Introductory Mathematical Methods for Biologists
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Lecture - 08
Computing Derivatives of Curves

Hi, welcome to this lecture on Mathematics for Biologists. In the previous lecture, we learned how to compute derivative and the idea of derivative.

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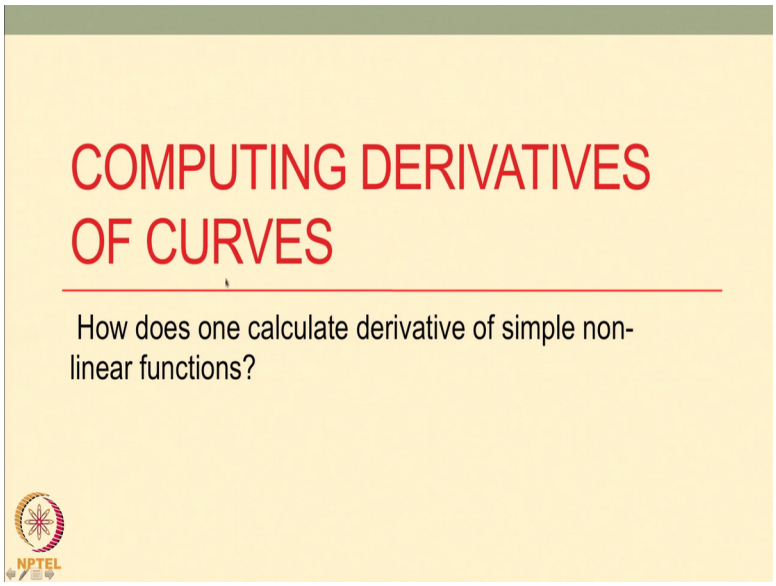

And how to compute the derivative for simple functions like linear functions like y is equal to mx. So, today we will discuss computing derivatives of curves how does one calculate the derivative of simple non-linear functions. So, we will talk about derivatives of curves various curves; we will start with the simplest curve we can think of and then we will learn how to compute derivative?

We discussed in the last lecture that computing derivative is equal to finding slope.

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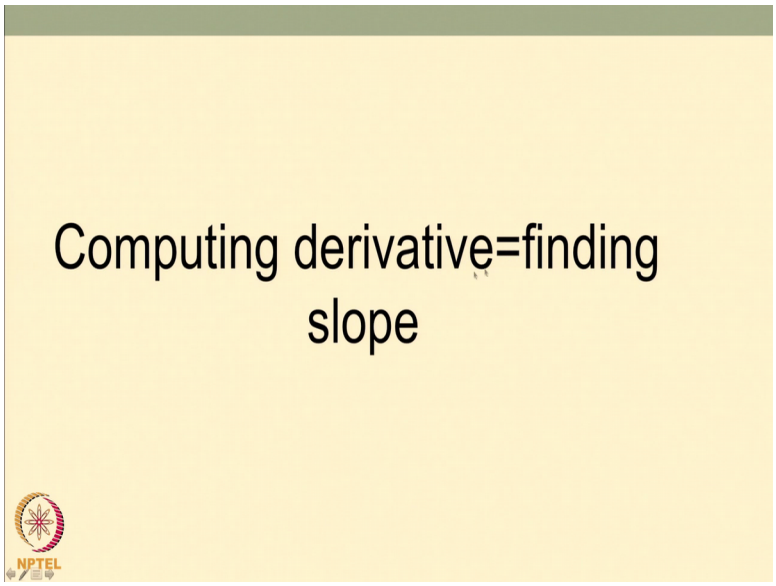

So, what by computing derivative; we know what do we mean by computing derivative; is essentially finding slope if we can find the slope that is same as computing derivative.

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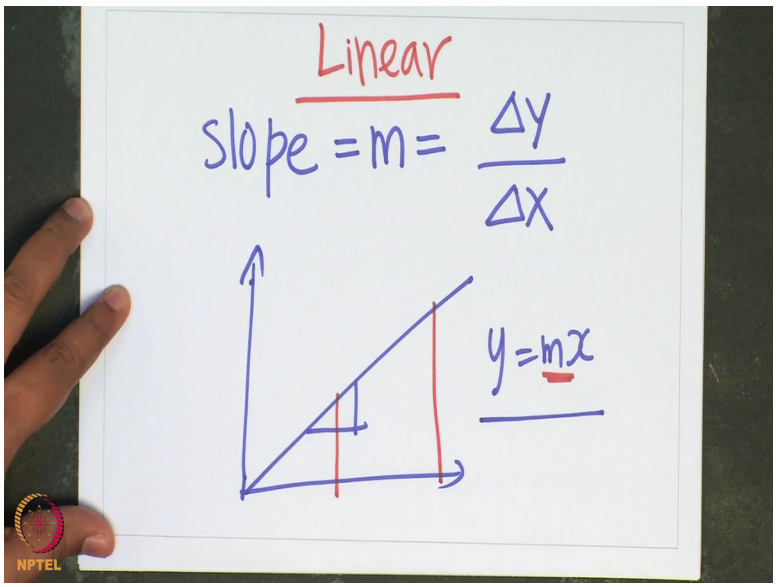

So, we said that slope which we call m which is same as delta y by delta x; we said this is something that we said in the previous lecture. And if you have a straight line and then the slope will give you the derivative which is y is equal to mx the slope is m.

So, this is something that we learned; we also learnt that if you have a curve, we briefly mentioned that if you have a curve y is equal to x square curve.

