Heat Transfer
Prof. Ganesh Viswanathan
Department of Chemical Engineering
Indian Institute of Technology, Bombay

Lecture – 03
Energy Balance

Alright, so we looked at we stopped in the last lecture by observing that there are ways to quantify the conduction process. So, we are going to go deep into that today and look at what are the ways to quantify and look at certain examples and certain types where we can look at the quantification process and try to understand what is the temperature distribution.

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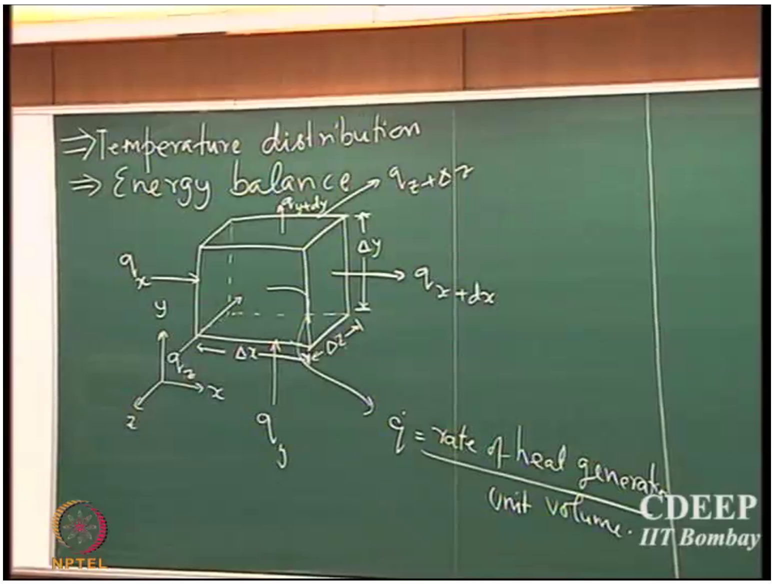

So, we stop by observing that we need to find the temperature distribution. So, in order to quantify the system we need to find the temperature distribution, so if we know the distribution obviously we will be able to find out the gradient that is the d t by d x the temperature gradient at any location in a system and if we know that we know the transport rate, we know the flux and we multiplied by the transport area we will know the heat transfer rate.

So, we need to write energy balance, so which means that we need to write a energy balance. So, let us take a general differential element; so x, y. q z what comes out is q z plus to delta z, q x, q y, so those are the rate at which heat is entering different surfaces of the element and we could add a we can say that q dot is the rate of heat generation per unit volume. So, in principle something could be happening inside for example; there could be electrical heating which leads to generation of heat inside the system that you are considering. So q dot could be the volume at rate of heat generation that is the amount of heat that is generated per unit volume of that system.

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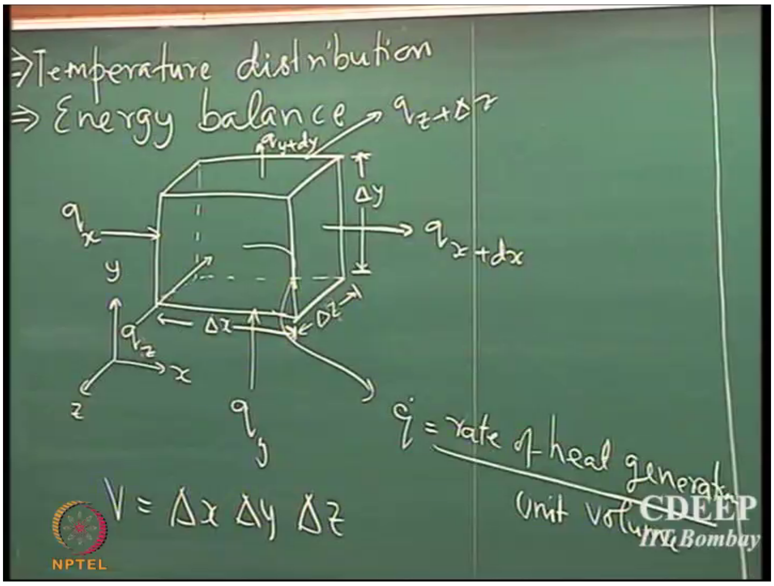

