**Lecture - 34**
Matrix Conditioning and Solutions and Linear Algebraic Equations (Contd.)

So in last class we were looking at matrix conditioning. How to classify whether a system of linear algebraic equations is well posed problem or ill posed problem. This boils down to looking at matrix A and we will come to that. I was just talking about a motivation to look at this problem. I showed a simple system in which reordering of calculations can change the results, another system in which inherently bad ill condition.

A small error on the right hand side can change the solution drastically. So the idea was to analyze errors of this type.

(Refer Slide Time: 01:08)
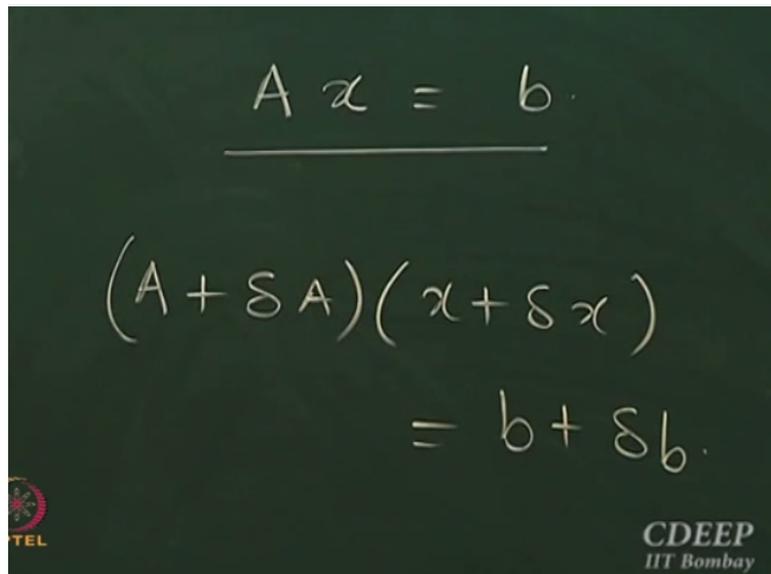
So we are looking at solutions of Ax=b and as I told you invariably when you solve a problem using computer you will never solve this original problem. We always solve a problem, which is A+delta A x+delta x=b+delta b. We can never solve the original problem except some very, very simple systems. In general, for most of the problems I mean let me just give you a simple example.

(Refer Slide Time: 01:51)

$$\begin{bmatrix} \pi & -e \\ e & \pi^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \sqrt{2} \\ e^2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{\pi^3 + e^2} \begin{bmatrix} \pi^2 & e \\ -e & \pi \end{bmatrix} \begin{bmatrix} \sqrt{2} \\ e^2 \end{bmatrix}$$

You can let say I have this problem pi-e let us say I want to solve this problem using computer okay. I cannot represent pi exactly; I cannot represent e exactly. Actually, for this particular problem you can find the exact solution analytically. What is the solution? You find a determinant and then cofactor inverse okay. So the true solution to this problem would be x1 x2.

Can you tell me what is the determinant of this? 1/pi cube+e square right and what is the cofactor? Pi square e-e pi*root 2 e square right correct. This is the inverse matrix okay. This matrix this solution which you get here or this original problem can never be represented in a computer because pi is not a rational number and when if you write a rational number, approximations will creep in because of finite procedure used in computing.

So this is the true solution I would say. What you get in the computer is the approximate solution of this problem okay and now the real worry is how bad is the approximation okay. When can computing make things wrong okay? I will give you a simple example now which is little more involved. This looks like a very simple matrix I have given this in the notes okay.

(Refer Slide Time: 04:00)

This matrix is 10 7 8 7 7 5 6 5 8 6 10 9 7 5 9 10. The reason for giving all these examples is because unless you know motivation is clear if I just do the raw theory it does not make sense so this is my A matrix okay. I am choosing x1, x2, x3, x4=1 1 1 1 and then if I choose this, I get 32 23 33 and 31. This is A matrix, this is x, this is the right hand side okay. 1 1 1 1 I have chosen it is a non-problem to you. If I ask you to solve for x, we should exactly get 1 1 1 1 okay.

