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NATIONAL PROGRAMME ON
TECHNOLOGY ENHANCED LEARNING

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ADVANCE
PROCESS CONTROL

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Lecture No. 05

Topic:
Development of Control
Relevant Linear Perturbation
Models (Part 1)

Sub-Topics:
Introduction to Stability Analysis
and Development of Output Error
Models'

So we have been looking at linear perturbation models which are developed from mechanistic models by linearization. So you have a good mechanistic model, you would use it for control, we have chosen an operating point where you want to control the system , lineraized the differential equation, discretize them and come up with them, discrete states first model, we may not know the first principle model accurately.

In that case I said well, you probably know at least the structure of the linearized matrices, and then you can identify elements some kin d of an optimization problem. And the structure is known a priori for simple systems, you can come up with the structure by and do it from your engineering knowledge about the process.

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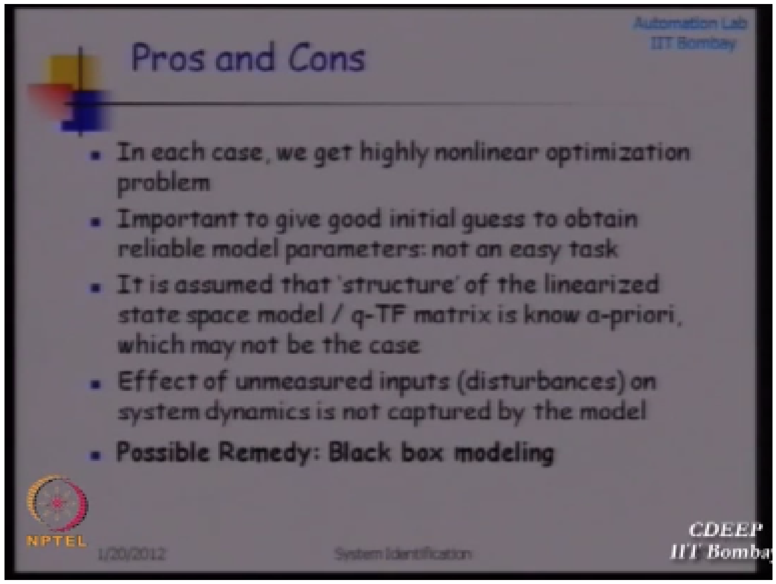

So the system has some three or four states and two inputs, two outputs or three inputs, three outputs, three measurements. Input systems it can argue from physics, a very complex system would become difficult to with log of feedback loops, it becomes very difficult to come up with the structure which is – anyway the other problem we are just looking at deterministic effects what about stochastic effect, what about effects which are disturbances which are unmeasured which are not exactly quantifiable through our first principle model. How do you model them, and I said the possible remedy is a black box model.

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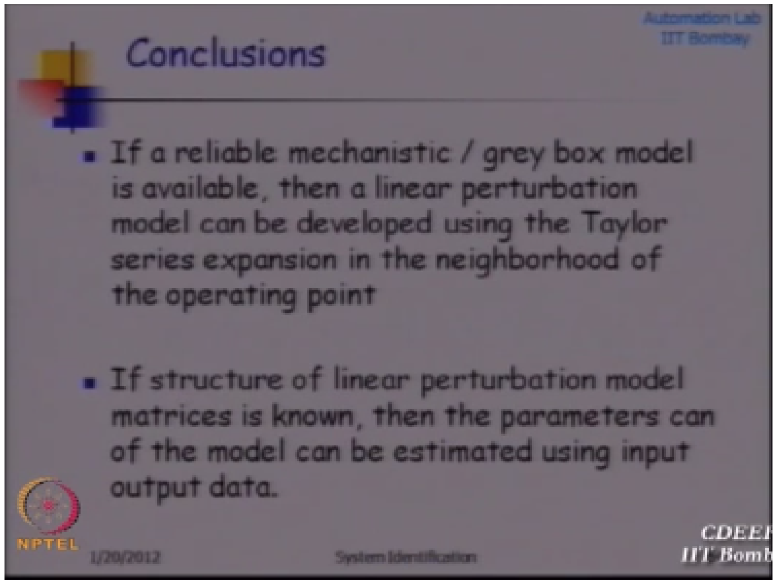

So in conclusion before I move on to next spot, but I will just say that a reliable mechanistic model is available, then one can always linearize, develop a perturbation model and use it for further controller synthesis. If you do not know all the parameters, if you just know the structure you can develop perturbation model from data. Now I want to move to this new topic, th e new topic is estimation of models directly from data without knowing any structure okay.

So I am going to call it completely black box model, and the analogy I gave is to a correlation okay. So effect correlations we are actually not using in physics, we are just saying that this is a dependent variable, this is a independent variable that correlate it through some polynomial. Likewise, I am going to say measurements are dependent variables, inputs are independent variables.

And they correlate through a differential equation or a difference equation. What is a difference equation is what I want to find out okay. Before I move into that, I need to do lot of preparation, one of them is to say a few things about stability okay.

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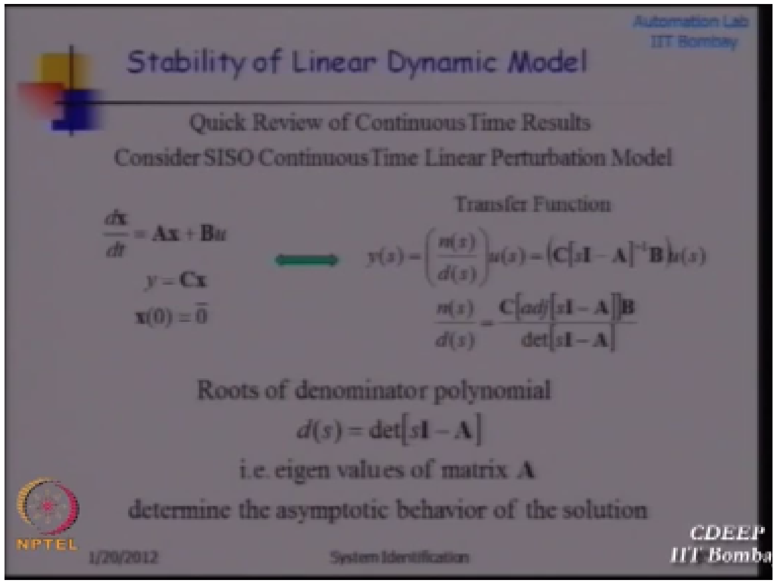

So I am just going to do a quick review of stability of linear dynamic systems. I am going to revisit this again after sometime, but here this concept one particular concept I need when I move to these time series models, and that is why I need to quickly review some concepts that look. Now all of you are done first course in control right. What is the condition for stability, if you are given a transfer function, show of the denominator polynomial, this is what is the behavior of the solutions.

And then what is so powerful about transfer function idea? Is that just look at the roots and you can say how the things are going to have behave asymptotically. You just look at in a looks and say that if there are vectorial part, you know you will get asymptotically behavior. If you have one of the roots which is on the right half plane in the S plane you have trouble, the solutions will go to ∞.

And then there are imaginary stables still imaginary stable systems on the imaginary axis, this all of you know about this, I just want to quickly review this collecting with what we have done just now it takes place models. And then I also want to relate this to discrete time models, because some stability ideas we need as we move on to, and as I said after I have done with modeling I am going to revisit the concept of stability again.

Because very, very it is the central concept to control okay. So this is what I am going to do that of very quick review of stability ideas. Now you consider the single input, single output system sees how is single input and single output, one measurement, one input simple system okay. we know that the transfer function for a single input, single output system will be given by some numerical polynomial.

See here all this numerical polynomial by denominator polynomial, and I want to relate this to this states place model, we develop a linearize state place model if you remember right. So X here other perturbations in the states, A is the coupling matrix, B is the input coupling matrix, A is the coupling matrix with the current states, I bring in the information about the past. And what is the relationship between these states place model and transfer function model, I have just given here on the right hand side.

The states place model is related to the transfer function through $C(sI-A^{-1}B)$ okay. I just relate in this as C(A(sI-A) cofactor transpose right into B, this is the numerator, this will be a sing le polynomial, why it is a single polynomial? You have one output, one input okay, what will be the denominator polynomial? The denominator polynomial will be determinant of sI-A okay.

So if a transfer function relating Y and U okay, roots of denominator polynomial is nothing but eigenvalues of, you see that, what is determinant of sI-A=0, the roots of the characteristic polynomial which is same as eigenvalues of matrix A. So the roots of DS, denominator polynomial which is same as eigenvalues of matrix A okay will be determined how the solution YT behaves asymptotically. If you random between this right okay.

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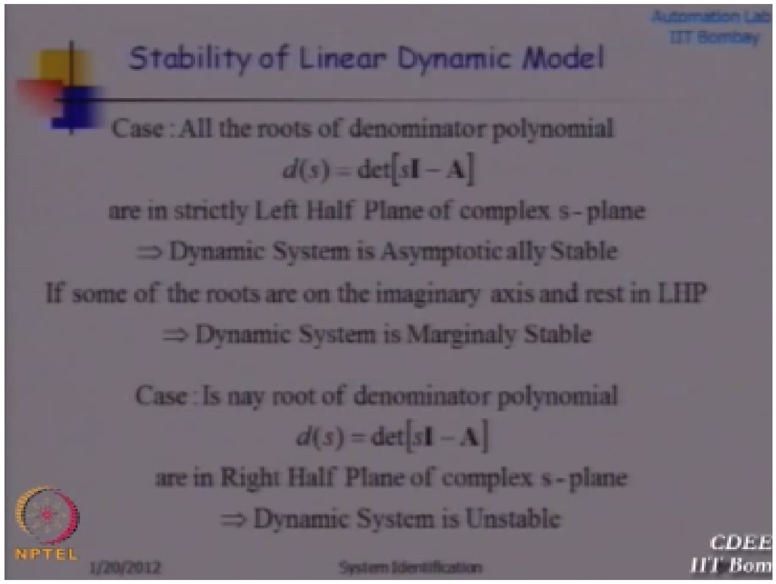

Let us quickly review what we know, what is the first case, one case is that all of you said this, that if the roots have negative real part, the roots can be complex, eigenvalues can be complex, roots can be complex okay. So we have to work in complex S plane and we have to locate where the roots are lying. If the roots are real okay, you may have and negative, you will have exponential decays.

If you have roots which are negative, but complex, negative real part, but complex, you will have oscillatory decays all of this you have studied in your course in control okay. So first thing is of course if the roots are strictly left off plane or LHP o f, then the transient of the system are asymptotically stable. Well what if some roots are on the imaginary axis, or it is a marginally stable system.

System may have transients which never die, but which they will grow okay. So you have bounded transients okay. So such systems are marginally stable systems. And this of course you know that, if anyone of the roots is on the right of plane, you have trouble okay. Please keep in mind, this is analysis of a linear perturbation model, linear perturbation model is an approximation in neighborhood of some operating point okay.

Well, I am showing same here like the solutions go to ∞ does not mean unstability actually of a linear system, you can see that the solution will go to ∞. In reality this is not going to happen, all it means is that a system has a tendency lot to stay at that point, turn away from that point, that is how it should interpret, take this going to ∞ with a pinch of salt.

So unstable means, you know it is like the pendulum, a pendulum has a stable state if it is this case, a pendulum is moving like this either stable state here, but talk about inverted pendulum okay, inverted pendulum this is an unstable state, you try to pertubate from this, it will try to go to the next stable state okay. It is not the solution truly does not go to ∞, what it means is that locally the solution has tendency to move away from that point okay.

If you at that point you will stay at that point, but which is very difficult okay. What is analog of this when you come to discrete times you okay?

