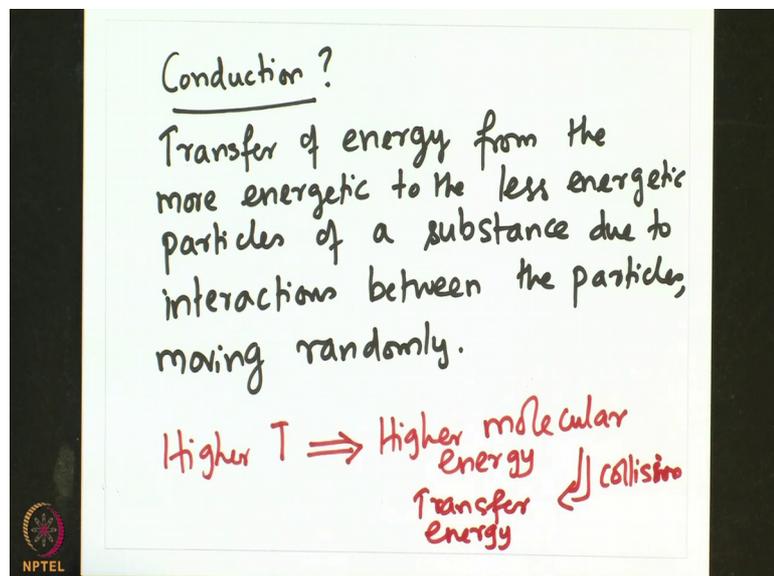


Heat Transfer
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Lecture – 02
Introduction to conduction

In the last class, we defined heat transfer, we defined the objectives of the course and what are the different modes of heat transfer. Let us start with conduction from this lecture.

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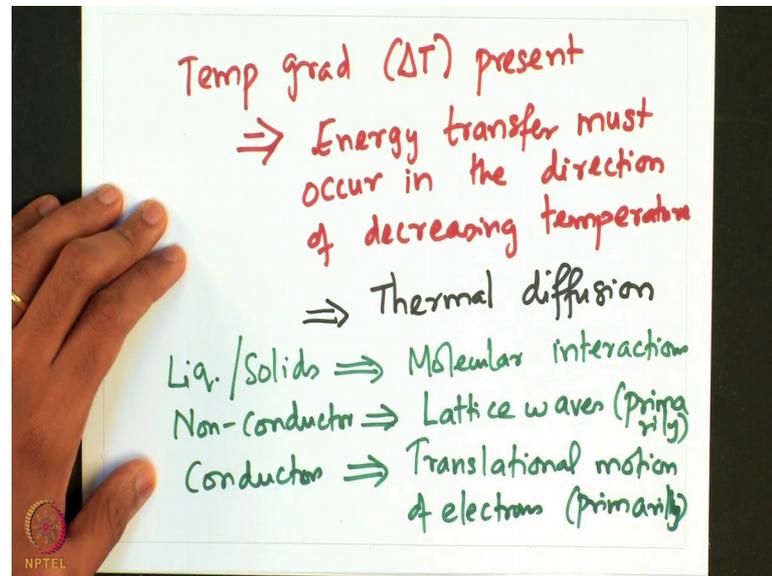


So, what is conduction? Conduction refers to transfer of energy from the more energetic to the less energetic particles of a substance due to interactions between the particles moving randomly. So, the more energetic particles would actually have the tendency because of this interaction between them to transfer the energy to a less energetic particle. So, this process is what is referred to as conduction mode of heat transfer.

So, suppose a particle is at higher temperature, then they naturally have higher energy, higher molecular energy and then due to collision they may experience collision and due to collision they actually transfer energy to those particles whose temperature is

actually smaller than the temperature of itself. So, this process is what is called as conduction.