Now what if I perturb this matrix a little bit okay. I will show you what happens. If to this matrix I add +delta A I add delta A okay. Now my delta A is going to be 0 0 0.1 0.2 0.08 0.04 0 0 0 -0.02 -0.110 and 0.01 0 and -0.02. Instead of taking this A matrix okay I am going to solve for A+delta A okay. This is my A matrix, this is my original x okay and this is my right hand side.

I am keeping the right hand side same, I am changing the problem now to this problem. I am changing A+delta A. I do not know what will be x and I am keeping the b same okay. Now if you see there my matrix delta A contains very small perturbations compared to you know the elements 10 7 8 my perturbations are 0.1 0.2 0.01 0.02 small perturbations in A matrix okay. You might imagine well if these kinds of small errors will not change the solution drastically.

You would expect the solution. What you would expect? X+delta x to be typically small perturbation in A. You know my x should change a little bit, it does not happen here.

(Refer Slide Time: 07:46)

$$x = \begin{bmatrix} \vdots \\ \vdots \end{bmatrix}$$

$$\underbrace{x + \delta x} = \begin{bmatrix} -81 \\ 137 \\ -39 \\ 22 \end{bmatrix}$$

If you solve this problem now okay, x turns out to be -81 137 -39 22, with such a small perturbation in A matrix my x changes from 1 1 1 1 to this vector okay. A small error, a tiny error okay in A matrix can cause solution to change so drastically that you cannot even recognize okay. There is something fundamentally wrong about that matrix. You make a small error in the representation, your solution can be substantially different.

It will not even resemble you know this original x, sorry this is x+delta x, should write x+delta x. So x+delta x is this and your x just compare your x was 1 1 1 1, these 2 are significantly different vectors okay. Well I will slightly make a difference okay. Instead of solving this problem, I will solve this problem.

(Refer Slide Time: 09:12)

$$Ax = b$$

$$A(x + \delta x) = b + \delta b$$

$$\underbrace{x + \delta x} =$$

A+x+delta x b+delta b okay. I am going to introduce a slight perturbation on the right hand side, A matrix nothing is changed. You have represented A matrix correctly okay.

(Refer Slide Time: 09:33)

My right hand side it changed to my b+delta b now is 31.99 23.01 32.99 31.02 okay. Look at the original b vector. It is 32, I perturb by -0.01 okay, this side I have perturb by +0.01, this is -0.01 this is 0.02 very, very small perturbations on the right hand side okay.

How does x change? If I introduce this perturbation my x+delta x becomes 0.12 2.46 0.62 1.23 look at the solution where is 1 1 1 1 and where is this solution or where is 1 1 1 1 and where is this solution. These 2 solutions with slight perturbation on the right hand side or a slight perturbation on the left hand side in the A matrix or b matrix is changing your solution so drastically that you know there seems to be something funny about this matrix.

You just change something a little bit, make a one small error, your calculations are going haywire okay. You would expect a small error committed in A or b would result in a small error in A or in the solution x that is not happening. So this particular matrix seems to blow up even a small or tiny error in the representation okay. This is the background; this is the motivation.

And then when you solve partial differential equations or boundary value problems, you have large matrices. You know depending upon how you discretize, how you create grades or number of collocation points or whatever you know method of least squares whatever you are using you have large number of points and you make small errors there. If those matrices are

ill conditioned, you can get answers which are wrong even when you have a very good program.

Nothing to do with how good your program is written, you know if you have best numerical package, you can end up doing or getting upside answers. So now I want to come up with a method of you know analytical tool by which I can say which matrix is good, which matrix is bad and that is what is going to be my theme for this particular lecture okay. So let us look at this case first.

I think with this numerical example at least you have motivation for why you are looking at this particular problem. Now I am going to do these 2 special cases of this derivation because they will give you insight. It is possible to do a more general derivation, but I think this specific derivation gives more insight. So let me look at this case first okay. I wanted to solve Ax=b.