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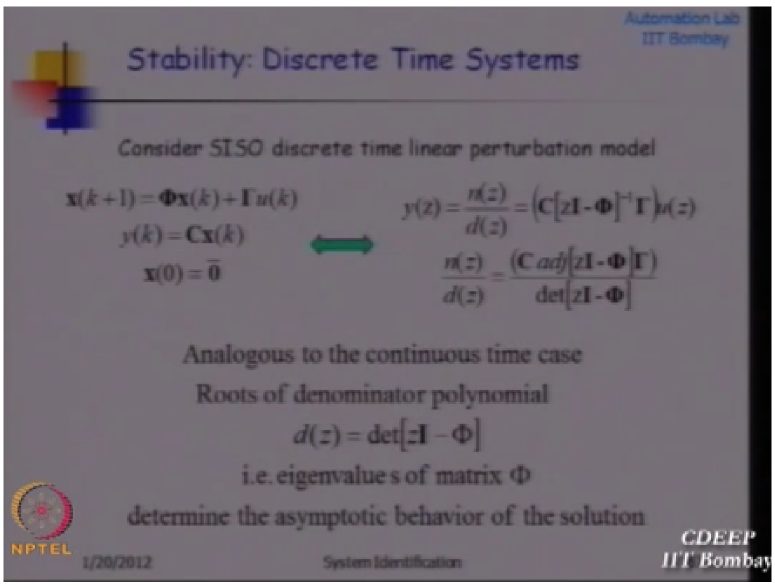

Now well some of you probably have done this course on discrete time systems for them it is just quick review now just consider a SISO discrete time linear perturbation model okay we have done this already right we have seen that this state space model which is actually discredited from the continuous time model okay is equivalent to this transfer function here okay can you says something my analogy now.

What would determine the dynamic behavior determinant of okay determined of zI - ϕ which is the denominator polynomial okay roots of this denominator polynomial will determine how asymptotic behavior of this dynamic systems okay so this is right now purely by analogy I will try to give you logic why this is a okay so just if I draw apology then I should just look at the roots of the denominator polynomial.

Okay let us try to see why dynamics is governed by Eigen values of so what does it what does govern the Eigen values of the system sorry what governs the stability of okay so how do Eigen values of ϕ influence the dynamic behavior okay.

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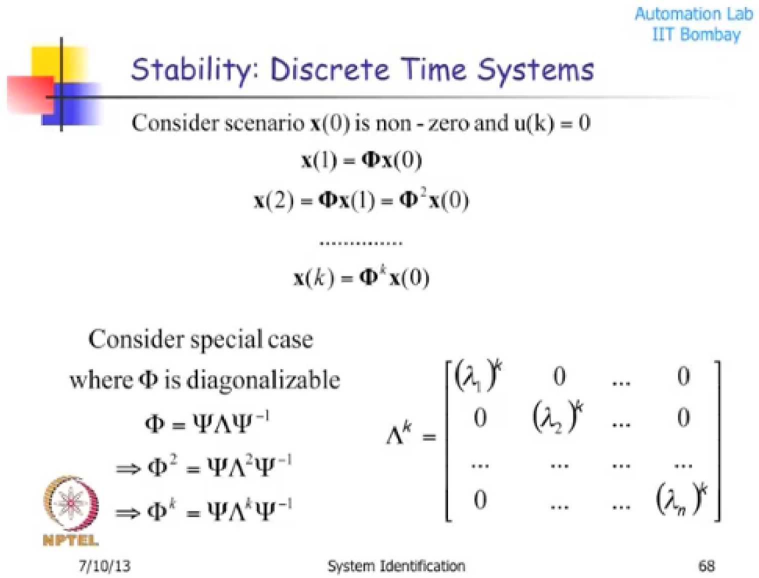

So let us look at a scenario I am looking at a scenario where x0 initial state is not equal to 0 but inputs are 0 okay let us look at a simple very simple scenario all the inputs from time 0 to infinity are 0 okay initial state is not equal to 0 okay it is saying have a model for a pendulum okay 0 means this stable state okay what up states means you are here okay so I am now looking at a system which is per term okay.

And then how is it going to progress in time okay is it going to go and die or the oscillations are grow okay that is what I am interested in knowing okay.

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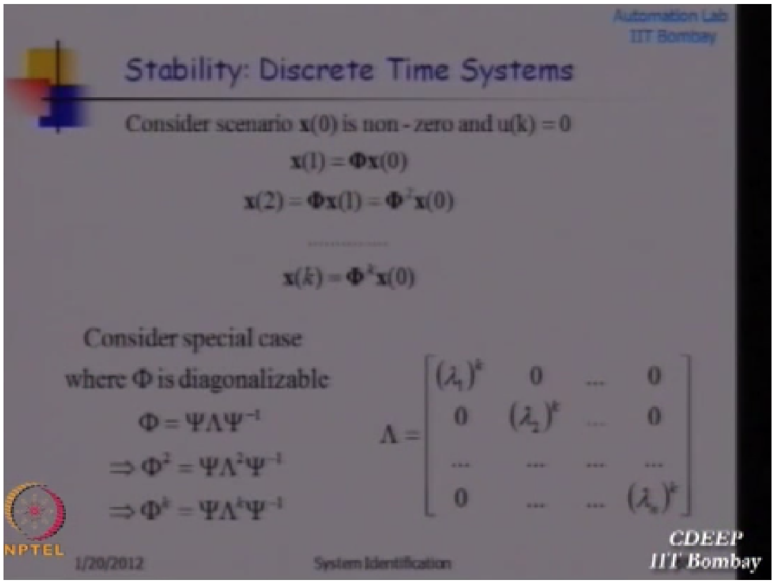

So or let us g back to our quatderable tank okay you have a scenario that there is some the system was perfectly at steady state okay the system was perfectly at steady state and then somehow for some reason the inputs are constant for some reason the level are brought up from the initial steady state okay I am holding the inputs constant somehow the somehow the levels have become brought from the initial steady state.

Question is okay if you have brought out from the initial steady state will you return to the steady state okay or will go away from the steady state okay so that is governed by Eigen values of ϕ.

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Automation Lab
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Stability: Discrete Time Systems

Consider scenario $\mathbf{x}(0)$ is non - zero and $\mathbf{u}(k) = 0$

$$\mathbf{x}(1) = \mathbf{\Phi x}(0)$$
$$\mathbf{x}(2) = \mathbf{\Phi x}(1) = \mathbf{\Phi}^2 \mathbf{x}(0)$$
$$\ldots\ldots\ldots\ldots$$
$$\mathbf{x}(k) = \mathbf{\Phi}^k \mathbf{x}(0)$$

Consider special case where $\mathbf{\Phi}$ is diagonalizable

$$\Phi = \Psi \Lambda \Psi^{-1}$$
$$\Rightarrow \Phi^2 = \Psi \Lambda^2 \Psi^{-1}$$
$$\Rightarrow \Phi^k = \Psi \Lambda^k \Psi^{-1}$$

$$\Lambda^k = \begin{bmatrix} (\lambda_1)^k & 0 & \ldots & 0 \\ 0 & (\lambda_2)^k & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & \ldots & (\lambda_n)^k \end{bmatrix}$$

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Why is it governed by Eigen value of ϕ now see look at model here I am starting with time 0 x0 the initial state which is non zero what will be at time 1 sampling time 1 ϕ times x0 right what will be at sampling time 2, ϕ times x1 which is actually ϕ^2 times x0 okay likewise if you just write this equations you can very easily show that at any time instant k okay the state at time instant k is ϕ^{k-} x0.

Solution of linear difference equation okay I am just recursively solving substituting one after another I will get ϕ to the power is everyone with me on this ϕ^k okay so now we have to understand how does ϕ^k behave okay now I am looking at now look at this here let us look at a special case right now well general case of course the slight modification the same idea works let us take a special case where ϕ is diagonalizable.

What do you mean by ϕ is diagonalizable Eigen vectors are linearly independent okay Eigen vectors are linearly independent okay so I can write my ϕ has ψ λ ψ inverse this we have done earlier right and then ϕ^2 becomes ψ λ^{2-} ψ inverse ϕ^k becomes ψ λ^k ψ inverse right so these are linear difference equations well if you have done stability around this is in course on numerical methods now we have done this in numerical methods we had a similar equation except in time we have iteration index right and we where analyzing error of iterative systems we are exactly same equation okay so what is λ^k λ is the diagonal matrix okay so λ^k is simply $\lambda 1^k$ $\lambda 2^k$ $\lambda 3^k$ so what is going to really see ψ is not going to change.

Once you decide the Eigen direction ψ and ψ reverse are constant once you choose Eigen directions or Eigen vectors these two matrix rises ψ and ψ inverse they remain constant so has k has time progress what changes λ^k right λ^k changes as time progress okay so now.

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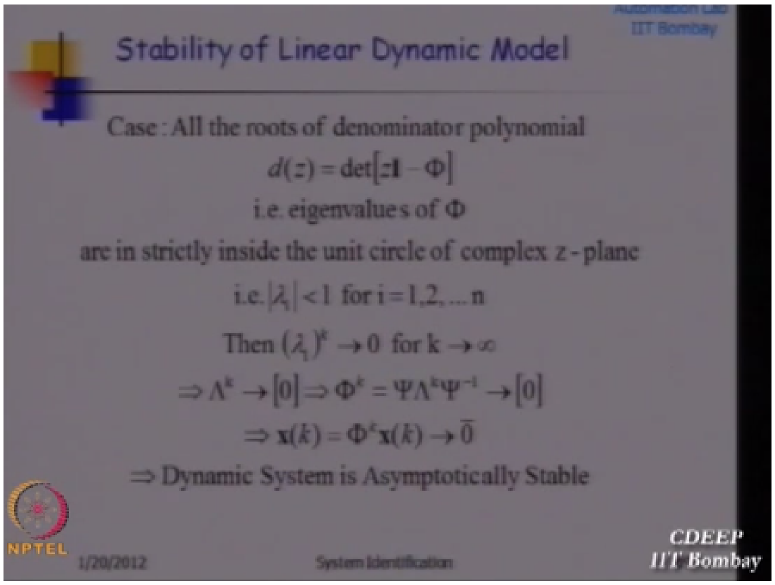

First case is that you know all the roots of the denominator polynomial okay are strictly inside unit circle okay are strictly inside the unit circle in the complex theory what happens if λ is strictly less than 1 is λ less than 1 λ^k as k goes to infinity see if my λ.

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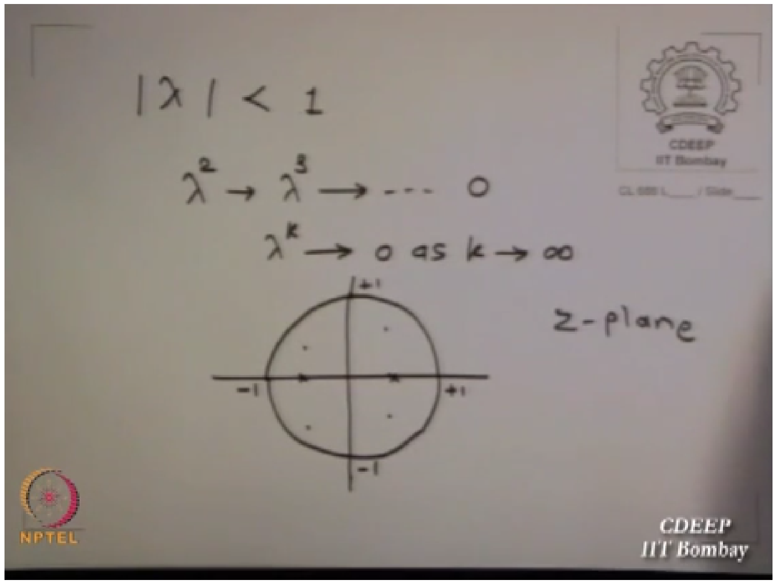

Is mod λ is strictly less than 1 then what I know is λ^2 λ^3 progressively this will go to 0 as okay so λ^k will tend to 0 as k goes to infinity okay λ^k will go to now what will happen if all the Eigen values are inside the unit circle what is this unit circle actually I am looking at z plane complex z plane and I am looking at this unit circle okay so this is +1 this is -1 this is +1 this is -1 okay this is the unit circle.

If λ is lying somewhere here or here or here or here or here or here where ever if it is inside the unit circle λ is a number lying inside the unit circle λ^k is always going to.

