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ADVANCE PROCESS CONTROL

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Lecture No –21

Soft Sensing and State Estimation

Sub-topics

Kalman Filtering (contd.)

So you have been looking at Kalman filter and in my last lecture I did derivation so those are few could not attend it should have a look at CD which seen through the lecture so what I will just take a very quick recap of what we have shown in the last lecture.

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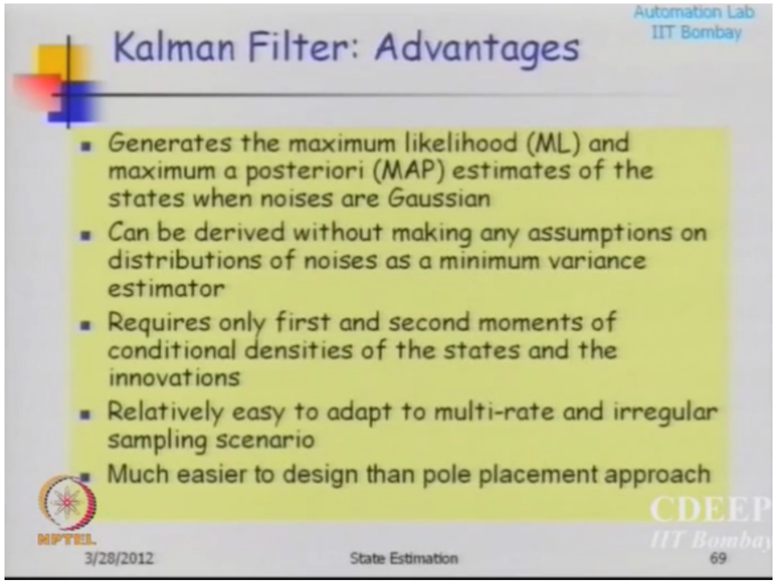

The one thing is if you make an assumption that state noise measurement noise and the initial guess for the state all of them have Gaussian distributions state noise and measurement noise are Gaussian white noise processes and initial estimate at time 0 the estimate have a Gaussian distribution then what we have shown is that Kalman filter gives you maximum likelihood estimate.

It also gives you a maximum estimate the value that are the vector that maximizes posteriori density function for nice thing about Gaussian distributions is that if state noise and measurement noise are Gaussian white noise processes and initial condition of the white noise then all the conditions are density of Xk given yk-1 or xk given yk.

Their also Gaussian processes so they also Gaussian densities they are not Gaussian process but they have Gaussian density you can show that distribution of them in Gaussian then Kalman filter can be derived without making any assumption about density so you do not have to make an assumption about Gaussian you can derive Kalman filter just using arguments of minimum variance okay.

A minimum co variance so trace of co variance matrix to minimize with respect to the gain matrix and we still get the same thing so one of the duty of the result is that it can be derived by the multiple view points and you get the same level if you start from this optimization view point just minimizing a scalar objective function minimize the trace of the covariance matrix.

You get exactly the same estimate which you would get if you have to start with Gaussian assumption and then looking at you know which estimate maximize the likelihood function or which estimate using maximum of posteriori density all of them just give you same result so different viewpoints just collapsed into one signal region okay.

Why this result is important because it is recursive it is systematically takes into an account how the noise uncertainty in the state dynamics and uncertainty in the measurement a systematically take into account yeah, here we have to be when we come to the stationery observer we need some assumptions of that is all.

But here the stability can be shown without requiring the observer assumption so observity assumptions are required few account if you look at stability while not ignoring have look at stability only one limited view point only in the initial state I think when you go to input to state you may require observity condition or absolute condition may be required for the existence of the Kalman get that we have to check.

Where the observity condition can be function so nice thing about Kalman filter is that only you work with first and second movements of the distribution whether it is Gaussian or non Gaussian you just keep working with first movement second movement and it is easy to do the calculations and this is much easier than doing pole placement.

Because how do you choose the poles if you happen to know wk and vk to be Gaussian white noise processes is not clear okay so that part is not clear so this give you very systematic way of handling.

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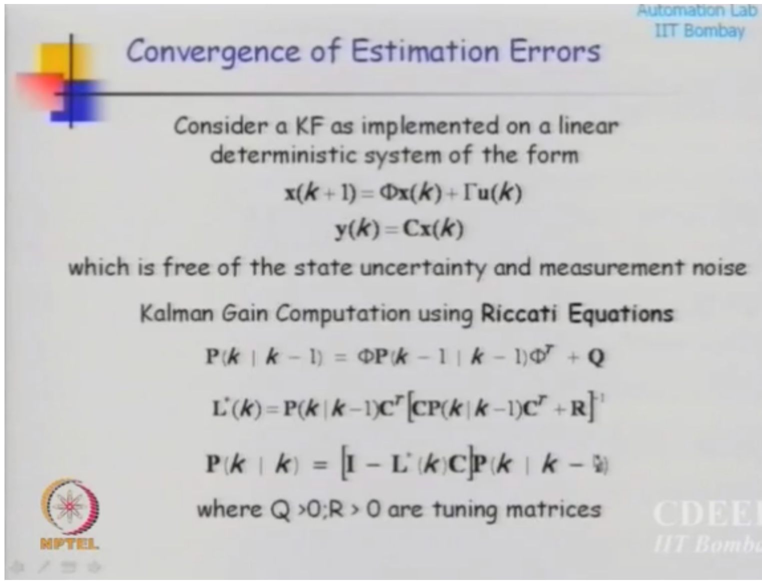

You also looked at convergence of errors and then look at very deterministic linear system and then what I showed was that you can construct the Lyapunov function and you just consider the error in the initial condition and it will converge the difference between the true state and the estimator state will assemble it go to 0 and this we did using you know this is the just summary of the Kalman filter but here. I am not looking at wk and vk yet I am using same update equation which are used for the Kalman gain here you can look at Q and R to be two matrices which are possible definite okay.

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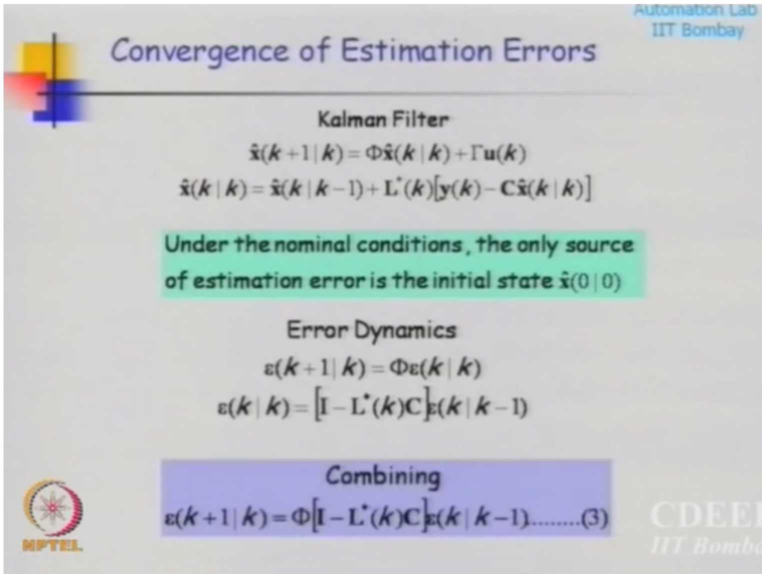

And then you can actually show that one can construct the error dynamics is given by this equation this we already derived earlier and this difference equation actually assembly stable.

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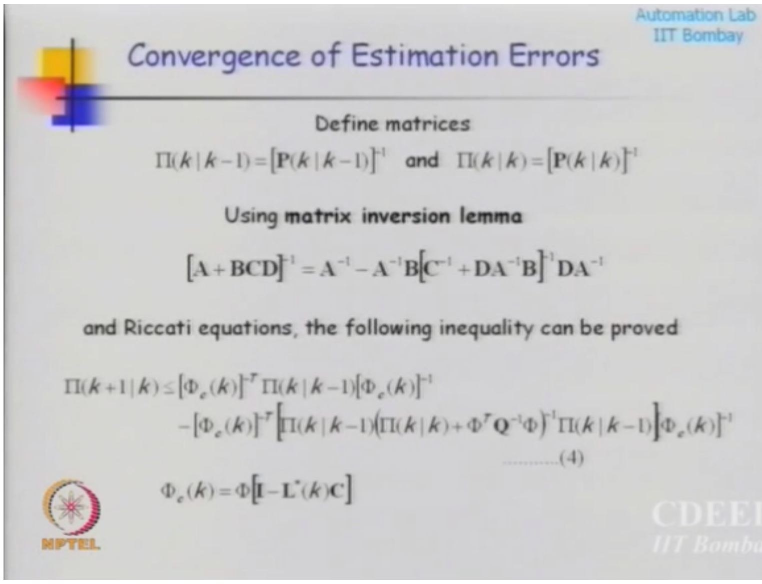

That I have shown using this matrix inversion lemma.

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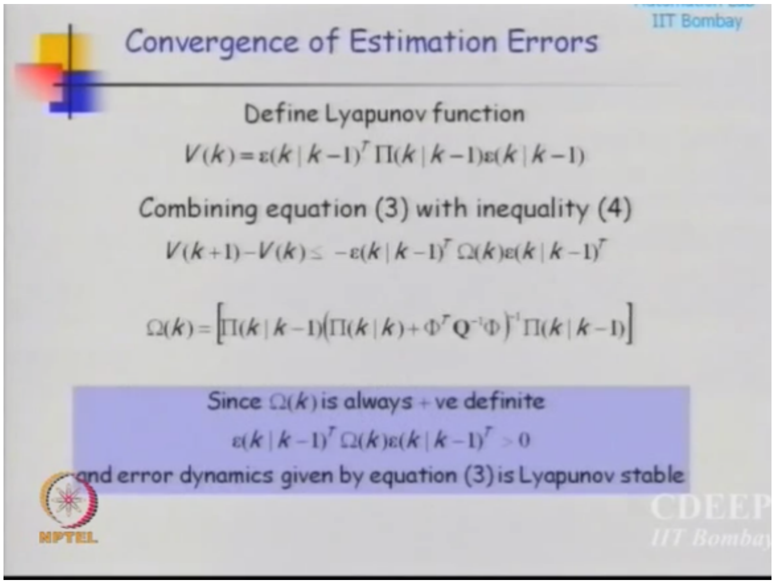

And then constructing Lyapunov function which is so we have define this πk here, πk is inverse of covariance matrix using inverse of co variance matrix we have constructed Lyapunov function and then shown that this Lyapunov function is negative definite moreover and a certain assumptions you can show that.

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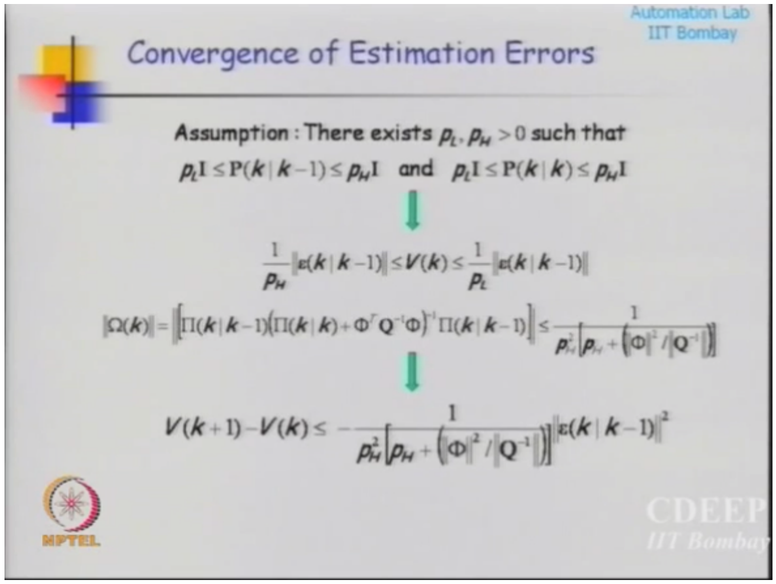

You can see the asymmetrically stable system so observer stable okay so actually in this case we designed the observer first for the performance how was the performance defined, performance defined using minimum variables and then now I am showing that the designed observer tells out to be stable okay just going back here to this difference equation why did I have to use I have to analyze the stability of the difference equation.

