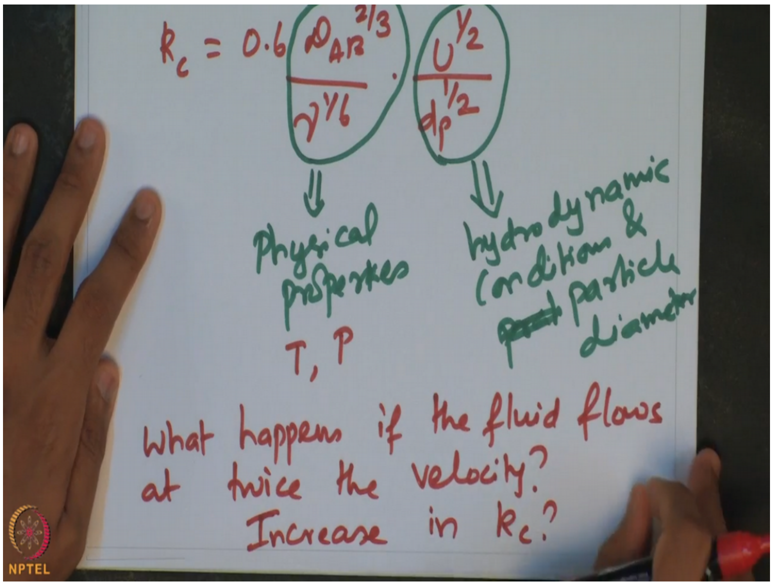
So, let us look at the Sherwood number, Frossling correlation of 1 by 3. So, from here if we substitute the expressions for the Reynolds number and Schmidt number, we will find that this is = 2 + 0.6 into U d p by nu to the power of half into Schmidt number which is nu by D

A B to the power of 1 by 3. And this is nothing but h, this is nothing but mass transport coefficient k c into particle diameter divided by D A B.

So, from here we can write that k c is = D A B by d p into 2 + 0.6 into D A B by d p into U d p by nu to the power of half into nu by D A B to the power of 1 by 3. So, if I estimate the mass transport coefficient and if the Reynolds number is reasonably large. For this system we can clearly see that the Reynolds number is of the order of 2000. So, the Reynolds number is, if we see for this problem, we will see that the Reynolds number is of the order of 2000 which is actually sufficiently a large value.

And so, if the Reynolds number is sufficiently large, than k c can be approximately written as 0.6 into D A B by d p multiplied by U d p by nu to the power of half into nu by D A B to the power of 1 by 3. So, we can now rewrite this slightly in a different fashion. So, if we now regroup the terms that is present here, we can rewrite this expression as:

(Refer Slide Time: 22:51)



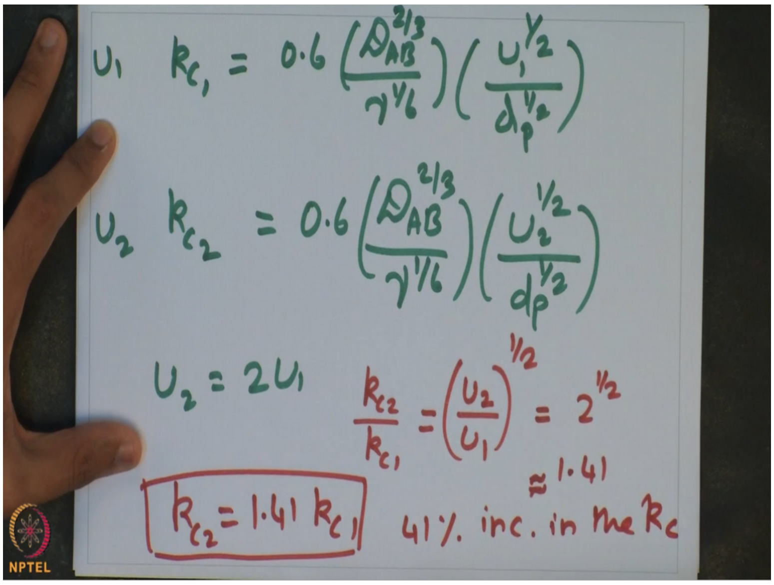
k c is = 0.6 into D A B to the power of, D A B to the power of 2 by 3 divided by nu to the power of 1 by 6 multiplied by U to the power of half divided by d p to the power of half. So, what is interesting in this expression is that this first term here D A B to the power of 2 by 3 by 1 by 6. This essentially captures the physical properties of the species and the fluid that is actually carrying the species, the diffusivity of the species and the kinematic viscosity.