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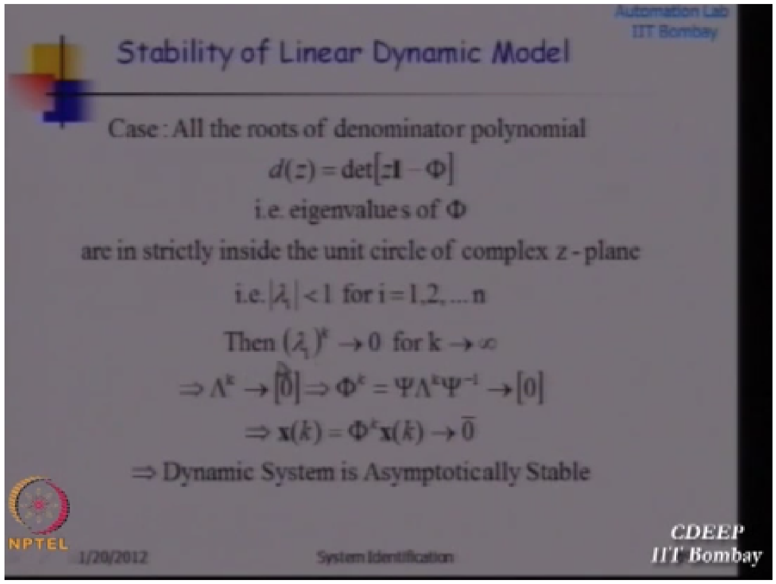

Diminish as k increases okay you are always to going to diminish as k increases okay so what happens λ^k is going to go to 0 as k goes to infinity so which means matrix λ^k is going to go to 0 null matrix is going to go to null matrix as times goes to infinity well which means ϕ^k is go to tend to null matrix as time goes to infinity okay so if my system is asymptotical stabile that is if consider a 4 tank I am just brought up from the initial point my inputs are constant okay.

The system is asymptotically stable if all Eigen values of the discrete time model are inside the unit circle then you know the levels are going to decay and settle at original steady state okay they are going to settle at the original steady state this will happen provided all the Eigen values of matrix ϕ or inside unit circle in the complex z okay so this is very critical such a system you will call as asymptotically stable system.

So what is an equivalent thing what is an equivalent stability condition for a z transfer function the roots of the denominator polynomial if it s a transfer function it should be inside unit circle in z plane or discrete time state space model Eigen values of ϕ should be unit circle 2 things are 1 and the same because transfer function is only a representation in z domain state space model is representation in time domain okay so what Eigen values convey you there in time domain same thing is convert to you by the roots of dz okay in the z domain okay.

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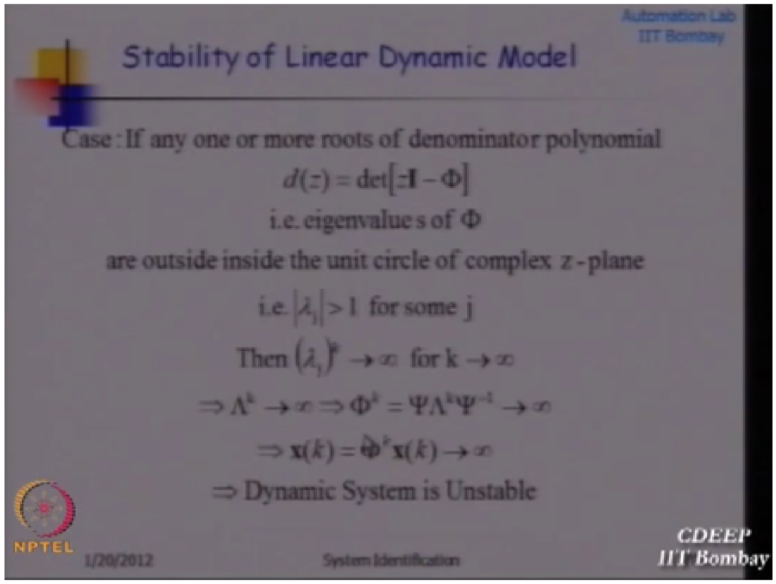

Okay, thanks for this correction so ϕ^k which is ϕ^kx0 this will tend to okay with this the yeah, ϕ^kx0 will tend to 0 vector okay, and the perturbation will die so the stability is decided by roots of the characteristic equation or roots of the denominator polynomial okay, which is same as Eigen values of migrates ϕ okay, you are look at the relative location. See the same power which you have in continuous time what is the power of analysis in the continuous time just look at the Eigen values you can say how it is going to behave same thing is here.

Just look at the Eigen values of ϕ or look at the Eigen value of the denominator polynomial it will tell you how the dynamic system is going to behave asymptotically okay, as time goes to infinity how will the system behave okay. Now what is you know you have scenario.

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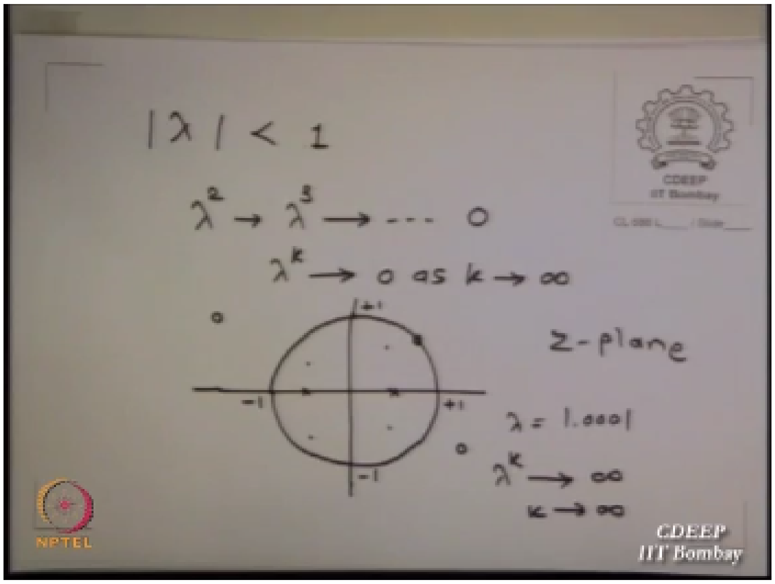

Where Eigen value is located here or it is located here somewhere outside and if it is 1.0001 okay, if it is 1.001 suppose λ=1.001 then λ^k will go to ∞as k goes to ∞ okay even if the value slightly greater than 1 okay, that to the power k will go to ∞ so the solution will diverge okay, the solution will diverge even if one Eigen value is outside unit circle. What about if the Eigen value is on the unit circle it will neither diverge nor converge it will be bounded the system is marginally stable, okay.

So equivalent to the continuous time case now we have result which talks about a discrete time case stability okay, roots of the denominator polynomial if you have a transfer function look at Eigen values of ϕ if you have a state space model. The relative location inside unit circle or outside unit circle or on the unit circle will tell you how is the dynamic system behaving, okay. Okay, let me just quickly do one more thing.

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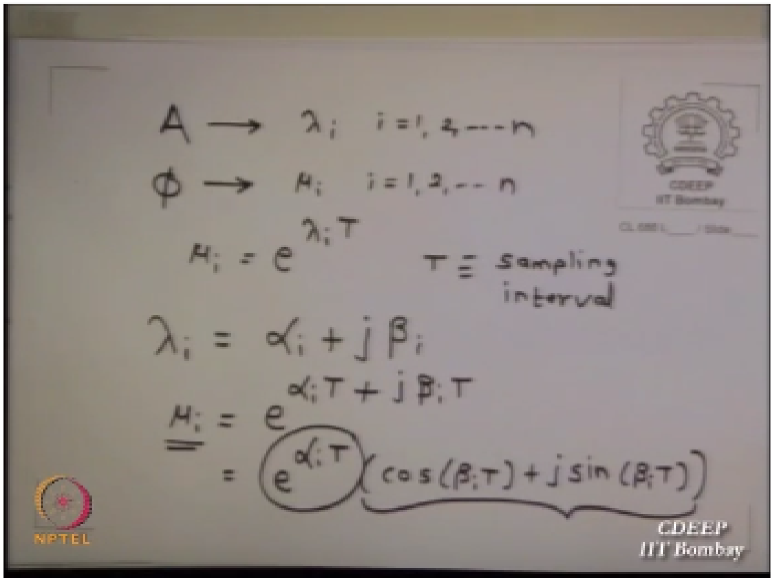

What is the relationship between a pole in the continuous time, what is a pole roots of the denominator polynomial in well you might have created different terminology we call it poles okay, but they are nothing but roots of the denominator polynomial okay, which are Eigen values of ϕ matrix or A matrix or whatever the case you wish. Now we are looking at continuous time pole so suppose you have this A matrix which is coming from a continuous time system and it si Eigen values are λi, i=1,2,n okay.

And you have this matrix ϕ let us call it is Eigen values as μi, i goes from 1,2,n okay, what is the relationship between Eigen values of A and Eigen values of ϕ okay. first of all you should understand one thing if your original system is stable in continuous time it will be stable in discrete time stability asymptotic behavior or solutions is a fundamental property that is not going to change because you change you know from differential equation to difference equation, okay.

If a system is stable it is stable if it is unstable it is unstable okay, that is not going to be changing because of change in the you know mathematical description of the system continuous time discrete time these are mathematical convenient descriptions under different context in a computer control systems we use discrete time representation that is why it is convenient to work with, okay. So the relationship between this is μi the exponential λiT, where T is sampling time sampling interval, okay.

Now how does this relate to the stability aspect see my λi let us say is equal to $\alpha i+j\beta$ say in general a Eigen value λi can be a complex okay, it can be a complex so $\mu i=e^{\alpha iT+j\beta iT}$ everyone with me on this okay, so which is equal to $e\alpha iT$(cos) is that okay. Okay, what can you say about this quantity modulus of this quantity is always bounded between + or -1 sin and cos okay, it is never going to exceed + or -1 okay, so how does μ behave will be decided by $e^{\alpha iT}$ okay.

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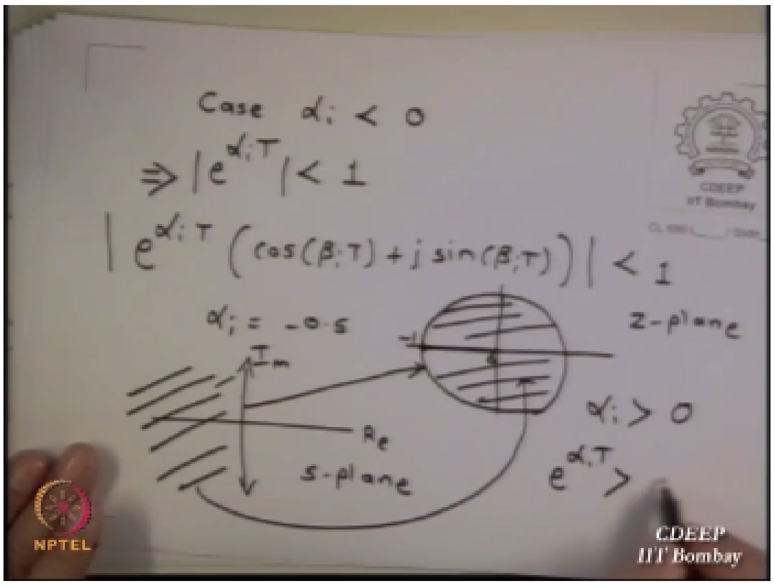

Do you agree with this okay, is if αi is strictly less than 0 okay, then $e^{\alpha iT}$ for any positive T of course, any positive sampling time is never going to be negative okay, you cannot sample with negative sampling time you will have to do a sampling time positive so if αiT is negative $e^{\alpha iT}$ will be strictly less than 1 or mod of actually $e^{\alpha iT}$ is the positive number I do not know it would mod, maybe strictly less than 1 okay, and so what happens here is you know $e^{\alpha iT}(\cos(\beta iT)+j\sin(\beta iT))$ mod of this is also strictly less than 1, do you agree, okay.

So a stable Eigen value in continuous time maps to a stable Eigen value in discrete time very, very important okay, fundamental characteristics of stability does not change because you found go from continuous time representation got me, always. Well, convention there is some problem because discrete time systems are some values for which there is no equivalent continuous time system okay, so see what is, what if $\alpha i=-0.5$ you cannot find an equivalent continuous time representation on particularly see the trouble is here.

You take this unit circle okay, and what you can show see this is my z plane okay, and let us say this is my s plane, so what I am saying is that is just left of plane it will actually maps inside the unit circle, the left half plane entire left half plane maps inside the unit circle okay. But there is one problem, problem in the sense that this line this part 0 to -1 okay, there is a problem you cannot find a continuous time analog which will map to a pole on 0 to -1, e^{RT} is never going to minus right.

So rest of the plane will map everywhere except that, except this segment okay, so if you have a discrete time system which has a pole here you cannot find a continuous time analog but okay, so otherwise it maps inside here in this continuous time stable part maps inside here. Where does the imaginary axis map, see this is the imaginary axis, this is the real axis circle the imaginary axis maps to the circle boundary and where is the right half plane map outside the unit circle okay. If α I > 0 then e αi > 1 okay, so this is just a brief review of stability of dynamical system will visit stability again.