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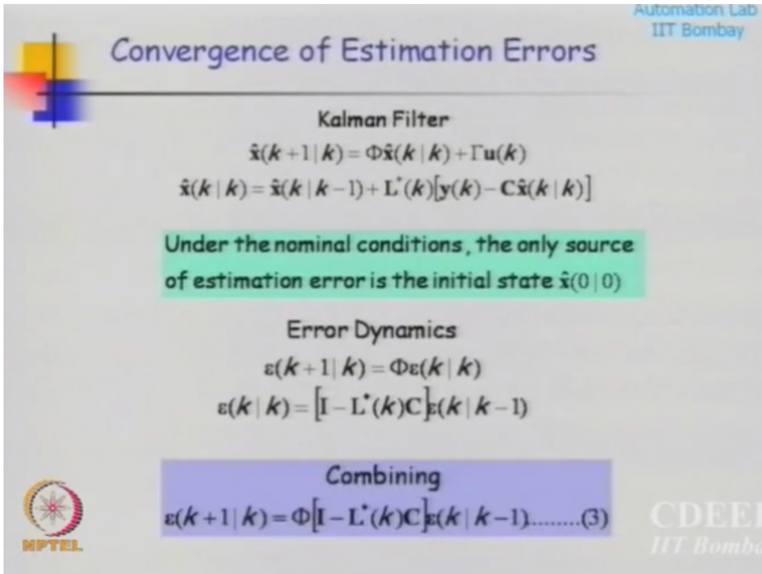

This particular difference equation here combined difference equation now for this combined difference equation I could not use the condition that I get the values inside the unit circle why what was the problem why I could not do that because it is time varying system it is matrix here Φ times LK C is constant Φ is constant Lk is not constant, Lk is time variance.

So this is our linear system but bit time varying matrix Φ, Φ means if you call this whole thing as Φ this combine matrix which you cannot look at either value because it is not constant it is changing so you cannot use either value along with here I had to result to use of Lyapunov functions okay indirectly it also demonstrate how Lyapunov functions can be used very, very powerful theory so that is why, that is why I did not talk about spectra radius less than 1 either value less than 1.

Because here you know it is this matrix combine matrix Φ times i-l*kc it changing the time okay so each one of them without different values and then you cannot say that if all of them are inside units circle you cannot prove that then system is stable that cannot be prove okay so such a condition does not exist okay.

If this matrix is constant it does not change between time you can use this condition of inside unit circle that is that condition holds for linear time and variance discrete times system okay so that is why I has to result to Lyapunov functions and then I just showed that I can construct Lyapunov function using co variance inverse or distance measured using co variance inverse.

And then if you see this, this vk is error transpose co variance inverse into error actually this term appears also in Gaussian density function e to the power same, same term so I just showed that this is a vk+1-vk is always definite and this means that the error assemble goes to 0 so we have at least under limited condition.

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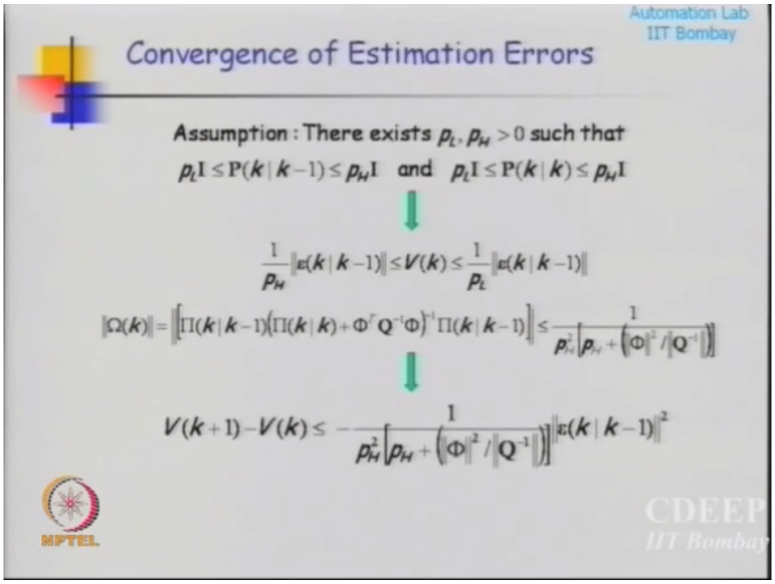

We have showed asymmetric convergence of Kalman filter okay I have not that step on here that happens I made a simplifying assumption that wk and vk are 0 to treated under Lyapunov framework when wk and vk re not 0 is possible under certain modifications there is you can in Lyapunov function theory or Lyapunov stability theory there is something called input to state stability.

So you can also talk about unforced see this is the you talk about unforced stability here you can also talk about stability when there are external disturbances right wk nd vk but then the max becomes much complex I do not want to get into that right now so at least for the limited phase I have shown that it is stable okay.

So now I just want to move right from here and what I want to do now is to make a connection with time series models okay again a when a started looking at these two things Kalman filter and time series models well somehow the book certain on time series models and book certain on Kalman filter are but two different proponents one kind of people who more believe in you know first principle models and then Kalman filtering is used there with mechanistic models whereas time series models come from computed different.

And there is a different connections and many times this is not highlighted in the sufficiently in many of the books but actually if you see what is the connection then you will be able to you know appreciate what is happening much better if you remember when we develop all those ARMAX, ARX you know those kind of models attempt towards to capture effect of unknown disturbances unmeasured disturbances okay.

In Kalman filter to you have two unknown disturbances wk and vk okay and attempt ahs to capture effect of wk and remove vk and know so something similar is happening we need to connect and see what is the connection okay now I want to first talk about this concept of stationery Kalman filter now stationery Kalman filter is what you people see that when k goes to infinity this pk given k-1 and pk given k do not change with time they go to a stationery function okay and the stationery solution I am denoting it by P ~ because this k given k-1 I am calling this as P infinity okay.

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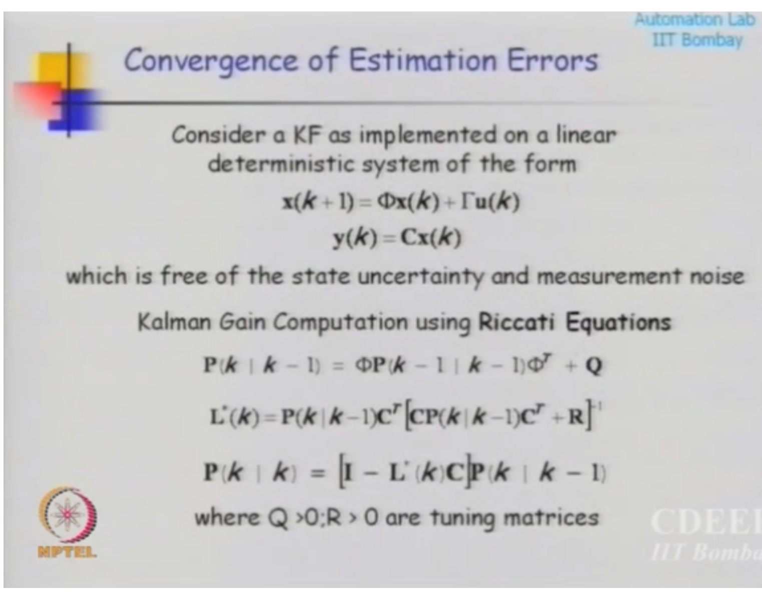

So those see these are when you go to Kalman filter these are difference equations if you look at these equations these are difference equations okay these are connected difference equations this and this the difference equations same time okay and then just like a linear system will have a stable solution this is a linear difference equation.

Now it will have a stable solution this is the linear difference equation now it have a lecture linear difference equations and it actually assemble to the certain conditions the solution converge to steady state okay.

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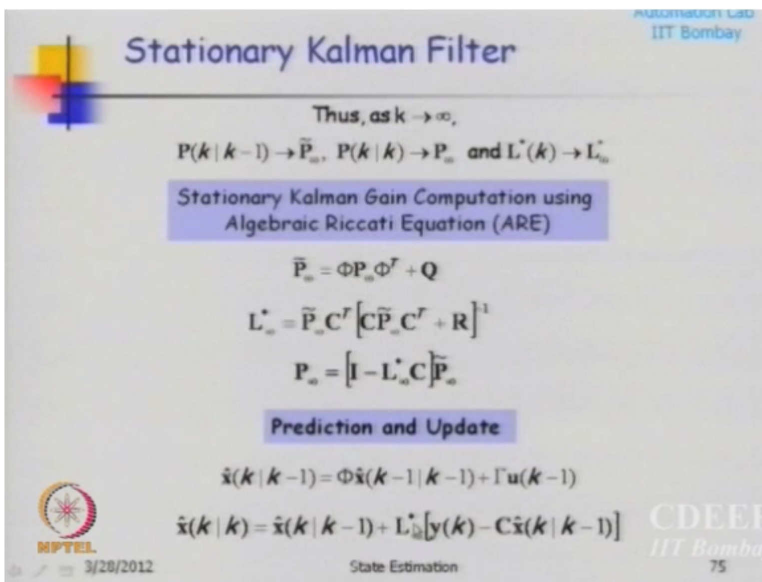

The steady state I m denoting by P ~ infinity P infinity and L* infinity now L *infinity is called as L *is called as steady state Kalman gate which means after some time when you look programming now my next computer assignment is that for the given system you have suppose to implement given by the observer and Kalman filter input to be given is pk okay start from my initial condition which is for the observer start from my initial condition different from the plant.

And see whether the observer convergence is the truth okay so these are the next assignment I will put it on the now the idea is that once we have the simulator you can do everything that we learned in the class so just now next submission is implemented remember from observer and Kalman filter on the particular system on which you are study okay.

And there will notice this program see that if you actually keep track of how gain is changing after sometime this gain becomes constant okay gain does not change the time and that particular gain is this now actually you can write down the equations the steady state equations okay these are called as algebra Riccati equations.

The original equations which we used to find the co variance matrix and the Kalman gain are called as Riccati equations and these are called as algebra Riccati equations because there is no time involved here in algebra Riccati equations I am just writing the same equations at the steady state so these are coupled matrix equations okay.

And there has been lot of literature how to solve this particularly when dimension become large it becomes complex problem and probably last part of 60's 70's lot of work went on how to solve the algebra Riccati equation how to solve so actually now my prediction and update is prediction of course is given be this there is no gain involved here update is given by constant matrix which is.

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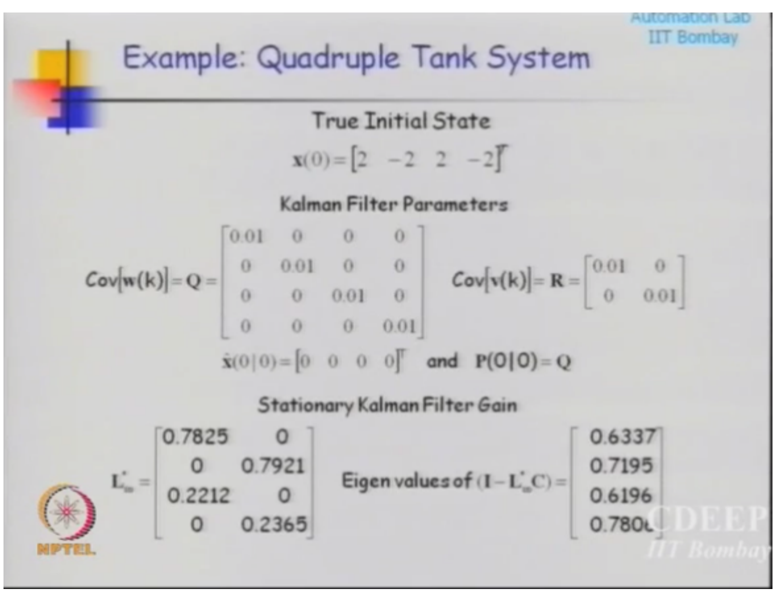

Now I am just showing you quadruple tank system which you are looking at theme example the I have taken co variance Q to be .01, .01 so Q is a diagonal matrix I have shown that there is uncertainty in each of the states not entering to any input but general uncertainty of the states whose co variance is given by this particular matrix and R is the there is the error in the measurement whose cp variance is again be .01, .01.