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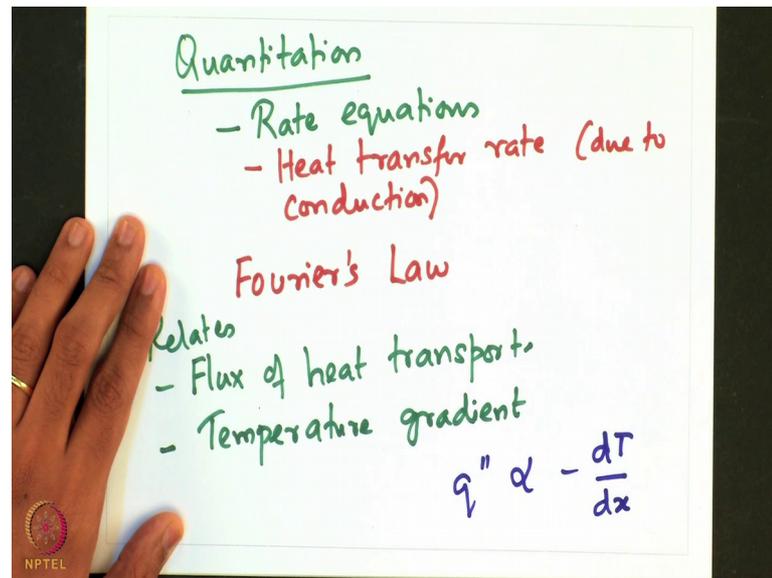


So, now, in the presence of a temperature gradient, what happens is that the energy transfer must occur in the direction of decreasing temperature. So, direction of energy transfer is very important while quantifying the heat transfer process. So, it is important to recognize that when there is a temperature gradient the energy transfer must actually happen in the direction of the decreasing temperature. So, in various quantification processes we will actually take into account the direction in which the heat transfer occurs and that actually is crucial in quantifying the heat transport process and the various methods that we will develop in this course.

So, this process of energy transfer that occurs in the direction of decreasing temperature is what is called as thermal diffusion. So, the conduction or the thermal diffusion in liquids or solids, it typically occurs through molecular interactions. However, in a non conducting material, the typical transport occurs because of the lattice waves. It is primarily because of lattice waves the primary mechanism that governs the thermal diffusion in a non conductor is the lattice waves and in the case of conductors it is essentially governed by the translational motion of electrons that is actually present.

So, different types of materials, the mechanism of thermal diffusion is actually different and accordingly the properties of these materials would actually be different. So, the next aspect we are going to look at is how to quantify the thermal diffusion process.

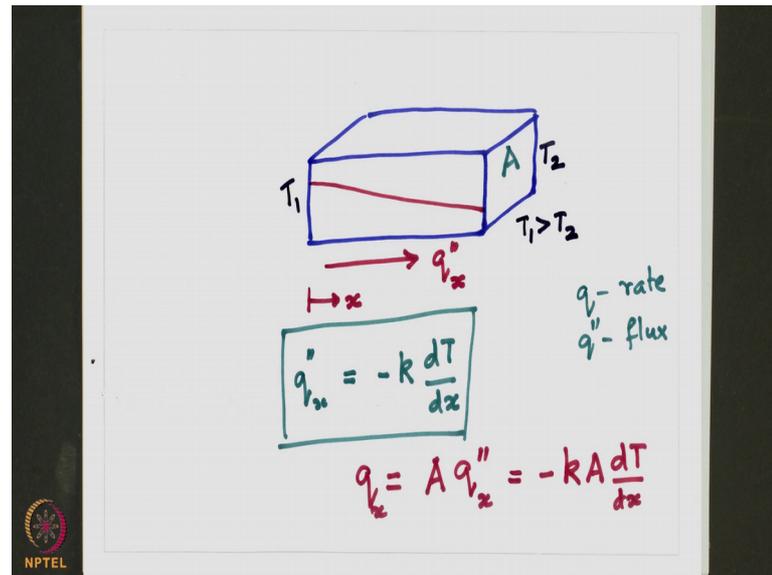
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So, we are going to look at quantitation of thermal diffusion process. Now, in order to quantify, one need to look at the rate equations. So, what we are going to see is we are going to develop, how to write the rate equations for these processes. In particular, we are referring to how to estimate the heat transfer rate in a given system due to conduction. So, suppose I want to estimate a heat transfer rate for a particular system, then we need to have a certain governing loss that connects the rate or that connects the rates with the corresponding driving force and other properties.

