Advanced Numerical Analysis
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Lecture – 29
Iterative Methods for Solving Linear Algebraic Equations: Convergence Analysis using Matrix Norms

We are looking at analysis of convergence or convergence analysis of these methods.

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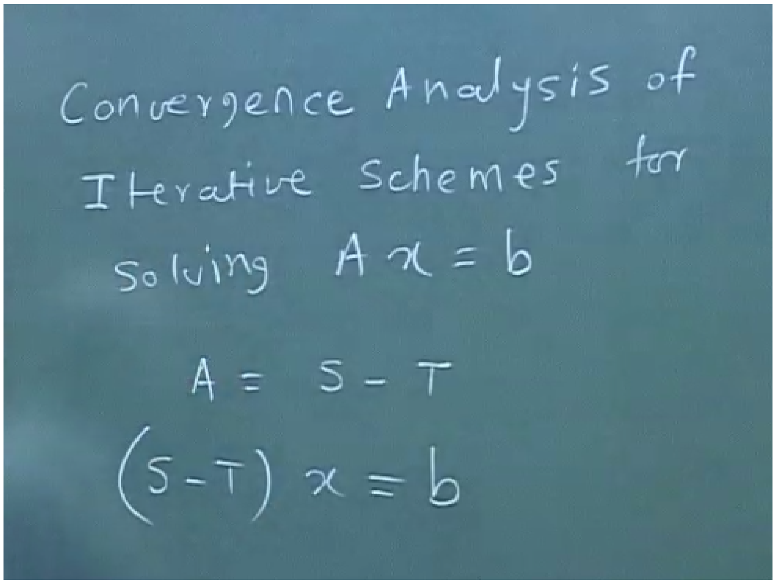

So Ax = b is a set of linear algebraic equations and what we have looked at till now is forming you know iteration schemes. So iteration scheme was formed by splitting this matrix as S - T and, then you come up with an iteration scheme by rearranging this equation that S - T * x = b. This you rearrange and in vector form for the purpose of analysis, we can rewrite this equation as x.

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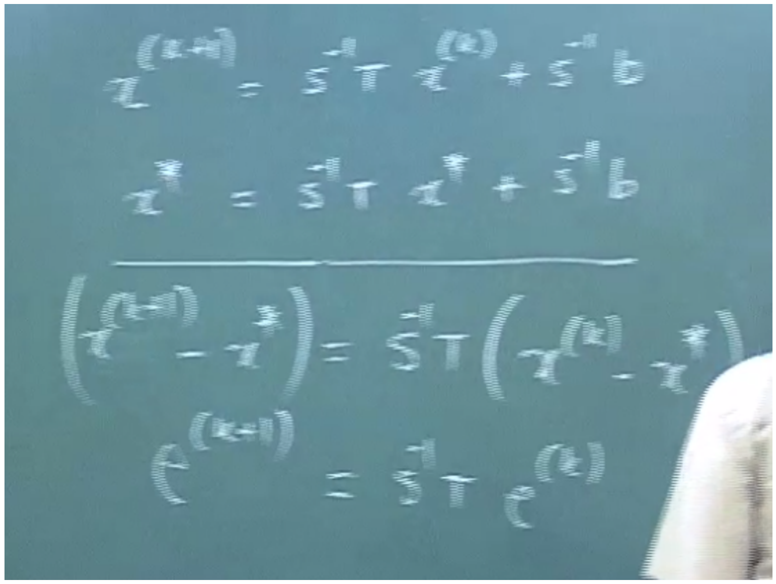

So my iteration scheme in general is x k + 1 = S inverse T xk + S inverse b. So, this is the way we have written this for the purpose of analysis. Different choices of S and T will give you Jacobi method, Gauss-Seidel method, and relaxation method and so on, and when you actually reach the solution what do you get is, x * = S inverse T, S inverse b. Let us say, so this is a solution of the system of equation of Ax = b. If I subtract then I get this equation.

So we have been looking at this general equation error or distance from the true solution = S inverse T * ek where k here is the induction of iteration. So, I start my iterations from some arbitrary x0. This is my iteration equation. X0 will give me x1, x1 will give me x2 if you substitute in this. We will get a sequence of vector x1, x2, x3, x4 starting from x0. The question is under what conditions will this sequence convergence to the true solution under what condition this error will go to 0.

That is what we are looking at. So we abstracted this by a general linear difference equation. We said that this exactly looks like.

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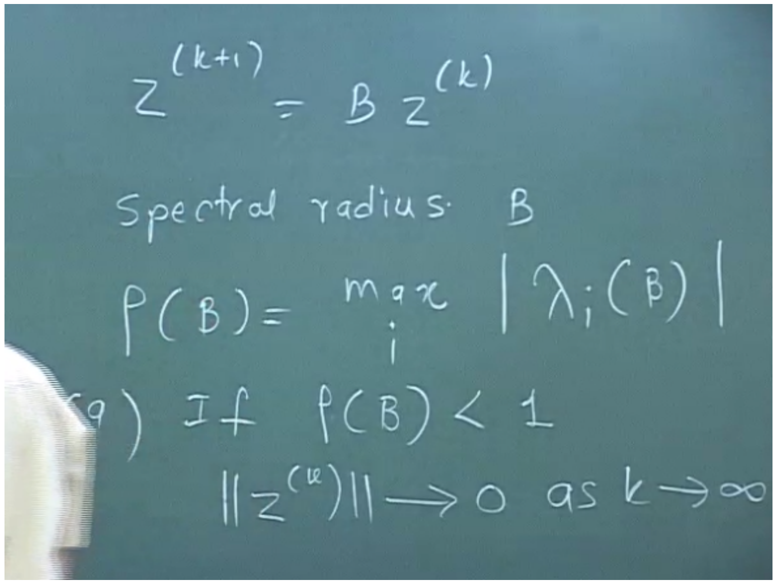

Z k + 1 = Bzk. And in my last lecture what I have done is derived conditions under which, this particular equation will actually behave as we want that is zk will go to 0 as k goes to infinity as k increases. So I have derived general conditions for this linear difference equation. We just mapped this specific equation that we are looking at to this general equation and then we derived certain conditions.

So these conditions are as follows. So now what I am going to do is I am going to define this spectral radius of a matrix. So, spectral radius of matrix B is defined as. So here lambda I represents ith Eigenvalue of matrix B. I am looking at mod of that magnitude of this ith Eigenvalue and maximum over that. So spectral radius of a matrix is defined as maximum over I Eigenvalues 1 to n whatever Eigenvalues that we have and this is over maximum magnitude Eigenvalue is called as the spectral radius of matrix B.

So the conditions that we derived was something like this that if A, if spectral radius of B is strictly < 1 then what we have shown is that zk will go to 0, norm of zk will go to 0 as k goes to infinity. As k increases, what I mean by k goes t infinity as k increases? Norm of zk goes to 0 if all the Eigenvalues of B are strictly inside the unit circle. If all the Eigenvalue are strictly inside the unit circle. The second condition that we derived was something like this that.

