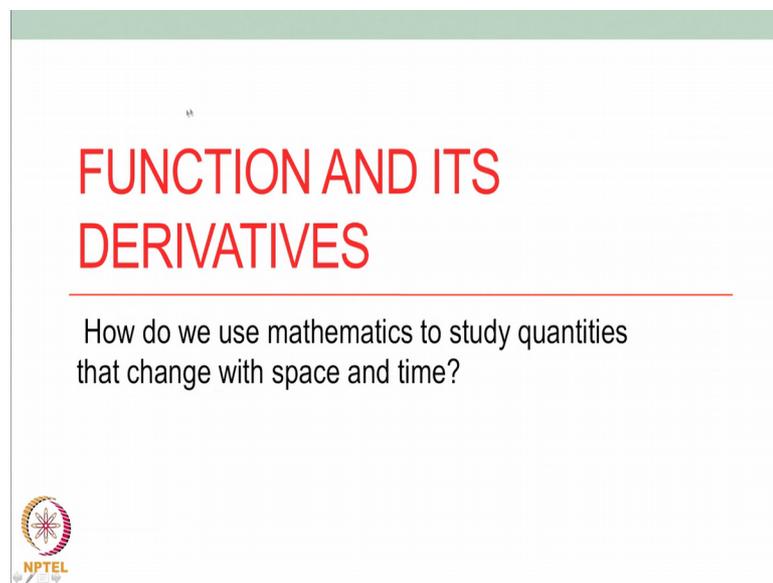


Introductory Mathematical Methods for Biologists
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Lecture – 07
Functions and its Derivatives

Hi, welcome to this lecture. We have been studying about various functions and today what we will learn is function and its derivatives.

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So, the question that we will try to answer or begin to answer in this lecture is the following; how do we use mathematics to study quantities that change in space and time? How do we use mathematics? So, let us read it again; how do we use mathematics to study quantities that change with space and time? In biology, we will often see or not only in biology; in our day to day life, nothing is static; things change in time, maybe of course all the time, there is also all the curves that we saw are increasing or decreasing with x or y .

So, this increasing decreasing with x and y ; we can learn more about this and try to and any data that we get will be increasing and decreasing or with x and y . So, if we want to get more information from this data, if we want to learn more things from this data; we have to use some new idea called idea of derivatives. So, there is a mathematical

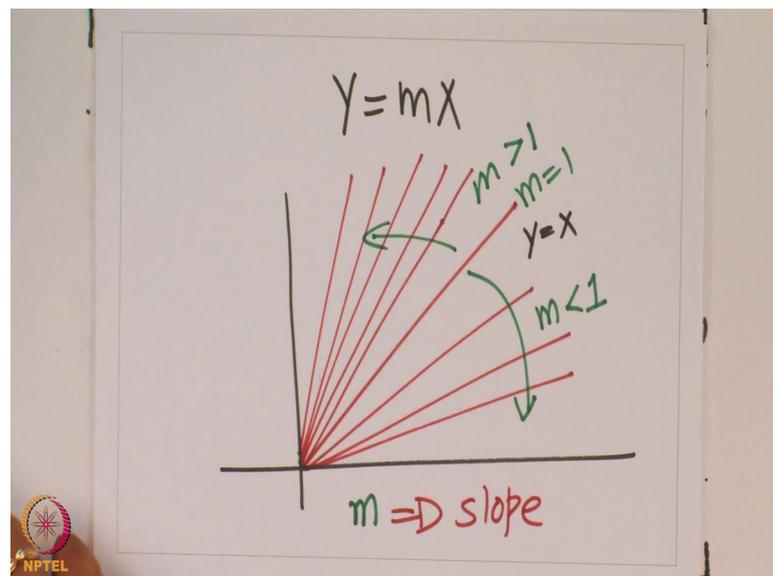
technique that one can use to extract more information and learn about something beyond the function and that is what we would discuss; the idea of derivatives.

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So, I will tell you what a derivative means, but before that let us recall the Y is equal to mx graph that we learned.