I just taken some sample 1 it is not that so that sufficiently there is large unknown component now just put that final steady state solution because it is difficult to show all possible Lk varying a function of time there are many Lk varying a function of time as K goes to infinity you get this probably we can see that it actually Kalman gain goes to.

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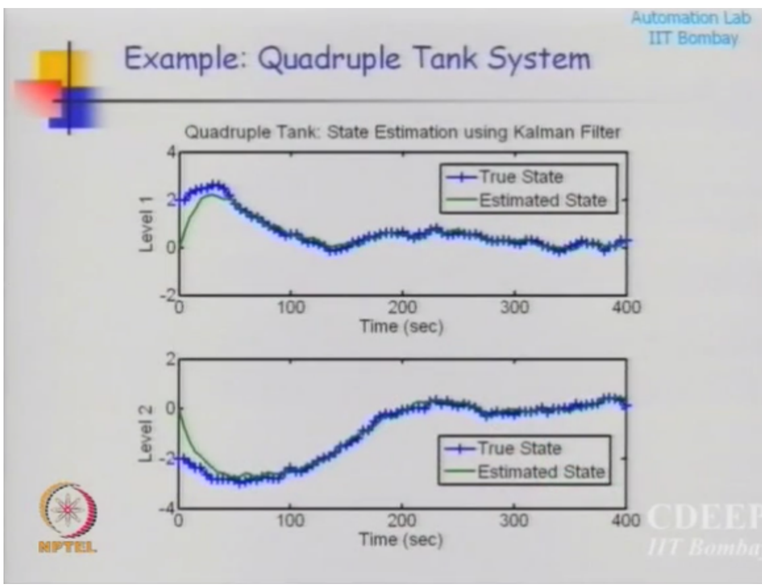

Let me just first show you that these results you can see here that this is the estimate this is the quadrapations model initial estimate I gave for the quadrapations model to be 0, 0 okay the true state is to -2 see I have the true state is 2, -2, 2 ,2 -2 okay initial value or initial guess for the observer I have to give initial guess for the observer I have given it to be 0, 0 okay.

The noise is there okay and then I have to you know I have to compare since I am doing the experiment in computer I can compare the truth with the estimate okay now these tow level 1 and level 2 are measured level 3 and level 4 are not measured okay so level 1 and level 2 you know there is no wonder that this starts following each other.

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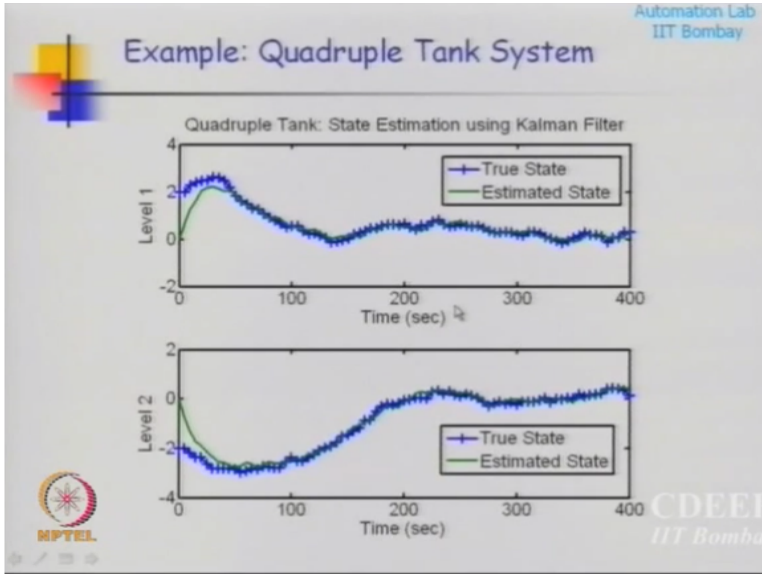

But look at level 3 and level 4, level 3 and level 4 are not measured so this is the comparison between the estimate this green is the estimate and this is the truth okay so this gives you least square estimate you have to understand you are never going to recover the truth this is the estimate that minimizes variance or co variance matrix okay.

So the green is an estimate of level and level 4 if I want to use it for the control I can use it because after some time the mean value is exactly following you see this so here you get least square estimate minimum variance estimate so it is not true value okay.

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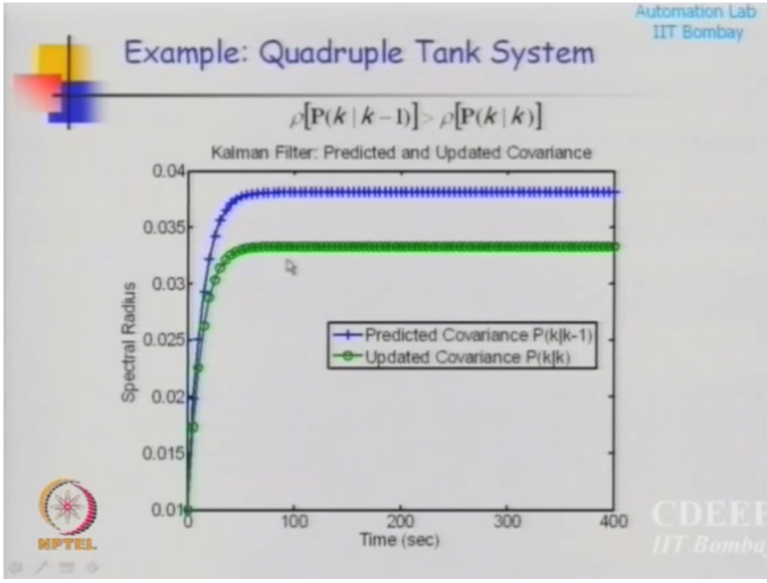

I just told you that co variance when you do n update the co variance of the update is state is smaller than the co variance of the predict state will that I have done here is I have plotted the spectral radius of the co variance matrices updated co variance matrix and predicted co variance matrix okay you can see that when you do an update the co variable reduces spectral radius reduces the spectral radius reduces.

If spectral radius reduces which may the co variance reduces okay because there are forecast four matrixes to show that reduce is difficult so I use a spectral radius of the measure because this is the positive matrix or I can put the positive okay smaller I can smallest spectral radius means the co variance is reduces.

So this is the demonstration of let us see how it looks like and let me see if I can demonstrate the Kalman filet is actually goes to steady state after sometime P matrix goes to steady state so now for the observer case I am not going to forced by code you can see it here but you have to write it on your own understand and write it okay so my Kalman filter is slightly older program.

But what you have to understand is that at Kith instant when I get an measurement I do all the calculation for Kalman filter so this is my pk1 is my predicted co variance update this is my Kalman gain calculation pk1 *C mat inverse of vk then I do a correction here this is the prediction I compute the co variance I compute the innovation ek, yk is a measurement which is coming from the plant and C mat*x^k is the estimate.

And this Lk times ek Lk is the Kalman gain which has been computed here Lk times ek is the where we are doing the update and this is the prediction this is the update co variance pk is function of pk-1 so these are the calculation of certain done for Kalman filter okay this is see you got to understand how the digital control system works to implement this.

At kith instant when I receive a measurement immediately I would do these calculations it is assume that it is almost instanteously at least relating instanteously compare to the gap between two samples okay so these calculations have to be done and then I send a whatever input that goes to the plant this is the plant simulation and this is the observer simulation this is observer and this is plant okay.

And then we can actually check how these matrices change as a function of time so let us start tracking them so let me say here to begin with we will put a or we will do it from the here that is Pk1 then pk and Lk and I will put a pause a so we will see that as we done this program okay it is the observity matrix and it rank is 4 okay.

So I am just showing you how pk changing in time and k changing with time you see that right now pk okay let us do it let us put them side by side pk and pk-1 let us put them show that okay you see how this co variance is change the time so Q and R matrices are given and then okay so this is my this part is my initial pk given k-1 this is pk given k this is updated co variance this part and this part is predicted co variances.

And Lk is a Kalama gain now you will notice that this value will start changing as a function of time so this again this is the predicted co variances this is updated co variances and so on and asymmetrically this equation will if you see the values here let us look at a diagonal values the diagonal values are reducing and that to diagonal values of the predicted co variances are higher than the diagonal values of the updated co variance okay so every time you update the co variances reduces the error reduces okay and Kalman gain is changing error function of the time.

But after sometime it will go to the steady state now if you see this matrix and this matrix just look at the diagonal values they are not different from previous time this LK is not changing so within some 20 samples the Kalman gain as converge to Lk* or Lk infinity okay co variance are converge to their respective steady state values okay.

So this is so after sometime see this change in co variance is occurring on the initial period after some teem you know it is going to steady state so one might say that why do you want to program you know if I running the Kalman filter for very long time only suppose initial 100, 200 samples Kalman gain is going to change okay.

Instead of doing this programming for you know time varying Kalman gain which is not going to change only for this takes the steady state value and you get practical approach you can do it you can just solve the algebra Riccati equations get the steady state Kalman gain and use it and you will get good okay.

So instead of doing this programming in which co variance change and then go to steady state you could directly choose to solve steady state Kalman gain equation the mat lab gives you solution which you want for the steady state algebra Riccati equation and you can directly get the Kalman gain and use it for so let us move back to is everyone clear about what is done so the observer is run when you quote when you right program is order to run at kith instant.

You have to get an measurement right the observer then you move from k to k+1 then this measurement is the loop this is the loop this loop comes back to observer again okay new measurement comes you do estimate and know it is running parallel to the plant okay that is what that is how you implement this okay and that you can see from this drops also see what is the meaning of if you see here initial period this spectra radius is changing after some time it has stabilize okay.

A spectra radius is not changing which means pk given k-1 nd pk given k they are not changing in time they have to stabilize okay we have stable solution okay.

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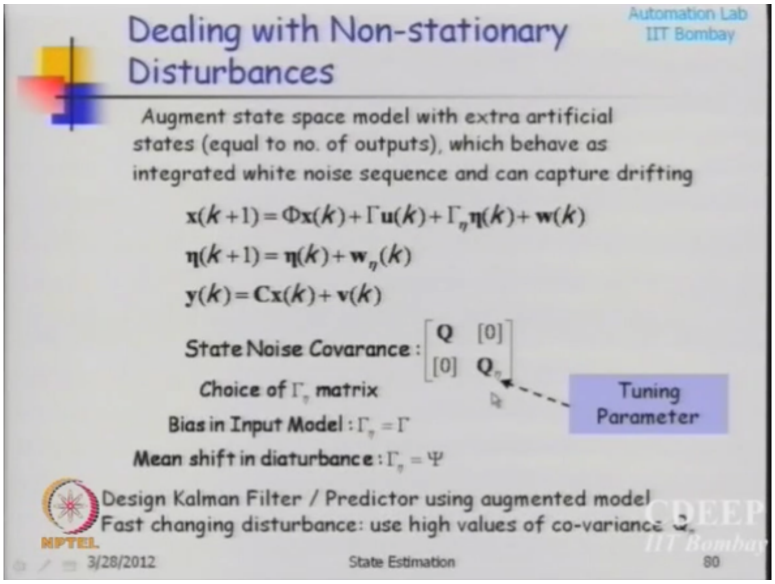

Now what is your model is not correct there is a mismatch in the model what is there is some disturbances or one of the simplest problem is suppose A and Φ are correct and C matrices but and the measurement noise typically is a Gaussian noise measurement noise measurement noise is not a problem but the state noise bring a Gaussian white noise process or white noise process sis the very simplest equation sometimes or often times.