And if I now look at this second term here, this essentially captures the hydrodynamic conditions and the particle diameter. So now, I can actually ask a question. How should I change the velocity in order to change my mass transport coefficient? So, I can specifically ask even a better question where; Can I do something to this term, to physical properties in order to change the mass transport coefficient?

And; Can I do something to the hydrodynamic conditions to change the mass transport coefficient? Now, what do these physical properties depend upon? These depend primarily on the temperature and pressure. So, if I operate at different temperatures, let us say I increase the temperature with which the, at which the reaction is being conducted, at which the diffusion of the species is happening, then I can essentially increase my diffusivity of the species.

And thereby, I can actually increase the mass transport coefficient. But I, much easier way to do this would actually be to change the hydrodynamic conditions. Because changing hydrodynamic conditions would actually require pumping the fluid at a higher flow rate, which results in a higher velocity, that is different value of U. So, let us look at what happens if the fluid flows at twice the velocity. What is the enrichment, what is the increase in the mass transport coefficient that can be achieved? So, we can answer this question. So, suppose let us say we define k c 1.

(Refer Slide Time: 25:43)



Let us say we define k c 1 as the mass transport coefficient, at let us say velocity U 1. And so, that is essentially given by 0.6 into D A B to the power of 2 by 3 divided by nu to the power of 1 by 6 multiplied by U to the power, U 1 to the power of half, into d p to the power of half.

So, if I now just increase the velocity by keeping all other the properties constant, such as the diffusivity or the kinematic viscosity or the particle diameter.

Let us say that I calculate what is the mass transport coefficient at another velocity let us say U 2. Which is given by 0.6 into D A B to the power of 2 by 3 divided by nu to the power of 1 by 6 into U 2 to the power of half, divided by d p to the power of half. Suppose, let us say that I am actually I am actually finding out what is the mass transport coefficient at a different velocity.

Let us say that U 2 is = 2 times U 1. Let us say that U 2 is = 2 times U 1. Now, I can find out that k c 2 by k c 1. If I take the ratio of these 2 expressions here, this is essentially = U 2 by U 1 to the power of half. Because, all other terms are same, I conduct the reaction at the same size of the catalyst pellet. The only change that I do is I increase the speed with which the fluid is actually flowing. And actually, double the speed.

So, this is nothing but 2 to the power of half. And that is nothing but approximately = 1.41. So, what you essentially get is that k c 2 is = 1.41 times k c 1, which means that there is a 41% increase in the mass transport coefficient. So, by doubling the velocity with which the fluid is flowing, one can actually achieve a 41% increase in the mass transport coefficient, provided the Reynolds number is sufficiently large.

But, even if it is not sufficiently large, such a, what is the nature of increase in the mass transport coefficient can actually be estimated even if the Reynolds number is very large by using the complete Frossling correlation. So, what we have seen in today's lecture is, we essentially looked at how to estimate mass transport coefficient. And we essentially observed that we can capitalise on the heat and mass transport process analogy that is present in the boundary layer.

Because these 2 processes actually occur in a similar fashion. And so, the correlation which is available for estimating the heat transport coefficient can actually be capitalised and can be used in a similar fashion for estimating the mass transport coefficient. And we particularly looked at Frossling correlation which is meant for the estimating the mass transport coefficient for fluid flowing past a sphere.

And we also observed that if we use this correlation, we can see that there is an increase in the mass transport coefficient that can be achieved by changing the hydrodynamic conditions. That is the velocity with which the fluid is actually flowing. Thank you.