So, such a rate law is what is called as the Fourier's rate law or Fourier's law and the Fourier's law essentially relates the flux of heat transport. It relates the flux of heat transport and the temperature gradient, because the heat transport is occurring in the direction of the decreasing temperature, the Fourier's laws states that the flux of heat transport is proportional to the negative temperature gradient.

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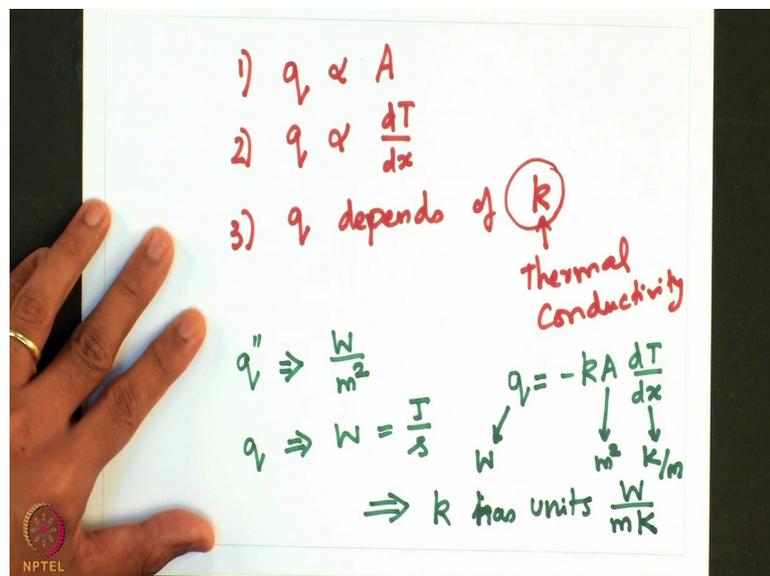
So, suppose we take a slab, let us assume that the slab is actually in a cuboid form. Let us assume that it is a cuboid slab and let us assume that the cross sectional area of the cuboid is A and let us also assume that the temperature of one end of the slab is T_1 and the temperature of the other end of the slab is T_2 and let us assume that T_1 is actually greater than T_2 .

So, what will be the temperature profile inside the slab? There will be a certain profile inside the slab and the heat is actually transferred from the end whose temperature is T_1 to the end whose temperature is actually T_2 which is lower than T_1 . So, this is the direction in which the heat is actually transported or the flux of energy that is being transported from one end to the other end is in this direction.

So, suppose if I draw coordinates and let us say that this direction is x direction, then the flux that is actually flux of heat that is actually transported can actually be written as flux in the x direction or generally referred to as q_x double prime and so, henceforth, in this course the quantity double prime superscript for q usually you will refer to as flux. And, so without a superscript would be rate and q with a double prime superscript would actually be referred to flux. So, the Fourier's law says that the flux of heat transport in the x direction for this case is actually given by minus k , which is the conductivity of the slab into the temperature gradient in that direction.

So, that is what is actually relation between the flux of heat transport. And the corresponding temperature gradient, which is the driving force. Suppose A is the area of the heat transport remembers that the heat transport is actually from one end to the other end which is actually across the cross sections of the slab. And, so, the rate of heat that is actually transported from one end to the other end via the cross section of the slab is essentially given by the flux multiplied by the cross sectional area. So, therefore, the heat transport rate q is essentially given by A times in the x direction, is given by A times $q \times$ double prime which is essentially given by minus k times the cross sectional area into the temperature gradient in that direction.

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So, from here we can clearly see that we can see that the heat transport rate q is essentially, proportional to the area of heat transport. It is also proportional to the driving force and of course, the rate depends on k , which is the thermal conductivity of the material. Now, note that thermal conductivity is actually an intrinsic property. So, the moment a material is specified then the thermal conductivity of that material has actually fixed. Thermal conductivity of that material is very specific to a given property and it is an intrinsic property of the system that is actually being considered.