The noise entering the state dynamics is colored okay it is like a drifting signal and so even though you are very elegant solutions when you are assume to be a white noise okay mathematically very convenience see many times be max simplifications so that you know you get some mathematically elegant solutions but then you start from there and you get some understanding there and then you start going towards reality.

So one of the things that is done is that suppose this wk we assumed to be white noise we modeled that unknown disturbances using two components we model this using another component here another additional I would say artificial state η here sometimes this is artificial sometimes it could never it to the real problem real disturbances.

And then this γ η is some kind of a tuning matrix so this ω is again is artificially introduced is ω is a white noise so what you are saying is that there is another disturbances which is integrated white noise, integrated white noise is random model so this is η k+1 is old value + white noise so this is integrated white noise this is like drifting behavior this is the most commonly used model when you want to model disturbances which are unknown but drifting.

A simplest model used okay and so this kind is used to take care of the fact that the state disturbances need not be white so this is white noise and this is called s integrated white noise why integrated white noise because this is the integrator η_{k+1} new value is old value + noise this is simple integration last value is equal to see integration I do not know whether you have done this derivation for you know PI controller recursive PI controller see is I suppose I am just giving you a parallel another example.

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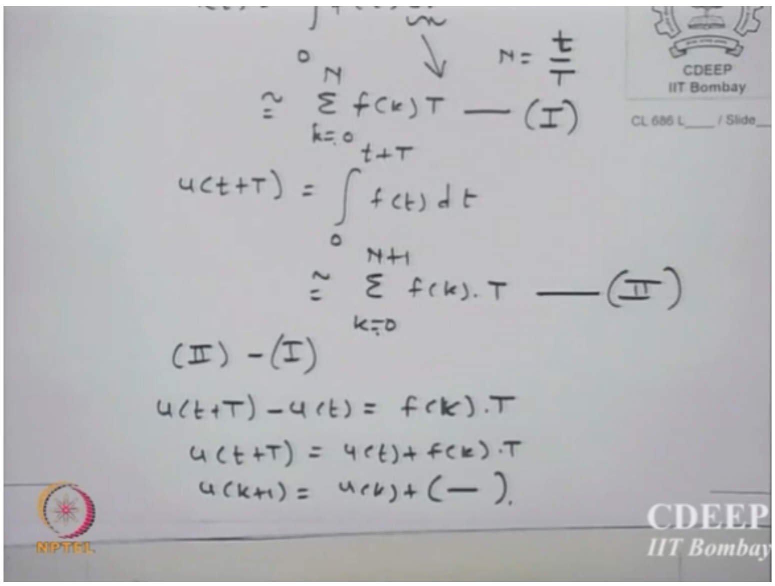

So let me appreciate why this is an integrator see suppose I have this term you know say ut= integral 0 to t, f(t)dt okay so this I can approximate as ∑I going from or k going from 0 to say n there are n samples from 0 to t sk and dt I will take sampling interval T okay so this integral I can dt is nothing but the sampling interval and fk is the value.

This function takes at discrete time points and if these samples are very, very close I can right this integral like this okay but then what you can do is that ut+ft is 0 to t+T which is f(t) dt and I can approximate this as ∑ k going from 0 to N+1 f(k)*T right I advance by one sampling interval okay T is my sampling interval or advance by one sampling interval so this is integral from 0 to N where N=t/T okay.

And N=n samples it time as small t and N+1 samples taken to time t+T okay so this is small t+ capital T so now one way to write this is to see suppose you take this as equation 1 and you take this as equation 2 okay I can subtract you know subtracting 1 from 2 you will get ut+ capital T-ut=see this is ∑ up to N, this is α up to N+1.

So if you subtract what will remain only one term will remain okay so that will be f(K)*T so I can write this equation as u(t+T)=u(t)+f(k)*T or in other words u(K+1)=u (k)+something so integrator in discrete time can be written like this okay this u is an integration of all the values in the past so I have writing as integrator in a different way.

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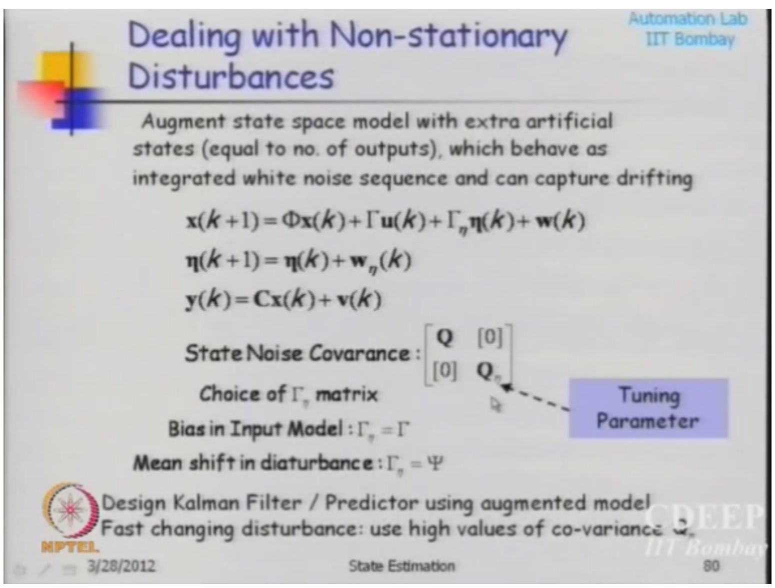

So coming back to our equation here this is an integrator okay η k+1= old vlue of η+correction now this correction is artificially introduce white noise so it is like a drift okay so drifting behavior so if I want to model state disturbances which are not simplistic white noise model then you can this is the common trick because you have to be able to model unmeasured disturbances there are ways by which you can do it slightly differently I will be seen that soon.

So this Q is when you know augmental system you have new states place model x k+ η k+1 together become a new state and then you can work with Kalman filter for the augment system so you can design a Kalman filter for augmental system and typically in such cases Q η is together tuning parameter to make sure that the drifting disturbances are model sand then a how to use this tuning parameter will require some experience it is not straight forward.

So w term will be there know w will get model through Q so I am saying that is addition of white noise+ integrated white noise w here that is the artificially introduced tuning white noise no it is not same as w yeah so this is something which is artifact of modeling that is just trying to capture a drifting noise as a combination of integrated white noise and white noise this is integrated white noise this is white noise and then attempt to merge the two into a okay.

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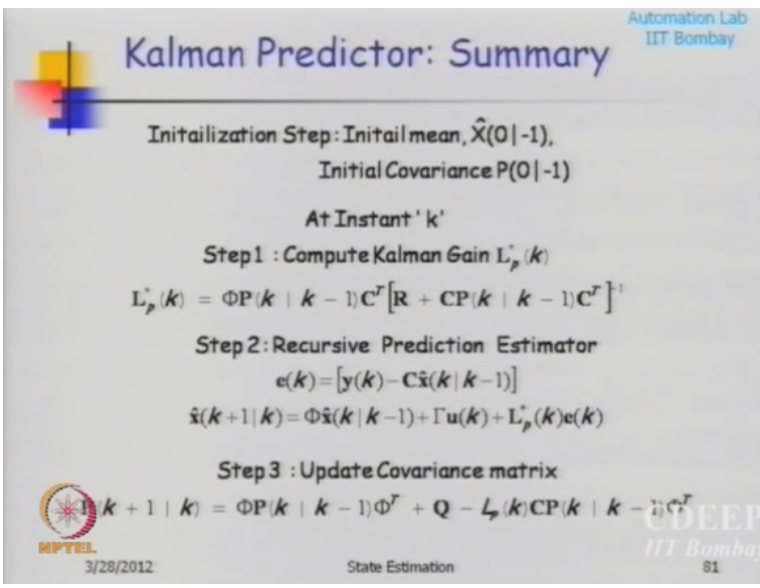

I just let go to Kalman filter summary we have to have initialization you have to have initial guess when you start you have to pick of your Kalman filter you need initial guess of the initial state you will also have to give a initial co variance this is typically given some α times I where α is large value okay you co variance large initial co variance means you are not sure about the state guess that you are giving you sure pretty sure about the guess that you are giving.

You can use a small value of this α times α can be small so this is the kind of tuning parameter when you actually start a Kalman filter what should be the initial co variance you have to give guess and this P_0 actually will tell what trust you place in your guess okay if you give large P_0

which means you do not trust your co variance if you give small which means you have a reasonable guess okay.

Then the at the Kith times step the thing that you to do is to compute Kalman gain there do this recursive estimation and one minute let us well some time back if you remember I talked about two different types of estimators we when we yeah.

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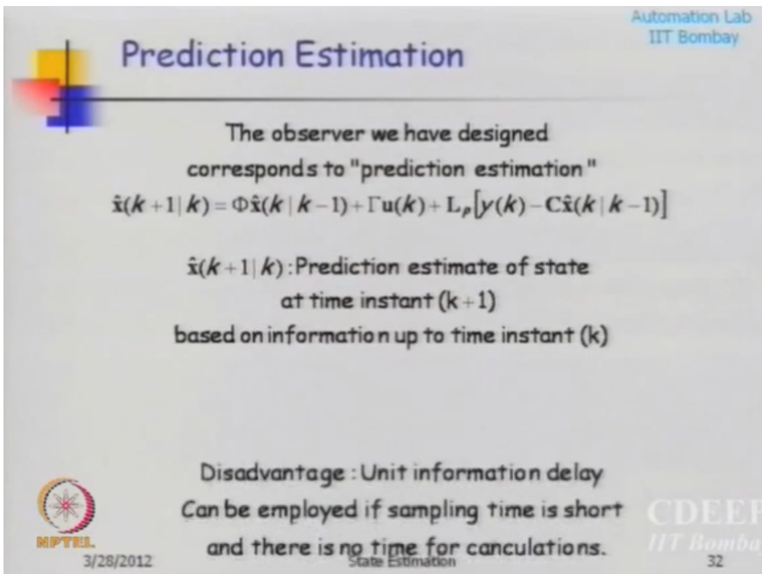

When we talked about remember of observer the first developed this prediction estimator if you remember and then I said filter estimator prediction estimator is using one there is one time delay between the estimate and Y okay.

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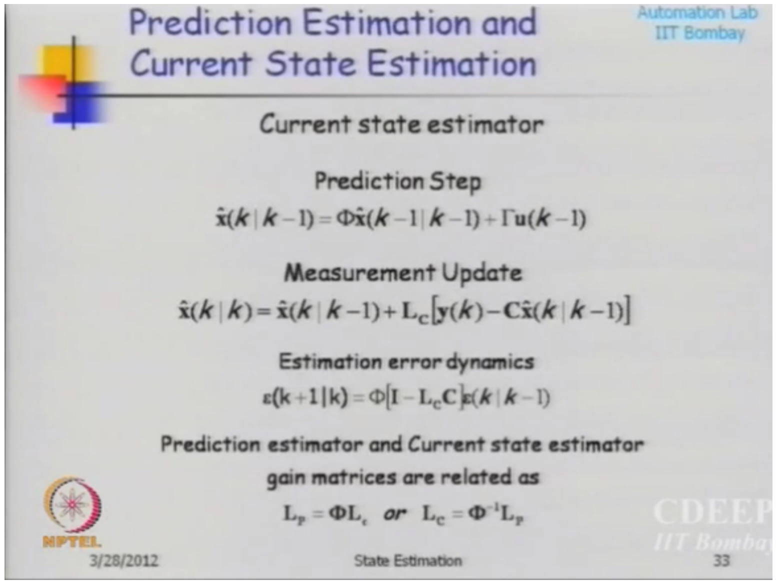

And then we had this current state estimator, the current state estimator was prediction step and then correction step right so there are there in the observer I first derived the prediction estimator and then I showed how you can use the filter estimator in Kalman filter I first derived the filter, Kalman filter I can also derive Kalman predictor once step predictor okay.