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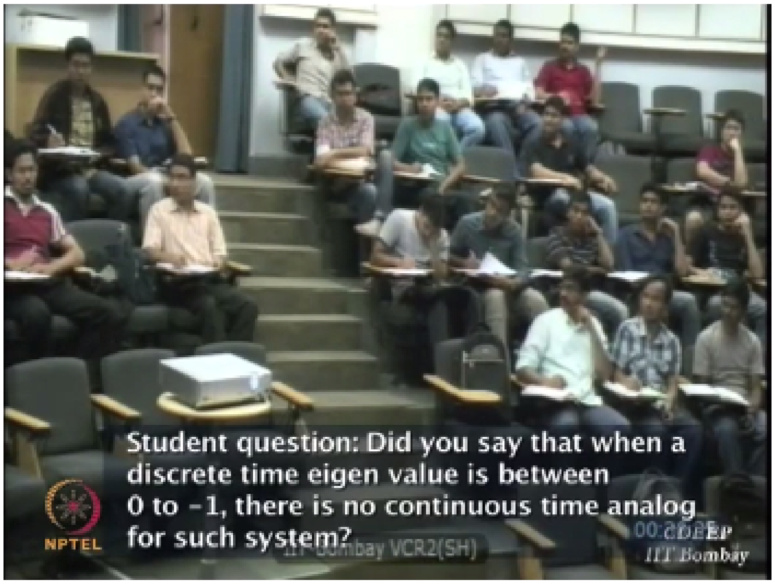

You cannot find at the qln continuous times is two images, yeah on the right on 0 to 1 there is no problem 0 to -1 there is a problem but complex time constant does not have meaning in the okay you will decide the complex ijh will map in to that that well single will not pare will.

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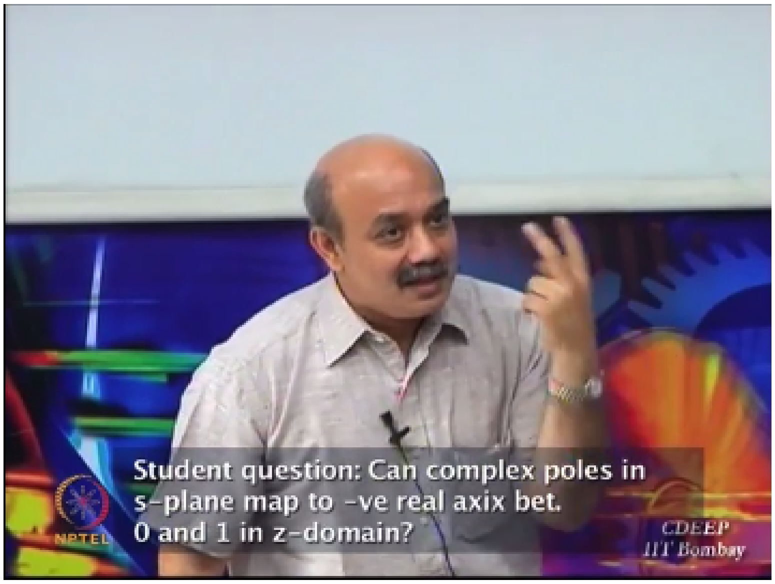

In continuous time you cannot system which are the single time constant which is complex if complex appear it will appear in pare so two can map but not one okay, a pare can map but single negative time constant mapping in to a single is not possible okay that is a trouble yeah.

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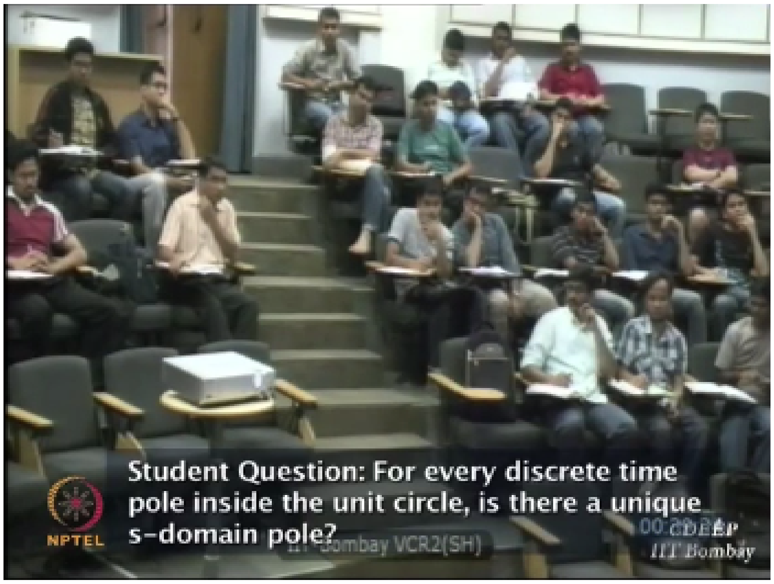

No that is that cannot be set the choice of different continuous time systems with it depends upon actually it is very that is good question with see let us take a simple system which is you know τ s + 1 okay.

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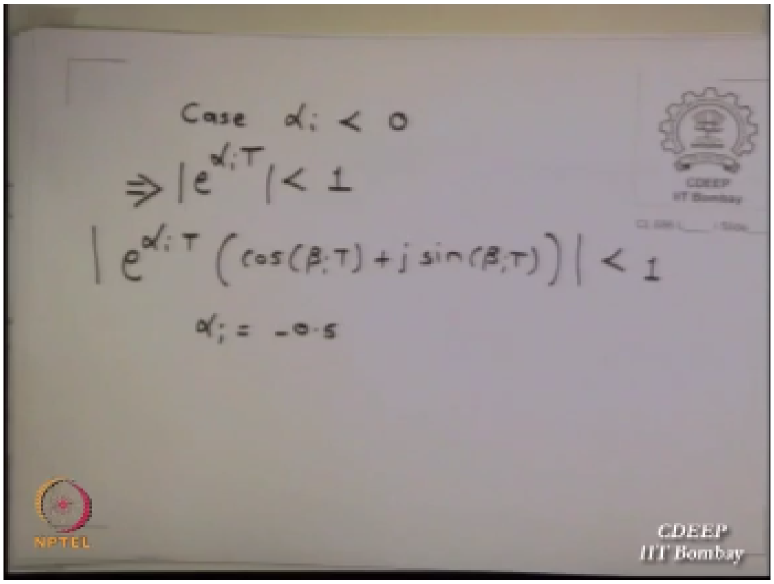

Let us take a simple system transfer function so gs = τ s + 1 so this will map in to a discrete time system okay where you know the discrete time pole is e –t / τ okay, time discrete time pole of if you convert this in to discrete time systems you will get the pole of the discrete time system to be this. now this ratio t / τ okay can come from see this ratio can come from different systems okay, see suppose this is one second and this is 10 second the ration will be 0.1 this is one minute and this is ten minutes ration will be 0.1 okay.

So multiple continuous time system is will map in to same discrete time systems the map is not unique okay, so when you go from its many to one map so when go from discrete time to continuous time there can be infinite possibilities if you have to fix something if you fix say sampling time then you get a unique okay, so the other way is very difficult to say when you go from discrete time continuous time function which is giving you this particular system okay.

Will have this kind of thing in the exercises so that is why I am not doing it here on the board so you will do it on the okay. So now we ready to move in to the next thing is everyone with me on this stability aspect okay inside you need circle \outside you need circle I will be referring to this inside unit circle business throughout okay because and you should not be confused, so let us move on to progressily the concepts are going to be fairly complex will well you keep asking doubts wherever you have problems just stop me okay.

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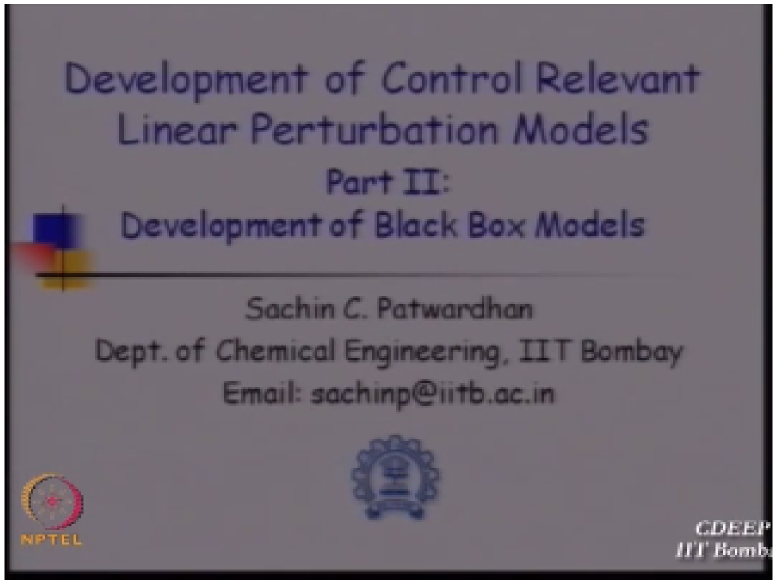

Because till now whatever we did was okay the certain thing which I am going to talk about you know are going to be fairly complex I am going to talk about something call stochastic process and the math's will not be fairly complex conceptually it is fairly difficult to understand what you have to do and I do not expect that only this piton lectures it is only sensitizing you we are just touching type of the ice-burg and eventually if you keep working in this area someday you will understand what all those things.

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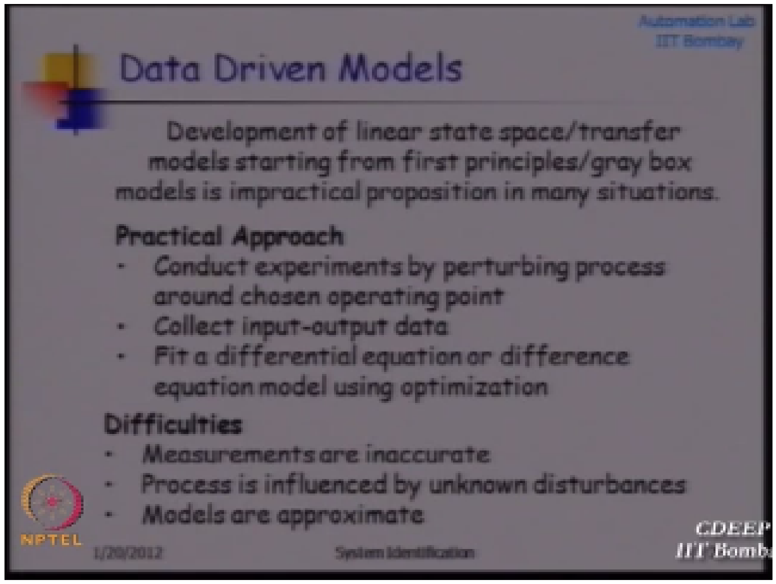

I am trying to going to make my attempt as simple as possible okay, now we are getting in to data driven models okay, why data driven models because well I do not know anything about the system I have gone to some industry I am given some plan I can play with it I can to wick I can change some input I can record outputs okay this thing of developing a control system for a car okay you have been ask to develop a control system for a car and you do not have mechanistic model for the car okay not quite for the I mean you may have some mechanistic model for the car duel in the literature but that model and your car are different right.

You have say Maruthi car and then tomorrow you may have to work with the Tata Indica and so each one will have a different model and developing a model for Suzuki will let us say take one year then another year will go in developing it for Tata Indica, so it is not obvious that if you have this model it will work there parameters are different how it behaves is different links are different and so on. So you want to develop a quick model not one year you want to do it you know one hour and do it from data.

So you might say well what is there you know fitting correlations you measure input measure output why are worried so much well the first problem is that I want the model of the car in dynamics okay while the car is moving, now what is the complication what are the inputs what are the multiplied variables in the car accelerator and brake okay and let us look at only one control output speed let us not worry about direction and all just look at speed and you know motion in one on one line you are going to deviate okay.

Now when a car runs on a track okay, it is going to subjected to drag because of the wind you are not measure wind you cannot effort to put wind measurement in your car that is too much okay, so but a winds are going to be you know they are going to be gust they are going to be steady state conditions steady air stagnant air condition then helping you sometime will sometimes it will being cross direction you do not know okay. When you do an experiment how the wind may behave no idea it is something which is outside your control it is disturbance but when you collect data let us say you put your pressure in the accelerator and you record data on the on board computer okay.