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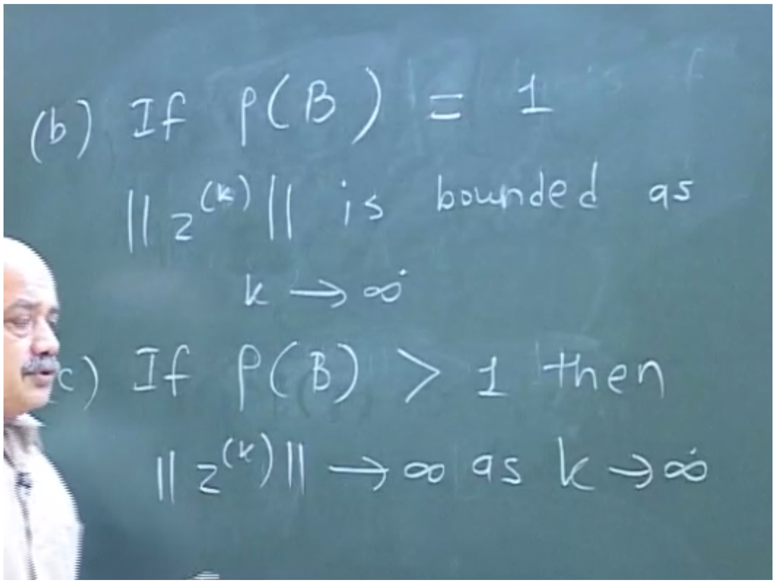

Instead of strictly < 1 is some Eigenvalues are on the unit circle. Then what happens is that norm of zk will not go to 0. What will happen in that case is that norm of zk remains bounded. So in this particular case, norm of zk as k goes to infinity and the third condition that we derived so what is important here is that without actually having to solve just looking at Eigenvalues or the spectral radius of matrix B we can tell how the asymptotic solution is going to behave.

We do not have to actually solve it for a specific z0 and say how the solution will behave. We just look at the Eigenvalues and we can tell how the solution will behave. So the last case was if spectral radius of B well I made a mistake here. Just make this correction. Spectral radius of B if this is strictly <= 1 then norm zk is bounded as k goes to infinity. So, spectral radius will directly tell you whether the solution is asymptotically bounded or asymptotically converging to 0 and the last condition was the spectral radius of B is > 1.

So if 1 of the Eigenvalues is outside the unit circle then norm zk tends to infinity as. So spectral radius of B is what critically decides how the asymptotic behaviour of the solution is going to be. If spectral radius is strictly inside the unit circle, then norm of zk will go to 0 as k goes to infinity. If the spectral radius is <= 0 which means if sum of the Eigenvalues are on the unit circle in some matrices in such cases the error will remain bounded.

Actually strictly speaking we should say it is strictly < 0 we can even replace this condition by spectral radius = 1 and the third case as spectral radius > 1. This will happen if 1 or more Eigenvalues are on the unit circle. The first condition will occur if all the Eigenvalues are inside the unit circle and this condition if any 1 of the Eigenvalues are outside the unit circle then so what is the implication when it comes to solving linear algebraic equations iteratively.

This is our iteration scheme and this is the error or distance from the true solution. The question is will the distance from the true solution shrink? Will you go towards the true solution as k goes to infinity or k increases as iteration increase will you approach x star? Now that will depend upon spectral radius of S inverse T. That will depend upon spectral radius of S inverse T. If you just apply these conditions what we would want is the first condition that is spectral radius of S inverse T should be strictly < 1.

Spectral radius of this should be strictly < 1 in order that the iterations converge to the true solution. What is important here is if spectral radius if I choose this S and T matrices, in such a way that this condition on spectral radius is satisfied then convergence will occur for any initial guess that you give here. See we are going to use iterative methods for solving Ax = b when A is a large matrix.

So very large matrix and how did you initial guess? It is very difficult. So because I do not know the solution so how do I kick start? So what this tells you that if Eigenvalues of S inverse T are inside unit circle, then I am guaranteed to converge to the solution from whatever initial guess you give. So you can give an arbitrary guess you are directed to convergence provided spectral radius of S inverse T is strictly < 1. So the condition that we get from this analysis for convergence of iterative schemes

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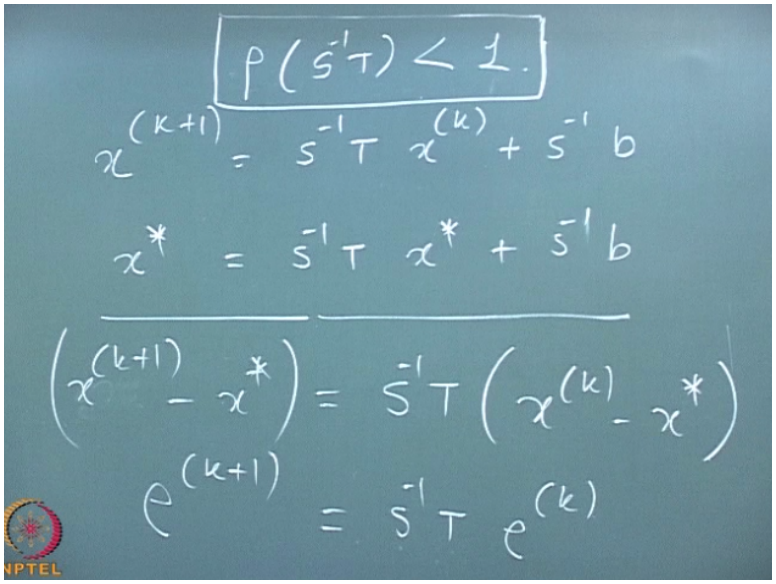

Is that spectral radius of S inverse T should be strictly < 1. So this is the condition that is important. This is the convergence criteria. So if I chose S inverse T in such a way that these kind of equations I just want to touch up on something else before I move to little bit of (()) (13:01) from what we have been doing. These kinds of equations do appear in other context when we are doing numerical analysis.

I just want to point to that before I move to you know something more related to this convergence analysis. In this particular case, we are getting this equation in the context of iterative schemes, where k here is the index of iteration and starting from some initial guess we are getting a new guess and so on and we are looking at the convergence of iterations. In some other context we get very, very similar equations except the k there will not be iteration index, it could be time. For example, let us look at this equation.

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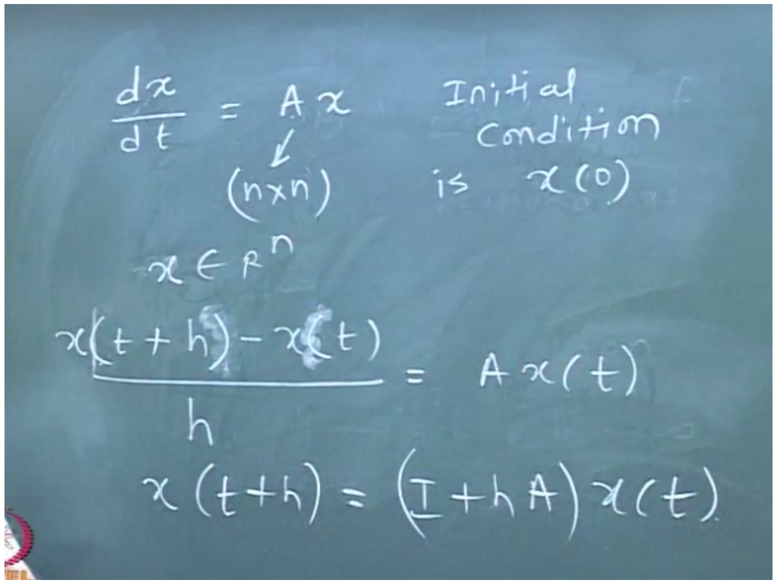

Dx/dt = Ax where initial condition x0. Here A is a n cross n matrix, x is a vector, x belongs to Rn and I want to solve this kind of an equation simply Euler method. So Euler method if I apply to this particular equation, forward difference method then you know I will get xt + h - xt/h will be = Ax at t. So this equation I can rearrange and write at x t + h = I + h * A xt and let us start now writing this solution starting from time 0. So initial condition is x0.