I was able to represent A perfectly. There was no problem. There was some error committed on the right hand side okay, b was represented wrong that is why the solution became x+delta x okay.

(Refer Slide Time: 13:41)
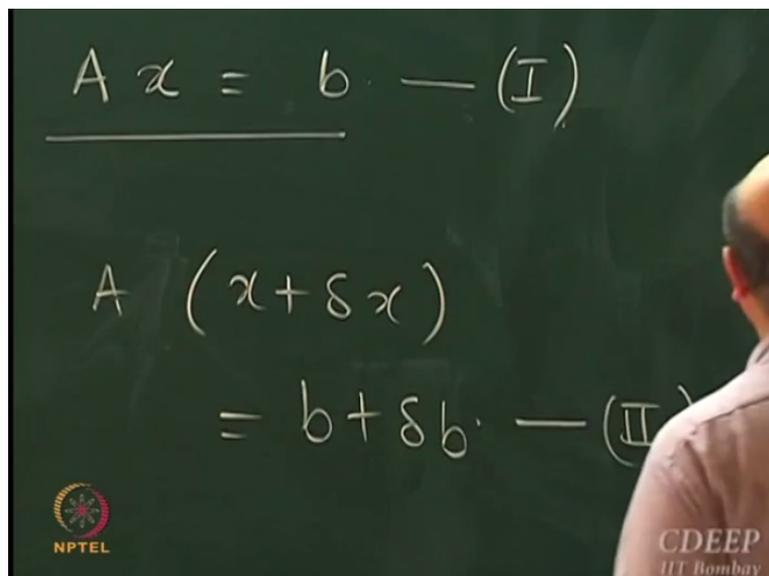
So this is my original let us say x is the true solution, A is the true representation of A matrix, b is the true representation of b vector and this is what you ended up solving in the computer okay. So this is my II okay.

(Refer Slide Time: 14:00)

What will I get II-I? So this is Ax+A delta x=b+delta b. If I subtract I from II, I will get A delta x=delta b okay. Let us for the time being assume that of course when you are solving for Ax=b, A is invertible. If A is not invertible, you are solving a problem which does not have a solution or which may have multiple solutions. We do not have a unique solution. Let us assume that we have a problem where A has a unique solution.

So I am not talking of a system which is just mind you I am not talking of a system which is singular. Singularity is not a problem okay. It is not a singularity, do not confuse singularity with ill-conditioning A singular system may not have a solution or it may have multiple solutions okay. Singular system may not have a solution or it may have multiple solutions depending upon whether b belongs to the column space of A .

And what is the null space of A matrix and so on. So that is the different story, this is not singularity. This is something different okay. So now delta x=A inverse delta b okay.

(Refer Slide Time: 15:45)

$$\|\delta x\| = \|A^{-1} \delta b\|$$
$$\leq \|A^{-1}\| \|\delta b\|.$$
$$\frac{\|\delta x\|}{\|\delta b\|} \leq \|A^{-1}\|.$$

Now I am going to use properties of matrix norm so norm delta x=norm A inverse delta b, which is<=norm A inverse using this is basic definition of matrix okay. I am just using properties of matrix norms so which means norm delta x change in solution due to change in the right hand side okay. This ratio is bounded by norm of A inverse okay. Well in general when you talk of delta x and delta b, these could be very, very small numbers.

And then their ratios sometimes do not help you to quantify everything. We need to talk about relative change okay. I would like to know about delta x/x delta b/b if I change you know percentage error with respect to the original solution. So this inequality is not sufficient, I need something more okay. So now I am going back to this first equation here.