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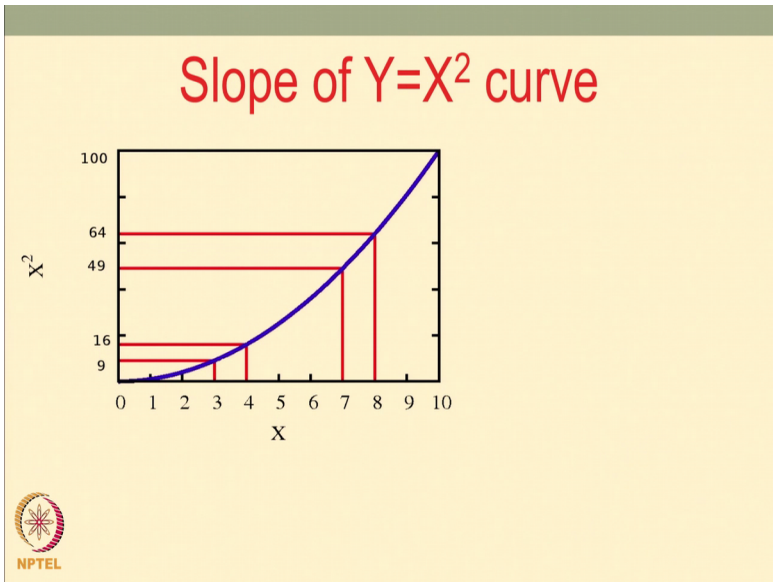

We can also find slope and how do we find the slope? Is something that we will see in detail today if it is not a linear function. So, this function was y is equal to mx is a line, so these are linear functions, these are line; so this is linear is like line.

So, this has the slope m which is the same slope everywhere the slope; you calculate here which is same as the slope you calculate here and all. For any value of X, the slope is m this is something that we learned. If this is not a line, if the curve is not a; if the function is not a line; what is the slope? Is something that we will learn today. So, y is equal to x square is the simplest example; where the function is a non-linear function; that is there is a square there.

Therefore, this is a curve is not a line and we will familiarize ourselves in computing slope of this function at various points. So, let us take y is equal to x square curve and let us have a look at it. So, here we will familiarize ourselves with a non-linear function; which is y is equal to x square, we will see how to compute derivative of this y is equal to x square curve. So, I will take this curve here; which is y is equal to x square.

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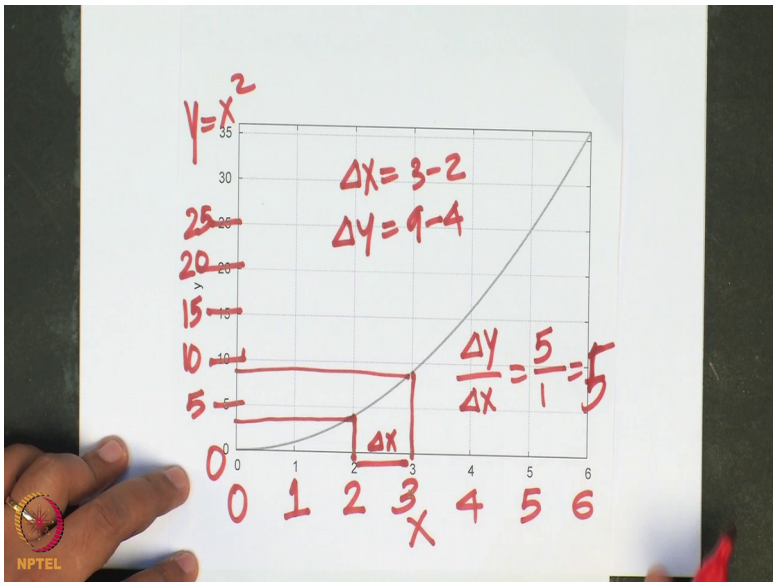

So, see this here what we have is in this axis; we have x and here you have y equal to x square; this is in the y axis. So, we have various values here 1, 2, 3, 4, 5, 6 and this is 0 correspondingly; we have 0 and what is marked here is 5, 10, 15, 20; so, this is red remark as it is; so this is 5, this is 10, this is 15, this is 20 and so on and so forth.

Now, if we look at the derivative at various places corresponding to 2; if I draw a line this is just below 5. So to; we although know that 2 square is 4; so, 2 square is 4 and 3 square is 9; which is just below 10. So, if I just draw; you will get a curve point, just below 5; which is 4 this is just below 10; which is 9. So, this one; this is delta x which is 3 minus 2; so delta x is 3 minus 2 and delta y is 3 square minus 2 square, which is 9 minus 4.

So, 3 square is 9; 2 square is 4; so this is delta y and delta y; by delta x delta y by delta x which is 9 minus 4, 5, 3 minus 2 is 1. So, you will get 1 here, so you will get 5 here, you will get 5 as the answer if you do.

Similarly, if you do between 3 and 4; so one can do similar curve between 3 and 4; let me draw that here.

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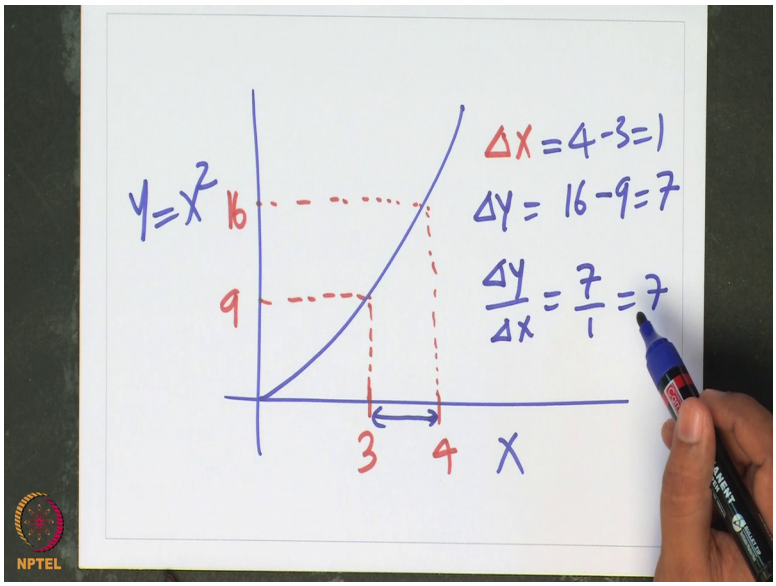