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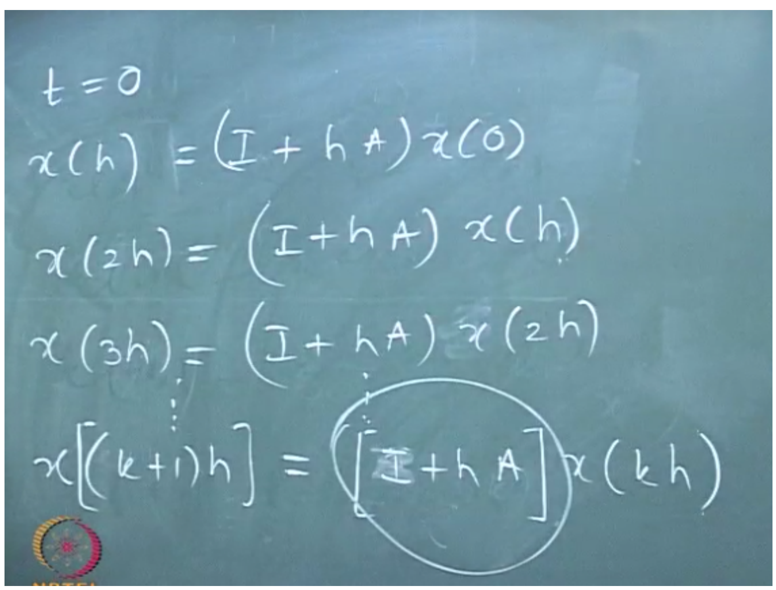

So at t = h we will get x 2h = I + hA * x. We will start at 0 so at x = h and t = 0 let us start from t = 0, so this will be xh = x0 then x2h = I + hA * xh. Then x 3h = I + hA * x2h. You can just go on writing the solution of this difference equation and then finally what you can show that this is x k + 1 * h = I + hA. In general, at kth instant time I can write this difference equation. Now this difference equation, is qualitatively similar to this equation, except when we looked at this equation, this k was considered to be index of iteration.

I could replace this I could reinterpret this as time. This is the same equation x k + 1 = some matrix * xk and if I want to analyze behaviour of this numerical iteration scheme I could look at spectral radius of this matrix. So, this is the very fundamental equation, and this appears in many different contexts for example I would want to decide how to chose let us say original system is asymptotically stable that means all the Eigenvalues of matrix A are in the left half plane and the solutions of that particular system asymptotically goes to 0.

Question is will the numerical solution behave in the same way. How do I choose this? You know integration interval h so that the approximate solution constructed by Euler method is same as the analytical solution. This question will be revisiting later in the course, but I am just preempting that a very, very similar equations arise at that point. So this particular matrix you can look at spectral radius of this matrix and comment upon whether this solution is going to go to 0 or other solution is going to go and explored.

So this particular analysis is very, very generic. Let us come back to our original case. So as I said we have been looking at this iterative scheme.

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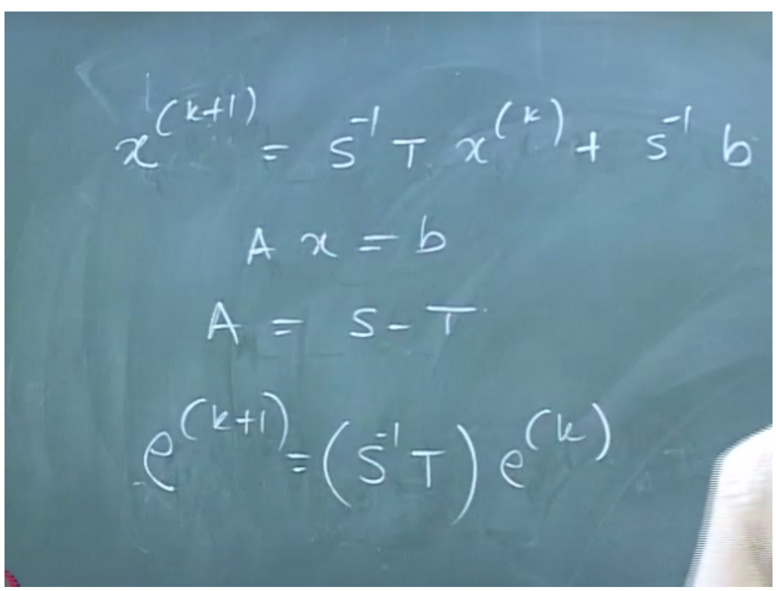

X k + 1 = S inverse T xk + S inverse b. This is for solving Ax = b and we have written A matrix as S - T and we have been analyzing ek + 1 = S inverse T. We have been analyzing this equation. Now this is nice that we got a condition which without having to solve this solution we can comment upon how the error is going to behave or how the distance from the true solution is going to behave as a function of iteration is xk just looking at spectral radius.

Just looking at Eigenvalues of S inverse T, but then is this something very easy to calculate the trouble here this is nice in the sense that it is a nice analytical tool we got an insight looking at Eigenvalues you can actually without actually solving the set of equations we can comment upon the behaviour, but the difficulty now is that I have to compute Eigenvalues of this matrix. Just remember that this A matrix could be a large matrix.

For a large matrix, to compute S inverse T and its Eigenvalues is even more complex problem. If I have computational resources to solve this, I might as well as solve this some other numerical method other than by some other method like you know usual technique Gauss-Seidel elimination the other than wasting my time in computing Eigenvalues to see whether the conditions are met or not.

So this is nice this gives us lot of insight, but we need something more than or some simpler conditions than this and this is where we are going to use induced matrix norm. So using matrix norms, we are going to or using relationships between the spectral radius and the matrix norm, we are going to derive some simpler conditions if those conditions are satisfied then we are guaranteed that the iteration method will converge.

So in order that we come up with these simpler conditions, I need to introduce the concept of induce matrix norms. Well we have been looking at vector spaces and then I had menti1d that set of matrices.

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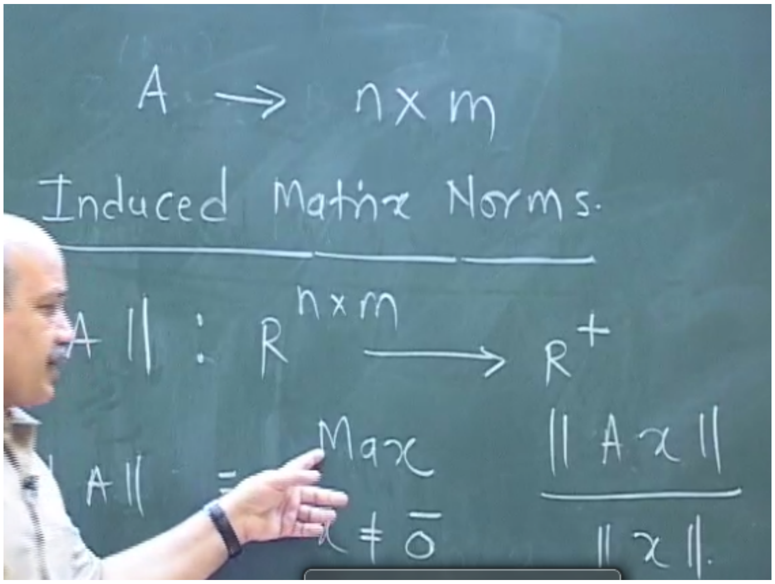

If I have these matrices A which are n cross m matrices it forms a vector space these are all real value matrices. It forms a vector space and I can define a norm on this vector space. So I can have a norm vector space or a norm associated with the vector space set of all matrices. Why this is s a vector space? This is a vector space if I add 2 matrices I will get another matrix. If I add 2 real matrices a scalar multiplication of any matrix will give me another matrix in the same set so this is the vector space and now I need to define on this.

