Biomechanics
Prof. Varadhan SKM
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture - 46
2R Inverse Kinematics

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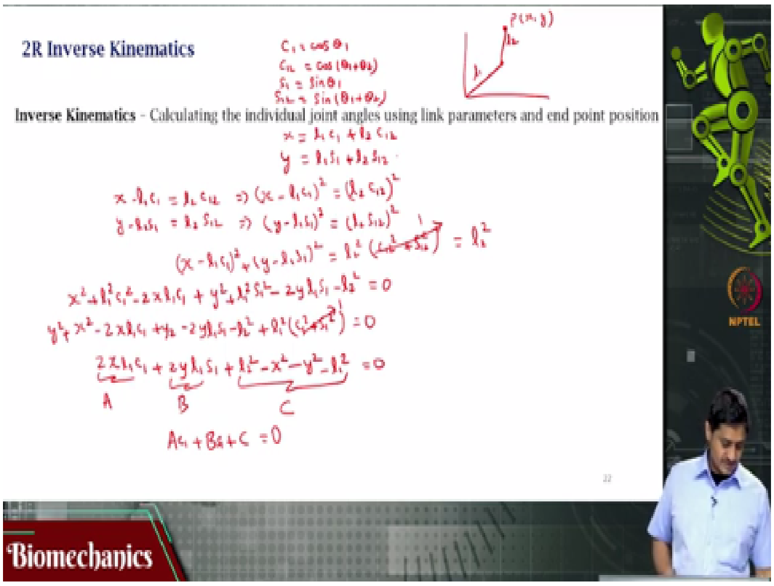

Welcome, to this video on biomechanics. We have been looking at kinematics. Specifically in the previous video, we looked at forward kinematics of a 2 link serial kinematic chain. In this video, we will be looking at 2R inverse kinematics or in other words, if I tell you the end point, coordinates of 2 link serial kinematic chain and the link lengths. Can you tell me what is the joint angle at each of the links? This is the question, broad big question.

It is a 2 link chain, planar chain no complications not 3 links not n link, there are exactly 2 links. There are 2 links and I am telling you the link lengths and I am telling you the end point coordinate, what is asked. Can you compute the individual joint angles that each of this link makes with the ground? This is the question. So, what we know from forward kinematics is suppose, I have these 2 links, with lengths l 1 and l 2.

If I were to express the coordinates of the end points, we did that in the previous case as x is l 1 cos theta 1 + l 2 cos theta 1 + theta 2. And y is l 1, sin theta 1 + l 2 sin of theta 1 + theta 2 this is what we saw. For convenience, I am going to call c 1 as cos theta 1, c 12 as cos of theta

1 + theta 2, s 1 as sin theta 1, s 12 as sin of theta 1 + theta 2. So, that means I am going to call this as l 1 c 1 + l 2 c 12 and y is l 1 s 1 + l 2 s 12.

Now so the equations that I have are, let us write this out again x is equal to let me erase this for convenience. I am going to erase this; I have already written this down so I am going to erase this. x = l 1 c 1 + l 2 c 12 and y is = l 1 s 1 + l 2 s 12. Now what I am doing? I am going to rearrange this equation slightly. I am going to write it such that the c 1 and s 1 comes to the left-hand side. That is x = x - l 1 c 1 is = l 2 c 12 and y - l 1 s 1 this is l 2 s 12.

Now I want to square these two equations. You are immediately wondering what is this person doing, why is it doing all these things? Because I am interested in getting rid of you know many complications there are theta 1 + theta 2. So, I am trying to have thetas, theta 1 and theta 2 in my equations. On first I find theta 1, which I will then substitute in the theta 1 + theta 2 equation to find theta 2 because I am interested in finding theta 1 and theta 2 from l 1 l 2 x y.

So, that is the problem. So, I am having this in my head and I am working towards that. This is why I am doing some manipulation, some arrangements consistent with normal loss of algebra. So, no cheating, I am just doing whatever algebraic manipulations that are valid. So, I am having this as x - l 1 c 1 is l 2 c 12 and y - l 1 s 1 is l 2 s 12. I am squaring this on both sides, so this will give me x - l 1 c 1 the whole squared is l 2 c 12 the whole square, is it not.