So, the volume of this small element v is nothing delta x multiplied by delta y multiplied by delta z so we know that. So, let us write a simple balance, so the general thumb rule for writing any balance whether it is heat or mass balance is that we say input to that element minus output plus generation term that should be equal to the amount that is accumulated that will be accumulation. So, you could call this I, we call this O, we could call this G and you could call this A. So, this is the manthra of all the balances that you would be writing in this course and perhaps in many other courses till you finish your B .Tech and in fact this is the manthra in almost all the balances that you would write in any engineering discipline irrespective of whether it is chemical mechanical etcetera.

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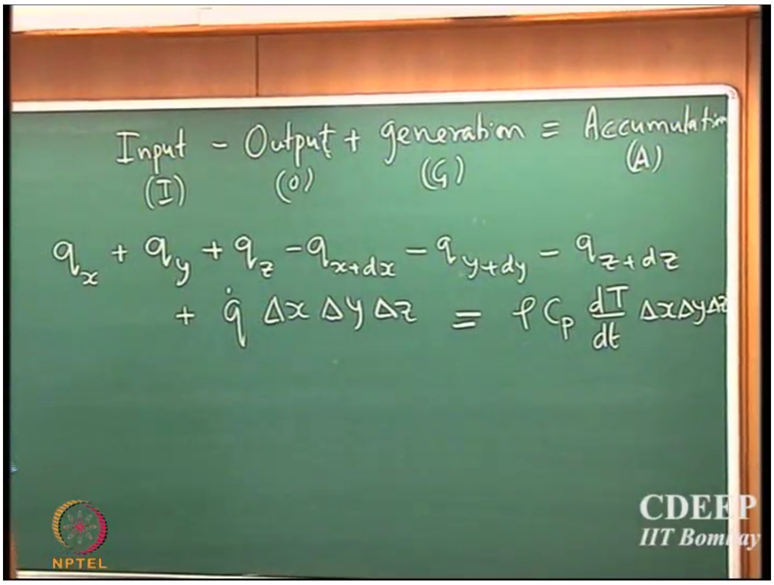

So, this is the general manthra and we would not go dwell more into this. So, we will use this manthra here today and then we are going to write a general balance for conservation of energy for this system and we will use that balance and try to simplify for different kinds of systems that we are going to look into in the future alright. So, what is the input to the system the amount of heat that is input is q x right, plus q y, plus q z. So, that is the total amount of heat rate at which heat is being input to that system that we are considering and what leads is q x plus d x, q y plus d y, minus q z plus d z that is the amount that is leading the element that we have considered plus whatever is being generated. So, you have to be very careful.

So, when you have a heat loss term, when you have a sink term or heat loss term, then you have to use a negative sign. So, you have to very careful about the sign convention that you use for any balance that you generally know. So, that could be q dot, so this is defined as rate of heat generation per unit volume, so therefore you have to multiply by the volume of the system that you are considering. And that is equal to the accumulation which is given by rho C p which is the density of the material that you are using multiplied by the specific heat capacity into heat temperature gradient with respect to time multiplied by the volume. So, that is the heat balance. It is very simple input what goes out we subtract that and whatever is generated you add to the system that should be equal to whatever is being accumulated.

So, this term tells you what is the amount of heat that is actually being stored, what is the amount of energy that is being stored by the system because of the heat transport process.

So, this tells you what is the capacity of the system that you are looking at and C p is the parameter intrinsic parameter which quantifies that capability of that system has to how much heat that it can store.