So there are various ways of defining norms on this particular case. I am interested in a particular set of norms which are called as induced matrix norms. So we are going to look at induced matrix norms for matrices which are n cross m. So this matrix norm, w e will denote this matrix norm by this symbol norm of matrix A is actually a map from R n cross m to R +.

It is a map from this set of vectors which is this R n cross m space to R +. R + is set of all positive real numbers. Because norm has to be positive value so this is set of all positive real numbers and the definition of an induced matrix norm is as follows. An induced matrix norm is defined as max over x != 0 vector, norm of it is defined as maximum of this particular ratio norm of Ax divided by norm of x. In some sense you can look at this as gain or amplification power of a matrix.

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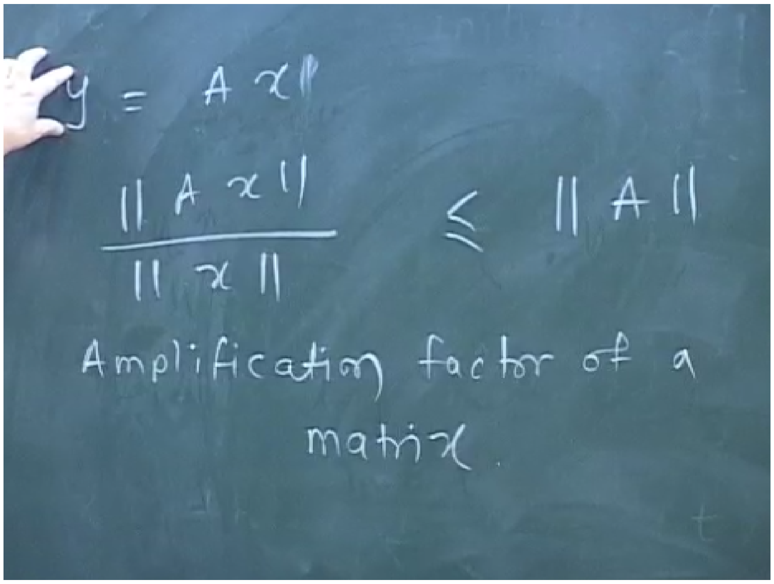

See if I decide to write this Ax = y, this is like this y is a vector which is obtained when operator A transforms x. See actually when a matrix operates on x it could do various things, for example it could rotate the vector. This new vector could be a rotated vector. I A is an invertible matrix in general this will be a rotated vector. It could be a straight vector, it could a contracted vector, it could be in a reverse direction, and it could be reflection.

It could actually have A matrix doing variety of things. It could be a projection. So this A matrix is like an operator which operates on x gives you y so in let us say control system terminology, this like you know this is input, this is output and then we could ask this question that you know what is norm of Ax/norm of x? You could look at this particular quantity norm of x. I am interested in finding a maximum value which bounds this ratio.

This maximum value which bounds this ratio is called us the norm. Of course I want it in a such a way that for some x this ratio is attained, which means I do not want to bound which is not reachable. For some value of x equality holds for remaining values, inequality holds so I want to find out that number. So this norm of A is nothing, but a bound on this ratio. So you could view this as amplification factor of a matrix or an amplification of power of matrix.

Now why it is an induced norm what is special about it being an induced norm. This norm is induced by see this is my domain, and this is the range. I have defined a norm on the range. I have defined a norm on the domain. The definition of norm on the range and definition of norm on the domain together will define an induced norm. So these two induce a norm which is this A.

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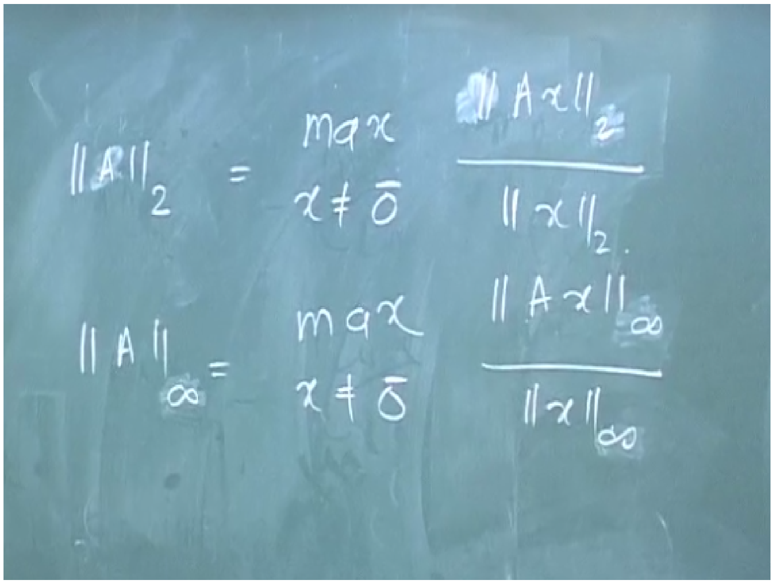

So for example, I could define a 1 2 norm. See this Ax is an element in a range space there could be a different norm associated in that range space. I might be using 1 norm in the range space. I have another norm defined in the domain that is 2 norm. So I could define a norm which is induced norm, which is induced by the norms defined on the domain and the range space and I could call this as A12. This is 1 norm and this is 2 norm.

Now this kind of funny definitions of norm are really used in practice. We do not do this. Normally what we are interested in so I am just telling you that this is possible this is not actually used. What we normally do is that we use 1 norm or 2 norm for example if I define 2 norm then this is 2 norm/2 norm. Then it will be called 2 norm of a matrix A. Similarly I could define 1 norm which is max x != 0, norm x. So this is 1 norm/1 norm.

I want to find out maximum of this ratio. So in particular we are going to use three different norms in this course one is 2 norm, other one is 1 norm and the third 1 is infinite norm. So the infinite norm will be simply defined as infinite norm here and infinite norm here. So I am interested in 1 norm, 2 norm and infinite norm and these norms will help us the relationship between the norm and spectral radius will help us decide behaviour of solutions asymptotic behaviour of solutions.

So far as good we have defined this norm. The question is how do I compute a norm? If I can compute a norm very easily then it makes sense to use norm instead of spectral radius as a measure of convergence of iterative schemes. Now what I am going to do is initially I am going to find the way of computing 2 norm. Well that is more for the reason of let us a (()) (31:28) reason that you know give you some insights into how the norm is computed.

Again but the problem is going to be with 2 norm is that we will get again Eigenvalues. So again to compute 2 norm you have to compute Eigenvalues. So it is not again convenient and finally we will move on to definitions of 1 norm and infinite norm which will be used subsequently in the analysis. Nevertheless, from the view point of getting insight I am going to first look at 2 norm. So what I want to do is to compute numerically this value for 2 norm of a vector.