So, if we had this curve y is equal to x square; so this is y is equal to x square and this is my X. If the value is 3 here, we know that correspondingly here; it will be 9 and if you have 4 here correspondingly, this will be 16. So, 4, 16; so delta x which is this value; which is 4 minus 3, which is 1 and delta y; which is 4 square minus 3 square; 4 square is 16, 9. So, this is 7 and we have delta y by delta x is 7; divided by 1, the answer is 7.

So, what we see is that depending on where we calculate, if we calculate between 3 and 4; we get the answer as 7, if we calculated between 3 and 4 we got the answer as 7. Previously, we calculated between 2 and 3; the slope we got the answer as 5; so, which one of it is correct? Or how do we actually compute derivative? We are assuming to get different answers depending on where you calculate.

So, the first point is that if you have a curve like this depending on where you calculate, you will get different answers. In other words, the slope depends on where you calculate the slope; depends on the x value at which you calculate. So, this is something that I wanted to specify very carefully here; so, if you look at this curve once more.

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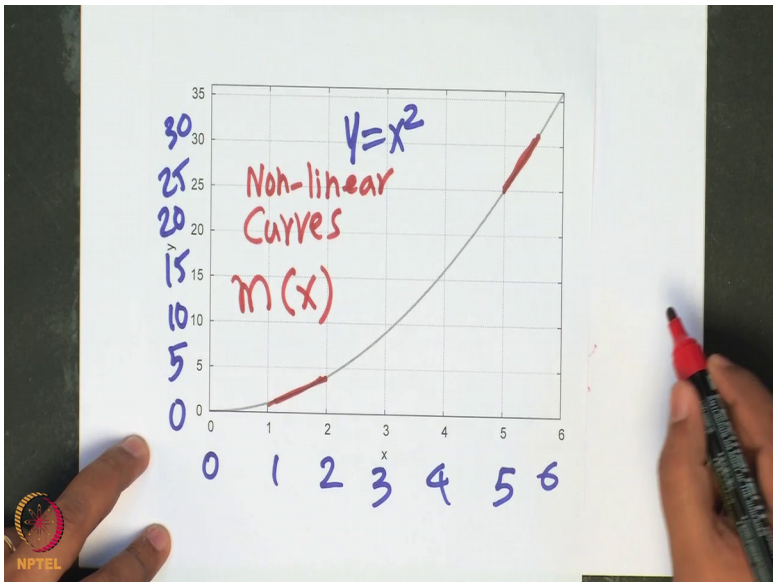

You have here 0, 1, 2, 3, 4, 5, 6 and you have 0, 5, 10, 15, 20, 25, 30 and you have this curve; which is y is equal to x square.

Now, the slope if you calculate here; the slope here will be very different from the slope here. So, depending on where you calculate, for what value of x you calculate; the slope will be different. So, this is a important point that all of you should know for curves; for line the slope was the same, but for curves which are non-linear. So, this is non-linear; it is not a line or non-linear curves slope which is; m is a function of x.

So, m depends on for what value of X; you calculate what x are? If the x is between 1 and 2; you will get different slope. If x is between 5 and 6; you will get different slope, if x is between 3 and 4; you will get different slope. The slope depends on the location where you calculate the slope. So, this is an important point that all of you should remember for non-linear curves; which is curve, which is not a line. The slope would depend on the x value; at which you would calculate it. So, this is slope is a function of X; so this is the first point you want to know.

Now, the obvious question will be; we had this calculation that we did, which is the slope between 3 and 4? So, we had 3 and 4 and we had 4 minus 3; 16 minus 7 divided by 1 and we got the answer is 7. So, if I take the interval between 3 and 4; if I take delta x is equal to 1, I get the answer 7. If I did the same between 3 and 3.5; if I took delta x is equal to

0.5; what will we get? So, let us think about this here we took delta x is equal to 1; we calculated the 3, 4 but we took delta x is equal to 1 around 3 near 3.

So, now let us calculate again here; in the same region just after 3, but taking delta x is equal to 0; 0.5 and let us see what we will get.

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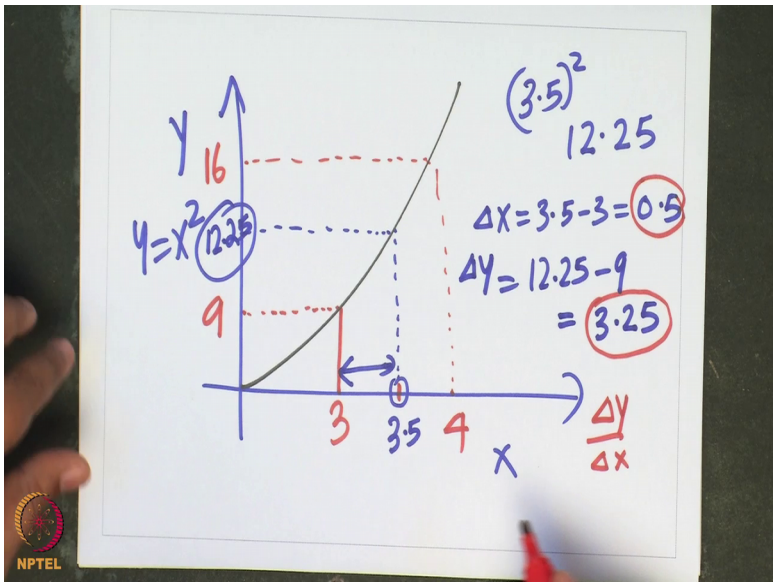

So, let me plot the same curve; so this is my X; this is my Y. So, y is equal to x square is what I am going to plot, so I am going to plot; y is equal to x square curve . So, now what I am going to draw, this is my 3 and the corresponding y value is 9; y is equal to x square; so 3 square is 9.