(Refer Slide Time: 17:18)

$$\|A x\| = \|b\| \leq \|A\| \|x\|$$

$$A x = b \quad —— (I)$$

$$\left[\|b\| / \|x\| \right] \leq \|A\|$$
$$\quad —— (IV)$$

$$A (x + \delta x)$$
$$= b + \delta b. \quad —— (II)$$

Ax=b so norm Ax=norm b which is<=norm A norm x. Norm Ax<norm A norm x. This is the fundamental inequality which defines the matrix norm okay. I am just using the definition of induced matrix now okay. So this particular inequality gives me here we are talking about b, norm b/norm x this quantity is always bounded by norm A okay. Now this inequality I am going to combine with the earlier inequality.

See this is the positive number, ratio of 2 norms, right hand side is the positive number okay. So now combining let us call this result IV.

(Refer Slide Time: 18:50)

Let us call this inequality as result III. So if I combine III and IV okay if I combine III and IV I can just multiply the left hand sides and right hand sides, 2 positive numbers which are<2 other positive numbers. So if I combine those 2, I will get if I combine these 2 inequalities, I get this relationship okay. If I do a little bit of rearrangement, you will see why I am doing this okay.

Is everyone with me on this this inequality okay? I am multiplying positive numbers on the left hand side, I am multiplying positive numbers on the right hand side, 2 inequalities are combined to generate this inequality. Now how do you derive something out of this?

(Refer Slide Time: 19:53)

$$\frac{\|\delta x\| / \|x\|}{\|\delta b\| / \|b\|} \leqslant \|A\| \|A^{-1}\|$$

Condition Number

So this gives me this fundamental inequality that relative change in the solution, I am looking at sensitivity right. Relative change in the solution due to relative change in the right hand side is bounded by this number okay. What does it mean? It means that maximum ratio of relative change in x to relative change in b okay=norm of A*norm of A inverse. What do you see here on the right hand side neither x appears not b appears, only matrix A appears okay.

So this you know if you have a slight error in representation of b, you know what is the maximum possible fractional error? This is something like fractional error right, norm delta x/norm x is something like fractional error right so what is the maximum fractional error that you get in the solution? This is bounded fundamentally by multiplication of 2 quantities norm of A and norm of A inverse okay.

This norm of A*norm of A inverse, this is called as condition number of a matrix. How do you evaluate this? You could use 1-norm, 2-norm, infinite norm whatever is convenient, you can use that norm and find out this quantity. Well finding out this for one or infinite norm has a problem because 1 or infinite norm would require computation of A inverse and many times A inverse is not comfortable.

Two-norm somehow happens to be convenient. I will give you a way of computing condition number using 2-norm so condition number using 2-norm is very, very you know is used very often but it does not mean that you cannot use the other way round, you can of course use the other definition okay.

So this in some sense gives a bound on the amplification factor or amplification of the error in the solution due to change in the right hand side or error committed. This error delta b could be committed due to variety of results. It could be because of representation. It could be while doing some computations. So this fundamental quantity which appears here I will show you that in some other context also.

When you perturb A matrix again same number will appear okay. So there seems to be something fundamental about this okay. Now let me analyze another case.
(Refer Slide Time: 23:07)

Now I want to solve Ax=b, this is my equation number I and then I end up solving A+delta A x+delta x=b okay. If I expand this, it will be Ax+A delta x+delta Ax+delta A delta x=b okay and then I want to rearrange this and say that delta x I am returning this, I am returning this delta x delta x on both sides here okay. The idea here is to give you the spirit of what is happening.

As I said it is possible to do a derivation where A+delta A x+delta x b+delta b, I am avoiding that general derivation. I am just looking at 2 special cases okay. Now this you get by subtracting so if I subtract if this is my equation V and if this is the same as equation I which we have written earlier so If I subtract from V from equation I okay b and b on both sides will disappear okay.

B will disappear on both sides Ax and Ax will disappear. What remains is this term this term and this term this will go and this will go if I subtract this equation from this equation okay

then what remains is A delta x=- delta A times x+delta x okay. All that I have done is I have expanded this and subtracted this equation okay to get this perturbation equation okay.