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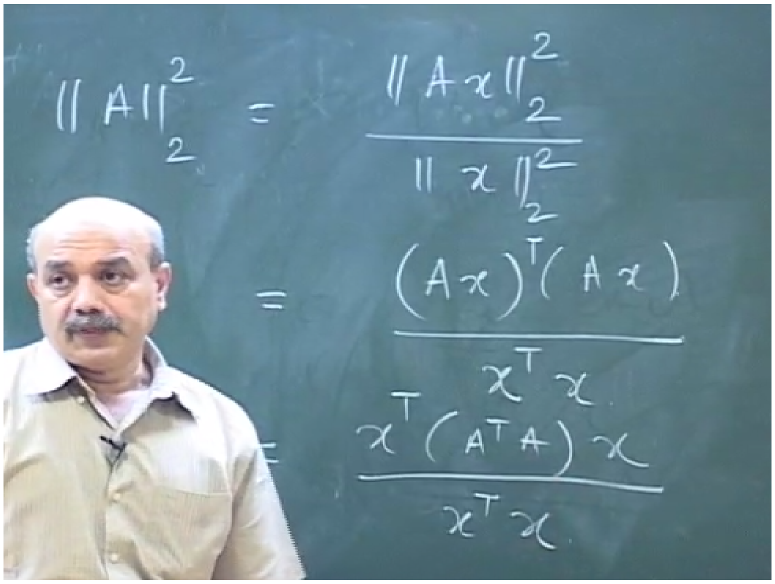

I want to numerically compute value for these 2 norm of a vector. What I am going to do is now is square these equations on both the sides. I am going to square these equations on both the sides. So 2 norm square = 2 norm square = 2 norm square. So this is Ax. What is 2 norm of a vector? This Ax is a vector. 2 norm of that is given by or square of 2 norm is given by Ax transpose Ax/x transpose x.

I want to find out this ratio which actually translates to finding out x transpose A transpose A x/x transpose x. I want to find out this ratio x transpose. So before we move to actually calculating the 2 norm, let me state few properties of norm. See when you call a function to be a function norm what are the conditions that it should meet? The first condition is that so we need to examine whether these ratios do they qualify to be called as norms.

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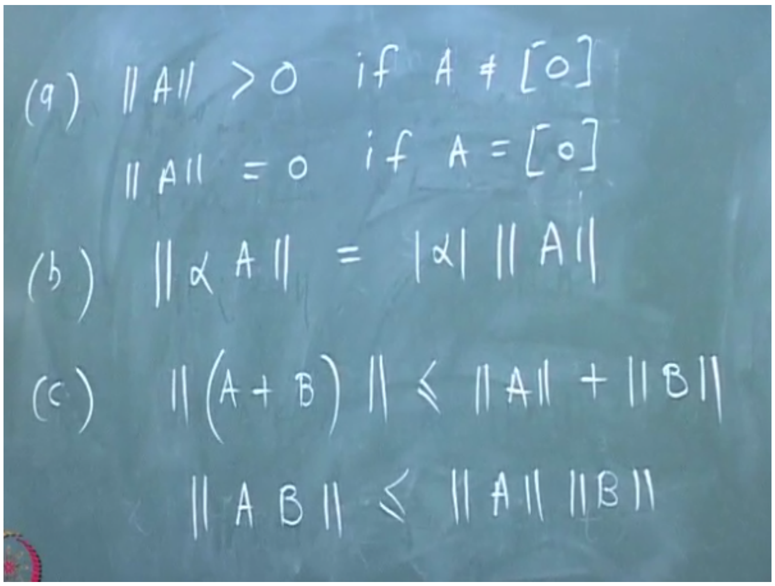

So the first condition for the first axiom for that a long function should satisfy is that norm of A should be > 0 if A is != the null matrix is != the 0 vector. 0 vector in the set of matrices is null matrix. So this should be > 0 if the matrix is != 0 and this A should be = 0 if A = null matrix. So this is the first condition that it should satisfy. The second condition that this function should satisfy is norm of alpha some alpha is some arbitrary scalar from the scalar field that is under consideration should be = mod alpha norm A.

It is very easy to check these conditions are satisfied by the given definition. The third condition is the triangle inequality. The third condition is norm of A + B if you take any 2 matrices A and B from this space or set of matrices n cross m matrices then what you can show that norm A + norm B. Norm of A + B is <= norm A + norm B. This is the triangle inequality starting from the definition it is very, very easy to show all these 3 axioms hold.

So these given definitions actually the given definition of induced norm actually satisfies all the 3 properties and it is indeed a norm and then there is one more property which is followed by all the matrix norm which is additional which is not part of the axioms is that if I have multiplication of 2 matrices AB then this is always = norm A norm B. This is always <= norm A + norm B.

This particularly would be useful for square matrices any arbitrary n cross m matrix you cannot probably. So before I actually do computations for 2 norm I want to give you a little bit of reinterpretation of definition of norm.

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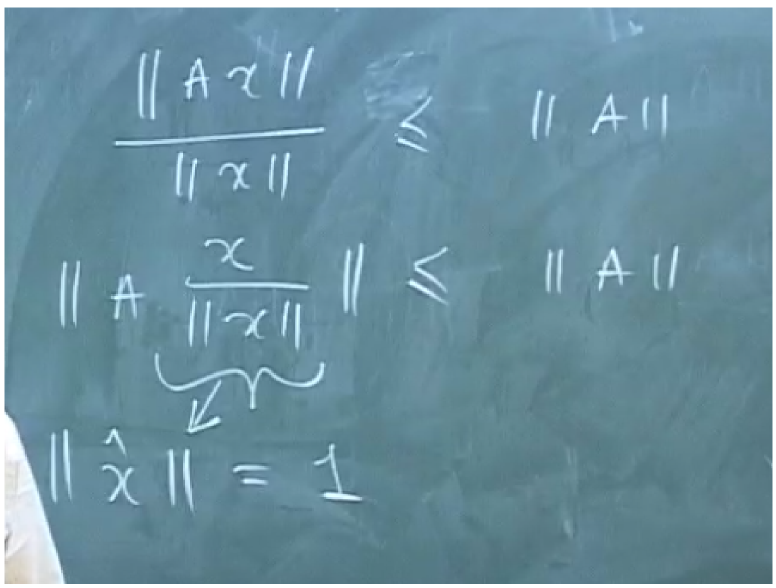

So we said that this ratio Ax/x is bounded by norm of A. So this norm of x is positive scalar. This x is != 0 so norm of x is a positive scalar and I can always rewrite this A times x/norm x<= A. What is this? This is nothing but if I call this as x cap, this is a unit vector such that this 1 nothing but a unit vector so for any arbitrary vector x x upon norm x is nothing but the unit vector so this will be x stat so norm of x stat will be = 1. So norm of x stat x upon norm x will always be = 1 and then.

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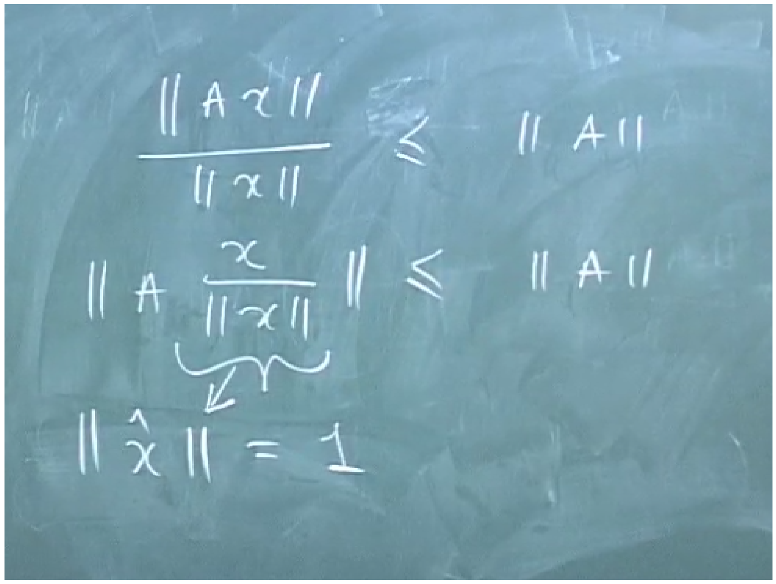

I could redefine norm of matrix A as set of you know max over. I could redefine my norm as max over x stat = 1 and of course this is for the set where this will happen, this will happen only when x is != 0 so that is implicit here where excluding the point origin that is implicit here so I could redefine by norm as x stat = 1. Norm A x stat is a unit vector. So if I take this let us say in 2 dimensions let us take 2 dimensions. If I take this unit circle in x1 x2 so this is x1, x2.