Now, if I just take 4; we already know if I take 4, we know the answer is 16; 4 square is 16. If I take the point in between which is 3.5, so let me just take this point in between which is 3.5 and let us see what is the corresponding. So, this point is going to be 3.5 square; so 3.5 square which we know is 12.25. So, this is going to be 12.25; that is what this value is.

So, now if I take this is 3.5; so if I take this as my delta x, delta x is 3.5 minus 3.5 delta y will be 3.5; square minus 3 square. So, which is 12.25 minus 3 square; which is 9 and this is equal to 3.25; 3.25, this is my delta y. So, what is delta y by delta x? So, delta y by delta x; this is my delta y, this is my delta x 3.25 divided by 0.5. So delta y by delta x; so I will be write it here.

So, let me write in a different paper.

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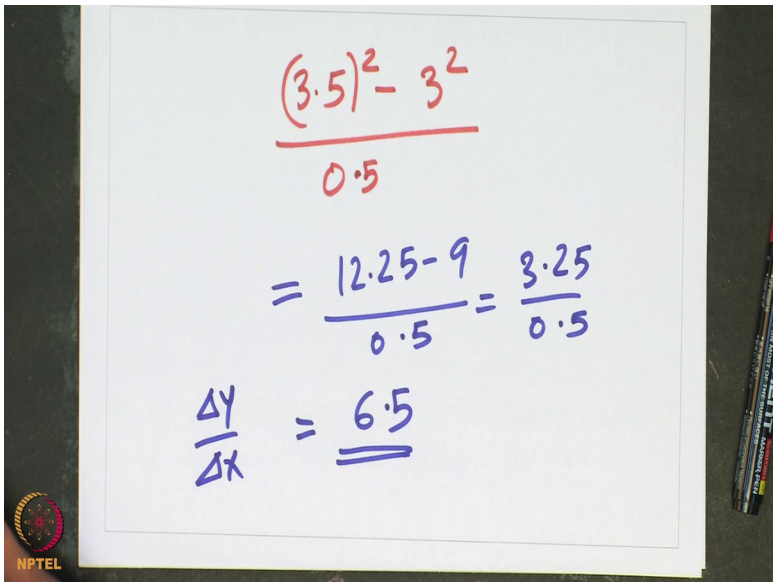

So, what we got is basically 3.5 square minus 3 square divided by 0.5; which is 3.5 minus 3 which is 0.5; so, this is going to be the answer is as we said 12.25 minus 9 divided by 0.5; which is 3.25 divided by 0.5; which is 6.5. So, delta y by delta x we got a 6.5.

Remember, when we had done between 3 and 4; we had got the answer as 7. So, when we did between 3 and 4; the answer was 7, when we did between 3 and 3.5; the answer is 6.5. So, the answer changed depending on where you; if I change the interval. So, when I did between 3 and 3.5; the answer turned out to be 6.5. So, if here the answer; if I took this interval, the answer was 7 when I took between this answer were 6.5.

So, let me write that here a little bit more carefully.

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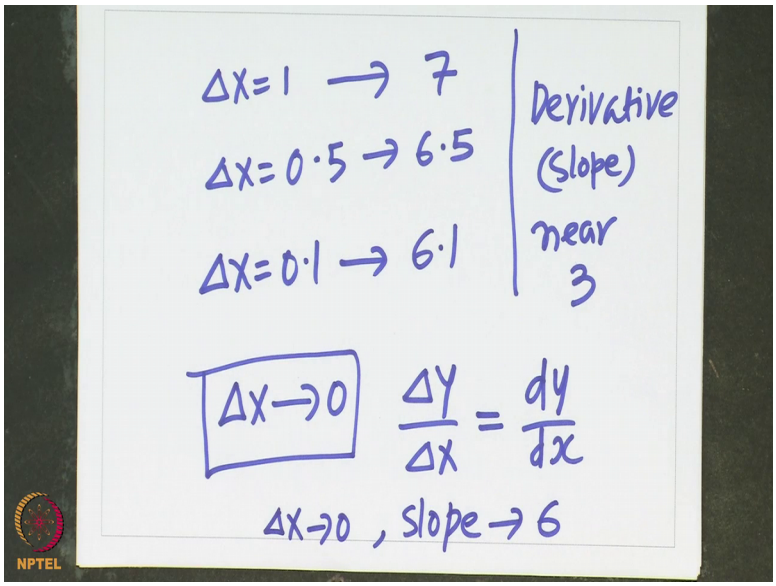

When I took delta x equal to 1; the answer was 7 when I took delta x equal to 0.5; the answer became 6.5. Same I calculated derivative, which is the slope or slope I calculate the slope near 3; I was calculating slope near 3, when I took delta x is equal to 1. I got the slope as 7, when I took delta x equal to 0.5; I got the slope as 6.5.

Now, let me just take delta x equal to 0.1; then what is the slope? That I would get; so let us see what would be the slope I will get? So, let me just do that.