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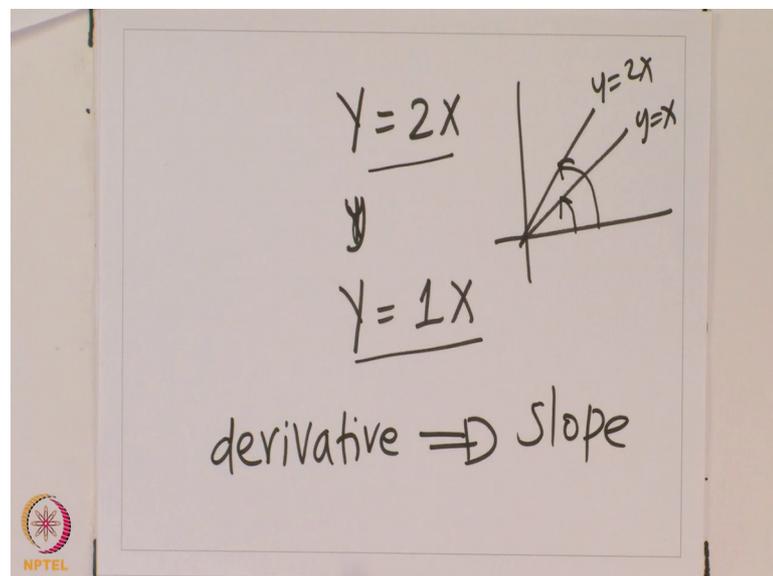
We had a simple graph Y is equal to mx for different values of m . So, let us try and plot that again; so when we had m is equal to 1; it was just Y is equal to X and then Y is equal to X would look like this. So, this is Y is equal to X ; if we had Y is equal to $2X$; we said

it might look something like this, Y is equal to $3 X$; it looks like this, $4 X$ and so on and so forth.

So, as I say go a larger and larger m value; so and a half X . So, if I draw like this, so this is Y is equal to X as we said; so here this is m is equal to 1 and as we go below m ; less than 1 and as we go this way; m will be greater than 1 ; what is changing is the slope. So this curve as a slant or slope, so the thing that we change m here represents slope; the more the slope; the larger the value of m ; the lesser the slope the lesser the value of m .

So, the first thing we want to say is that Y is equal to $2X$.

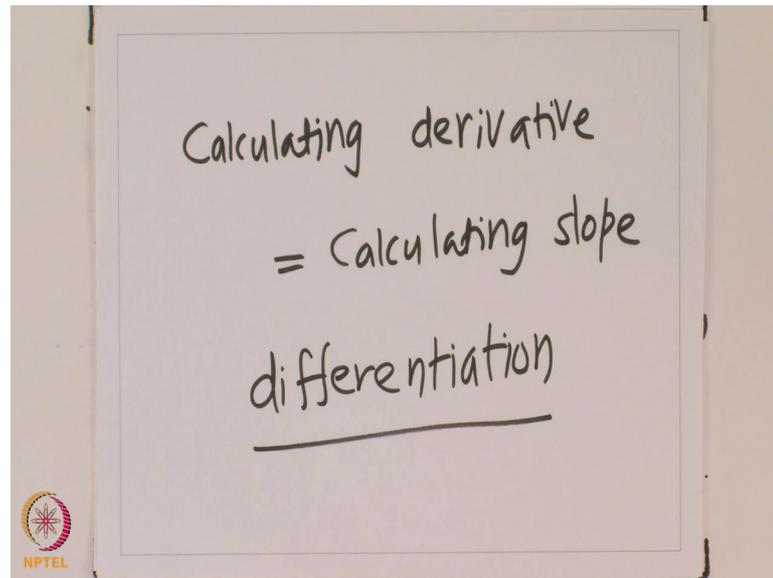
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So, if I just think of Y is equal to $2 X$ and Y is equal to $1 X$; this is $1 X$, then this is $2 X$. So, $2 X$ has more slope this is more slanted; so, this is Y is equal to $2 X$, this is Y is equal to X this is more slanted. So, this has the higher slope compared to this; so, this derivative when we say derivative, it is nothing but slope.

So, derivative is essentially slope and calculating derivative is equal and is nothing, but calculating slope. So, I want to say that in mathematics there is a process called differentiation.

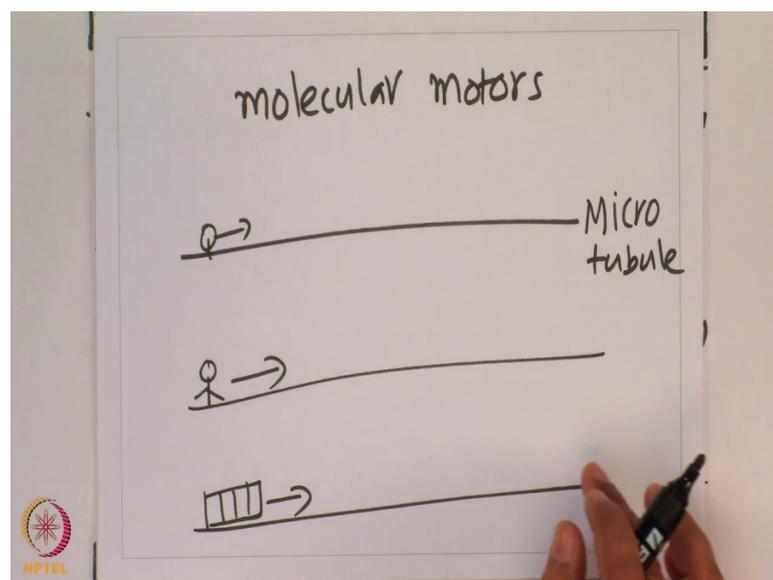
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So, that is calculating derivative; if you want to calculating derivative is same as calculating slope. So, now this process in mathematics is called differentiation; so why do we do this? Because it has; this is very useful calculating, the slope is very useful what when we calculate? When we think of this a two experimental data; think of two experimental data.