All the vectors can map on to this unit vector because you can normalize them by x by norm of x and then we are looking for and this is very easy to interpret we are looking for that value of ratio which is maximum when you move along this circle. When you move along the circle what is the point where you get the maximum of this norm Ax/x or maximum of the ratio when Ax cap or maximum of this norm Ax cap in the range space. So this could be another way of interpreting this induce matrix norm. Now let us now move on to computing the 2 norm.

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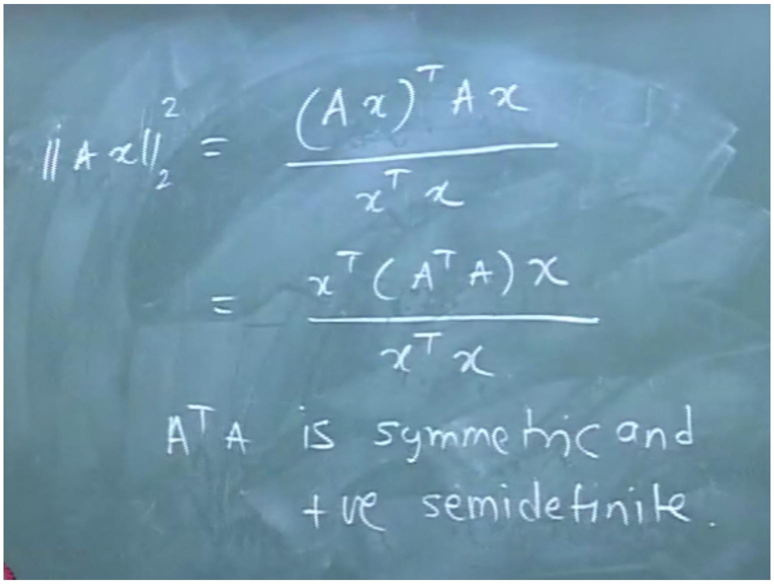

So for computing the 2 norm what I had said was norm Ax 2 square so I wrote this Ax transpose Ax/x transpose x which is same as x transpose A transpose A x/x transpose x. Now the question is how do I find maximum of this ratio? What I am going to do now is do some algebraic tricks and then find the maximum of this ratio. First of all, observe this matrix. This matrix is always a symmetric matrix. Transpose of this matrix is itself a symmetric matrix.

If A is a full rank matrix, A transpose A you know will what will be A transpose A? Will it be always positive definite or positive semidefinite? Depends upon what are dimensions of m and n. A transpose A is always a positive definite matrix or a positive semidefinite matrix. So this is symmetric matrix and it is always a positive semidefinite matrix. So A transpose A is symmetric is symmetric and positive semidefinite. Why it is positive semidefinite?

How do you define positive semi-definiteness? X transpose suppose you call this matrix B. X transpose Bx should be >= 0. Look here this is a vector in to this vector transpose this vector is always going to be positive or it is always >= 0. Even if x lies in the null space of A this will be this numerator can be 0 or it can be positive, it can never be negative. So that is why this matrix A transpose A will always be a positive definite or a positive semidefinite matrix.

Let us take a special case where A transpose A is a positive definite matrix. It is a symmetric matrix. Let us take a special case when it is a positive definite matrix.

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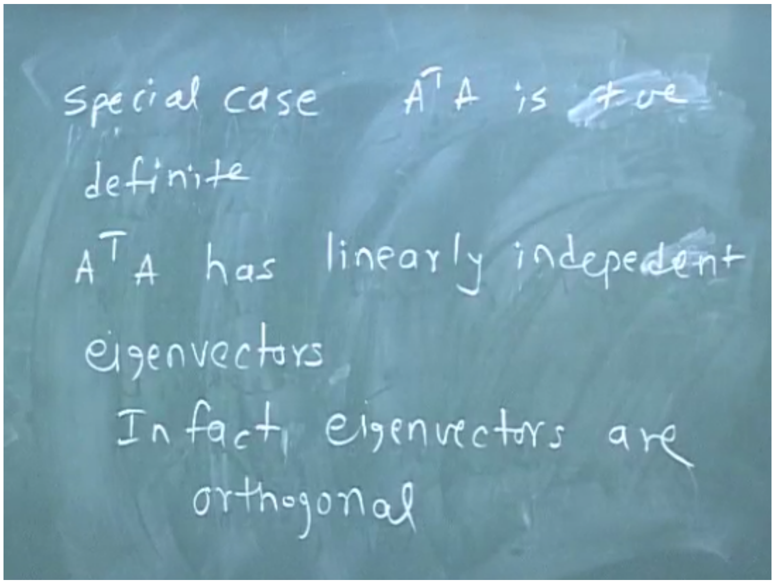

So it is a positive definite matrix. For the case of convenience let us further assume that A transpose A has linearly independent Eigenvectors. I am going to make an additional assumption that is this required? If it is a positive definite matrix symmetric positive definite matrix the Eigenvectors are linearly independent in fact something more.

They are orthogonal, symmetric, then they are orthogonal you can also choose them as orthonormal. So it follows actually from this assumption that A has linearly independent A. These are linearly independent Eigenvectors not only that so in fact the Eigenvectors are orthogonal. So symmetric positive definite matrix, the Eigenvectors are linearly independent and they are orthogonal that is much stronger property so I can write this A transpose A. I can write A transpose A.

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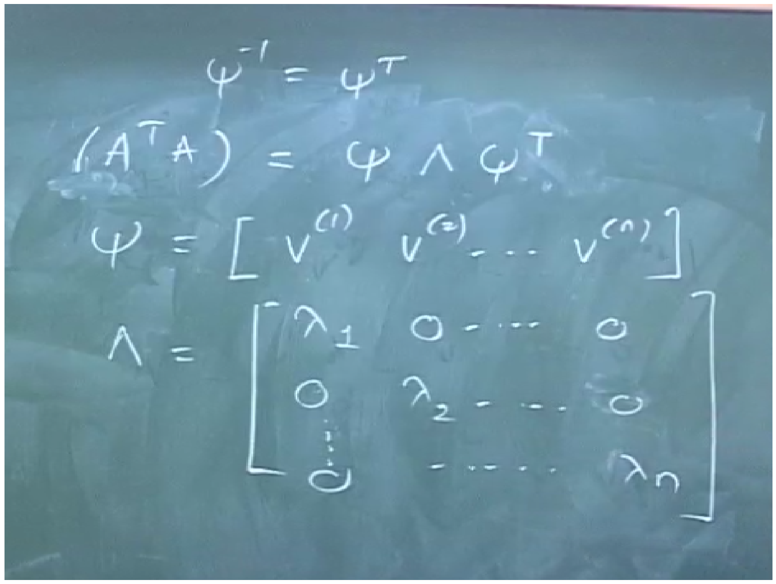

I want to split A transpose A as you know psi lambda psi inverse, I want to diagonalize this matrix. What is this psi matrix? This psi matrix consists of Eigenvectors of A transpose A. So let us take V1 is the Eigenvector of A transpose A, V2 is the Eigenvector of A transpose A. So these are n Eigenvectors of A transpose A then I can diagonalize this matrix A transpose A using Eigenvectors and this lambda is a diagonal matrix which consists of lambda1.