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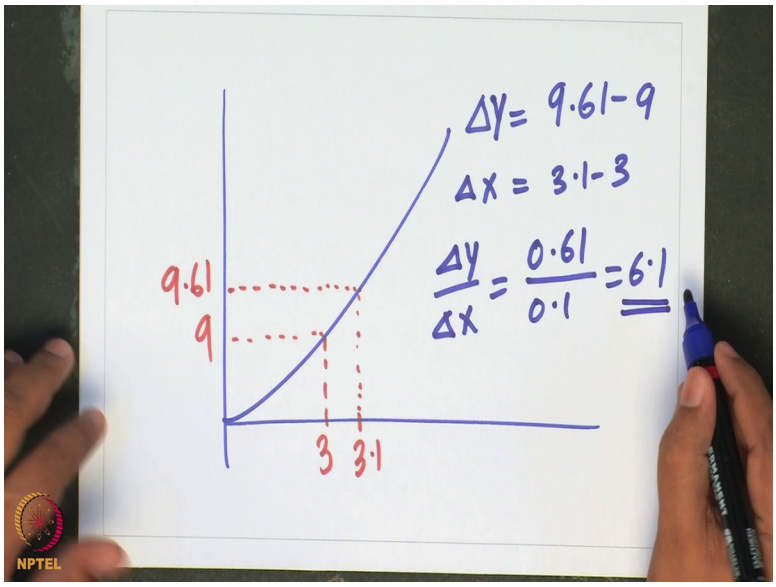

So, if I just take; I have this curve y is equal to x square and this is my point of interest; which is 3 and this is 9. Now, I am going to take a point just 3.1; what is 3.1 square; which is 9.61; 3.1 squared is 9.61. So, my delta y is 9.61 minus 9; my delta x is 3.1 minus 3. So that means, delta y by delta x is 9.61 minus 9; which is 0.61 divided by 0.1; which is 6.1.

So, if I calculated between 3 and 3.1; the slope turned out to be 6.1. So, now let us come back to this; when I took the delta x 0.1; when I took the x interval as 0.1; the slope turned out to be 6.1. So, as I decrease the delta x; the slope seemed to change. So, what is the correct value? What is the right way of calculating the slope? What should be the delta x that I should take to calculate slope around 3.

If I want to calculate slope around 3; what should be the delta x that I take? This is a obvious; this is a question that will be in your mind. And the mathematics; it says that once you take delta x, which is the smallest possible. So, you should take delta x tending to 0; the smallest delta x.

If you take the smallest delta x; delta x tending to 0, if I take the smallest delta x and then you calculate delta y by delta x; that is the derivative. So, this is if called dy by dx . So, dy by dx that is calculating the derivative of continuous function y will be calculating delta y by delta x by taking the smallest delta x which is delta x tending to 0 taking the limit delta x becoming very small and small and small like if you calculate this will become different then it turns out that near 3 near 3 if you calculate smaller and smaller and smaller and smaller this will come closer and closer and closer to six. So, as delta x goes to 0 the slope near 3 will goes to six in this slope near 3 will go to six

So, this is something as you say this is how mathematically.

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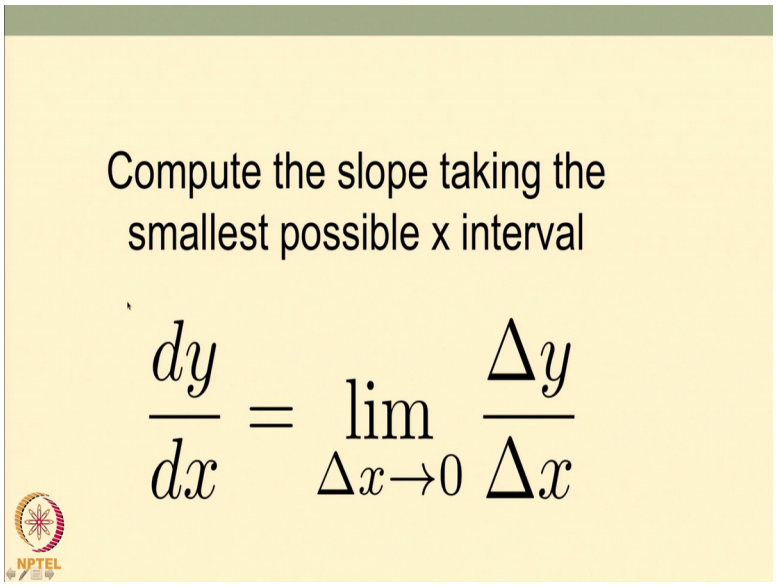

Then defines one defines compute the slope taking the smallest possible x interval that is the definition of our derivative. This is the mathematical definition dy by dx, which is the derivative of Y; as of with respect to x is defined as in the limit delta x tending to 0 that is the smallest possible delta x, you take and you calculate delta y by delta x. Then the value you get is the slope; the slope will be most accurate if you take the smallest delta x possible.

So, this is how you calculate derivative, so this is something that you would want to try for some other locations like x.