So, let us look at what are the units of these different quantities that we have looked at. q double prime which is the flux, is essentially the units of this is watt per meter square and the units of heat transport rate is one which is nothing, but joules per

second. So, now we know that heat transfer rate is given by minus $k A$ times dt by dx . So, if I now set the units here, the units of q is watts, units of A is area is meter square and the units of dt by dx is Kelvin per meter.

So, therefore, from here we can easily see that the unit of thermal conductivity has units of watt per meter Kelvin that is the units for the thermal conductivity of any material. Suppose if the system is non isotropic, there is no reason why the temperature gradient has to be same in all directions. The temperature gradient can in principle be different and different directions in a given material, which also automatically implies that the flux of heat transport in different directions can in principle, be different.

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Non-isotropic

$$q'' = -k \nabla T = -k \left[i \frac{\partial T}{\partial x} + j \frac{\partial T}{\partial y} + k \frac{\partial T}{\partial z} \right]$$

$$= i q''_x + j q''_y + k q''_z$$

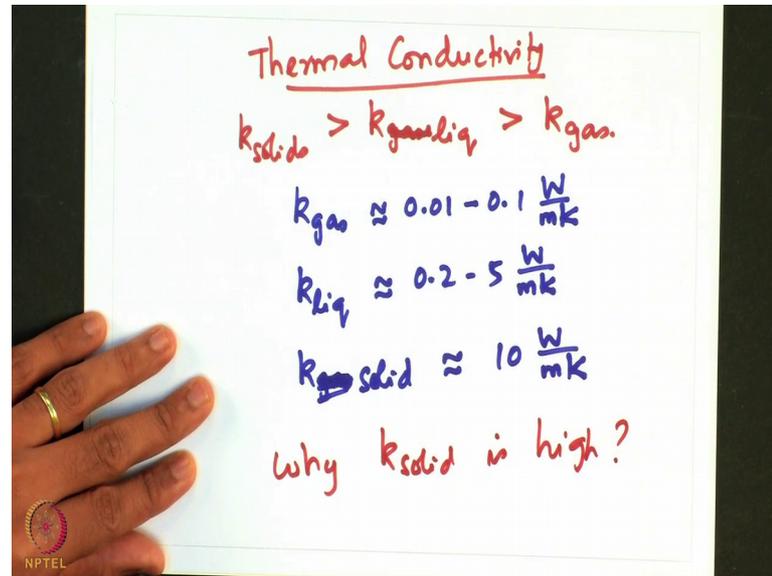
$T = T(x, y, z)$

So, suppose if it is a non isotropic system, then the flux of heat transport is essentially given by minus k into some $\text{grad } T$, which is the temperature gradient and that is equal to minus k into the unit factor in the i th direction, dt by dx plus j dt by dy gradient in the y direction and gradient in the z direction, which automatically implies that this is also equal to $i q''_x$ which is the flux in the x direction plus $j q''_y$ double prime plus $k q''_z$ double prime.

So, this is the flux in x -direction, this is the flux in y -direction and this is the flux in z -direction. So, from here it is very clear that the temperature in the slab or in the

material that is being considered is clearly a function of all 3 positions. So, it is a function of both x , y and z ; with x , y and z being the position inside the solid. Let us look at what is thermal conductivity.