So, you do an experiment; let us think of a simple experiment.

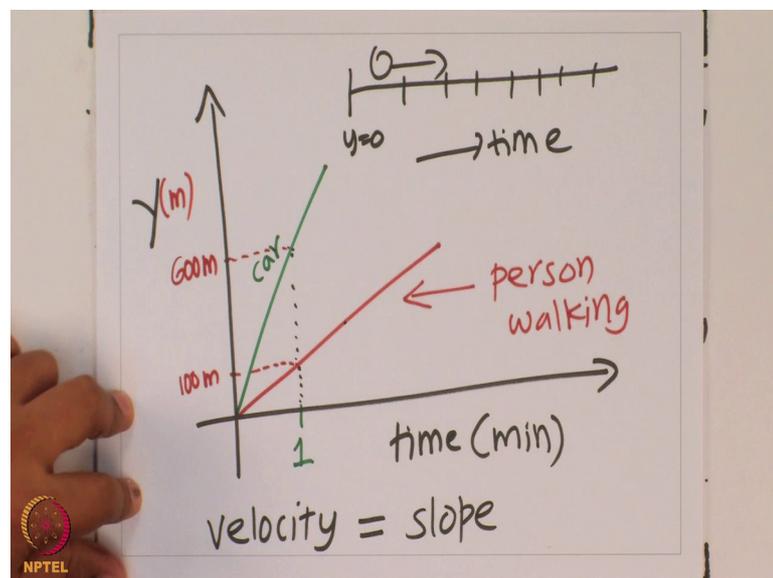
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Let us say in biological cell there are molecular motors some motors called Kinesin Dynein or and let us say they are walking along a track of microtubules. So, some motor is walking or even think of we walking ourselves from one place along this line. So, now if I do an experiment of we walking or a molecular motor walking; so molecular walk, motor walk along some microtubule in cells and let us say; we are walking along a track or even a train going along a track, if you have a bus or a train going along a track.

So, these are different experiments that you can think of, different machines going along a track. Then you can ask a question; how much did it travel in time after 1 minute? How much did it travel?

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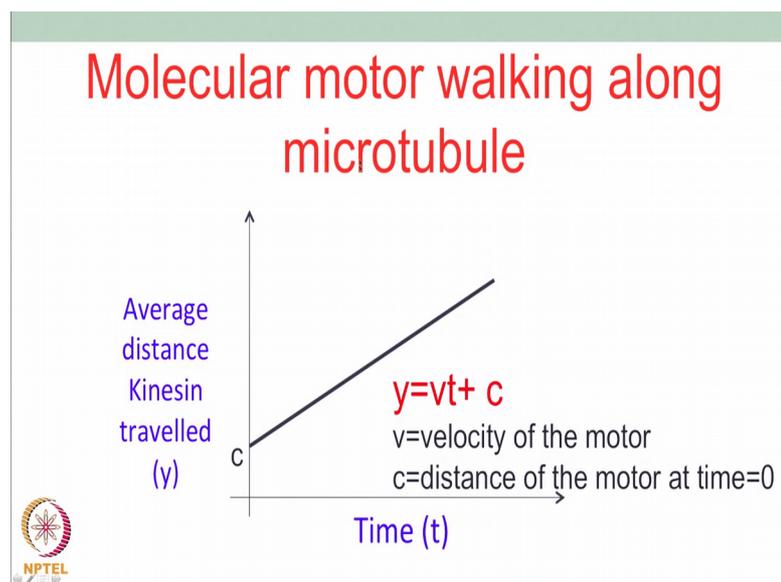
So, if we are doing this experiments and here a Y is the position along this track. So, this is Y is equal to 0; this is Y is equal to 1, 2, 3, 4 different positions as a function of time. This is moving along this track and we will get some data and if we plot this; we might get some 2 graphs let us say, I did one experiment.