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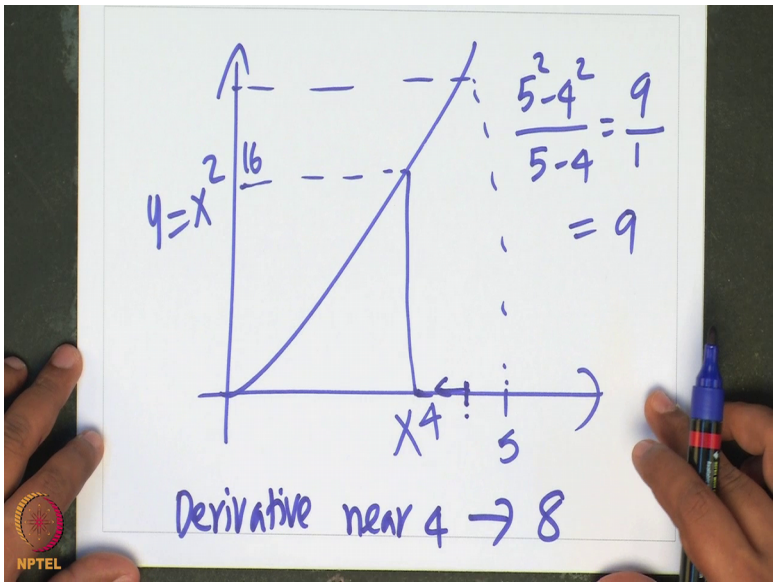

For example, you would want to take some other point 4 for example, you had this function y is equal to x squared and this is X; you would want to take around 4 and this is 16 and 5. So, if you take between 4 and 5; you have 5 square minus 4 square divided by 5 minus 4 which is 25 minus 16, you will get 9 is the slope.

Now, you can calculate between 4 and 4.5; just like we did. So, I urge you to do this calculate the slope between 4 and 4.5. So, you will get five square minus 4.5 square minus 4 square divided by 4.5 minus 4; you will get answer which is smaller than 9. And as we come closer and closer to 4; as delta x becomes smaller and smaller and smaller; you will get the derivative near 4. So, derivative near 4; will be 8 derivative, near for the slope, near 4 will be 8; as the delta x tending to a small, it will approach towards 8.

So, it turns out that for this function y is equal to x square; if you take this function y is equal to x square and calculate the slope.

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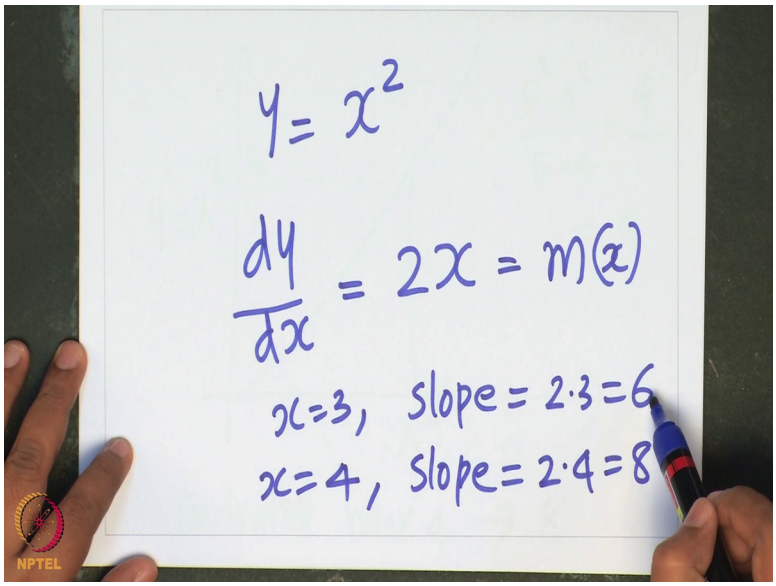

So, you have function y is equal to x square and you calculate dy by dx since it turns out that the answer is two x. So, depending on for what value of x you calculate you will get dy by dx which is a function of x. So, in other words the slope which is m is a function of x; if x is 1; if x is 3, the slope is 2 times 3; which is 6; that is what we did if x is equal; if you calculate the slope near 4 x equal to 4.

The slope will be 2 times 4 which is 8; so if you take this function y is equal to x square, you calculate slope near 3; you will get 2 x; 2 times 3; 6. If you calculate 4; 2 times 4, 8; so the formula for any value of x at; near a particular value of x, if you calculate slope; it turns out that the slope is 2 x.

Now, this is something that the bottom line; that all of you should remember is two things one slope.

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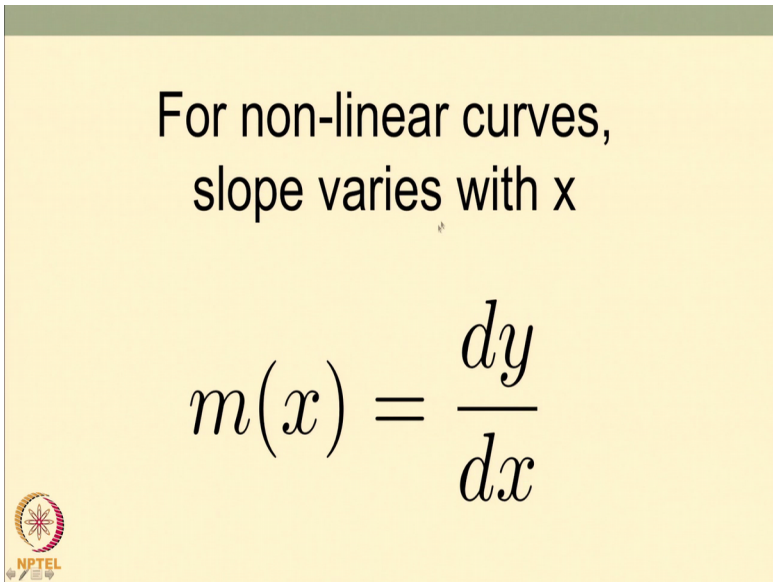

Dependence for non-linear curves the slope varies with x for depending, for various value of x. You will have different slopes and for a particular curve y is equal to x square, the slope will turn out to be slope is 2 x. So, why it is will x squared the slope is 2 x, the slope depends on x the slope varies with x.

This also means that if I take the derivative of m; so, you have m is equal to 2 x here. So, let us think about this a little bit; you have m is equal to 2 x.