The speed that you get is going to be a complex function of your accelerator, and there is one more disturbance the track well in our conditions that is a big disturbance track and we suddenly very good and suddenly you know the patrol and so there are several problems which are I am going to call them as unknown disturbances so the data which I get is always corrupted with unknown disturbances okay.

Whatever I do whatever experiments I do they are always some unknown disturbances okay, which is influence how the system dynamics behaves and problem in developing models completely from data black box models we do not have any structure about coming from physics okay is in such a situation is this unmeasured disturbances, if the input that I give to the system okay if that is the only thing which influences the system okay great you can model it very well okay, they are so many other influences is difficult to model okay like you know if I teach you and then I except should know something about parameter systems.

And then for that I would except their study but then you go to hostel which is a big I am measured disturbance for me and you know there are all kinds of attractions there and is difficult to model a system which as lot of unmeasured disturbances. The second problem is when you are developing these models these models are approximate okay, we are developing a model for a unknown dynamical system which is subjected to unmeasured disturbances okay they are going to make some assumptions you know this is like a linear perturbation model you know that really car is not a linear perturbation you know car dynamics in very complex linear perturbation model is an approximation for your convenience because you can do mathematics very well okay.

And if you want to control the speed is some small range this models will be very good okay, so that is the assumption, so models are approximate okay then there are unmeasured disturbances as if that was not enough the measurements are always corrupted with error your get perfect measure okay you have a transmitter you know it is you convert signals from you know one*other and finally you convert into some electric signal which is connected to the computer there hundred different places where can pick up some noise okay your data which you get I will show actual practical data.

In a bigger where the temperature of the water is perfectly constant I put thermo couple I get data which is fluctuating temperature is not fluctuating data is fluctuating because there are errors in the middle okay so there are two problems.

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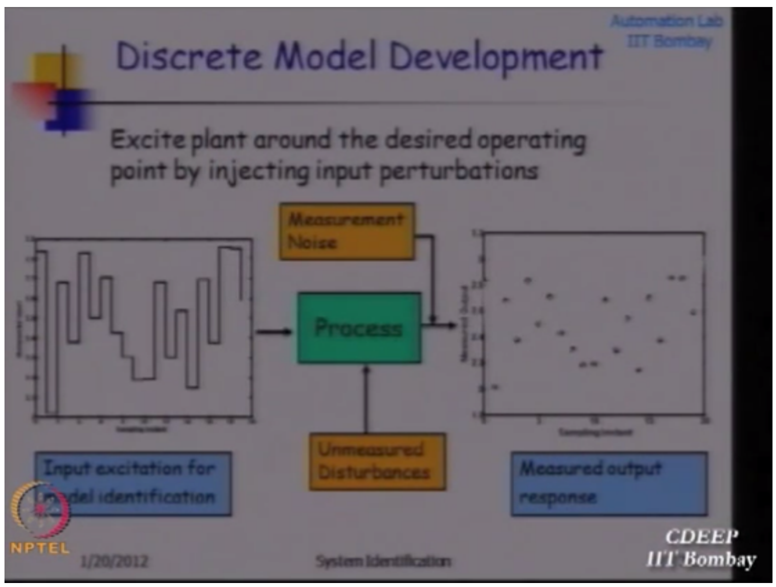

There are two problems now we have to develop computer oriented model I have freedom to, to experiment with the system so this is my experiment I am going to injected in excitation okay inside my computer I am going to get a sequence of numbers which is speed measurement okay this is my you know accelerated the way I have played with the accelerator and well there are always a measured disturbances wind is not going to be my control okay.

That track conditions or not in my control so there are unmeasured disturbances unknown sources we may not know even source every time okay and then the measurements are corrupted with noise okay so that trouble is that if you want to get a model that relates this input with the output then whether you like it or not you have to model this disturbances not a very strange thing.

You have a disturbance which is not measured which is not known to you and still modulate okay looks formidable or in fact impossible how can somebody model something which is not measurable okay and the sources not known how do I model that.

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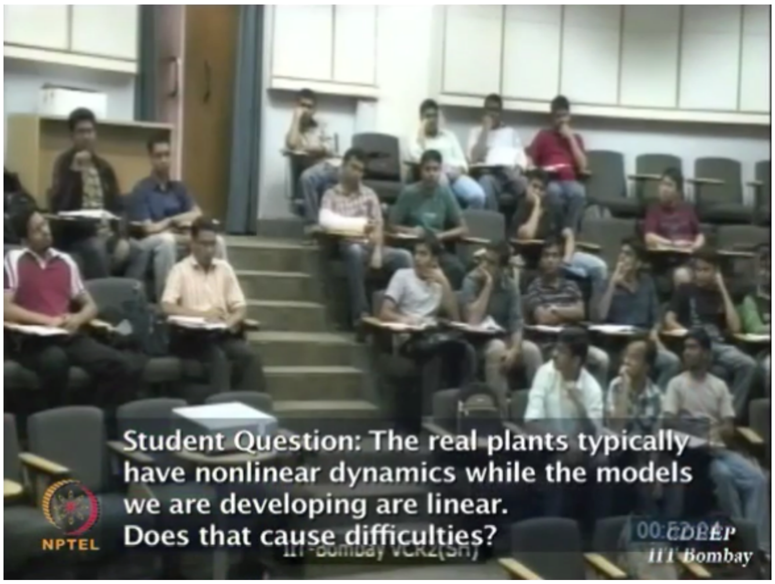

Because there will be an approximation error which is coping up because of linearization okay so that also when you are actually developing a models of this form that cause can be viewed as contributing to the unknown disturbance it is a kind off the disturbance which means from approximation levels okay.

So I would say there three kinds of a errors approximation errors okay measurement errors and actual physical inputs which are unknown okay so the data which you get as all of three you mean data will not be approximation errors, approximation error start cropping when you start modeling okay.

The data will have only measurement errors and okay approximation errors is something that cops of when you start model so data will have unknown disturbances okay.

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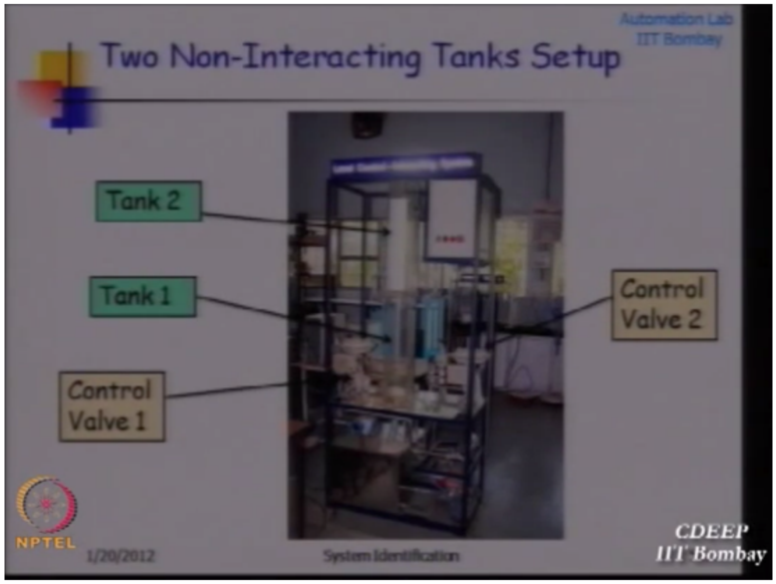

So let us not get into what are the types? Let us look at a practical problem okay so that now I am going to look at us simpler system I do not want to look at the quadruple tag there are two inputs and two outputs I just want to look at a simple system two tanks one place above which other okay and there is there are two control valves control valve 1 and control valve 2.

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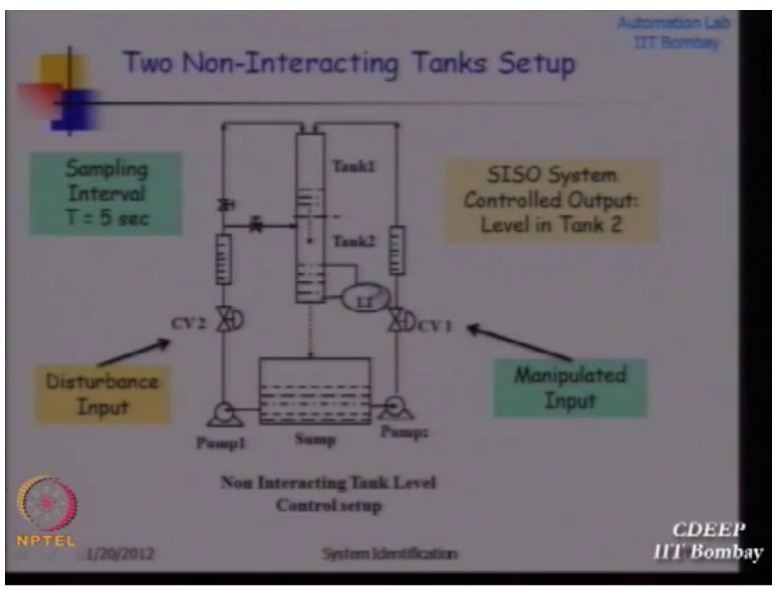

I show you schematic diagram of this okay this control valve 1 is used for manipulation it you pumping slows in tank 2 it is slows by gravity from tank 2 this tank 1 to tank 2, and then tank 2 to, to the some here simple process you can probably go to our under graduate lab this is undergraduate lab experiment in chemical engineering okay.

So what we do is this second valve we use it to introduce a disturbance if we want to okay right now I am when I operate this system this pump here is subjected to fluctuations in the power there is enough disturbance already in the system I am not worried right now I want to introducing another disturbance here so I am going to look at a simplest possible case this pump this second pump is this pump is off okay.

This pump is off only I am pumping liquid through this control valve 1 okay what I am going to measure with this LT here is level transmitter I am going to measure level okay I do not have a mechanistic model through the system I am just going to develop a model by doing quadrapation studies I am going to quadruple the control valve position.

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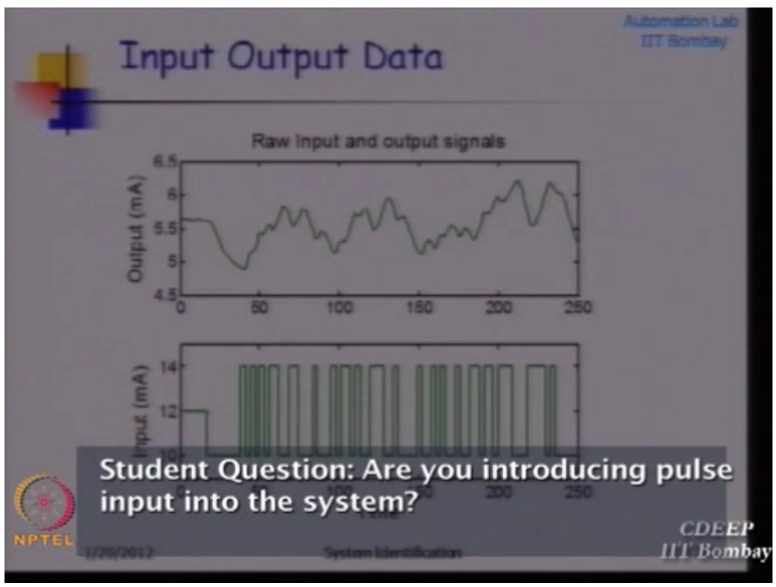

I am going to records the level okay I am going to record the level I want a model that relates that quadrapations in valve in yeah, not pulse I m giving sequence of pulses or sequence of steps actually there not.

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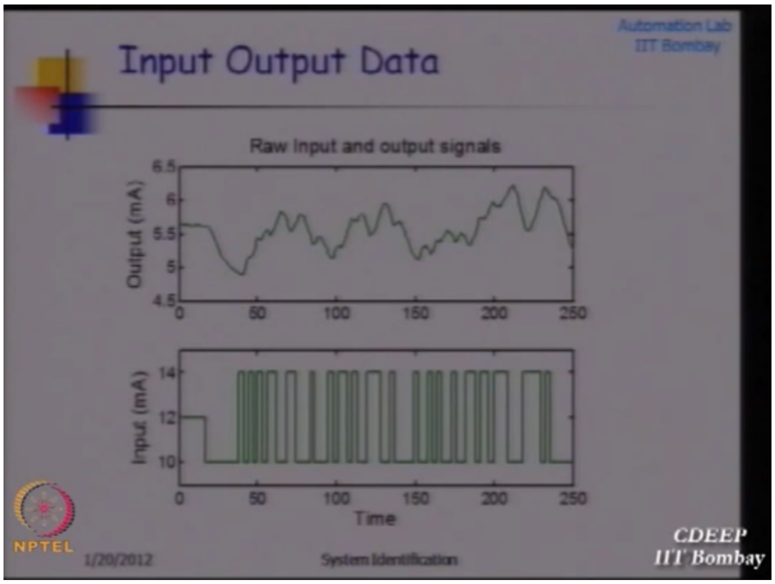

If you look at it there not really pulses there positive and negative steps now why I am doing this why positive or negative steps when I am linearizing a non linear model okay if you want to do an experiment that behavior on the positive step and the behavior in the negative step is going to be different is a behavior in the negative step and positive step is going to be different okay.