So, if I am go walking that is; I got this, if this is a car I might get something like this. So, this is time in the X axis; so I did experiment of a person walking. So, this is the person walking and the position of the person in the Y axis; as a function of time and this is a car or a bicycle or a motorbike; so what is the difference between this and this? Here after one minute, so, let us say this is 1 minute.

So, time in minutes; so this is time in minutes after 1 minute; if we look at it, how much did distance the person travel? So, if you consider this person; the red curve says that the person has reached only here. So, after 1 minute of walking; let us say the person has covered 100 meters. So, the person has covered 100 meters; the car or bicycle in the corresponding time, would have covered many 100 meters. So, this could be like 600 meters or so; this is Y is the distance in meters. So, this tells that the red curve has a smaller slope that we know than the green curve.

So, this is a smaller slope; this is a larger slope and this also tells that the red curve represent a slow moving object is a person, while the green curve represent a fast moving object. So, if you now want to say that this is slow moving and a fast moving thing; we have to use make use of this idea of derivative.

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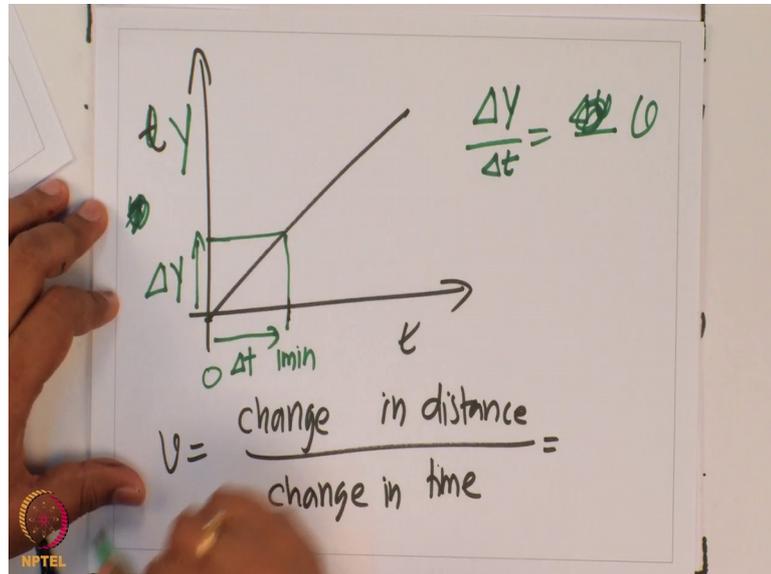


So, we will come here; so this is if you have a molecular motor walking along a microtubule, you can write typical equation like Y is equal to some vt plus c; where v is like the slope or m and t is like X. So, it is like Y is equal to mx plus c where the X axis is t and the Y axis is the distance the Kinesin travelled.

If you wish and this is give; this equation v is velocity of the motor, c is the distance at time t equal to 0. So, this is the graph that is plotted here; now what is v? We all know that v is nothing, but the velocity and the claim here is that velocity is nothing, but the slope. So, as we know velocity is equivalent of the m; which is the slope and the person

has smaller velocity compared to a car; this we all know. So, velocity is how do we calculate velocity? Is change in distance by change in time.

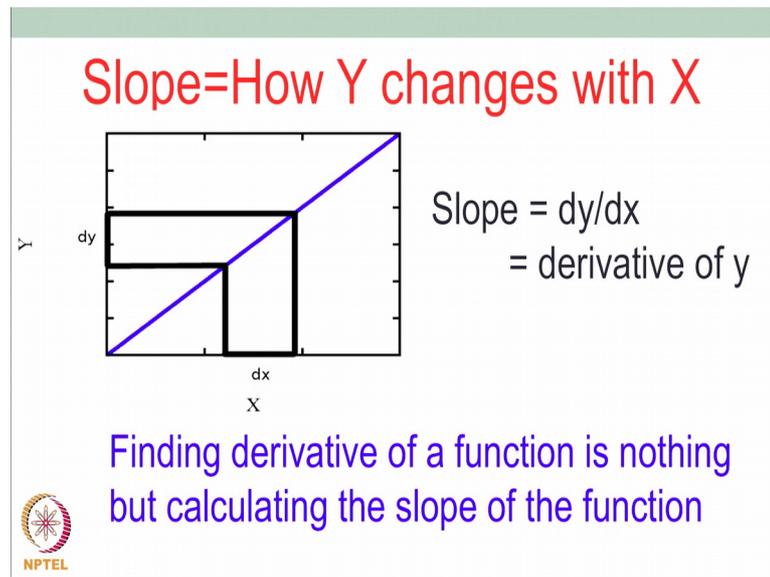
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Now, how do we represent that mathematically? So, I can represent if you have a y-t graph; this is t and this is y, I can write velocity change in distance by change in time that is; if I change the time in so at t equal to 0. So, this is 0 and this is 1 minute; in 1 minute how much did I travel? So, the x; the time changed by 1 minute, how much did the Y change? So this quantity which we call this; so this is my y, this is my time. So, this is delta y; how much did the Y change when the time change this much?