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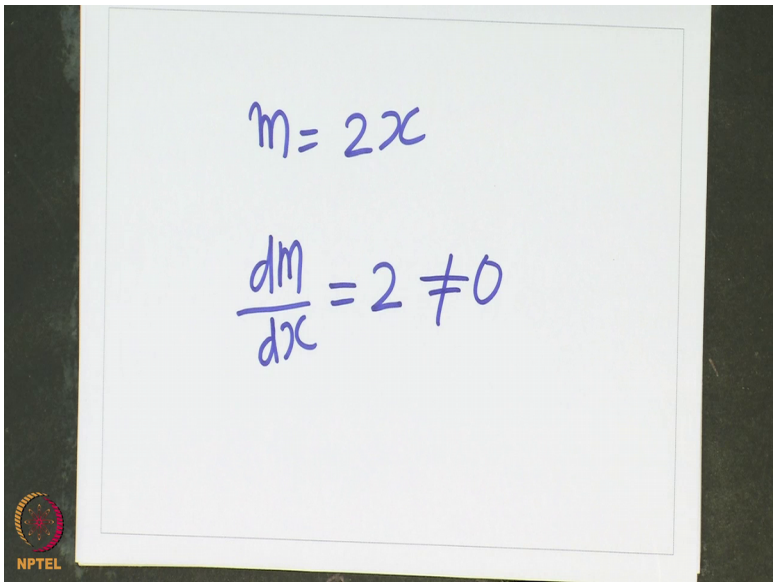

We have m is equal to x; which is a slope, you can also calculate dm by dx. How does the slope itself changes here? This turns out to be 2, we know the derivative of 2 x is 2; the slope is 2; this is like m is equal to 2 x dm by dx is 2, which is not equal to 0.

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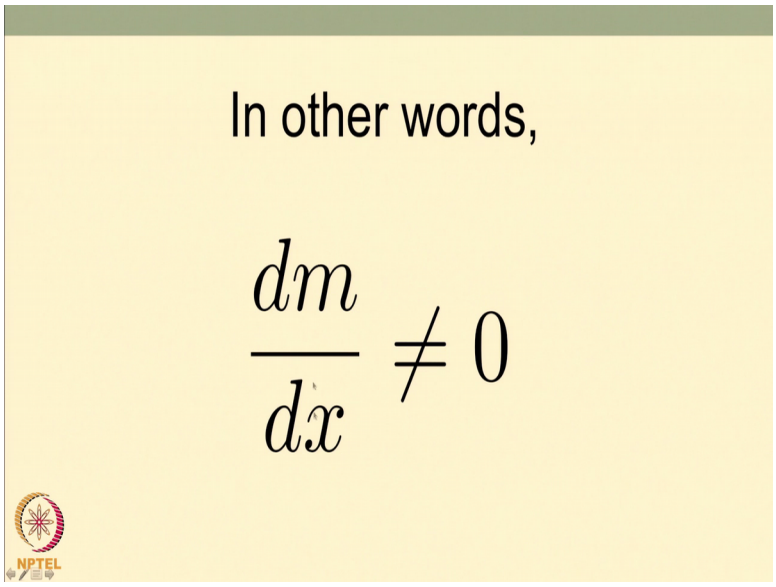

So; that means, the slope of the slope dm by dx is not equal to 0. So, this is also a definition of non-linear curves; the slope itself will vary with x. In other words, the change in slope; the slope of the slope itself; how does the m vary with respect to x? That is also m also will vary with respect to x; in other words dm by dx will be non zero.

This really come back to this idea; a little bit more as we go along but just keep in mind that for non-linear functions dm by dx; will be non zero slope, will depend on whatever value of x; you calculate. Now; if you take other function x cube.

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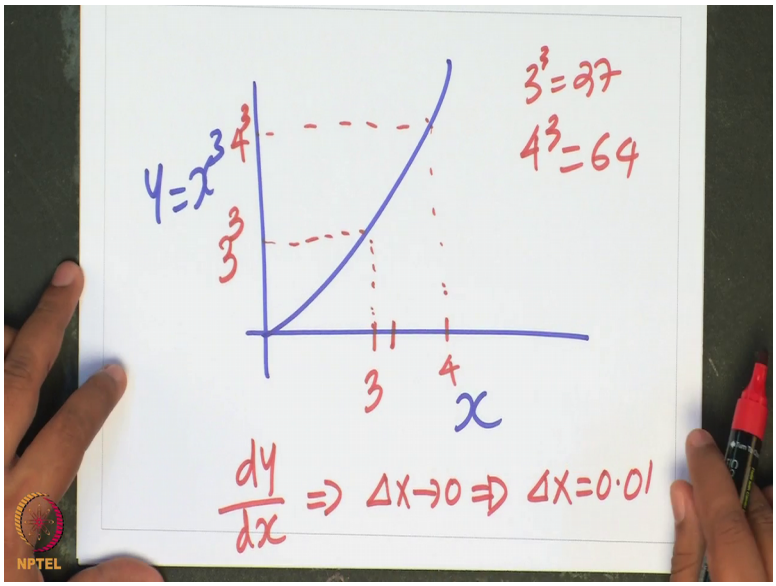

So, that is you can take y is equal to x cube and this is x you can take various value of x. If you take x equal to 3; we know that you will have this curve, y is equal to x cube and correspondingly we know that 3 cube; which is 27 and you can take 4 and you have 4 cube; which is 64.

Now, as we said to calculate the derivative dy by dx; we should calculate the smallest delta x possible. So, we want to calculate delta x; very small which implies, let us take delta x is equal to 0.01; which is small. So, if you take delta x equal to 0.01 that is 3 and 3.01. So, you take very close to 3; 3 and 3.01 and calculate the corresponding y values.