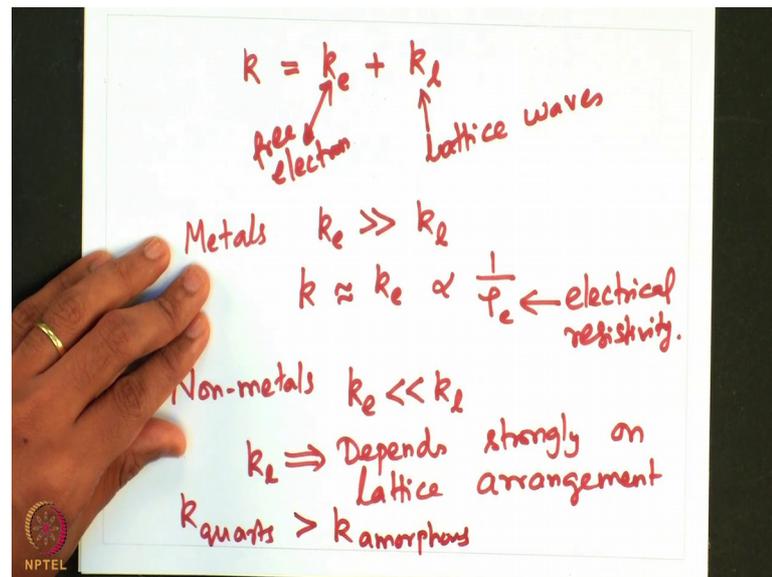
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So, thermal conductivity is a property which essentially captures the ability of the system to actually conduct heat. So, the thermal conductivity of solids is typically greater than the thermal conductivity of liquids which is typically greater than the thermal conductivity of gases. So, to give you a sense of numbers, the thermal conductivity of gas systems typically is in the order of magnitude of 0.01 to 0.1 watt per meter Kelvin and the thermal conductivity of liquid is typically in the range of 0.2 to 5 watt per meter Kelvin and the thermal conductivity of gas systems, solid systems is typically 10 watt per meter Kelvin.

So, one may ask a question as to why there is difference in the thermal conductivities across different types of the materials? Why solid thermal conductivity of solids is high? Why k_{solid} is high, what could be the reason? So, typically the thermal conductivity in solids has 2 components and note that in solids the molecules are actually packed very close to each other. And so, the propensity for these molecules to collide with each other is very high and therefore, the ability to actually conduct heat is at much higher in solid, which explains why the thermal conductivity of solids is much higher than that of liquids and gases.

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Now, the thermal conductivity in a solid typically has 2 components, k_e and k_l . So, this is essentially, the component that is because of lattice waves and this is essentially because of the electron movement, component that is because of the free electron movement.

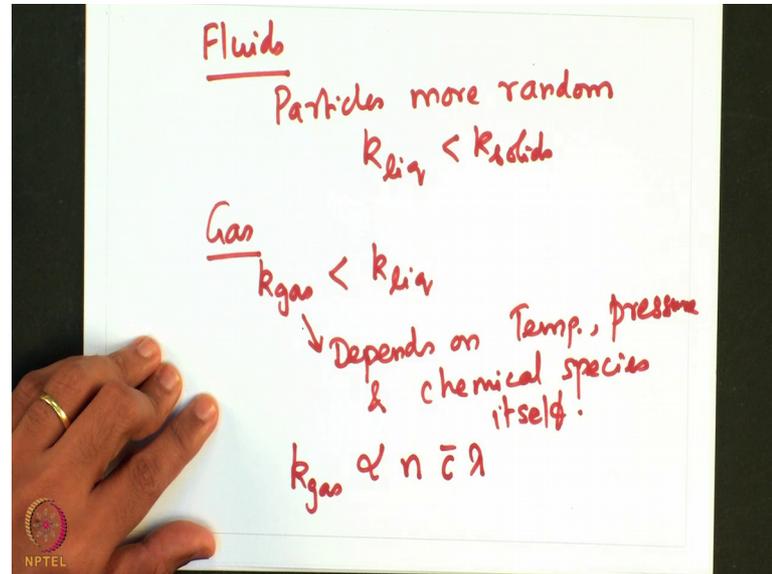
Now, if you take metals then the thermal conductivity component due to free electrons is actually much higher than that of the lattice. So, typically for such systems k is approximately equal to the component that comes from the free electrons movement and k_e which is typically a function of $1/\rho_e$ which is the electrical resistivity of the metal.

If we take non-metals; for example, ceramics, then typically k_e is much smaller than k_l and in fact, this is the reason why the insulation systems typically have low conductivities. k_l actually depends strongly on the lattice arrangement. So, one can play with the lattice arrangement of the non-conducting material in order to change the conductivity and thereby consider those materials as actually an insulating material.