So, delta Y by delta t or this is velocity; if Y is velocity here, this is delta v by delta t is velocity. So, this is velocity; so this is position of course, Y is position sorry; this is no velocity, this is position and the change in Y with time is called the velocity, so delta Y by delta t is velocity.

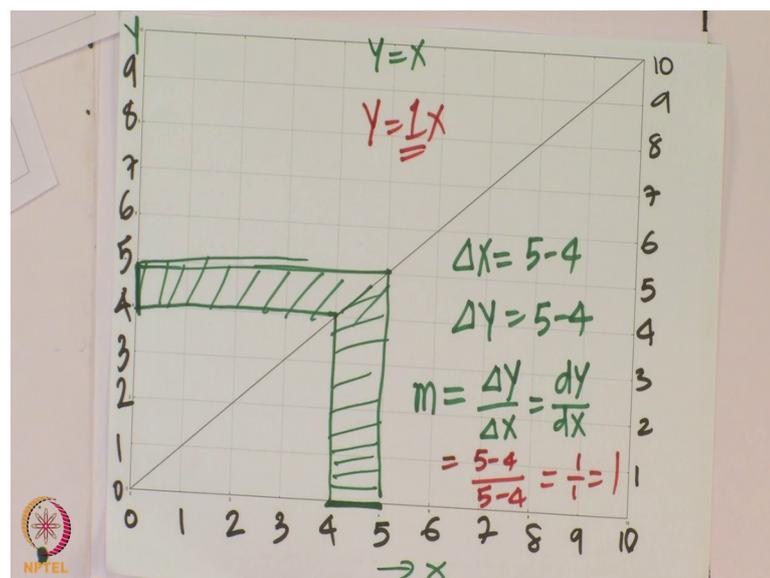
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So, let us see this; the slope how Y changes with x. If you have a small range of x, if I take how much did the Y change. So, dy by dx is called the derivative of Y, so finding derivative as a function of finding derivative of a function is nothing, but calculating the slope of a function.

So, let us think about the slope a little bit more by taking the curve that we all knew.

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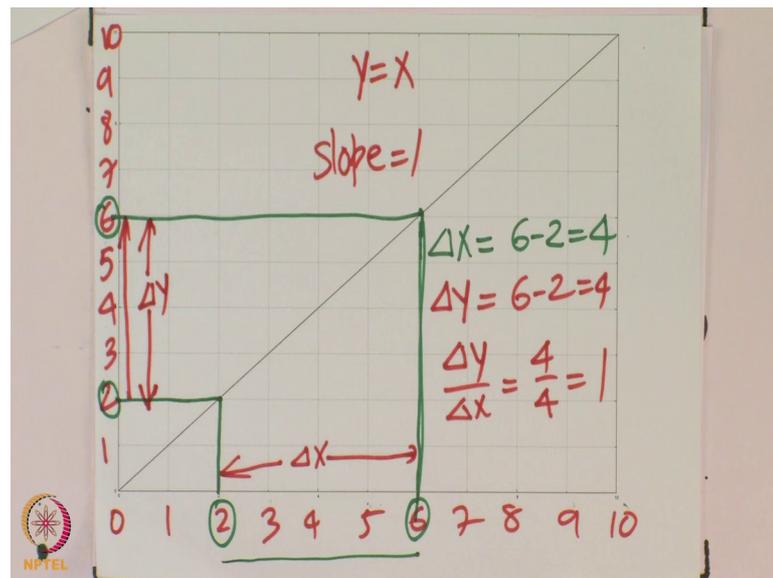
So, this is the simple curve Y is equal to X; so if I just take two values of X. So, X is equal to 3; so this is my X and this is my Y value; so if I just take X between 4 and 5. So,

the corresponding Y value it is also 4 and 5. So, if I just take delta X is this difference; so there is 5 minus 4 and delta Y is also 5 minus 4; this difference and the slope m is delta Y by delta X or is also; when delta X is very small; this can also be written as dy by dx; we will come to that.

But for the moment, let us consider these are equal things; same thing delta Y by delta X and dy by dx is the same thing. At the moment, we will discuss more about this later but this is 5 minus 4, divided by 5 minus 4. So, this is nothing but 5 minus 4; divided by 5 minus 4; which is 1 by 1; which is 1.