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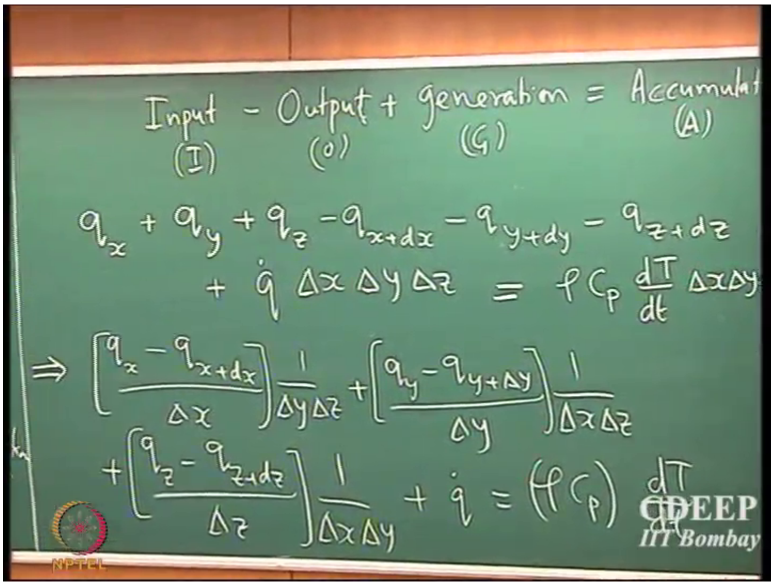

So, now we can simply divide these equations by the volume and so it will be, for the units matching here.

Student: (Refer Time: 06:57).

C p is for unit volume, are they units matching? Look at the units I want you to pay attention to the units, the units matching for all the terms here? You have to convince yourself when you are actually in the class everything that is happening is right is it right? No. What is wrong?

Student: (Refer Time: 07:24).

Which one?

Student: (Refer Time: 07:26).

Where this one here?

Student: (Refer Time: 07:29).

Yeah, why is that a problem here?

Student: (Refer Time: 07:34).

But it is already taken right watts per meter cube, it is the unit of q dot we multiplied by volume it becomes watts.

Student: (Refer Time: 07:46).

What is the units of q x? it is watts it is not flux it is transfer rate is rate of heat transfer, keep in mind the convention that we use, you should always remember this when I say q it is rate when I say q single prime it is rate per unit distance and when I say q double prime it is p flux that is the rate of heat transfer per transport area alright. So, now I divide equation by so q x plus d x divided by delta x into 1 by.

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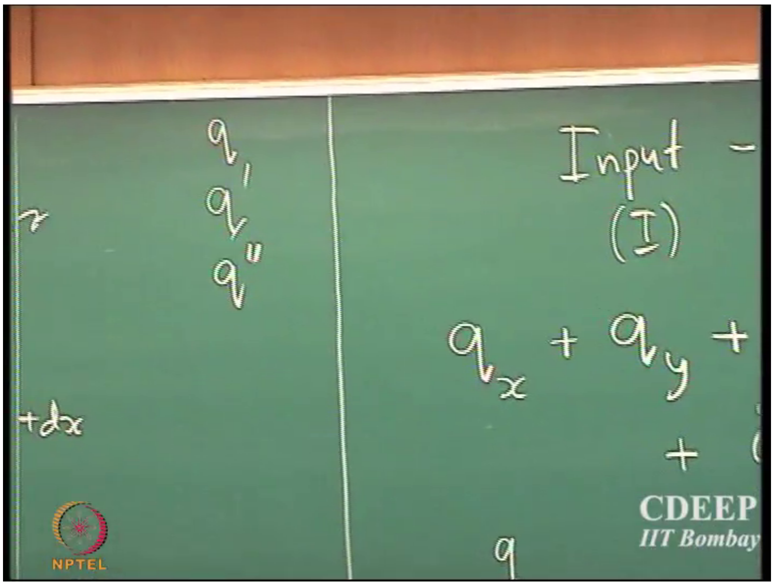

Student: (Refer Time: 08:29).