Likewise, y - l 1 s 1 the whole squared is l 2 s 1 to the whole square. I am adding these 2, why? Because l 2 c 12 squared + l2 s 12 squared. So, this will give me l 2 square times c 12 squared + s 12 square. This is sin squared a + cos squared a or sin squared x + cos squared x, that is 1. So, I can use that as part of my analysis. So, I am adding these two equations. What will I get? I will get, x - l 1 c 1 the whole squared + y – l 1 s 1 the whole squared is l 2 squared into c 12 squared + s 12 squared.

But c 12 squared + s 12 squared is 1. So, I will get this to be 1, or rather this whole left-hand side = l 2 square. But not just that I also have to expand the squares on the left-hand side, which I will do step by step. So, I am having this is a - b the whole square that would be actually a squared + l 1 square c 1 square, p square - two times a into b, is it not? Likewise, for the y coordinate y square + l 1 squared s 1 squared - 2 times a b.

Remember in this case, a is y, b is l 1 s 1. This whole thing is l 2 squared and I am bringing the l 2 square to the left-hand side which will make it - l 2 squared is 0. This is the equation.

Now I have l 1 squared s 1 squared + l 1 squared c 1 square. I can bring the two, so that would be; x squared - to 2 x 1 - 2 x l 1 c 1 + y squared - 2 y l 1 s 1 - l 2 square + l 1 squared into c 1 square + s 1 square. This is the do I have the same number of terms check 1, 2, 3, 4, 5, 6, 7, 1, 2, 3, 4, 5, 6, 7 correct. This whole thing is 0. Now, c 1 square + s 1 squared is 1 because of this, this whole thing will vanish to 1. So, I will have only l 1 square.

Now rearrange this slightly and multiply this whole equation by - 1. Just for convenience, I have this as 2 x l 1 c 1 + 2 y l 1 s 1 + l 2 squared - x squared - y squared. Here we have left out a y squared here, - y squared - l 1 squared is = 0. Now I am going to call this coefficient as, sum say capital A, and this coefficient as sum capital B, and this whole constant as sum capital C. So, this can be called as A c 1 + B s 1 + capital C is 0.

This is the equation that I have in terms of theta 1, that I need to solve because this is c 1 and s 1. That is cos of theta 1 and sin of theta 1, I have in terms of this.

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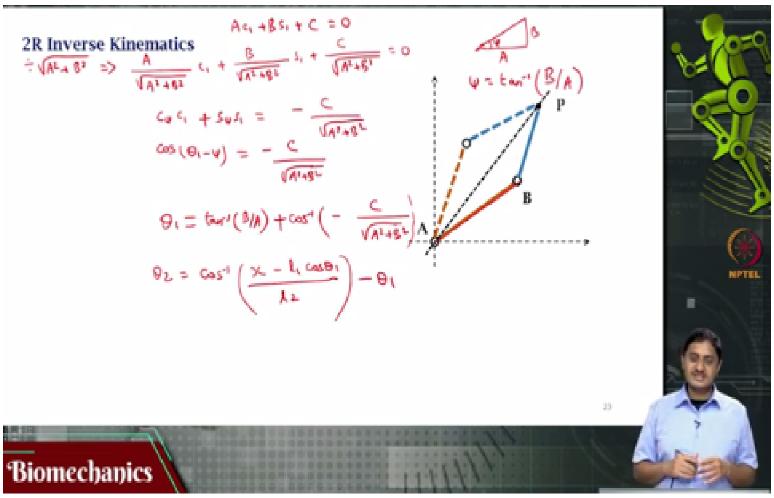

Now, let us proceed in the next slide. Now let us rewrite that original equation which is, A c 1 + B s 1 + capital C is 0, now this is the equation that we have. Now, divide this entire equation by square root of capital A square + B squared. What happens? That will give me, capital A divided by square root of capital A square + B squared times C 1 + capital B divided by square root of A squared + B squared times s 1 + capital C divided by square root of A square + B squared the whole thing is 0.