(Refer Slide Time: 25:40)

So this implies that delta x=-A inverse delta A or norm delta x=norm A inverse delta A x+delta x using the fundamental inequality of matrices, I can write this as A inverse norm of delta x is always less than. So I am going to rewrite this as norm delta x/norm of x+delta x. Is everyone with me on this? So this is something like relative change in the solution except here we have retained delta x on the denominator okay.

So this is something like relative change in the solution okay and right hand side is this okay. I am going to play a trick on the right hand side.

(Refer Slide Time: 27:04)

I am going to write this as this quantity on the right hand side I am going to write as norm of A inverse*norm of A*norm of delta A. Is everyone with me on this? Just check this. I am multiplying and dividing by norm A okay and then I am going to single out this quantity okay and then using this I am going to write or I am going to rearrange this inequality as follows.

(Refer Slide Time: 28:01)

Norm of delta x/norm of x+delta x okay. Relative change in the solution to a relative change in A matrix is again bounded by A*A inverse that is norm of A*norm of A inverse. Again this condition number reappears when you are trying to analyze a system in which A is represented slightly erroneously so this is again condition number. This is also condition number of A okay.

So general case where there is error in all 3 becomes little more complex and you can look at it in some of the text books on numerical analysis. What is important is that this condition number which seems to play the key role in how sensitive is your solution to the errors. See what you are looking at here how sensitive your solution? What is the fractional change in the solution to fractional change in A matrix or what are you looking here?

What is the fractional change in the solution to fractional change in the b matrix? Well we had made while deriving this we had made that A is perfectly represented. There we had made an assumption that b is perfectly represented, but A is wrong okay. So basically you just get insights into what really seems to be the key factor. This analysis of based on norms seems to suggest that this quantity is something fundamental okay.

And this determines how well-conditioned, how ill-conditioned a particular system is. Well to give you a thumb rule, if this number is large okay then what is large? Large is say about 1000 with this number appears this is more like Reynolds number below 100 if this quantity is<between 1 and 100 okay then matrix is well-conditioned okay. If between 100 to 1000, it is a grey area, you do not know what it is.

Beyond 1000 you can expect trouble okay beyond 1000 you can expect trouble and if it is 10 to the power 5, 10 to the power 6 deep trouble okay. Nevertheless, MATLAB is a wonderful software, it can give you reasonably accurate solutions to condition number 10 to the power 5, 10 to the power 6 but MATLAB starts breaking down if you give a matrix whose condition number is very, very large.

Now how do you get more insights into condition number okay. Will use here 2-norm to compute the condition number. So I want to get an estimate of this quantity right.
(Refer Slide Time: 31:27)

So let us do it using 2-norms for a matrix. If I give you A matrix okay we have done this earlier, 2-norm of the matrix is given by largest magnitude Eigen value of A transpose A okay. So this we have done earlier or max over i lambda i A transpose A, A transpose A is a positive definite matrix always or positive semi-definite matrix. All the Eigen values are 0 or >0 okay.

So non-negative Eigen values so you can always find a maximum so let us call this as lambda n. Let us say that we have arranged the Eigen values we have numbered them such that lambda n is the maximum magnitude Eigen value of A transpose A okay and it is very easy to

show that it is very easy to compute the 2-norm of A inverse without having to compute A inverse that is very nice okay.

You can compute 2-norm of A inverse, you use a fundamental relationship. If you are given a matrix okay is the relationship between a non-singular matrix okay. Its Eigen values and Eigen values of A inverse is the relationship.

(Refer Slide Time: 33:16)

No, see if A is a invertible matrix, no 0 Eigen value okay. Then Av=or let us take b here because we are talking about A transpose A. Bv=lambda V right okay. It is very easy to show if lambda is a Eigen value and if V is the Eigen vector okay I just have to do this, pre-multiply both sides by A inverse, so I will get V=lambda b inverse V okay or 1/lambda V=B inverse V.

So if lambda is Eigen value of B, 1/lambda is Eigen value of B inverse and Eigen vectors are same just see. Eigen vectors do not change, B inverse V=1/lambda okay non-singular matrix I am talking about non-singular matrix okay.