Excuse me, joule per kilogram kelvin and rho is k g per meter cube. So, that is why you have multiply by volume plus q y, minus q y plus delta y divided by into 1 by. So moment we formulate it in this form all I have done is I have just divided the equation by the volume and I have just collected the terms which corresponds to x y and z together. So, the important thing to observe here is that this delta y delta z which appears as a coefficient for the rate terms in x direction that is nothing, but the area of heat transport in the x direction. So, that tells you what is the cross sectional area at which the heat transport is occurring in the x direction, that is basically this plane here you see this plane here delta x it is the plane in the z and the y direction. Similarly delta x delta z is the cross sectional area of heat transport in the y direction and similarly for the z direction is that clear to everyone alright. So, we can rewrite this as; so when I say that limit delta x goes to 0, delta y goes to 0 and delta z goes to 0. So, we can write this as d q by d q by d x, 1 by delta y delta z plus it is a minus sign minus d q by d y delta z o t. So, with this can I get the temperature distribution yes or no? Can I get the temperature distribution with this?

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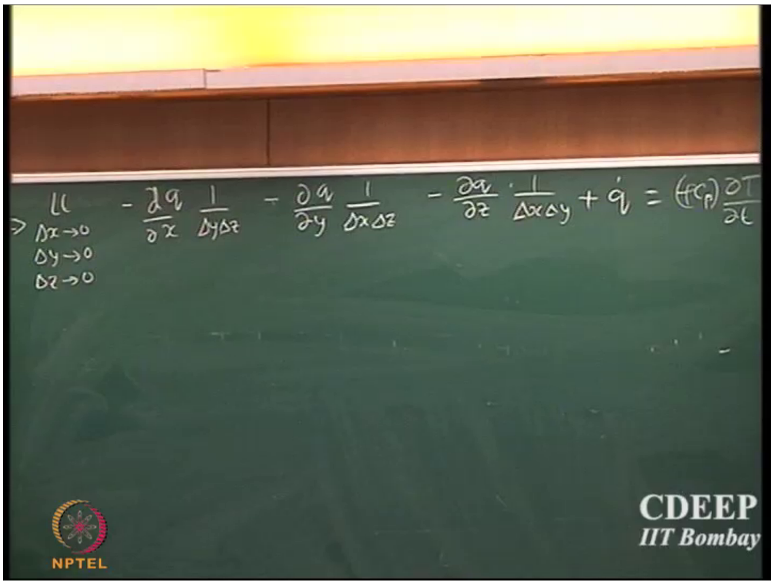

So, if we know what? Yes?

Student: (Refer Time: 11:31).

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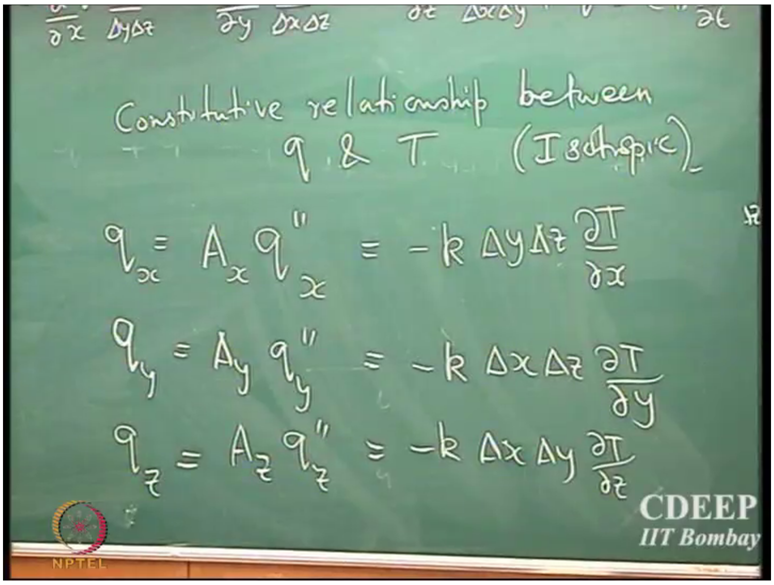

Is some constitutive relationship so we do not know what q is, so if there is a constitutive relationship if you have not heard this word called constitutive relationship, if we know the constitutive relationship between q and temperature we are done. So, if we know the constitutive relationship between the transfer rate and the temperature, then we are done we have completely described the temperature distribution we do not know the distribution because we have to solve the equations, but we have described it completely.