Then I need an average behavior on both the sides okay my operating point is 12 milliamps of series control valve X between 4 to 20 milliamps as the input 12 milliamps of fixed 50% opening 12 milliamps is 50% opening okay I am introducing the fluctuation okay which goes between 10 milliamps to 14 milliamps okay.

Above 2 out of 16 how many percent? 1/8 15% + or – 15% okay I am introducing a fluctuation now what is happening because of that the level which was initially steady the level that was initially steady okay it was steady at this particular 5.6 milliamps is some level corresponding to some level inside it.

It started fluctuating what is fluctuation? The other fluctuation is record okay this is the extreme I was allot to do now using this input and output data single inputs single outputs let us understand the simplest thing okay will move to the complex things later single input single output system I want to use this data I will come up with some models okay.

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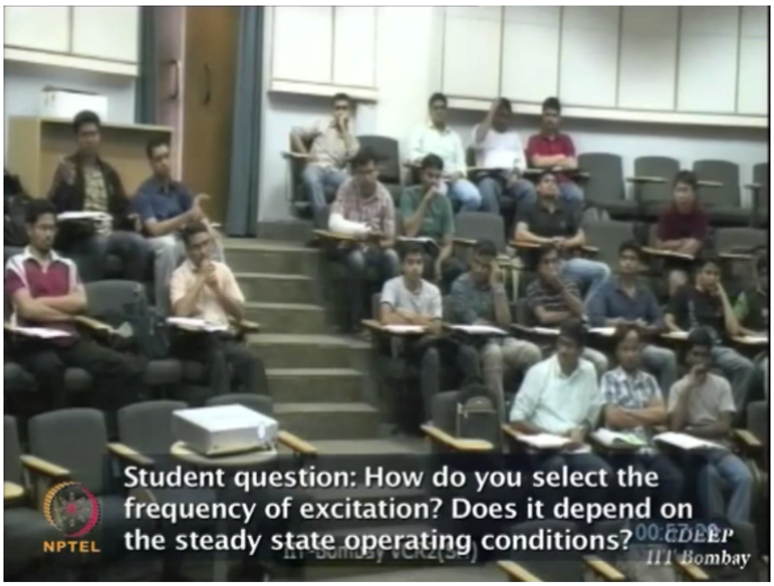

If I change my operating condition I well your question is important it is not that I do not want to leave your question but the way I introduce positive and negative quadrapations here or the fluctuations know I have chosen the frequency of fluctuations very carefully it is chosen in such a way that the plant is excited in the range in which it has a nominal frequency okay or close to the corner frequency okay.

I do not want to excited in very high frequency rate the system is never going to operate or very, very low frequency which is of important but which will get covered if I take up band okay so how do you choose frequency of this excitation is a important issue and but it will postponed it is I will brief it that is upon the some point later.

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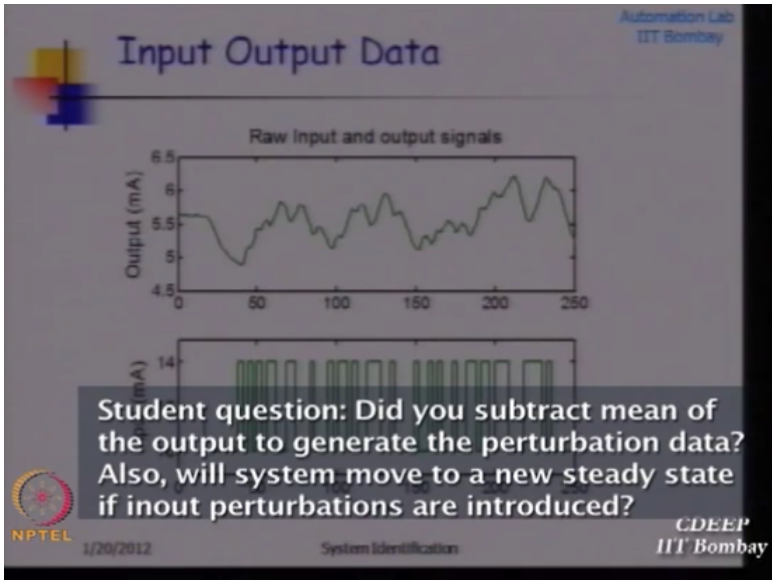

But how do you do this is, is you know you need to go through some yeah, values of not 0 will be close to 12 yeah, yeah so if I do quadrapation variable what I have done here is next I have this is an absolute data absolute values okay I am subtracting the steady state here I am subtracting this steady states here.

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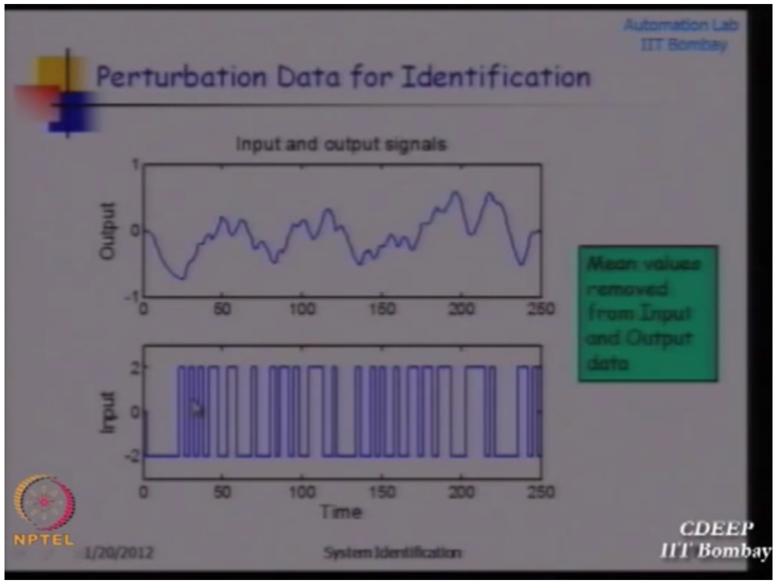

And then I get quadrapation data the quadrapation the input if you take a mean of the input it might be close to 0 no, no it may not be close to 0 yeah but if you keep doing this then you will get a which is fluctuating around some mean original mean value provided it will go to a new point provided estimate or unstable or measured stable okay.

It is not going to new point the sustainable stable okay this level tank is a stable system okay if you increase the level inside the tank by gravity the flow out increases know so the level starts so it is a self governing or self open the stable system any other question yeah.

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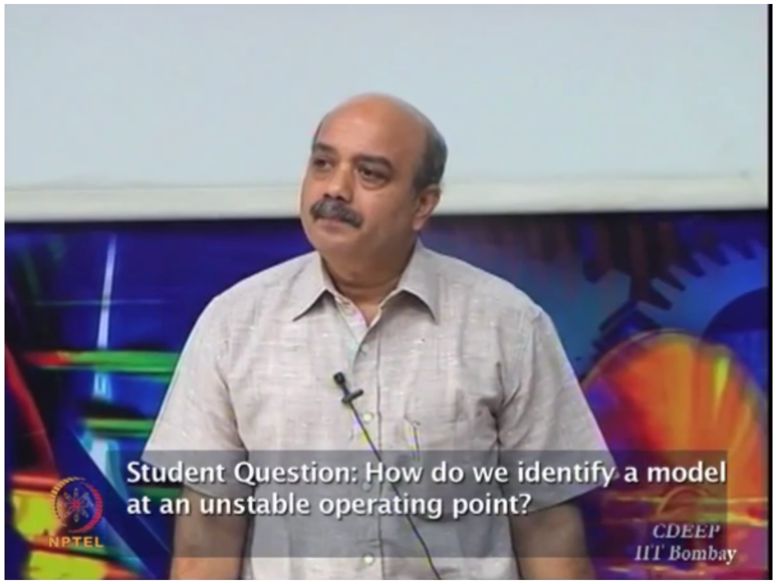

Or you want to directly ask me I am right now grade 1, you are asking me grade 10 question not even grade 10 actually how do I model unstable point my student is doing research on that okay so it is PH.D question so maybe we are right now in first grade okay.

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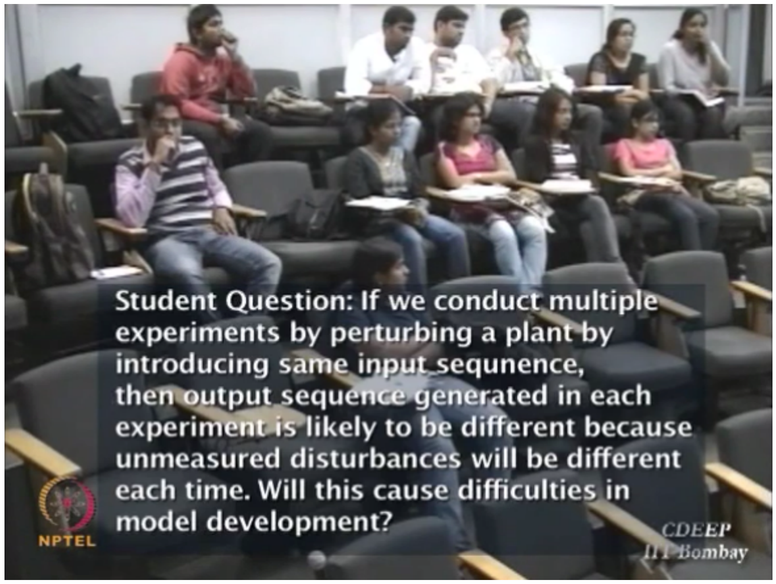

This is not this is known input quadrapation yeah, but when we assume that there fundamental so how do I capture fundamental characteristics of you have a very good point very intelligent question every time the disturbance will be different I am not talking wait okay.

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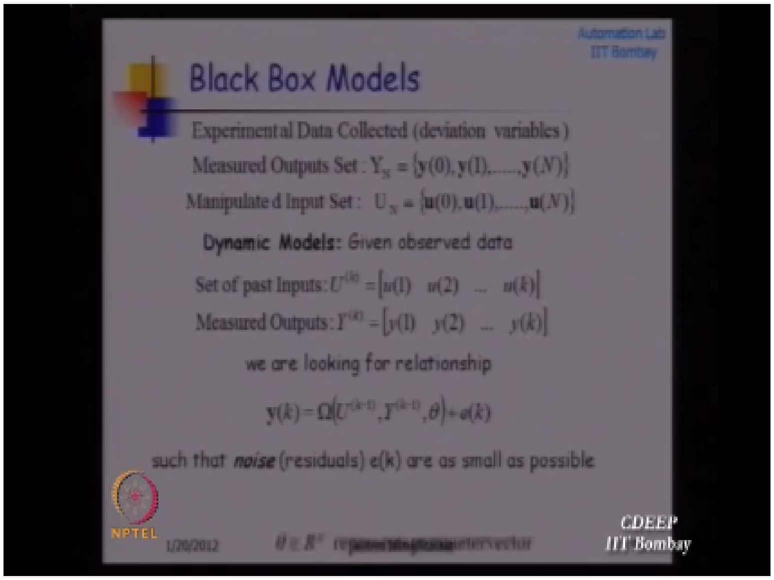

Okay I think my animation I forgot to put okay just do not look at this down part look at the 1st thing, so what I have done now I have collected the experimental data okay, I have set off inputs y and measurements and u which are known to me which I have given, now u here is what is going outside my computer control computer, y is the measurement t5aht is coming to my computer okay.

So computer oriented model, sequence of numbers that has been collected as measurements and piece wise constant inputs that have been sent to the system okay. I want to develop a model that relates the current output okay. I want to develop a model that relates current output with past y and past u, why past y and past u? You have seen how dynamics system behaves okay.