So, basically the slope of this curve is 1; which we knew Y is equal to 1 X is our thing; so m is 1. So, m is nothing but delta Y by delta x; so that is what I want to say here that the slope. So, now to calculate slope; let us little bit understand this little more carefully.

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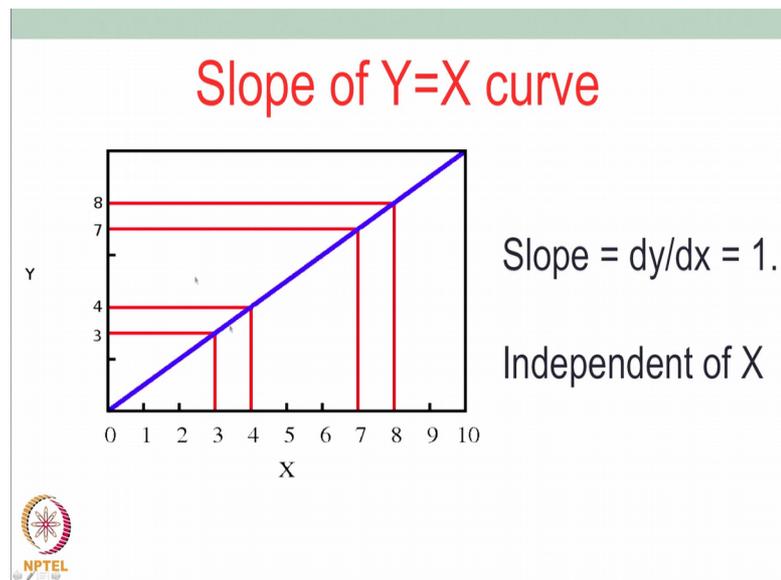


So, let us take this graph once more; so again this is same curve 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 and here is 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. So, this is the curve which is Y is equal to X; now I can take any two X values. So, how do we calculate slope? Take any two X values are you want; so let us say I want to take 2 and 6. So, I took two X values; any two X values, you want you take; the difference between this.

So, that is the delta x; the difference between these two X values which is 6 minus 2; which is 4, then draw a line touching the curve that you have and calculate the corresponding Y value and here also draw a line and calculate the corresponding Y value.

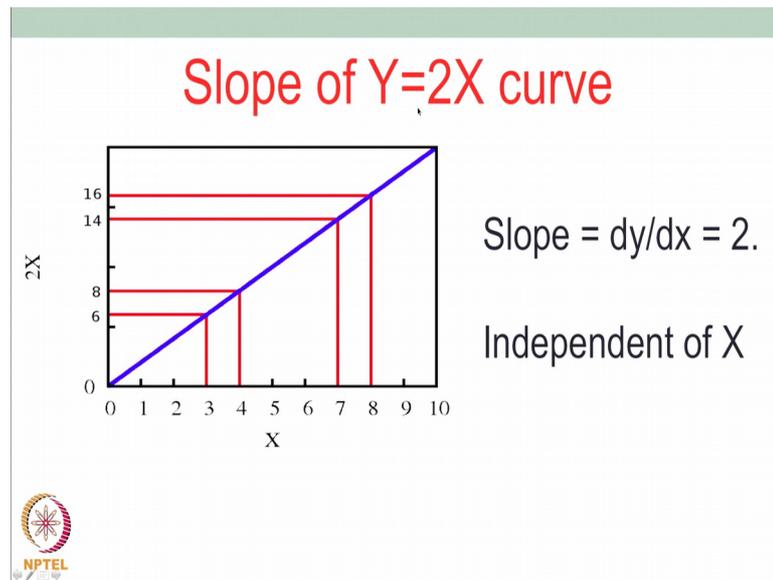
So, the corresponding Y value here is also 2 and 6; that is why this is Y is equal to X. So, delta Y, the delta Y is also 6 minus 2; this difference is 6 minus 2; which is also 4. So, delta Y by delta X is 4 by 4; which is 1, so this is how you calculate slope. So, this is all the slope is equal to 1; which we knew; so slope is take any two X value; calculate this difference which is here delta x. Calculating this Y difference, this is your delta Y; delta Y divided by delta X is your slope. So, if you take two values of this; any two values, you can take; anywhere you want and you will get a slope.

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So, let us look at here if you take 3 and 4; you will get 3 and 4. So, 4 minus 3 divided by 4 minus 3; which is 1, if you take 7 and 8; you will get 7 and 8. So, you will; this is 8 minus 7; you will get the value which is 1 by 1. Now, if we take a more complicated function which is let us say; Y is equal to 2 x. So, this is the function that we want to consider now; which is Y is equal to 2 x.