(Refer Slide Time: 34:18)

$$(A^{-1}A)^T = I$$

$$(A^T)(A^{-1})^T = I$$

$$(A^{-1})^T = (A^{-T})^{-1}$$

There is one more thing, one more relationship which I need to use is that A inverse, do you agree with me? A inverse*A is identity transpose=I okay. So A transpose*A inverse transpose=I. Is everyone with me on this? I can interchange inverse and transpose. See what I started with A inverse*A is identity okay. Transpose=identity see this is identity=identity that is what I am writing.

(Refer Slide Time: 35:26)

$$(CD)^T = D^T C^T$$

Then I am expanding this because if you multiply 2 matrices and take their transpose, it is you know this rule, CD transpose=D transpose C transpose. So I am just using that here okay and so A inverse transpose is same as A transpose inverse okay.

(Refer Slide Time: 35:50)

Or let us not confuse with that A here let us put this B. Notationally, I am just talking about general properties of matrices. We were talking about Ax=b, I am not talking of that A in general when A is a invertible matrix, you can write this, any square invertible matrix you can show that B inverse transpose is same as B transpose inverse, you can interchange transpose and inverse.

(Refer Slide Time: 36:30)

Now these are the 2 properties which I am going to use. I am leaving a few things for you to derive. I am not doing it on the board but norm A inverse 2 square=max over i lambda i of A inverse*it is A inverse see what was here? 2-norm of A is maximum magnitude Eigen value of A transpose A. In this case, we are talking about A inverse. So in a A inverse transpose A inverse okay.

I am just combining and saying this okay. Now Eigen values of A transpose A okay. A transpose A is a positive definite matrix. All Eigen values are positive okay. What is the relationship between Eigen values of A transpose A and A transpose A inverse? 1 upon right so if lambda 1 is the smallest Eigen value of A transpose A. What will be the largest Eigen value of A transpose A inverse?

1/lambda 1, so 1/lambda 1 is the largest Eigen value A transpose A inverse okay. This is the largest Eigen value of A transpose A inverse. Is everyone with me on this? So lambda n what is the largest Eigen value? We have numbered Eigen values. The largest one we are calling lambda n, the smallest one we are calling lambda 1 for A transpose A. You have some doubts?

A transpose A we have numbered the Eigen values, lambda 1 is the smallest, lambda n is the largest. We have chosen the numbering, we look at the Eigen values and we number them. Lambda n is the largest, lambda 1 is the smallest. Now if lambda 1 is the smallest Eigen value of A transpose A, then 1/lambda 1 is the largest Eigen value of A transpose A inverse. It follows from the first relationship okay.

So now what I need to do is okay so I am now going to combine these 2 things and come here.

(Refer Slide Time: 39:53)

I am going to combine these 2 things, so norm A2 square*norm A inverse 2 square seems to be=lambda n/lambda 1 okay or norm A2 norm A inverse 2 is square root of lambda n/lambda

1. Where what is lambda n? Largest Eigen value, which also called as singular value of A okay, largest Eigen value of A transpose A and what is lambda 1?

Smallest Eigen value, so condition number of this matrix is simply ratio of largest Eigen value of A transpose A to smallest Eigen value of A transpose A okay. Computing this largest or smallest Eigen values for a positive definite matrix not that difficult, not very difficult. Why positive definite? Because A transpose A is a positive definite matrix okay.

If you ask MATLAB, fit your matrix A and say COND it will give you condition number of matrix okay. Actually, it will compute this ratio and tell you what is the condition number okay.

(Refer Slide Time: 41:41)

So this way I would call this quantity as C2 of A okay I will call this quantity C2 of A. So condition number based on 2-norm of the matrix okay, likewise I can define condition number based on infinite norm or 1 norm okay. Though in that case, I will have to compute A inverse explicitly and of course we can do that for a simple matrix to get insights though for a large matrix that might be not so suitable.