And this is what is given by Fourier's law that we saw in the last lecture, so q is given by the area of heat transport in whichever direction you are considering supposing if it is x, it is the area of heat transport in x direction multiplied by the corresponding flux in the x direction right and so this is given by minus k so supposing I assume the system is isotropic if I assume that it is a isotropic system, there is no reason that you should not assume it but let us say that for simplicity we assume that it is a isotropic system, although all the framework does not change whether the system is isotropic or non isotropic but let us say for simplicity purposes we assume that it is isotropic.

So, there will be k into what s the area of heat transport in x direction? delta y delta z. So, we observe that so that is the coefficient that comes out in your energy balance into d t by d x, so that is the representation of Fourier's law in the x direction. Similarly we can write for y minus k, y and q z is k z, so we can substitute all those into the model equation now, so this is what is called the energy balance or model equation that is what is called the model equation.

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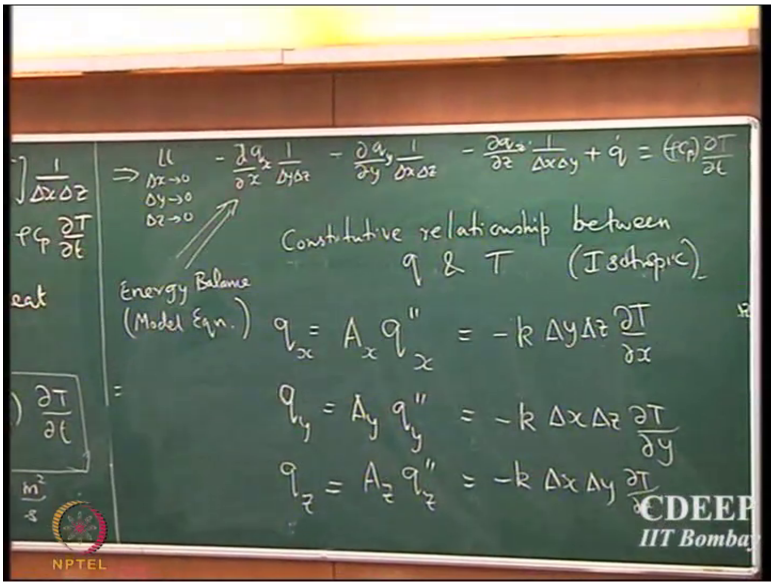

So, we can substitute these fluxes from Fourier's law into the model equation; d by d x into k into delta y delta z, plus d by d y, k into plus q dot equal to rho C p into tau T by. So, supposing if the area does not change with the cross section area is constant, supposing for a system in case of Cartesian coordinates we will see that the cross sectional area of heat transport remains constant.

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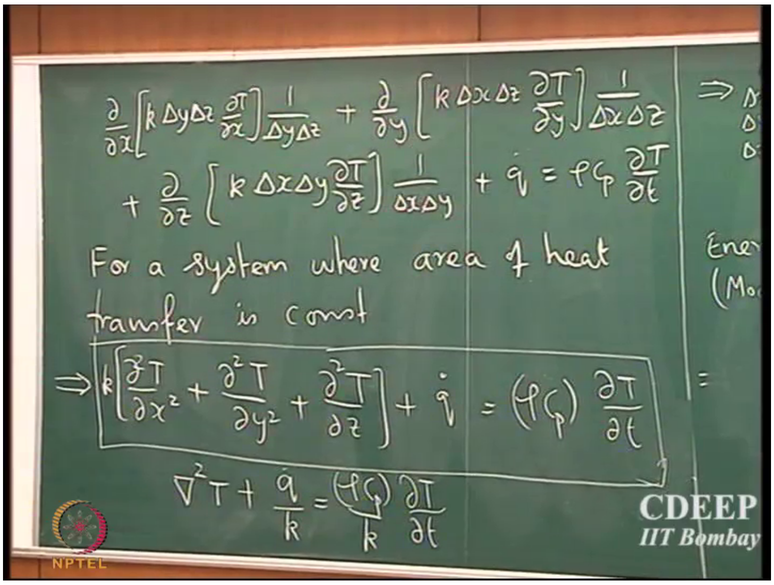