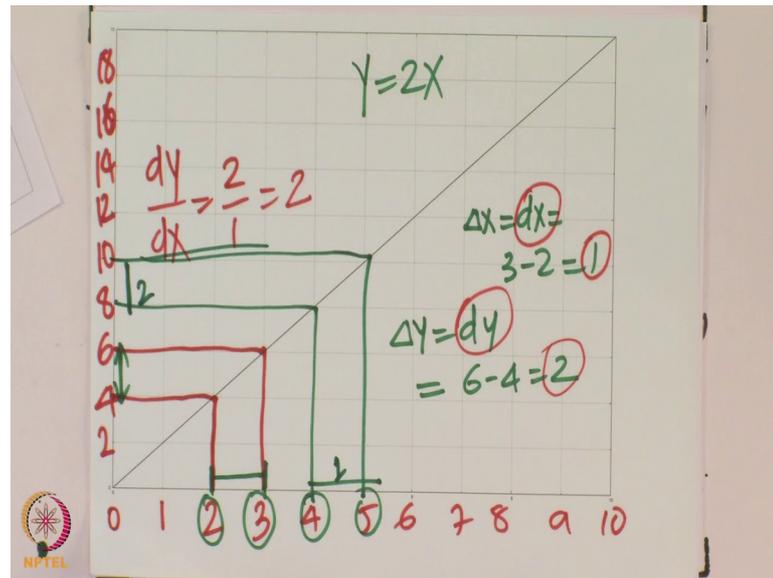
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So, if you look at Y is equal to $2x$; you have when X is 3; Y is 6, when X is 4; Y is 8. So, if I just take dx as this and dy as this; 4 minus 3 is 1; 8 minus 6 is 2. So, 2 divided by 1; so which is 2.

So, the slope is 2; so if you take 7 and 8, the 7 correspondingly 14; 8 would corresponds to 16. So, 16 minus 14 is 2 and this would mean 2 divided by 1; which is a 2 again, which is of course, this also shows that is independent of X. So that means, you calculate the slope here or here; you will get the same value and I want you to understand this little bit more carefully. So, let us think of a graph little bit more carefully; so let us take this graph and I want to put some values here, which is slightly different from what is printed here.

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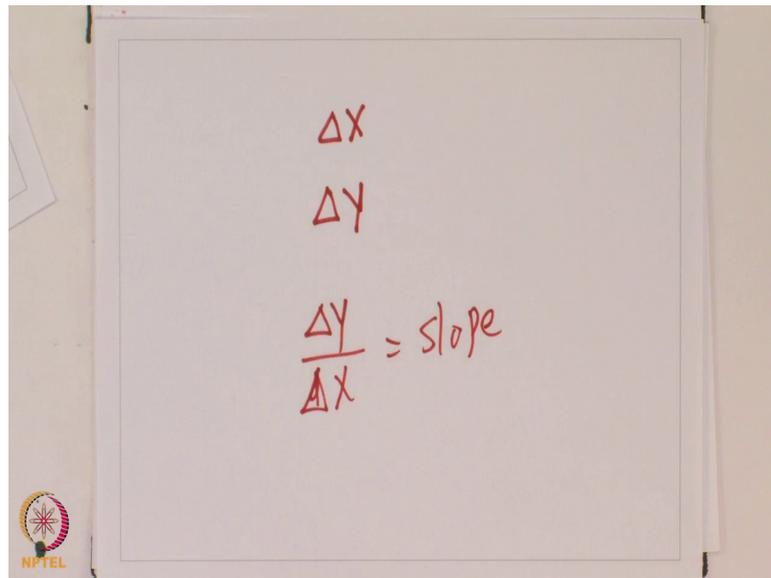


So, consider this graph; which is a straight line. So, I want to put some values here; this is 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 and here I want to put values 2, 4, 6, 8, 10, 12, 14, 16, 18 and so on and so forth. Now, this function then this plot will become Y is equal to $2x$; now as we just did to calculate the derivative of this, you want to take any two values; 2 and 3; find the difference between this X value. So, that is ΔX or dx which is 3 minus 2; which is 1.

Then find the corresponding Y values; the corresponding Y value here is 4 and the corresponding Y value here is 6. So, ΔY or dy is 6 minus 4; so those difference is 6 minus 4. So, 6 minus 4 is 2; so then if we calculate ΔY by ΔX or dy by dx . So, this is my dx which is 1 dy ; which is 2 dy by dx is 2 by 1; which is 2. So, the derivative of this is 2; that is what we saw. Now, if we were to take some other value of X and Y ; you will essentially get a similar number.