This is the diagonal matrix. I can rewrite this as A transpose A = psi lambda psi inverse. In fact this Eigenvectors of A transpose A they are orthogonal and it follows that this psi inverse is nothing but psi transpose we can show because the orthogonal property that psi inverse = psi transpose because the Eigenvectors are orthogonal you can choose them to be orthonormal. If you choose them to be orthonormal then this property will hold.

Psi inverse = psi transpose and then I can write A transpose A as psi lambda psi transpose. If you just want to see a quick derivation of this it would be, a very quick derivation of this would something like this.

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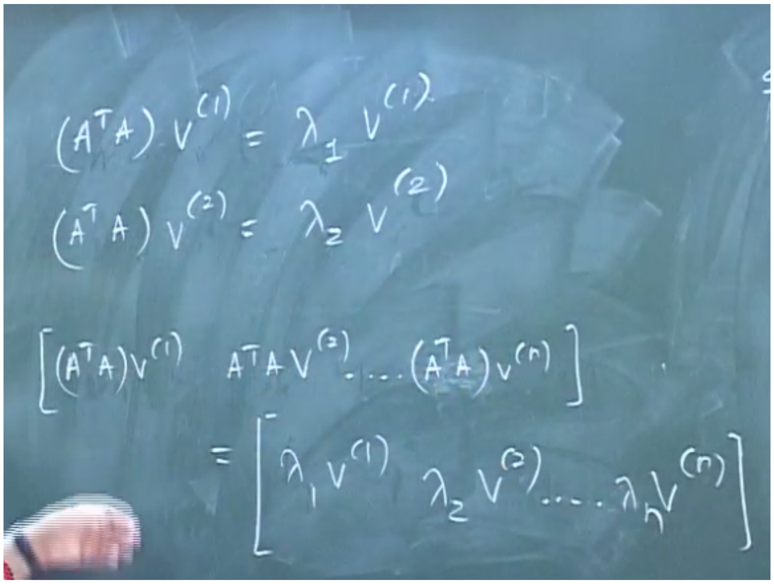

A transpose A * V1 = lambda 1 V1. This is the first Eigenvalue and Eigenvector combination of A transpose A. A transpose A V2 = lambda2 V2 and so on I can combine these equations as A transpose A V1, A transpose A V2. All that I have done is I have kept this vectors next to each other so this matrix = this matrix so I have combined all these equations into 1 single equation and then I can write this as.

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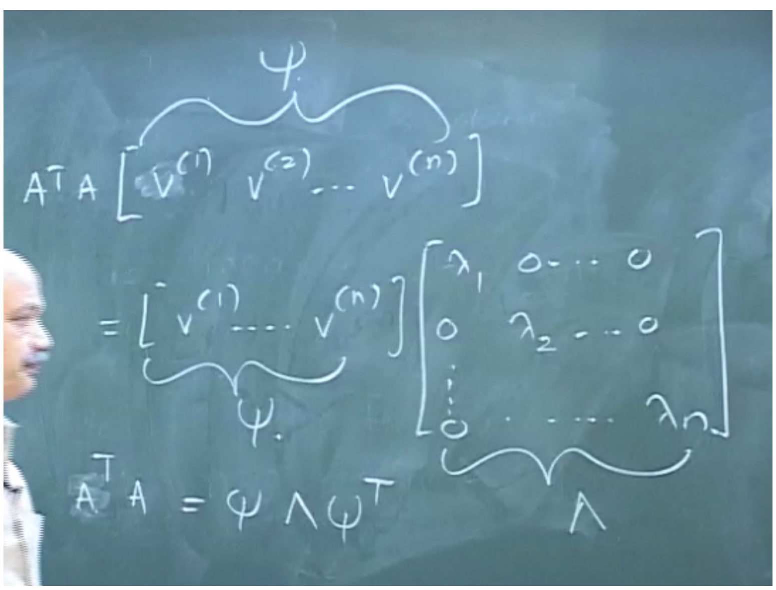

A transpose A * V1, V2, Vn = V1 to Vn * lambda1. I can rewrite this equation. This is my psi matrix and with this I can simply write because these are orthogonal vectors and orthonormal vectors and this is an invertible matrix. I can write so this is psi, this is psi so A transpose A = psi lambda psi inverse. I have chosen Eigenvectors to be orthonormal and this is my diagonal matrix lambda. So this derivation is very, very straightforward. So now having done this what I am going to do is I am going to use this to arrive at the norm of 2 norm of the matrix.

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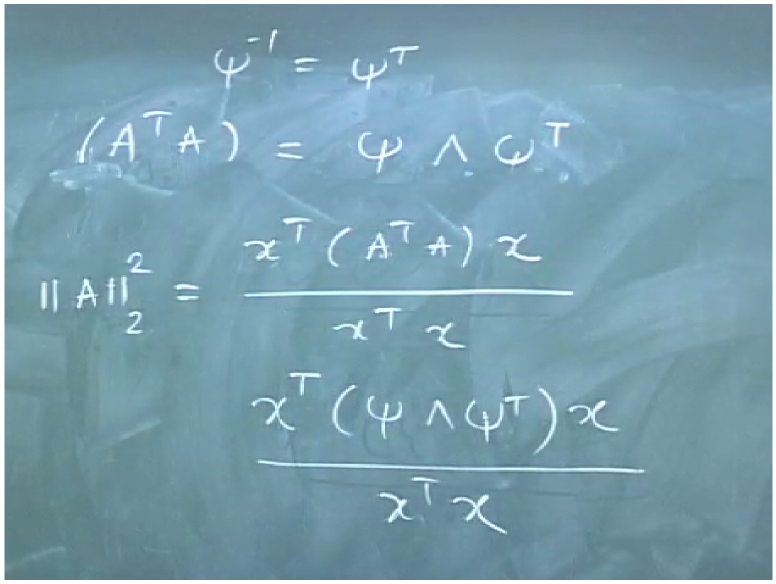

So coming back to our 2 norm definition we have x transpose A transpose Ax/x transpose x. What I am going to do here is to replace A transpose A by this term so this is equal to so this is 2 norm square = this term. So this is nothing but x transpose psi lambda psi transpose x/x transpose x. I am going to define a transformation now which is.

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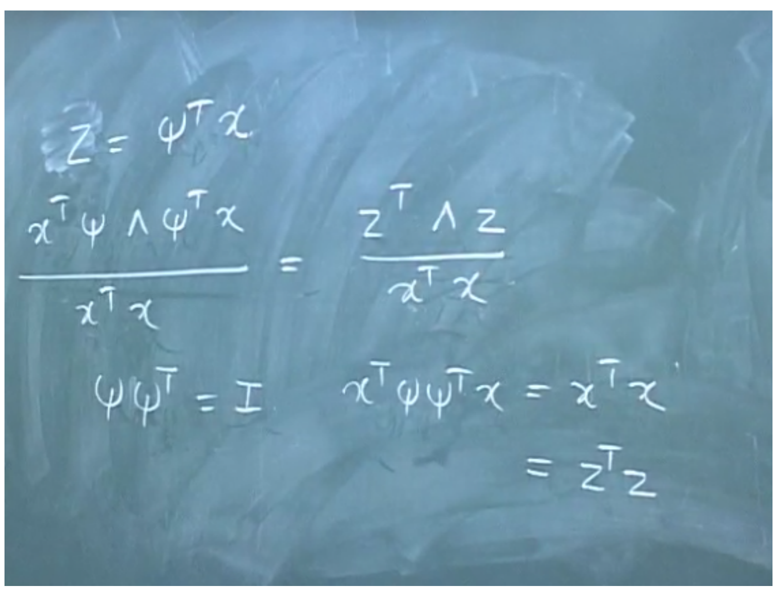

I am going to define a transformation z which is = psi transpose x. Using this transformation I can write x transpose psi lambda psi transpose x/x transpose x. This is = z transpose lambdaz/x transpose x. Now I want to play 1 more trick. I know that psi psi transpose = I. Using this identity, or psi transpose psi = I. Using this identity, I am going to write this as x transpose psi psi transpose x = x transpose x because psi psi transpose = I. So this is nothing, but z transpose z.