Remember, in this equation I have this right triangle with sides A and B, capital A and capital B with some side psi. So, this is the situation that I am describing. Now immediately in this from this right triangle A divided by square root of A squared B squared is cosine of psi, is it not? That is cos psi, is it not? In our notation that is C psi c 1 + B, divided by B is the opposite side to the angle psi B divided by the square root of A squared + B squared is the sin of psi, is not?

That is s i s 1 this whole thing is - C divided by A squared + B square C psi c 1 + s s i s 1, formula looks somewhat familiar is it not? That is, sin A sin B + cos A cos B is cos of A – B, is it not? So, that would give me cos of theta 1 – psi is this value - C divided by square root of A square + B square. Now psi itself is what? The opposite side by adjacent side is it not? That is tan inverse of B by A.

So, if I want theta 1, I will get it tan inverse of B by A + cos inverse of - C divided by square root of A squared + B square. This is the value of the first angle theta 1. Now I need to substitute this angle in the original forward kinematics equation which is you know this equation. For example, and find out the value of theta 2. And you will get, I will leave that as an exercise for you.

And you will get theta 2 as cos inverse of x - l 1 cos theta 1 divided by l 2 the whole thing - theta 1. This is the value of theta 2. This will give rise to a situation in which there are at least two configurations AB and BP that can come either as these solid lines, or as these dashed lines. So, there are two distinct pairs of solutions, or two sets of theta 1 and theta 2, from the inverse kinematics problem.

That we are showing here as the solid brown blue line and the dashed brown and blue lines. These two solutions are also called as the two branches of the inverse kinematics problem, or two branches of the inverse kinematic solution. Commonly in robotics and biomechanics, they are called as the elbow up solution and the elbow down solution. So, from this exercise what we have learned is that even in the simplest case of 2 link planar kinematic chain, the inverse kinematics is relatively complicated.

That even in the case where the 2 link lengths are given under 2 coordinates are given the x y coordinates are given on the link lengths l 1 and l 2 are given, the theta 1 and theta 2 are not unique. There are at least two sets. In the case of the 2 link chain, there are two sets of solutions means because, why is this happening? Because, of the non-linear nature of the kinematic chain or the way in which we describe this kinematic chain.

Because of the sins and cosines involved, sin inverse and cosine inverse can refer to two different angles. Because of this reason we have at least two solutions that will satisfy a given equation. This is for the simplest case. Now imagine, I have a large number of degrees of freedom. Many link, multi-link serial kinematic chain. Then, finding the inverse kinematics is a relatively hard problem to crack.

This is just an introduction to the topic, so you get an idea of the complexities involved in the biomechanical analysis of multi-link serial kinematic chains.

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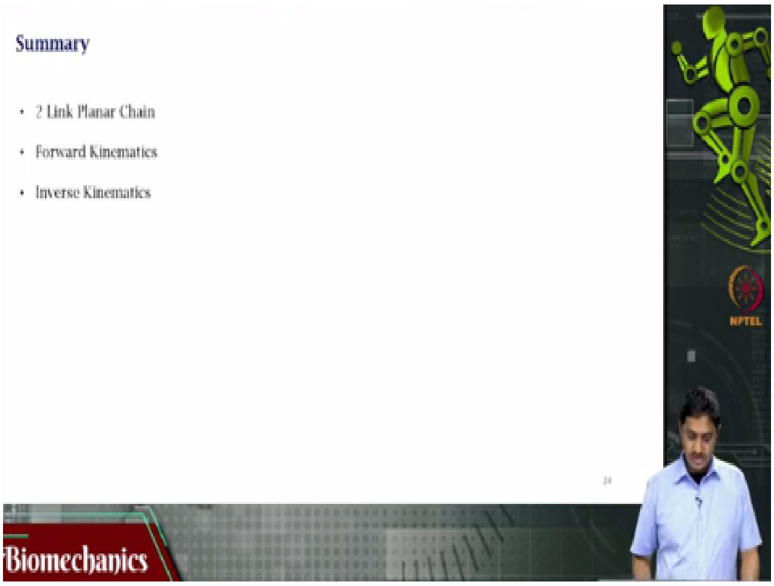

So, with this we come to the end of this video. In this video, we studied and analysed the inverse kinematics of a 2 link planar kinematic chain. Thank you, very much for your attention.