You have seen earlier how difference equation behave in dynamical systems current kevel will be function of past level and one level in the past, past inputs, we have seen this. I want to fit a difference equation here I have put it in little abstract form.

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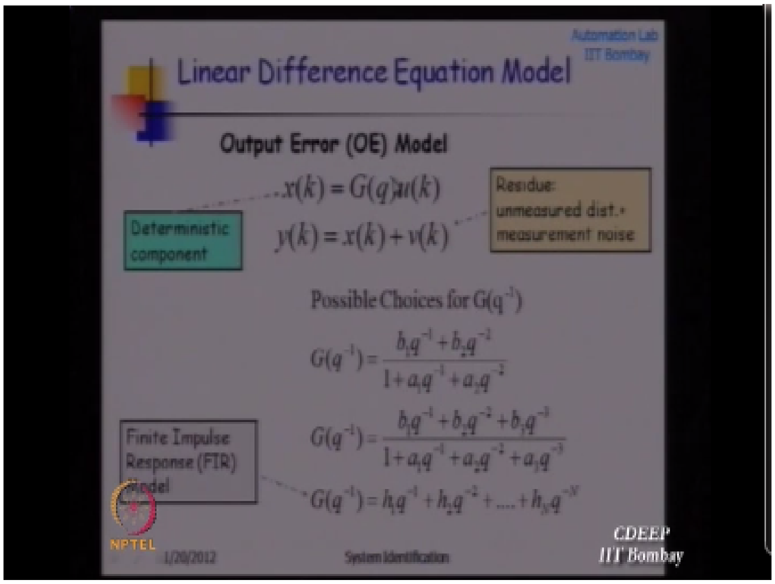

Now I am going to look at this model, I am going to call this model as output error model OE model okay. Why it is called OE model? Because okay everything that is unknown is blame to this error in the output, y is the measured output okay, what is x here? How do you interpret x? x is the effect that is present in y which is coming from u alone, inputs alone, is that clear. X here is a variable okay for which actually quantifies effect of u alone on y okay.

Whatever is the rest I am calling it has v here okay, this v here is the rest of the signal. So I am saying that the output signal consist of 2 sub components, what is the 1^{st} components effect of u alone okay. How is the effect of u coming through? It is coming through a transfer function gq okay. Effect of inputs on the output is coming through a transfer function.

Do I know this transfer function? I do not know this transfer function what is the order? I have to guess, it is a black box model, I do not know whether it is 1^{st} order or 2^{nd} or 3^{rd} or 4^{th}, I am going to try fit each one of them okay and see which is the best okay. so the way I am going to go about is by travel error, I have data okay, I will fit a 1^{st} order model, 2^{nd} or 3^{rd} or 4^{th} okay and then see which one of them gives me good fit and then decide okay.

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Yeah that is a good way of saying it, x is the prediction but not right now wait for u, will call it prediction little later. Right now x is the component which is there is the settle difference between and prediction okay just wait. Now how do I chose this gq I have given some 3 options here, can you see the options right now I have seen earlier my gq came from physics.

Now I do not know okay so it could be anything that I choose, I have given 3 options 1st option is a second order transfer function, the 2nd option is 3rd order transfer function and I have 1 more option do you see that, what I have done is, I have done a long division I have divided let say take this case, I have divided numerator by the denominator and if I do that I will get a model which is a series.

You just do the division just like the normal division upto to polynomial you can do polynomial division and then you trunked the order. So you can get variety of models we will look at the 1st one, and look at this model how I fit this 1st model okay.

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No we have pass that stage now just do not talk about the good physical model available, no do not confuse here right now I have no clue what the model should be, I have no clue about the parameters, I only have the data okay, so I am just looking at fitting the transfer function into this data okay.

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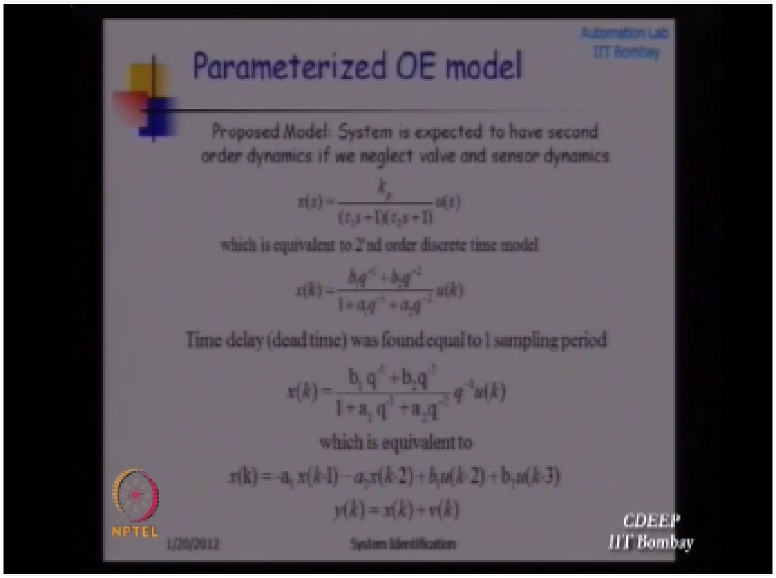

So all that situation scenario is but there, it is probably there in your research problem but not okay. now let us make an argument that there are 2 tanks so the model should look like a 2^{nd} order okay, so I am assuming that the model between x and u is the 2^{nd} order, I am neglecting wall dynamics, I am neglecting sensor dynamics.

So I am saying that 2 dominant capacities is one is tank 1 and other is tank 2, so my model should be like 2 orders okay. A 2^{nd} order model is actually a 2^{nd} order difference equation; see here there is one more thing which I want to add here, for this particular system okay. We know that there is the time delay of one sample okay.

So the model that I have proposed here is actually 2^{nd} order model which one more time delay with, you see the time delay here okay, so if I convert this 2^{nd} order into a difference equation I get this linear difference equation everyone with me on this, this is my model which is proposed model okay.

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Not straight yeah, yes no there is always one in a time delay in any discrete time this additional time delay which will give 2 units okay. Any discrete times is still what do you do now will have influence after 1 sample okay most of it yeah.

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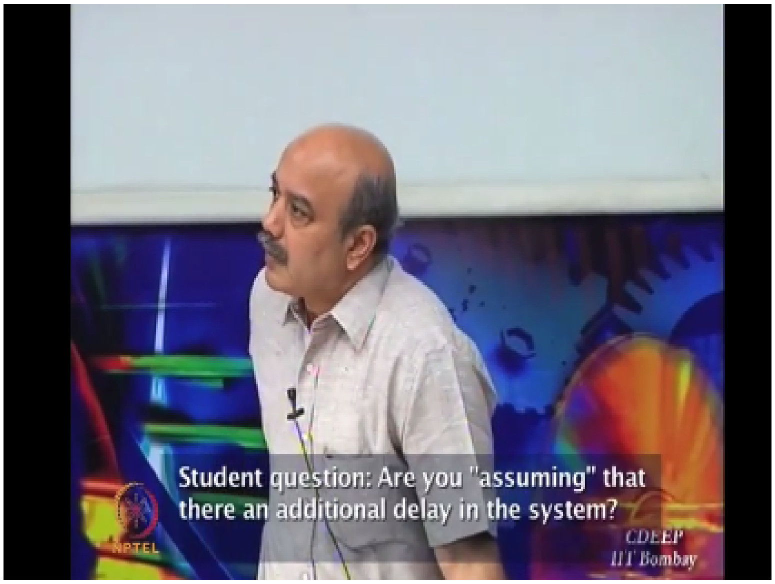

No I have assume I have found that I this particular there is one time delay, this is actually usual lab set up so I have found that there is the unit time duration okay, that is why I am adding it, if I have found experimental 10 sec right now sampling time I think is 5sec if I have = 10 sec I have d^{-2} okay, time delay = 1, I have independently found that there is the unit time delay in addition to the time delay which is because of the discrete.

See you look at the discretion x k + 1 is function of u k, discretization process itself introduces a unit delay in addition to that there is one more delay here which is okay. now I want to estimate model parameters what are the model parameters a1, a2, b1, b2 right these are 4 parameters I need to estimate the trouble is do not know x I know only y, y as x is hidden inside y but I do not know what is the distribution between x and v I have no clue okay.

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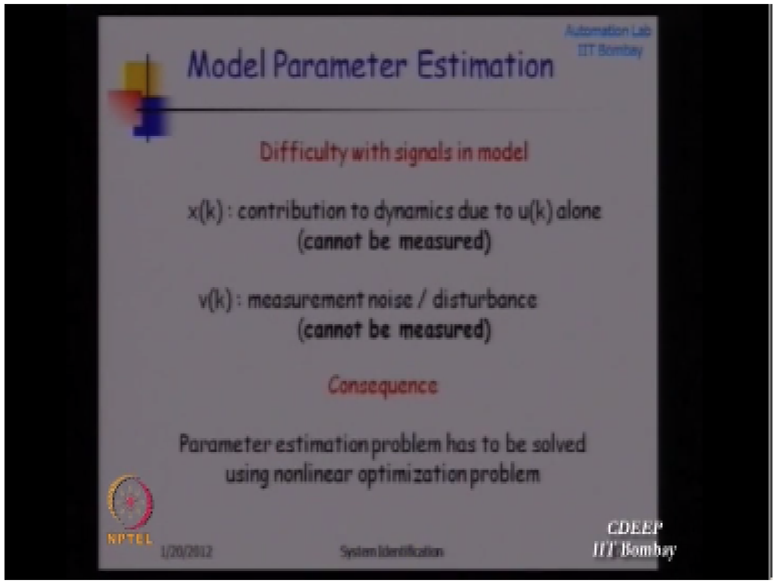

This cannot be measured the contribution to y which is coming from u alone cannot be measured though we have measuring a separate in the level you cannot have two level transmitters one which gives you only effect only effect of you that is not possible okay so this is the fundamental problem vk is the unknown disturbance off course unknown disturbance can never be mutual okay unmeasured disturbance can never be measured so now you have to do some kind of optimization and come up with.

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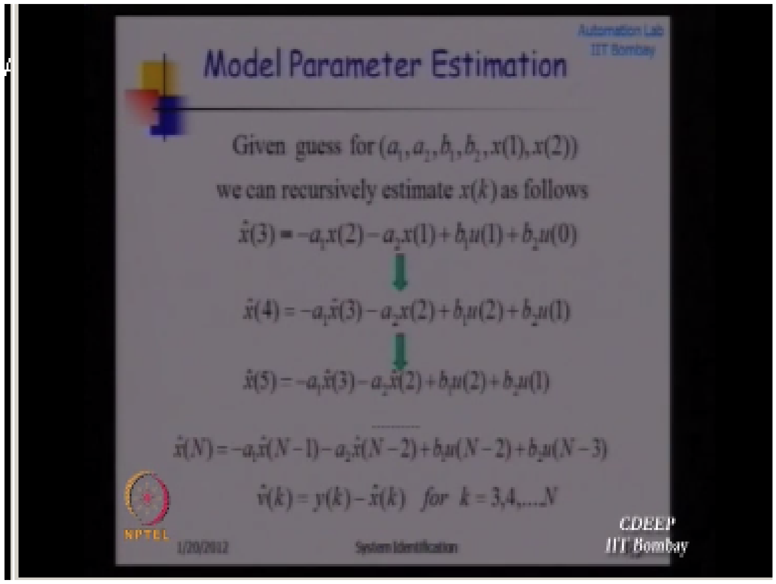

Okay now if I am given a guess okay if I am given a guess let us okay.