So I am not giving you a derivations I am directly giving you a final form here you can look at derivation if it is run very, very similar to what we have done slight algebra is different but the final form is more or less the same I mean in the ideas in the derivation more or less same and final form of the equations is slightly different okay.

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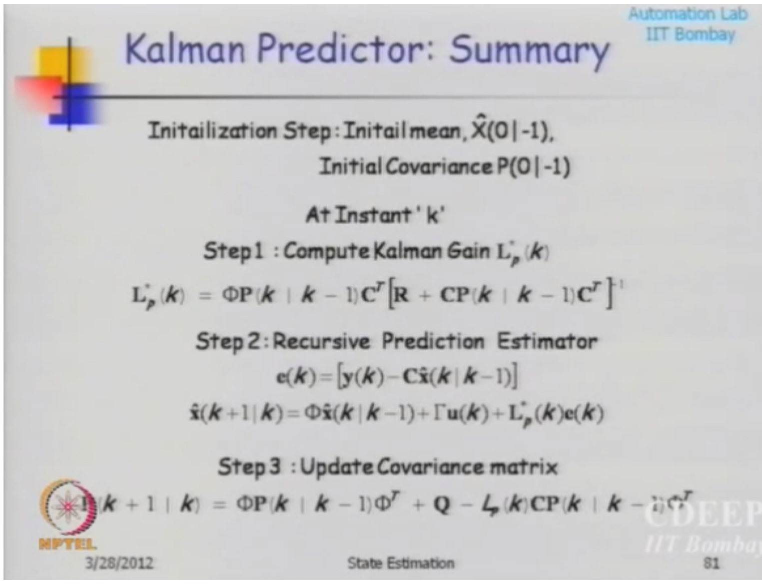

So this is in this case prediction estimator I compute the Kalman gain okay I have a prediction estimate which means k+1 is Lp start as like prediction remember the observer this is the prediction Kalman filter Kalama predictor okay and the update there is only one co variance update there are no two different there is nothing like you know prediction correction this is only one step.

So again you will ask why will you use this and why not the Kalman filter it depends how much computation time you have you have to understand that you do is calculation of matrix multiplication is and you know gain update inverse or whatever is involved in real time and it depends upon which system you are trying to use this observer.

If you are system is very, very pause if we do millisecond calculations and if you have a processor which is you know ongoing processor is not so powerful then you might result to prediction because in prediction you can do this calculations between the two samples keep again ready and when the measurements come you can just all can at a instant when you want to use observer.

You can just use the prediction estimate and do not wait for calculation at a given sampling at instant so that is where the prediction estimator is useful another reason I am talking about prediction estimator will become clear very, very soon but is the idea clear is what just like we had prediction given by the observer what is the prediction? Kalman predictors okay this Kalman predictor and there is only one co variance.

Because we do not have a prediction correction is only prediction step okay and this is the this is how you update the this is the new co variance from the old co variance this is how the you calculate the Kalman gain for this case but Kalman gain and for the prediction case and the Kalman gain for filter case only differ through this Φ okay if you look at the if you compare the value if you have the notes you can find out and see the how the values compare only this Φ will come extra here.

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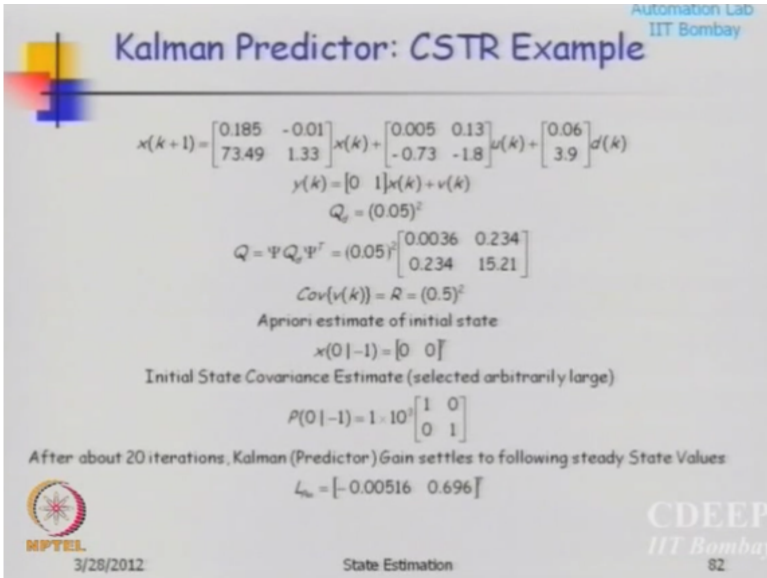

So I am just showing you how Kalman predictor would look like this is for the CSTR example the reactor example this is my model I have developed a Kalman predictor here in the steady state predictor will look like this.

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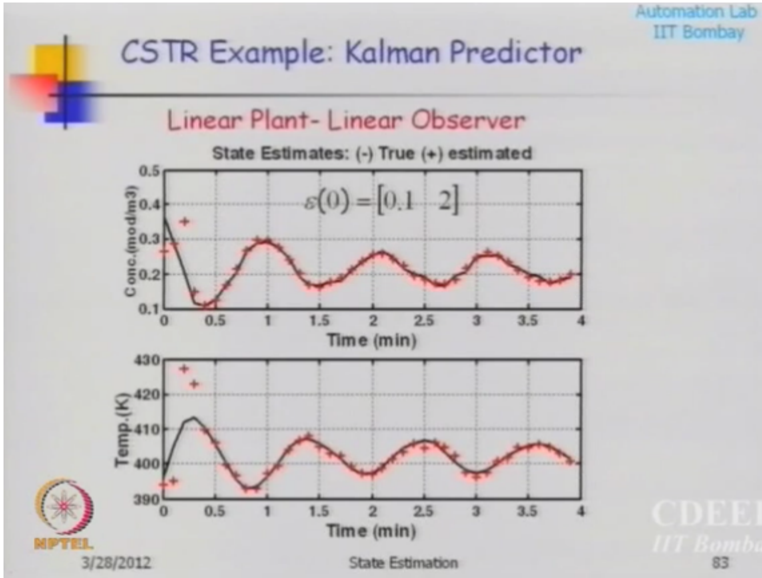

And I am trying to estimate concentration from temperature and very quickly the observer error goes to 0 and since I am doing simulations I can actually compare the truth with the estimator value I can see here that the estimator concentration and the true concentration almost setting on the top of the thing so you will get very good estimation very quickly.

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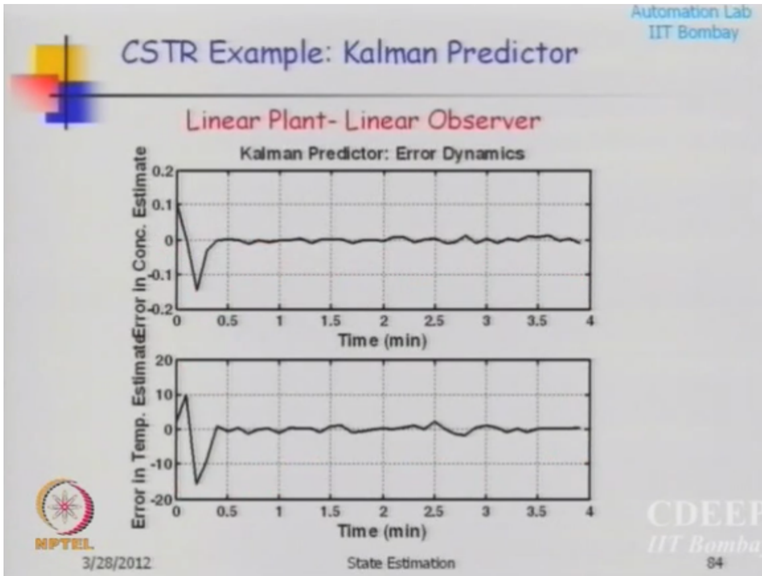

So the error is you know initial error is large but after that it converges this is like 0 error and you get mean value estimate through this so the point is see now in the reactor I want to control up composition okay now once this observer is settled I can develop even before you go to the advance control I can develop a simple PID controller okay.

That will measure concentration what do you mean a measure concentration actually physically you are measuring a temperature then you give the temperature measurement to this observer, observer will be reconstruct concentration estimate and then based on the concentration estimate you will do the control so you course on the loop using estimated state okay so this Kalman filter is actually acting s a soft sensor okay.

So actually in some sense after this micro process have become you know so powerful and you can use calculation the combination of real sensors which give you, you know fast measurement of some physical variable which can be measured together with these model based sensor you know as expended the possibility of measurement for beyond what possible say 30 40 years back when you could not use on board computers now you can put a on the computer you know on a device and you can have these soft measurements coming up.

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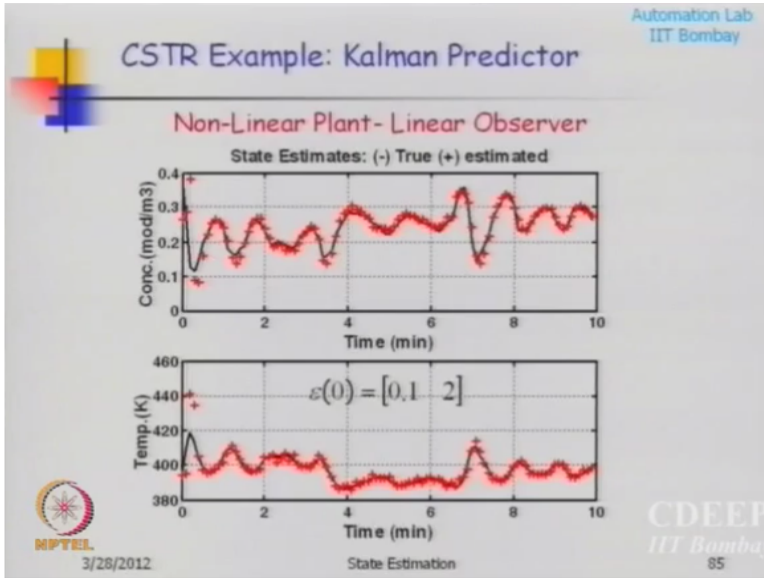

So model based sensing I would call it model based sensing is a very, very rich area used in many different fields of engineering okay.

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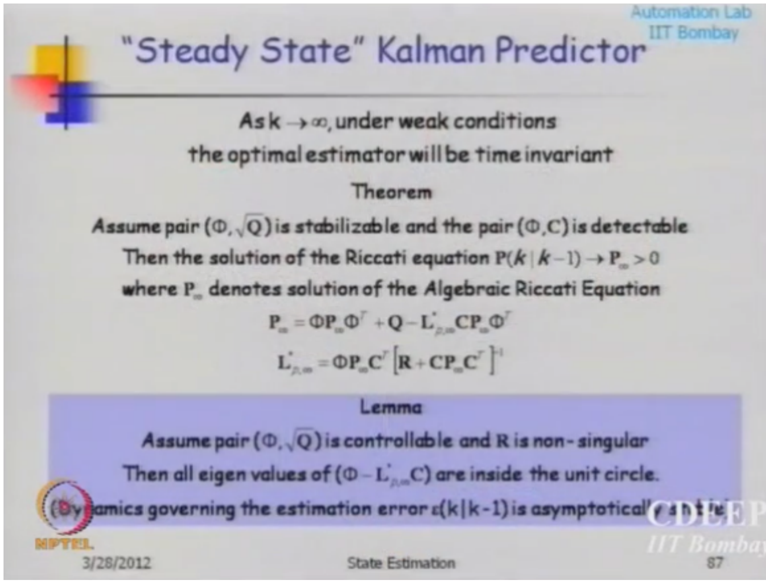

Now question is under what condition the solutions of Riccati equations exist and when you will get stable solution so you can show that under weak conditions this pair Φ and square of root of Q is stabilisable now what is stabilisable? I m going to talk about it right now just accept this theorem and if Φ and C are detectable be care of observibility.