So, for example, the conductivity of quartz which actually is a more regular lattice arrangement is actually much higher than the conductivity of let's say amorphous graphite, so, that is because the arrangement if there is regular arrangement then there

is a strong potential for conducting heat better than if there is lattice arrangement is regular.

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In the case of fluids, the particles are much more random and therefore, the conductivity of the liquids is actually smaller than conductivity of the solids or else the propensity or possibility of the molecules to collide with each other actually goes down because they are more randomly packed and therefore, the conductivity of liquids is actually smaller than that of conductivity of the solids.

So, let us consider gas systems. So, in a gas system the conductivity of gas is once again much smaller than the liquids, that is, because the particles are even more randomly packed and typically depends on temperature, pressure and the chemical species itself. So, the conductivity of the gas is typically proportional to the number of particles per unit volume that is present. The mean molecular speed \bar{c} and the mean free path λ and the conductivity of the gas are typically a function of these 3 and different gases will have a different mean free path and the different molecular speed. So, as a result different gases will have different conductivity.

So, we said that the conduction occurs through the process of energy transfer in the decreasing temperature direction is typically what is called as the thermal diffusion process.

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Thermal diffusivity

$$\alpha = \frac{k}{\rho C_p}$$

Thermal conductivity
Density
Heat Capacity.

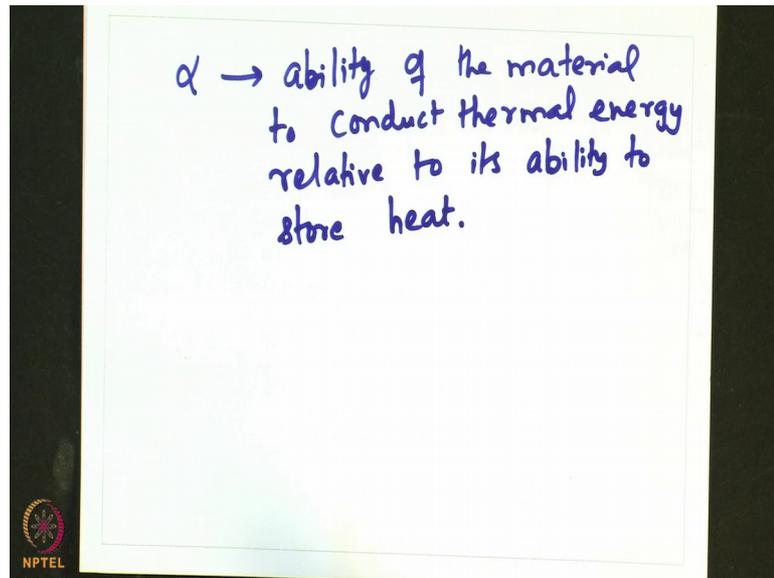
$$\alpha = \frac{\text{W/mK}}{\frac{\text{kg}}{\text{m}^3} \frac{\text{J}}{\text{kgK}}} = \frac{\text{J/s/mK}}{\frac{\text{kg}}{\text{m}^3} \frac{\text{J}}{\text{kgK}}}$$

Unity of α is $\frac{\text{m}^2}{\text{s}}$

So, the thermal diffusion is actually characterized by a quantity called thermal diffusivity and thermal diffuse, the symbol that is typically used is alpha; which is given by k by ρC_p , where k is the conduct thermal conductivity of the material, ρ is the density of the material and C_p is the specific heat capacity of the material. So, what will be the units of thermal conductivity? Thermal conductivity units are alpha is, the units of thermal conductivity is watt per meter Kelvin, units of density is kilogram per meter cube and the units of specific heat capacity is joule per kilogram Kelvin, which is equal to joules per second meter Kelvin divided by kilogram per meter cube joules per kilogram Kelvin.

So, if you cancel out the like terms, the unit of thermal diffusivity is meter square per second. So, what is thermal diffusivity?

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Thermal diffusivity, essentially characterizes or captures the ability of captures the alpha essentially captures the ability of the material to conduct thermal energy relative to its ability to store heat.

So, with these definitions what we will start from the next lecture is, looking at how to write heat balance in order to estimate or find the temperature distribution in the system that is being considered.