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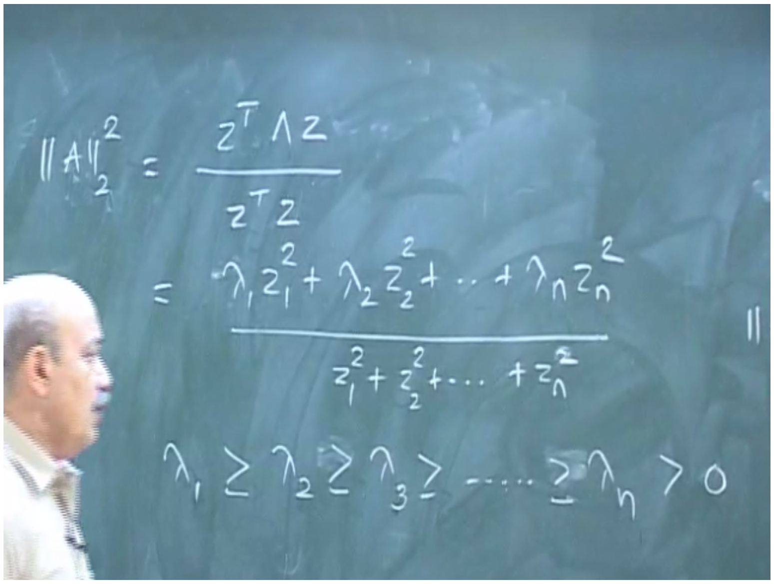

Now with this transformation my 2 norm square this becomes z transpose lambda z divided by z transpose z. Now this is the diagonal matrix. So this is nothing but lambda1 z1 square + lambda2 z2 square, lambda n zn square/ z1 square + z2 square to zn square. So I have expressed square of A norm of A in terms of this ratio of 2 polynomials. What you can see very easily that the numerator is always a positive number because A transpose A is a positive definite matrix.

All the Eigenvalues of A transpose A are positive. So this is always a positive number divided by this which is a positive number. Now let us order the Eigenvalues. Let us number the Eigenvalues. We can number the Eigenvalues the way we want. I am ordering them in such a way I am numbering them in such a way that lambda1 is the largest magnitude Eigenvalue. Lambda1 is the largest magnitude Eigenvalue.

What you can show is here? lambda1 is the largest magnitude Eigenvalue. So I can replace lambda2/lambda1, lambda3/lambda1, lambda4/lambda1.

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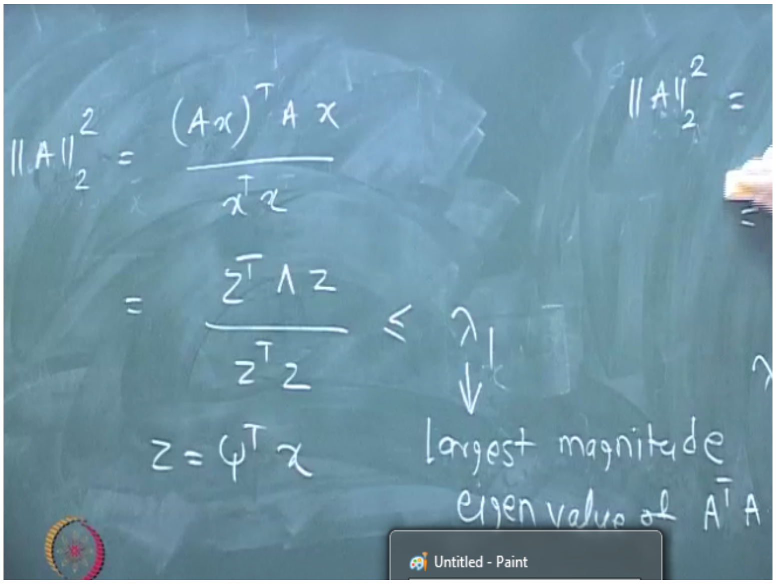

What I can show is that this ratio here is always <= lambda1 square z1 square + lambda1 square z2 square. So lambda1 * lambda1 * z1 square + lambda2 what I have done is in this particular case I have just replaced lambda2/lambda1, lambda3/lambda1. So this inequality will always hold to this is equal to lambda1. So lambda1 is an upper bound on this ratio. So what we have effectively shown is that I just move here what we have effectively shown is that norm A2 square = x Transpose x/x transpose x.

This is = z transpose lambda z/z transpose z which is <= lambda1. This is the largest magnitude Eigenvalue of A transpose A. So this is where z = psi transpose x. Psi consisted of orthonormal Eigenvectors of A transpose A. So this here is largest magnitude Eigenvalue of A transpose A or the largest singular value of A in other words. So what it turns out that if I come back here it turns out that for a particular value of z the bound is attained.

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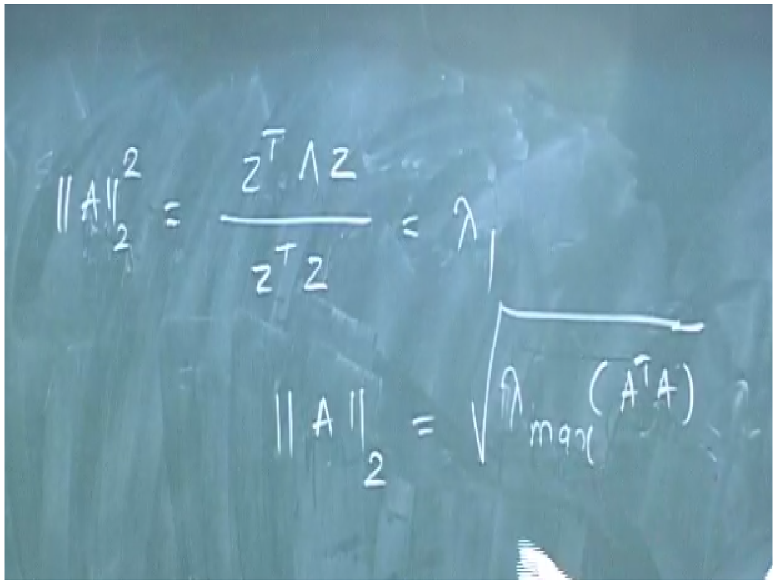

So we can actually show equality and then further we can show that 2 norm of A is nothing by square root of what we have proved effectively is that 2 norm of A is nothing, but maximum magnitude Eigenvalue of A transpose A. A transpose A is positive definite. Even if it is semidefinite this equality will hold I have just done everything for a simpler case. So this is what we have proved that 2 norm of a matrix can be computed as maximum magnitude Eigenvalue square root of the maximum magnitude Eigenvalue of A transpose A or square root of the maximum magnitude singular value of A.

It is also called singular value of A so but again there is a problem. Eigenvalue. So 2 norm we wanted to computed something which is computational is simple for S inverse T but again we are stuck with Eigenvalue. So this is not convenient. we will move on in the next lecture to finding out 1 norm and infinite norm which are more convenient. Nevertheless, this derivation is quite gives you insight into how the norm can be computed.

And that is why for the more the (()) (57:37) reason I have done this derivation, but more useful thing is 1 norm and infinite norm which will be discussing in the next lecture.