Then you can showed that the solution of Kalman predictor always exist and you will get a this is about the steady state solution you can actually come up with algebraic criteria this is for the Kalman predictor just like we can know stationery Kalman filter you can know stationery Kalman predictor and this gives you conditions under which the solutions re you can find the stationery Kalman predictor which is stabilizing them the error will go to 0 okay.

So right now all the things there are needed to understand the but I am setting for the completeness may be as the progress it may able to collate and understand this reason I am talking about the steady state Kalman predictor is.

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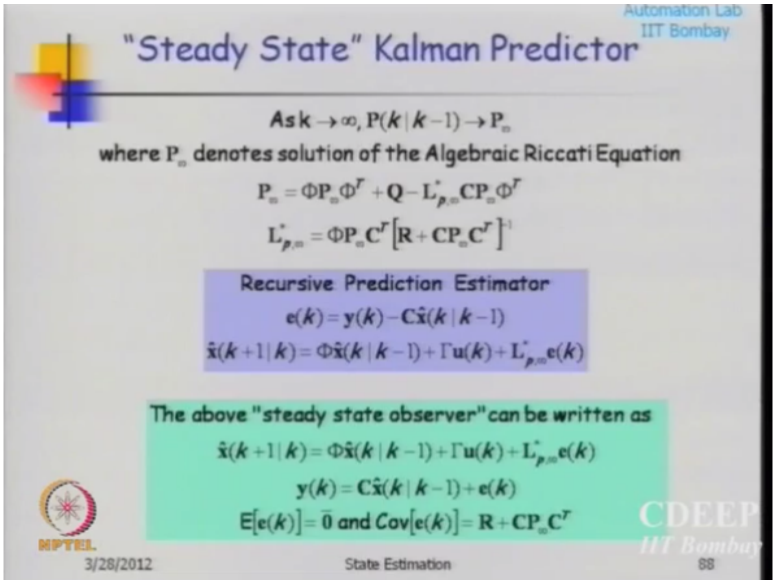

This is my steady state Kalman predictor as k goes to infinity pk given k-1 go to p infinity let us call the p infinity here this p infinity is the solution of this algebraic Riccati equations these re to coupled equations if you see here this p infinity is a function of p infinity +Lp but Lp itself is the function of p infinity okay you can actually eliminate and combined on equation one big equation and solution of this is the steady state Kalman predictor and steady state Kalman predictor.

When does the solution exist for this particular this is our tough equation to solve because just remember these are matrices all these are matrices okay and then you have problem here because use this p infinity nicely multplicated but when you go to Lp there is an inverse of p infinity coming up here okay.

So solving this problem getting solution for this problem is not tribal okay fortunately now mat lab gives you solution of I think there is sub routine called A or E algebraic Riccati equation okay so you just Φ and C and say A or E it will give you Kalman gain okay now to solve those algebraic Riccati equations you just give P Q ΦC okay and say A or E and if you do not have mat lab you can use Φ lab so which is the public domain software.
And you can install it on your machine and then you can get going you can design it so it is not a so any one of these standard or you can write your own I need quote you saw the red goes to steady state so even if you do not have ARE okay you write your tiny quote for the observer after some hundreds samples you will get the steady state okay no problem.

You are in the business you just have some computer with you and well these things I mean these things can become complex when you minims you want to have embedded observer it can be complex because you know it being on the computer assume that inversion can be done very easily if e want to do it on some micro controller writing an were 14.you know computation support may not be available.

Or you know it may not be so powerful then doing matrix of computations then doing matrix inversion is not good using computer but computers are relatively slower okay and difficult to use for the very fast systems so well so embedded computing even though things are very simple on the computer PC laptop they are not easy when you go to embedded control but if to see what is the advantage of designing this lp ∞? What is the compression advantage? Online there is no every time calculation, you compute this off line, you design this lp0 this step involves this computer.

So online matrix computation all are gone and anyway you know after sometime it stabilize to lp use that lp ∞ and you know solve the problem, so straight way of. So this is much more practical in to have stationery Kalman and use it but in the assignment you are not going to use it stationery.

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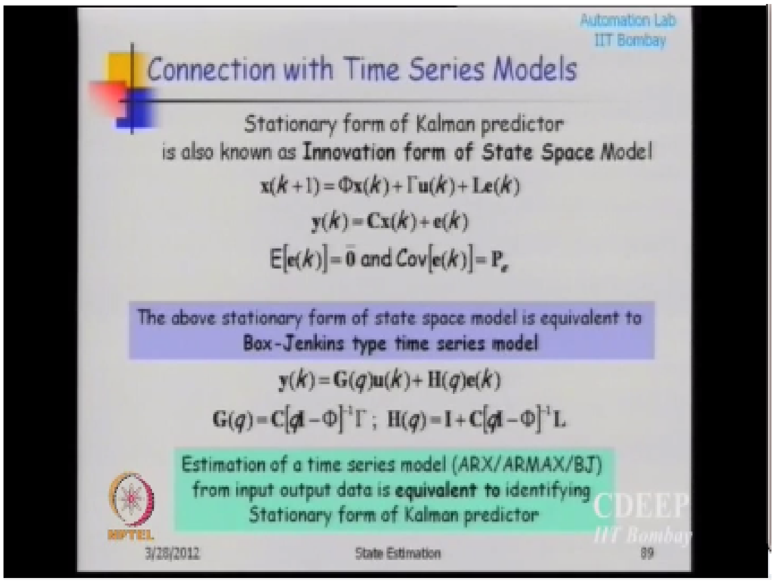

Now I what I want to do is, I want to connect, I want to show that stationery Kalman predictor is nothing but a form of boxing fix model or time series model. So we just close the loop and other way of looking at this is at boxing fix model or ARMAX model is a parameterization of stationery Kalman filter that is under view point okay. so look at this stationery form of the Kalman filter.

Now instead of calling it L* I am just see this is what we had right to look at this last equation x^ k + 1, φ x / k -1 γ uk + some constant k x ek, where ek is actually yk – x hat / k -1, so I have written it an different form in a same equation okay. Now I am going to just remove this you know k + 1 / k – 1i am just going to remove that entire complex notation. I am going to use the same notation that we use at the end of systematic identification okay.

So this actually this form of model is called as innovation form of state space model here you have φk x k + γuk + L times ek is defined by this equation yk = Cx k + ek this kind of model form is called innovation form of state space model. It is driven by we have shown that ek is the white noise sequence and ek in our case are innovation we have shown that innovation is a 0 mean white noise sequence we have proved that earlier okay.
It is 0 mean white noise sequence with known co variance we also had estimate of P how to compute P so if I just take Q transform of this I will get nothing but you know we have done this earlier, I will just take few transform of this, this is nothing but the Box Jenkins model okay. These two are one in the same, so I have arrived at completely differently.

Earlier what we have done we developed box Jenkins model first and then state realization, now I am coming to Box Jenkins model to another root. I developed Kalman filter then I developed stationery kalman filter and I said if I take it is transform you get Box Jenkins model okay. so it is the loop once I started from this end and I came to this end, now I started from this end and I am going back here okay.

So I am just showing that stationery kalman predictor is nothing but a form of Box Jenkins model okay it is the form of Box Jenkins model. So there is the deeper connection between time series models particularly when you are modeling unmeasured disturbances okay and now will you understand it whether this course, no it is very difficult.

You will take some time to seek in it took many years for me to understand this connection, unfortunately the books we talk about time series modeling do not talk about kalman filters. The books which talk about kalman filter do not talk about time series modeling. And then you have to read both the connections. Dooms book it has connection but he makes it an exercise so I am just going to happen and solve emphasis problem you do not see those connections.

Just keep this in mind that when you are developing a time series model actually it is the way of developing a form of kalman filter and other way around a kalman filter stationery kalman filter can be showed related to Gaussian models. So I am saying this is generalized form it could be ARMAX it could ARX so all those are parameters equation. So identifying these models is somehow equivalent to.

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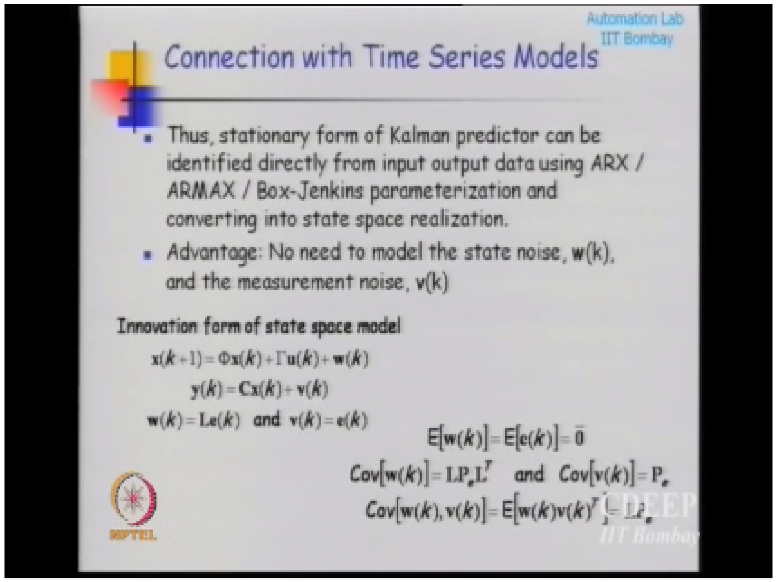

So one could say that kalman predictor could be directly identified from data and I do not have to go through 1st principle wk vk characterization then kalman and I can directly Box Jenkins and the I have a kalman filter, so that is another view point it is completely data driven getting a kalman stationery filter. So you can always instead of doing this you can always directly identify from time series data.

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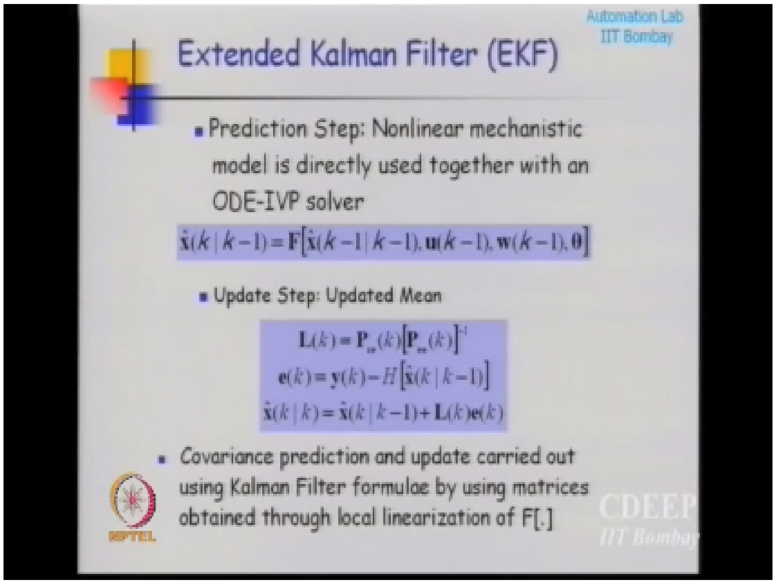

What do you do when the model is not non linear okay that is done using that I am just going to mention it in 2 minutes, we do not have time to go through this, if you have see we did a linearization okay and then we developed a observer using linearizing models but in reality most of the systems are not linear they are non linear and then you need to use non liner model.

So actually kalman development I have uploaded by the way those papers the seminar paper which is given it this entire area. So kalman paper talk about linear system that problem was I will not call low hanging it was very difficult to come to that point where you can come up with those recurrence solution, it has so elegant but after that the trigger research on what you do when the model is not linear.

So the simplest thing that came up first was or the extended kalman filter, where you try to use same updated formula or gain calculation formula as kalman filter but through local linearization of the non linear model and I have just summarize it here very locally. So you just do the, these equation for the gain calculation remain same expect the φγ is computed by locally linearizing the non linear at every time point.

So it is kind of merging the two by using some of the approximation and I have uploaded one very nice Chen which actually talks about entire development of state estimation from say 50s to now. A very exhausted way on that probably you should attempt if you want to do research in that area one of the very nice way.