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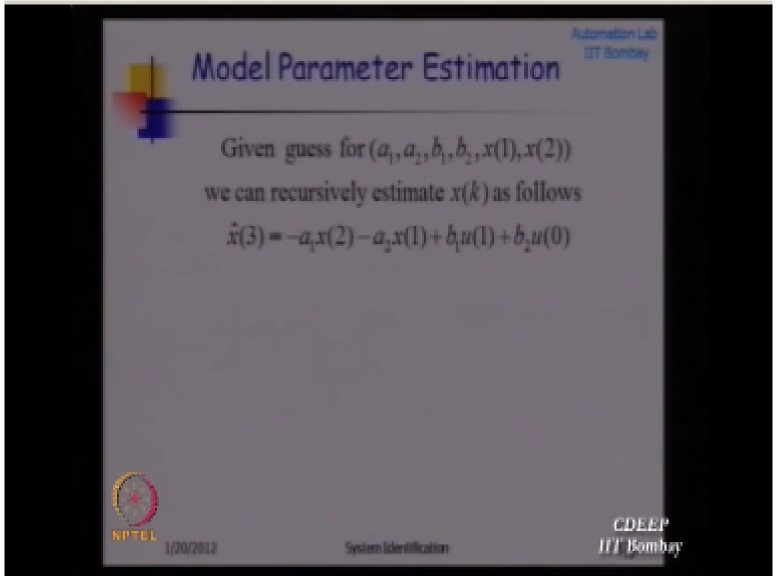

So what I am given I have this model let us say you give me a guess, for even a1, a2, b1, b2 okay I have no idea about what is x1, x2 okay we can make a simplify assumption x1 x2 is 0 because you are starting from a steady state your data in which you started from a steady state so the input was zero effect of input on y was 0 at time 0 you can make sampling at 0 okay but right now, I am saying that you give me guess for 2 initial points x1, x2 okay if you give me guess for initial points x1 x2 a1, a2, b1, b2 I can predict okay I can predict x3 this is prediction okay prediction is different from true x prediction of x is not equal to the true x okay.

So this is my prediction is everyone let me on this I know inputs I know entire input sequence u is known to me okay now, you have given me a1, a2, b1, b2 and x1, x2 given this information I can use the difference equation in time go progressively predicting okay so if I predict x3 using x2,x3 I can predict x4 right is it clear I can predict x4 because, I know u2 I know u1 okay parameter guess I been given to me okay.

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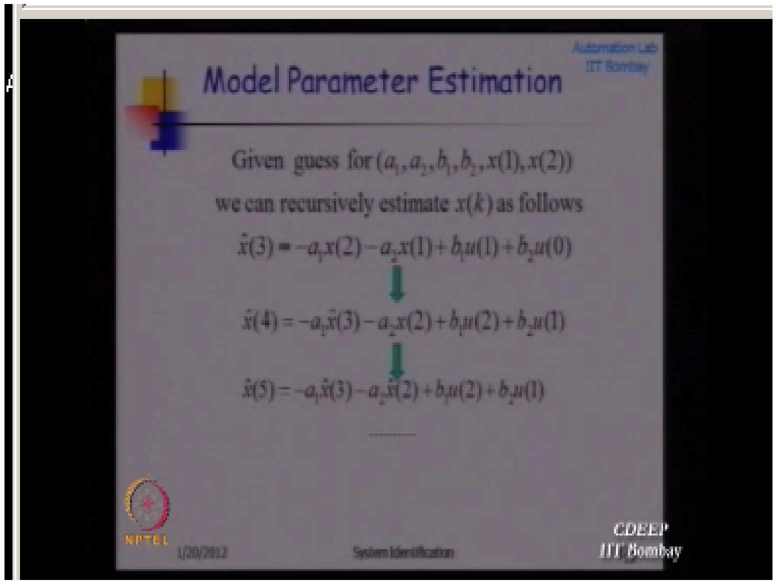

I can just go on from if you give me this 6 things 4 parameters and 3 initial states guess for two initial states I can just go on solving the difference equation I mean how will you do it in the program put a for loop okay and you go matching in time so you just going marching in time to go from x2 to x3, x3 to x4, x4 to x5 you just go marching in time okay it is very simple so in general I can go up to time n okay.

So this is where I begin when I move on to x4 which is function of u2 and u1 and x at 3 which was so x3 was computed here that is used here okay so move on to the x at 3 x at 4 while you computed here there used here okay and you think u2 are known to you okay so you just go on doing predictions these are predictions for that given guess these may not be equal to truth they will be equal to the truth only when you have correct values of a1, a2, b1, b2 well in the real system what is current it is difficult to answer.

So if I have all these predictions I can also predict v right I can predict v if I have predictions of x gap 2 x cap 3 x cap 4 x cap 5 I can predict v cap I am not calling it v I am calling it v cap because it is a prediction it is an estimate it is not the truth okay so that is why I am calling it v cap okay now I am going to estimate these parameters through non linear optimization problem optima ion is great thing what gauss did is a invent this wonder technique and you know all of us are in business so, you have to solve the problem through optima ion what I have to do is minimums some kind of an objective function what I am going to do is these square estimation okay I want this vk what is vk?

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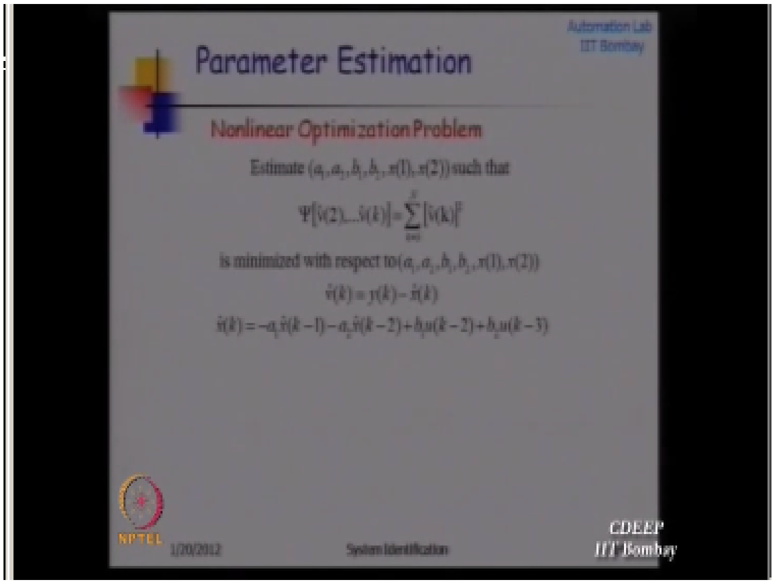

Vk is difference between y majored and x what is x effect of input u alone I want to have difference between y major and x okay as small as possible how do I say this okay I say this by putting in objective function some of the square of the vk should be as small as possible well y some of the square y0 absolute value you can choose absolute value if you choose square problem is simple to solve relatively okay we work with two knob.

Because you know there are many, many properties of two knob which you can explode and come up with analytical expressions for you know understanding the behavior of optima ion problem and so on so two knob is very, very convenient mathematically so this is optimization problem in which so what is uptimes going to do every time guess even a1, a2 ,b1, b2 x1, x2 okay.

And compute the objective function if it is you know you have given some tolerance if it is greater than the tolerance it will take a new guess by a gradients or whatever method conjugate or you know Newton's methods and then it will find a new guess it means stop only when you get some of the square of error minimize in some sets okay so what is get after that is an estimate

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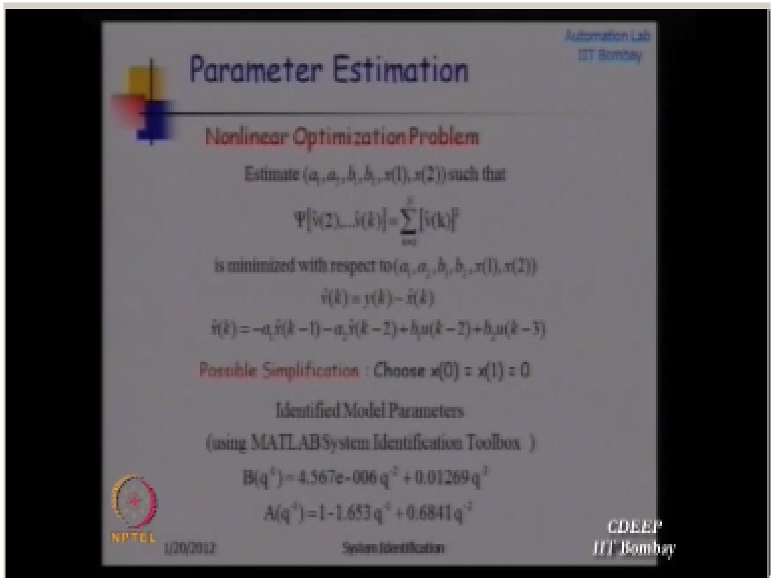

You can possibly you can choose if you do not want to guess every time x1 x2 you are saying that you are starting from initial steady state initial states are zero that reduces the problem to only four guesses even a1, a2, b1, b2 not an easy thing how to do guess a1, a2, b1, b2 for a discrete time system if I do this using met lap system identification tool box I am going to demonstrate this two box at some point then this is the module that I get okay I get optimum values these are estimates of a1, a2, b1, b2 for the two time system which is showed you how good is my model okay

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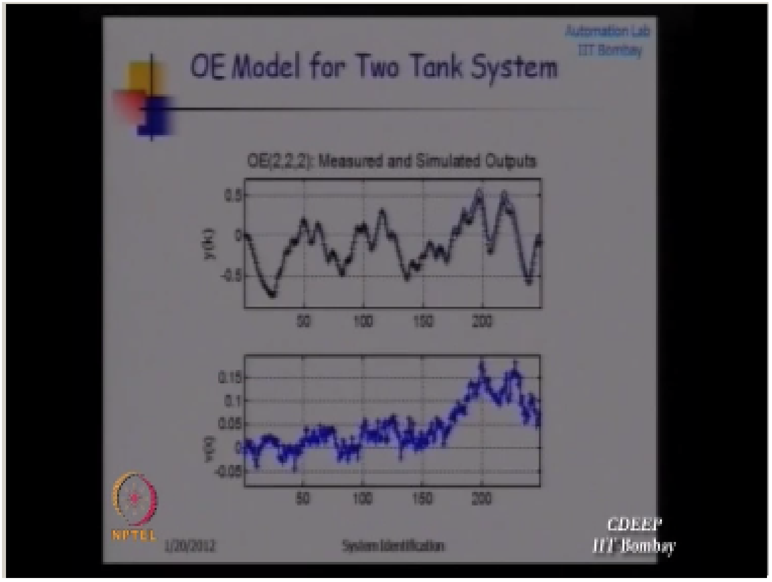

Look here plus our model predictions blue line is the correct actually data which I have collected from the system okay I also plotted optimal estimate of vk which I got after I predicted the model okay you get c that there is significant differences to well this is significant it is almost explaining everything well I am also worried about you know net picking and modeling even this significant for in significant difference.

Okay my next task is to going to be how do I model this funny signal can I develop a model this is coming from unknown sources now it has three kinds of errors measurement error in the transmitter it has unknown disturbances third approximation errors all three of them are present in the signal okay but you can see the model is put in of right I can use this model of control I do not if I just have to design a controller this model is a very good model it predicts almost everything that is there in the data there is some minor predict left okay will off course what you about modeling but this is a good model.

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V of k is differences between measurement and effect of u on the level v is everything measurement noise plus disturbance plus approximation error everything combines together is b.

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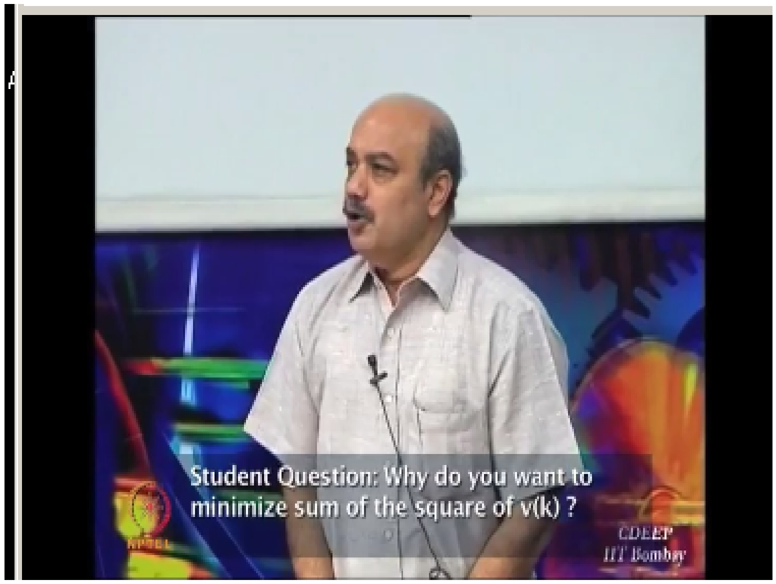

You want to have that part as small as possible because you are saying that most of it is coming through u yeah it is a model so do not confuse a model with reality this is a model is this a correct model will question that is output a error model is zero correct model I will question that the next thing I am going to do is to question is error output model is a good model or can I improve on it I am going to do that okay.

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