(Refer Slide Time: 42:27)

$$C_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$$

$$H_3 = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix}$$

So I can define C infinity of A as infinite norm okay depending upon way you choose norms, you can define the condition number okay. Now let me complete some story which I began long time back. I never told you exact reason. I kept on saying that polynomial approximations okay. Polynomial approximations give rise to difficulties because polynomials higher and you know fourth order or fifth order you know you get ill-conditioned matrices okay.

Something may appear disconnected now okay but I want to complete the loop now and go back and say why polynomial approximations higher than you know certain order okay. So why does it happen that polynomial approximations create problems? I am going to analyze this in condition number okay. Sometime back I had put one more quiz in the puzzle.

I had showed you that when you try to do polynomial approximations, you get a matrix called as Hilbert matrix H right. Actually in MATLAB, there is a command called hilb, if you say hilb 3, it will give you 3 cross 3 matrix, which is Hilbert matrix. Hilb 4 will give you 4 cross 4 matrix and you can just do this once. Just go and create Hilbert matrices and start looking at a condition number.

Condition number is let say this okay. Condition number of Hilbert matrix are notoriously bad okay. What does it mean? It means that if you make a slight error in the representation of numbers okay these ratios are going to be you know see this condition number tells you worse case error. It is not that for a particular case, it will happen but if it happens it can be very bad. I have shown you some examples right.

You perturb A matrix slightly your solution just goes you know out of box 1 1 1 1 it goes somewhere else, you know it is not in some small neighborhood of 1 1 1 1, it goes to 83 and some 52. So solution can be completely different with a small error okay. So let me take this Hilbert matrix. H3 is this Hilbert matrix, Hilbert matrix has a very nice structure 1/2 1/3 1/4 1/5.

And I kept on telling you that second order or cubic polynomial is okay but fourth order, fifth order, sixth order, seventh order polynomial okay become bad to solve.

(Refer Slide Time: 45:36)

Why becomes because you get a situation, you get H times theta=some U, U is known on the right hand side, H is the Hilbert matrix, theta are the parameters to be estimated, you got this kind of an equation okay. Theta are the polynomial coefficients okay, H is the Hilbert matrix and U is the right hand side, whatever is the right hand side what is important is how well-conditioned is H matrix because you are solving Ax=b this is another form right.

Estimating coefficients of a polynomial okay now for this simple matrix you can show that I am going to call this as H3 because it is 3 cross 3. You can show that it is 1-norm is same as x3 infinite norm, this is 11/6 and you can actually for this simple matrix you can compute the inverse and you can show that H3 inverse 1-norm=H3 inverse infinite norm=408. You just compute H inverse for 3 cross 3 matrix, you can do it by hand also.

(Refer Slide Time: 47:17)

You will get exact solution and you can actually get these numbers and then what you show here is C1 H3 is same as C infinity H3, which turns out to be 748 okay. The calculations are not bad; condition number is 748 okay. The worst case error that can happen is of the order of 1000 times okay, not so bad not so bad, just let see what happens if you want to fit a sixth order polynomial okay.

(Refer Slide Time: 48:01)

In a sixth order polynomial okay but even in this case for H6 would be 1 1/2 1/3 1/4 1/5 1/6 1/2 1/3 and so on okay up to 1/6 1/7 okay H6. If I want to fit a sixth order polynomial, then I will get H6*theta okay H6*theta=U okay. I will get H6*theta=U and what you can show is that C1 H6 condition number based on 1-norm which in this particular case turns out to be condition number based on infinite norm also same for a sixth order polynomial okay.

Just look at this, condition number is so bad okay whatever you try to do you will not going to get reliable solutions, 10 to the power 7 a small error can get amplified in certain directions in very, very bad measures. What are those certain directions? Those are related to the Eigen vectors of A transpose A. In the directions in which A transpose A has maximum magnitude Eigen value, the Eigen vector corresponding to lambda n will amplify your error worst okay.