So, if we assume that the for a system, where area of heat transport is constant we will see in future we will see few examples of area is not constant and how these model equations will change, but let us say to start with we assume that the area of heat transfer is constant, which means we can plot delta x delta y etcetera from the derivatives inside and so this will simply become k into d square T by d x square, plus d square T d y square, plus d square T by d z square, plus q dot equal to rho C p rho T, so this is the energy balance. What is this term in the bracket called as classically? Called the Laplacian, so one could rewrite this expression as del square T, plus q dot divided by k equal to rho C p divided by k into, so this is the a cute way of writing the energy balance. What is this term does anyone know?

Student: (Refer Time: 17:18).

Yeah.

Student: (Refer Time: 17:21).

(Refer Slide Time: 17:23) number is non dimension, it is a dimensionless number this has a dimension, rho is k g per meter cube, C p is joule per kilogram kelvin and k is watt per meter kelvin, so what are the units of this quantity.

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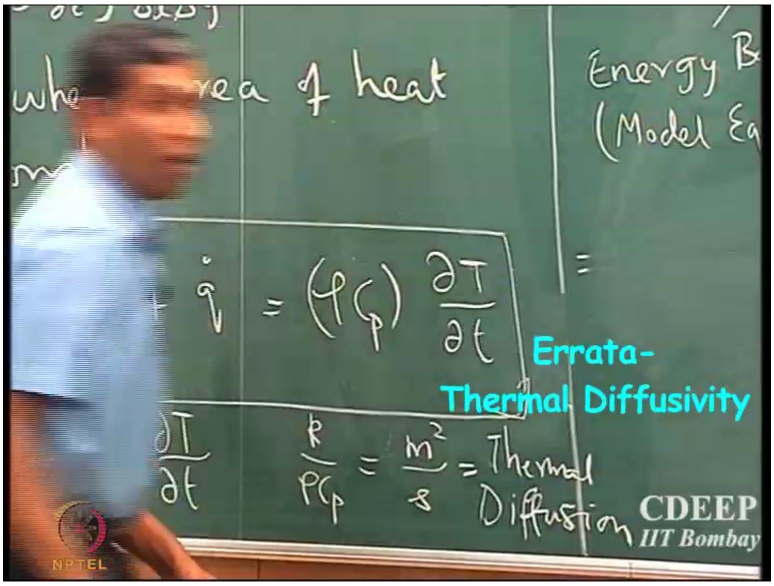

So, the units are it is k by rho C p the unit is meter square per second yes?

Student: (Refer Time: 17:52).

Because in principle, the system need not be isotropic right, so if it is not isotropic then you would expect that this k's will be different and of course the gradients can obviously be different. So, therefore you have to distinguish the fluxes in all three directions and the transfer rate in all three directions.

Student: (Refer Time: 18:14).

It is oh it should be q x, q y sorry thanks for pointing that out please correct your notes which should be q x, q y it is a (Refer Time: 18:56). So, q x, q y and q z are the transfer rates in the corresponding direction, alright so the units of this k by rho C p is meter square per second, can you guess what it is? What this quantity is from the units?

Student: (Refer Time: 18:53).

It is diffusivity. So, it is called; so this quantity is called thermal diffusivity. So, what it signifies is k is the thermal conductivity and rho C p is the capacity of the system right.

So, this is the ratio between the ability of the system to conduct it versus it is ability to store the energy and so rho C p remember what I told you a few moments ago, rho C p is characterizes it is in the intrinsic property that quantifies the ability of the system to store heat within itself, store energy within itself. And k is the intrinsic property of the system which characterizes the ability of the system to transfer heat from one location to the other via conduction. So, this is the ratio between the ability of the system to conduct heat versus the ability of the system to store heat within itself and that is what is called thermal diffusion. Yes?

Student: divided by (Refer Time: 20:05).