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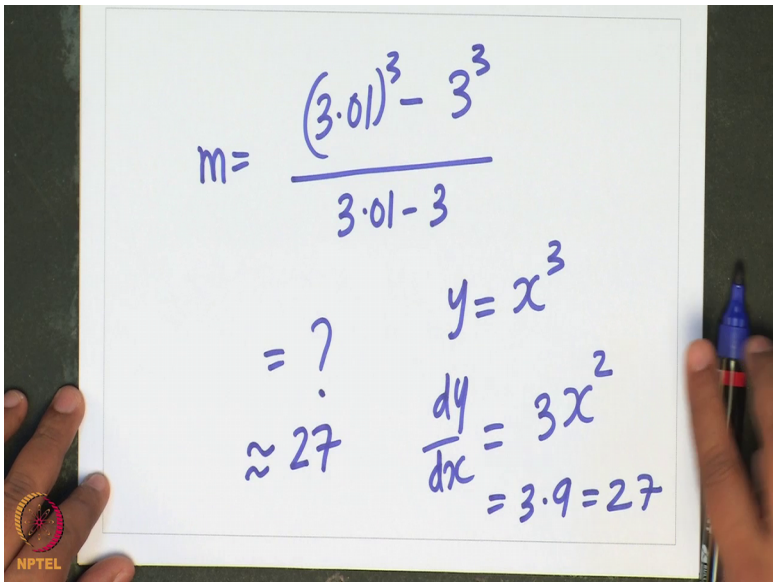

And if you calculate the slope; I urge you to do this, that is I urge you to calculate m is equal to 3.01 cubed minus 3 cubed; divided by 3.01 minus 3.

So, if you what would be the answer? I urge you to calculate and it turns out that the function y is equal to x cube; the derivative is 3 x square. So, if x is equal to 3 x square is 3 times 9; which is 27; so 3 x square is 27. So, the answer will be close to 27; the answer will be close to 27; please do calculate this and see this.

So, this is similarly you can calculate near 4 and 5 and all that and you will see that the answer depends on where you calculate and the answer you will get is y; dy by dx is equal to 3 x square. So, for various curves you will get various slopes at different points and the way to calculate the slope is calculate delta x; take delta x very small and calculate the delta y by delta x. So, that is the thing that we have to learn from this lecture.

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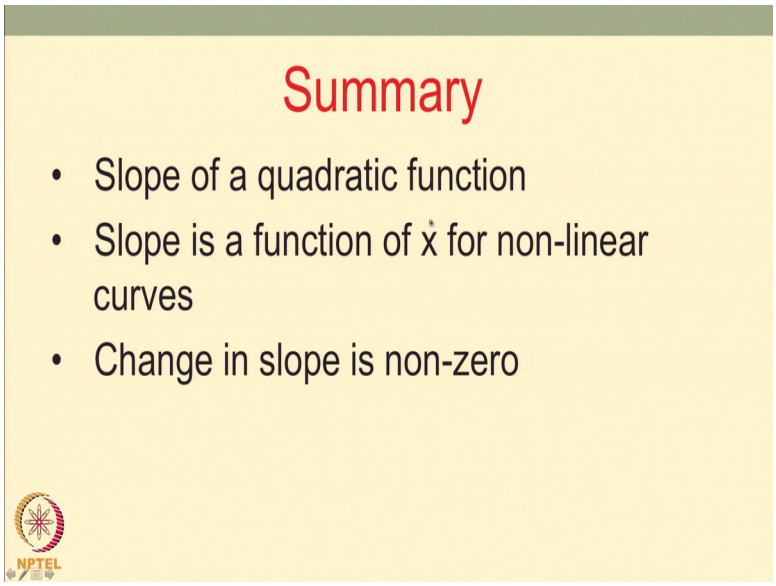

So, to summarize we learned how to find the slope of a quadratic function. We learned that slope is a function of x for non-linear curves; we also found that the change in slope is non-zero dm by dx is non-zero. And the way to calculate slope is take the smallest delta x possible and then calculate d delta y by delta x. So, the biggest takeaway from this lecture is the following.

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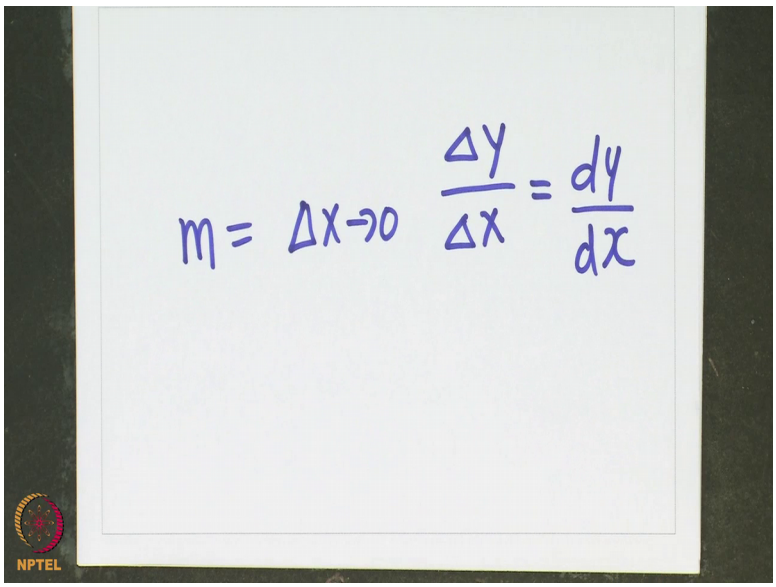

The slope is take delta x very small; delta y by delta x, this is what the slope is; this is defined as dy by dx. So, this is the way to calculate dy by dx; this is the takeaway from this lecture and with this, we will stop this lecture and continue in the next lecture.