So ek will show you an example if I do eks now ith linear model what happens is you can use it only in the small range around the operating point, now linear model you can use everywhere I am just showing you here.

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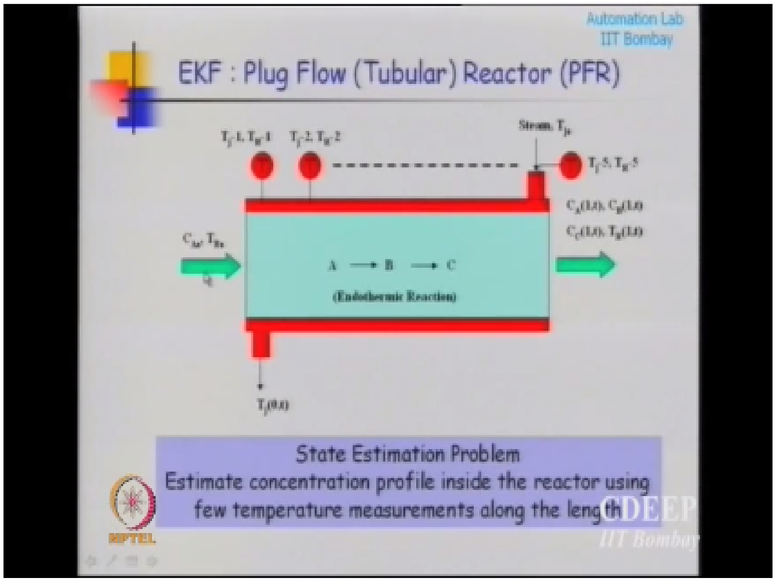

Over a wider range I can use state estimation using this extended kalman filter, in extended kalman filter I am going to use non linear differential equation directly in my prediction step, in the update step looks similar. See what is different the update step is same okay, the prediction step here in extended kalman filter is nothing but directly using non linear differential equation okay that is what it means.

I have written it in little bit in abstract form but this prediction step instead of using the linear difference equation we use non linear differential equation directly OD 45 whatever. So this is estimation of another example a tubular reactor this is the system in which a reaction is carried out from A goes to B goes to C and typically in such system what do you want to know is how the concentration is changing as a function of length and as a function of time, this is the distributed parameter system.

Concentration of the reactants A, B and C are changing as the function of time, you can only measure temperature okay but temperature is also changing as the function of time along the length, so this is the problem which as more complex than the 4 time problem. So you can place only few temperature measurements along the length of the reactor, it is like a think of a tube in which the reaction has been carried out.

Now I want to know how the profile inside changes as the function of times I can use partial difference equations in my prediction step.

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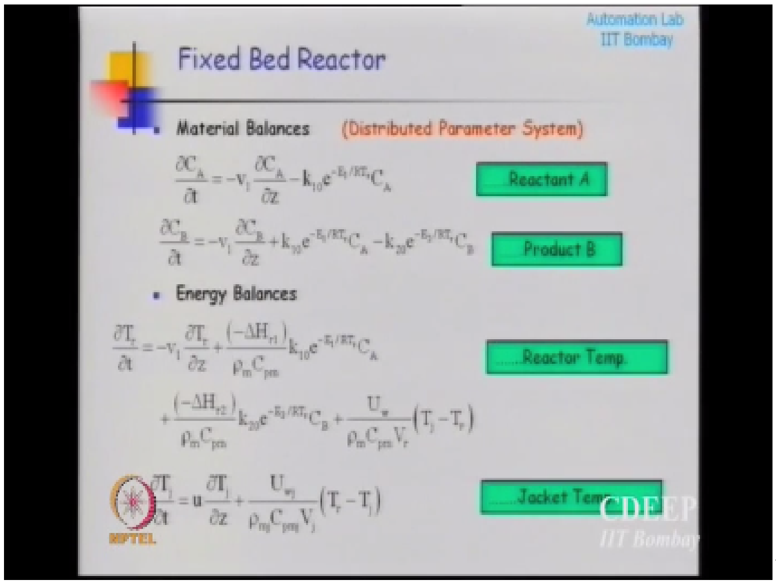

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These are couple partial differential equation that talks about how concentration and temperature are related and so on if you solve them together you can develop a software sensor that actually shows you the profile inside just based on temperature measurements okay. So I am merging temperature measurement data with the model predictions online in a computer okay and then this is some m tech students' work where we actually use this to.

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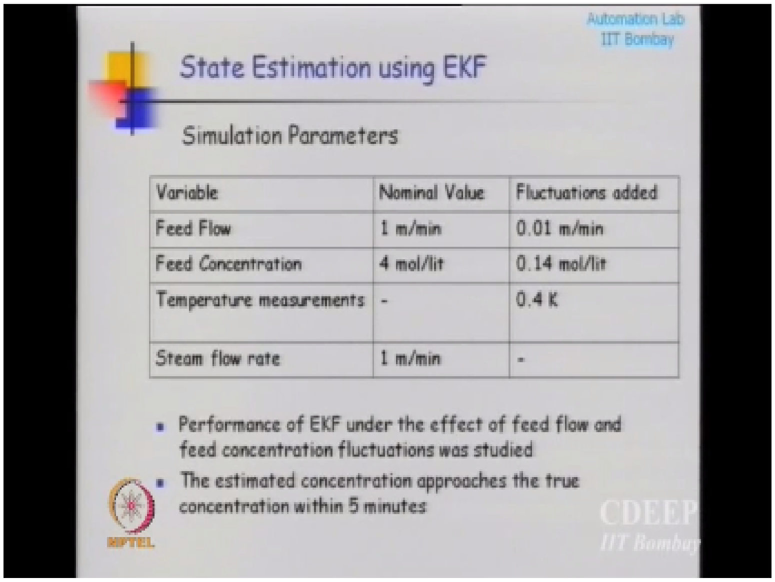

Variable	Nominal Value	Fluctuations added
Feed Flow	1 m/min	0.01 m/min
Feed Concentration	4 mol/lit	0.14 mol/lit
Temperature measurements	-	0.4 K
Steam flow rate	1 m/min	-

Generate concentration profiles inside.

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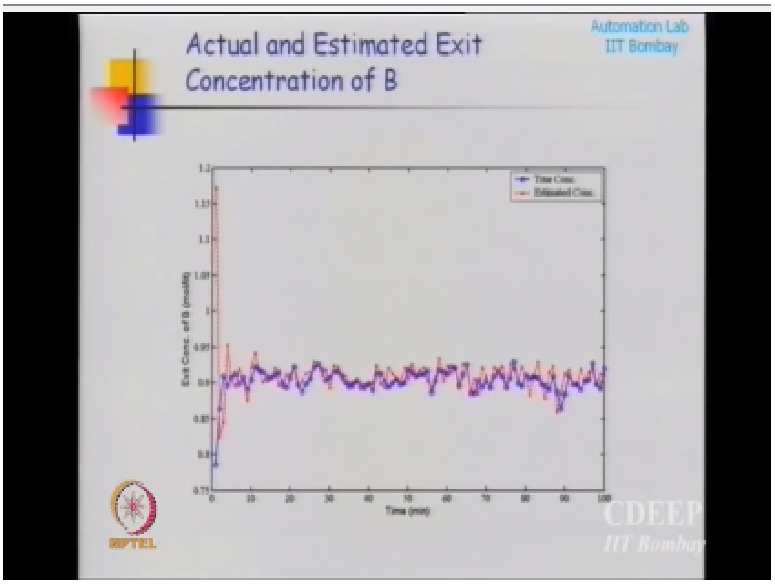

Some details are given here and there are unknown disturbances in the feed concentration and then this is the exits concentration I am interested in what is the purity of the product but I can only measure temperatures I cannot measure concentrates. So I have used these partial differential equations online, did the temperature measurements through kalman filter.

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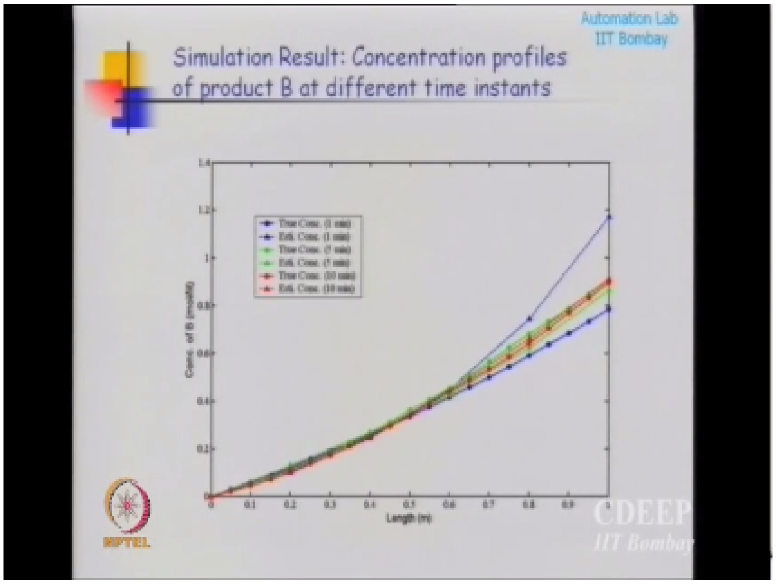

This is how the profile of the concentration changes in function of time, so this is that time 1, 5 minutes and profile goes to steady state space after sometime. So using this model based technology you can actually see what is happening inside, you can reconstruct a state.

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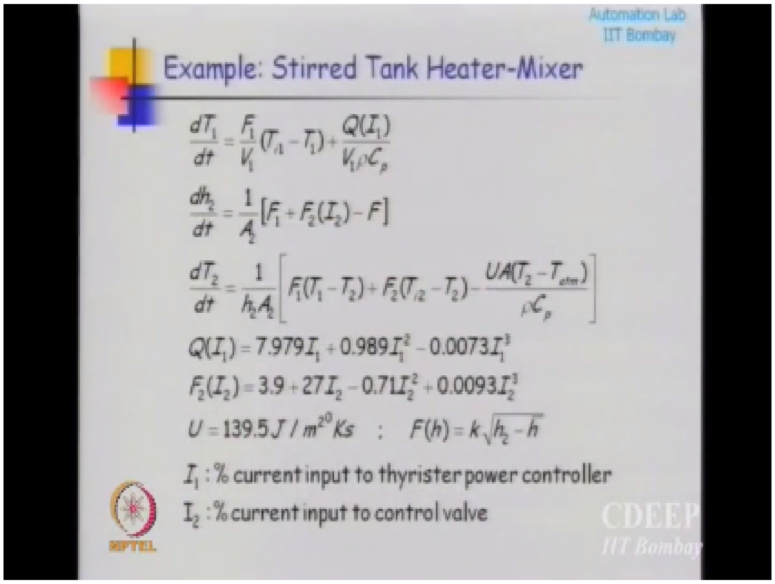

See ultimately all these things are used in all kinds of image reconstruction and you know because you only get certain measurements, you have the model and you reconstruct what is inside okay. So this is one example where we have you know this I have shown you earlier there are 2 tanks in series there is the heater here and mixing hot water and cold water this is the simplest system you will had in your bathroom.

And you want a particular temperature for your water for bath okay, so this is a my problem is to control temperature at level in this particular tank, the problem is here one problem we faced while developing a state estimate at here is that the heat transfer from this tank wall to outside changes as a function of time in the day time it is different, night time it is different, it is different when the pan is working.

The heat transfer to the atmosphere is different okay and the model changes, so what we did was we wanted to monitor online the rate at which the heat transfer is changing, it is not a directly measurable parameter okay but through model what we have done is that, see this is the Q transfer we have developed under one particular condition.

We multiplied by effectiveness factors some α, and we say that α can change with time, extra state that can be estimated using these measurements and then I am showing you here, this is effectiveness factor heat loss parameter we called it okay I can track okay. so when the heat loss parameter was wrong there was the difference between the estimation temperature and the true temperature.