So, this is the so what happens; so there are two processes which are occurring simultaneously, one is the energy is being transferred because there is collisions between the electrons or there is lattice waves which is happening and so there is transfer of heat from one electron or one molecule to the other molecule. Now when the temperature is increased the energy state of that molecule also is going to be increased right. So, the rho C p it signifies the ability of the molecules to store heat because of higher temperature and the interaction between them is the one which is going to signify the ability to transport the heat. So, this ratio is basically the thermal diffusion and the reason why it is ratio is thermal diffusion is? Supposing if the material has a very capability to store heat then the amount of heat, that it can transfer because of the interaction is going to be very small.

So, it is a competition between the ability to transfer heat because of the interaction versus the ability of that system to store the heat in the same location with the same molecule and that is why it is called diffusion, in fact that is the definition of diffusion. Any other questions ok? So, is it enough to just write the balance, have we completely described can we solve this equation now? Is it possible to solve? What do you need this is a partial differential equation what makes a p d e complete?

Student: (Refer Time: 21:49).

You need boundary conditions right. So, let us now discuss for the next few moments as to what are the different types of boundary conditions? What are the general classes of boundary condition?

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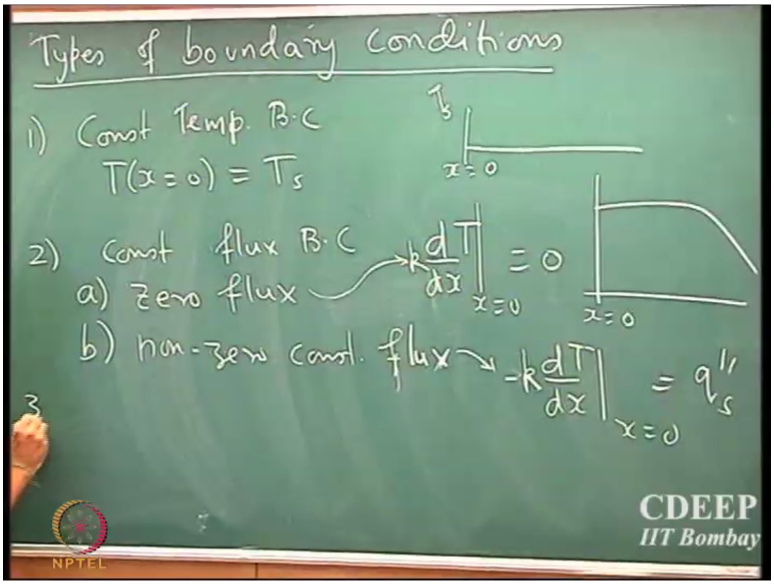

So, there are three basic classes of boundary condition or three types of boundary condition; one is the constant temperature boundary condition.

So, for example; if you take a system ok, so let us say this is x equal to 0, one possible boundary condition is you could fix the temperature of that particular boundary, we can say that the temperature is fixed at some constant. So, typically I use subscript s for surface so it could be surface temperature. So, we can say that the temperature at x equal to 0 is maintained at a certain constitutive temperature T x, so that is one type of boundary condition.

So, there are several examples of this; so supposing if you want to let us say you have you have water geyser right so we have geysers at our residence in our bathroom where we get hot water right, so the temperature of the surface so you see there will be a heating coil, so the basic geyser works is there is a heating which is electrically heated and there is a thermostat which maintains a certain temperature of the heating coil. So, note that it is not the temperature of the water that it maintains it is actually the temperature of the heating coil that it maintains.

So, if you are trying to write a model of this system to find the temperature distribution of water then one of the boundaries to which the coil is exposed to which the water is exposed to a coil will actually be maintained at a certain constant temperature. So, it is a constant boundary condition, constant temperature boundary condition is what you should use at that time. Another type of boundary condition is called the; constant flux boundary condition so this is the second type, there are two different sub types in this case; one is 0 flux, so note that 0 is also a constant, so 0 flux and then non 0 flux non 0 constant flux. So, the way we look at it is, supposing if you have a 0 flux at x equal to 0 if you have 0 flux, then you would expect that the; so if I look at the temperature profile inside near x equal to 0 because the flux is 0 at that location you will see that the temperature is going to be flat it is going to be parallel to the x axis.