So it depends upon how is your B is aligned you will get so this particular matrix. As you start increasing polynomial order, you will get Hilbert matrices of higher and higher order. If you go to tenth you know 10 to the power 12 so a small error committed can create a havoc okay. I will just illustrate how things can be different even for this H3 okay. Then you can judge what will happen for you know sixth order polynomial or seventh order polynomial.

Why we do not get good results. Why we need cubics line? Why we need polynomial interpolations which are you know piecewise polynomial interpolations? Why do we really go for that?

(Refer Slide Time: 50:49)
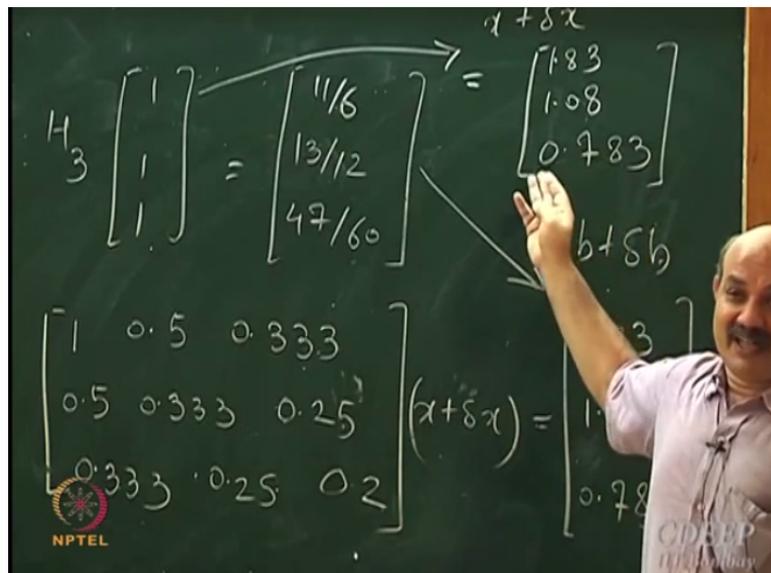
I just complete this one example. See if H3* 1 1 1 will give you 11/6 13/12 47/60 okay. Now what I am going to do is that instead of solving for this problem, I am going to round off this matrix okay. I am going to solve for H+delta H. What is my H+delta H? Okay I have rounded off this right hand side perfectly okay right. This 2-digit approximation very, very often we do in calculations.

And this you might see nothing wrong you know 1/3 being represented as 0.333, I am truncating okay, nothing wrong. My solution here was 1 1 1 okay, right hand side was this, left hand side was that. I just A+H+delta H and this is b+delta b. How much does the solution change? Okay this x+delta x turns out to be 1.83 1.08 0.783. Just imagine tiny error in every number okay.

Trying to fit a third order polynomial okay, you are trying to go from that matrix to this matrix, you might find perfectly reasonable okay. I will get this solution. A small perturbation for a matrix whose condition number is only 700 okay, not very bad okay gives me so much difference in the solution. If I decide to represent this by this and that matrix by this matrix okay.

You see why condition number is so important when you want to study matrix computations okay. Which solution is the correct solution now? The correct solution is 1 1 1 right. What you are getting here is completely different. Just imagine what will happen if condition number is 10 to the power 6 or 10 to the 4 or 10 to the power 5 okay. So the solutions which MATLAB gives you not MATLAB I should not singular MATLAB.

Any software will give you for a matrix with high condition number is likely to be a garbage and you should know this okay when the solution is garbage and when you have committed mistake. If a matrix is well-conditioned and if you are getting garbage, you have made a mistake in programing. If a matrix is ill-conditioned and if you are getting a garbage, not that the software is wrong, not that the program is wrong, it is inherent problem okay.

Just see here 700 condition number, small change in the right hand side, a very small change in the left hand side A and B matrices, you get drastically different solution. Even for third order polynomial you have this situation that is why we do not try to fit high order polynomials. We will continue this story and this is end of this series of lectures on Ax=b. A little bit of it is remaining, will complete it on the next lecture.