After sometime the observer is able to estimate things that are not directly measurable that is what I want to point out here, so there was the mismatch here and after the mismatch was closed okay. After I corrected for the wrong parameter in the model online I could get good estimates of temperature.

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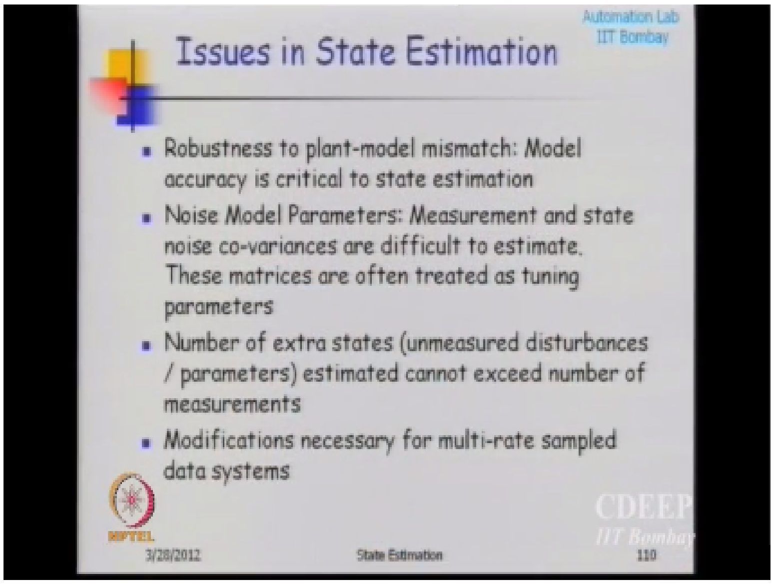

So there are many issues in the state estimation what to do how to deal with the par model mismatch. I am doing all this things under the assumption of the model is perfect typically the model is not perfect, now what to do even if it is perfect in the beginning when you start your observer after sometime the model can become bad. See for example in chemical plants what happens is that there is fouling okay some deposits take place.

If you have used the emersion heater in the beginning it is you know very nice and no deposits but after sometimes if the water is bad there are deposits and rate at which the heat is transfer to the water changes okay so there is the model plan mismatch. If you develop a model in the beginning that will not work after sometime so you need to change the model.

How do you estimate the measurement noise and the state noise golden question? Recently one of my students completed his theory on this part, how do you estimate noise from data, then there are issues like, how many extra parameters you can estimate from given set of data, what if the data is missing okay. What is there is multi ray data, in wireless censor packet loss appear okay.

In some situations for example some concentrations they cannot be measured at the same rate as you can measure temperature, so there are multi ray systems okay. so you can modify all this for begging with multi raid systems, all these things can be done using filtering ideas and it is a vast area of research. Even though the research is going on in this I do not think we have the solution, there is scope for lot more.

There are some fundamental results at least for linear system there you can prove it, but non linear case there is not proves, because there is rank condition which violates if you have one measurement and system is observable then you can only estimate only one more extra step, you cannot estimate. I can give you a reference where, but how does it translate when you go to a non linear system is not clear.

Local observability what does it means in terms of global observability of a non linear system is not varying, perfectly linear system, you can prove this that you cannot estimate. Particularly when you model the extra states at a integrated whiteness, if it is not white noise I do not have a clear answer, if it is integrate white noise that is how the typically extra state, the parameter that we estimated we model with the integrated white noise.

So we have two measurements, we can estimate two extra parameters and not more than that, if you try to estimate in sense what you can uniquely estimate. See if you start estimating more parameters you will get some value but it is not unique, the answer is not unique I am talking about the unique estimation. This is the independent of whether it is unknown filter this is the filtering issue.

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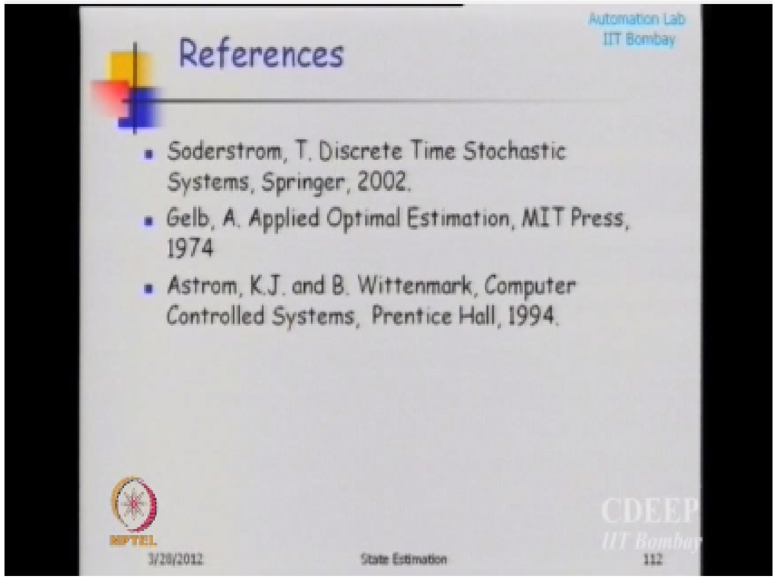

First of all dynamic based observer can be used to reconstruct unmeasured states from frequently measured outputs, and kalman filters of course one of the most elegant solution when you know the noise model, it can be used extended kalman filtering can be used when the system is not linear and actually these are belong to class of so clad Bayesian it is a wide class of observers. The new names and the new buzz words in this area are particle filtering and symbol filtering and so on.

These are used everywhere, there are applications, so actually what I have taught you here is not just limited to process control or to control, this is developing an soft sensor or state that cannot be directly measured but you have the model for it. How do you reconstruct? How do you merge? Observations which are available online with a model and create a software sensor or model based sensor popular to the industry will be a software sensor.

For you guys it should be a state estimator or an observer okay, so well these are the nice references soderstrom if you want to go deep into this area soderstrom book is excellent, the book by Gelb is also one of the standard references for kalman filtering and Astrom Witternmark of course. So this brings us to end of these lectures on state estimation and I just preempt you now with where I want to move next.

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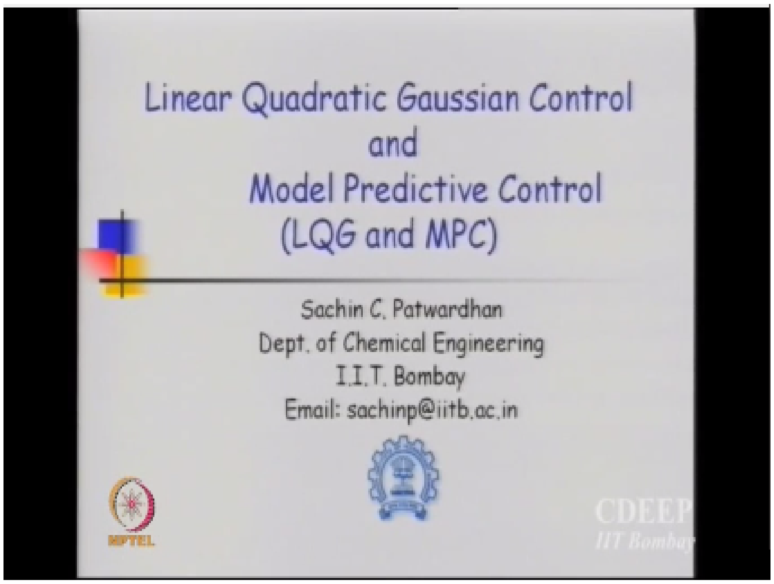

Well where I want to move next just take 5 minutes to talk about this will start these all the lectures from the next time, I am going to talk about a control. So now just take a look at what we have done, we have started by saying you know we want to develop a control relevant models, we have two ways of developing controlling models.

1 if I have a mechanistic model I linearize and develop a control relevant model which is linear difference equation, if I do not have is start from data and then derives a difference equation states place model which I can use for developing a controller. In between before we went to control design we talked about this observer, observer is using just the measurements reconstructing the state vectors.

Now I am going to complete this, so remember one thing when you did modeling you had the interface with the real world because model is you know trying to limit the real world, you are in the world of models this is the imaginary world okay. Here first of all you are looking at linear models okay or liner results are valid, so it is a very nice universe where you can do manipulation you can place poles.

And then you design a controller in this linear model space and the problem comes is how do you implement this that is realization through hardware that is the control computer. So you go back to real world by implementing the observer controller into, so now I talked about observer and controller afterwards. I am going to talk about what is called as straight feedback controller, so state feedback controller assumes that is the state measurement develops.

In practice what is available is only some measure values but then I have shown you to reconstruct the states, so now assume that the state can be reconstructed I have state measurements available now how do I design a feedback controller if all the states were measured okay, that is the next question that I want to answer okay.

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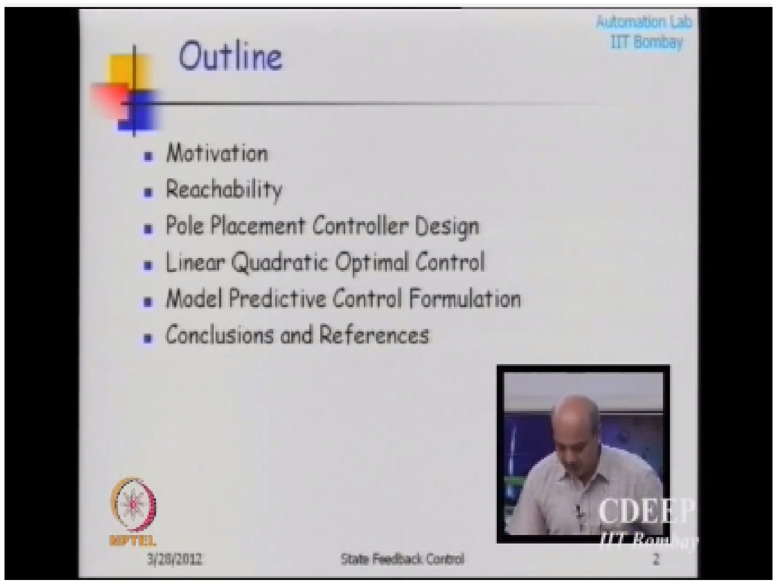

So what is the outline of this I want to talk about motivation then there is going to be this part I can go a little faster than observer because you will find this is like a to mirror images, there I talked about here I will talk about something called reach ability okay, which will be a very similar concept. There the idea was can I estimate state from the measurements now I will talk about can I take the state from anywhere to anywhere.

Then we talked about pole placement observer I will talk about pole placement controller same idea, transform this system to controller there we did observer form. Well when we went to multiple inputs multiple output systems we did not pursue with pole placement, we went to optimization formulation, I am going to optimization controller design which is linear quadratic optimal controller.

So just one to one mapping you can do and then I am going to talk about this technology which is the practical technology which is implemented in the plants and we will also see whether this is my friend who actually implemented this, he can give one or two lectures of actually how he does it practically or the real very complex plants.

So this is complete cycle, so with this course you should know at the end starting from scratch even if you do not any model, you can develop model from the data, you can develop estimator combined with a controller and then you know how the whole thing is done, the entire cycle. So we are not missing any point in the life cycle of controller from model development to controller.

We will do all that and the we will start talking about these developments so only one slide I will show here before I will close this, so given this state model and controller design is that we assume that all the states are measurable okay and design the controller then we design the stable estimator actually we will take the step 2 1st we derive a estimator and then what we do is we combine the two.

So estimator a reconstruct with states using reconstruct states you will implement the control, so and then we will have what is called the separation principle where you can show that you can independently design observer and controller to be stable with joint system it will be stable, so that I will show. Why I am working with the state space because I can work seamlessly talked about signal inputs everything can be done under one frame work.

So it is very easy to, so this is what is going to be my next say 3 or 4 lectures I will complete this and if you ask me what is the next computing assignment we implement kalman filter what is the next assignment. Whatever we do we just go and study on that particular system.

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