So, we could describe this by an expression we can say d t by d x at x equal to 0. So, we can describe this boundary condition by the following expression; d t by d x at x equal to 0 equal to 0, so that is the no flux boundary condition or 0 flux boundary condition. And then you can have a constant flux where you say d t by d x at x equal to 0 is some constant oh there should be a k.

So, the flux is k into d t by d x and with the a minus sign, so keep in mind that you should not forget the minus sign why is there a minus sign? Because the energy transfer is in the negative temperature gradient, so minus k d T by d x at x equal to 0 is equal to constant flux.

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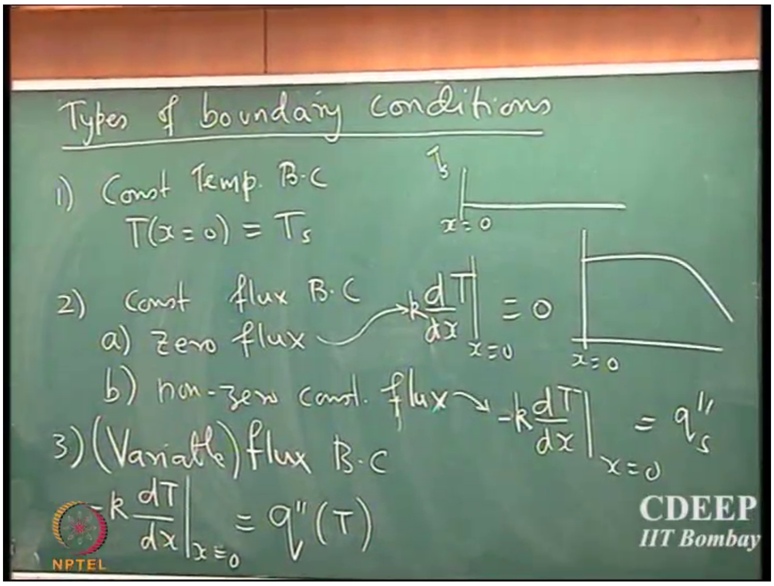

And then the third type of boundary condition is the variable flux boundary condition or it sometime simply called as flux boundary condition, but really it is variable flux boundary condition just to distinguish between the second type and so that is simply given by minus k d T by d x at x equal to 0 equal to some flux which is a function of the local temperature.

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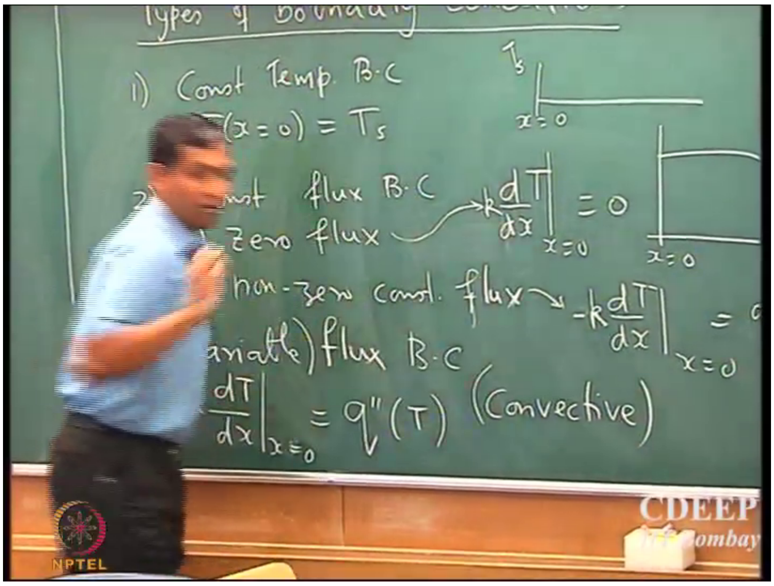

So, how can we define this flux? Is there a way to quantify this variable flux? Is there a way to quantify this flux? Remember that for conduction we said the quantification is done by Fourier law what about this any suggestions? I will give you a hand it is also called as convective boundary condition or convection boundary condition.