If now you can think of doing the same thing take 4 and 5 correspondingly; we will get 8 and 10. So, this is 1, this is 2; so you get 2 and this is 1, 2 divided by 1; you will get again the derivative is 2.

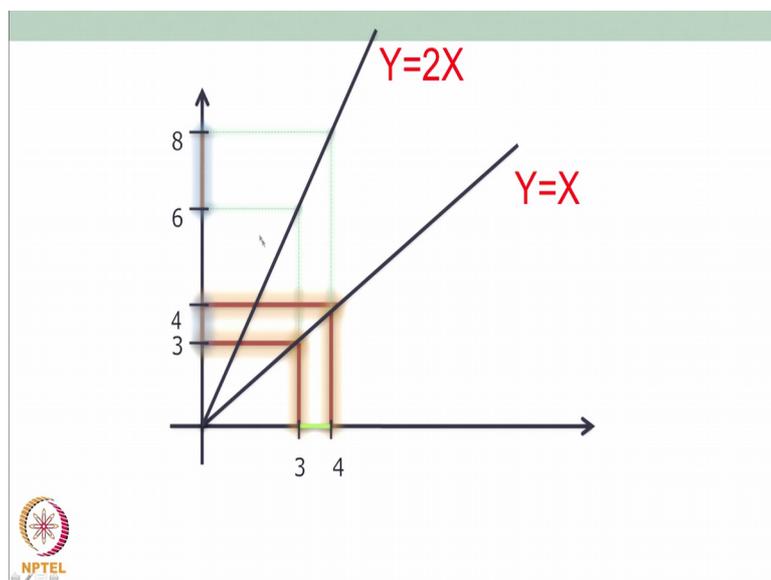
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So, the point I wanted to make here is; this is very simple that to calculate the derivative what you need to do; this take any 2 X values and calculate delta x, take any two values and calculate delta Y and delta Y by delta X is my slope.

Now, let us think of doing this for a curve; so before that let us see that.

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So, look at this video; so I am going to start with 3 and 4 and I have 2 curves here; which is Y is equal to X and Y is equal to $2x$; this is Y is equal X and Y is equal to X and I am starting with 3 and 4. So, my X 1; so 2 X values are 3 and 4; so the delta X is 4 minus 3

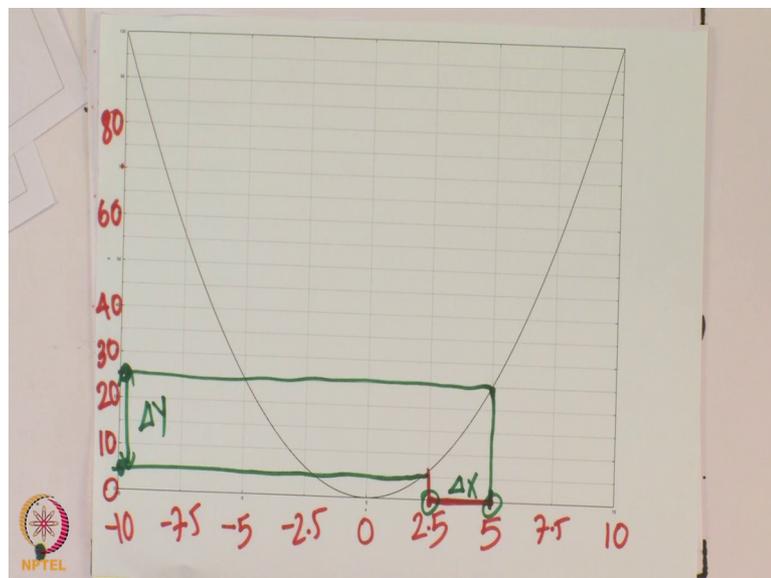
which is 1, then I draw a curve to this; Y is equal to X graph and I look at the corresponding Y values which is 8 and 6, but if I look to the Y is equal to X graph; those are 3 and 4.

So, here the difference is 4 minus 3; which is 1 here, the difference is 8 minus 2; which is 6 which is 2. So, this has even though ΔX is 1; which is the same for both the Δy . For the Y equal X graph is different from the Δy ; for the Y is equal to $2 X$ graph. So, therefore, the slope of this is different from the slope of this.

So, I want you to understand this; for which I will play again this once more; I take some $2 X$ values 3 and 4, the corresponding Y values for the Y is equal to $2 X$ curve is 8 and 6, but for the same Δx ; the corresponding Y value for this Y is equal to X curve is 3 and 4. And therefore, the ΔY by ΔX will be very different and I want you to think about this.

Now, we want to quickly understand this for a curve. So, instead of you go beyond a straight line; if you consider a curve.

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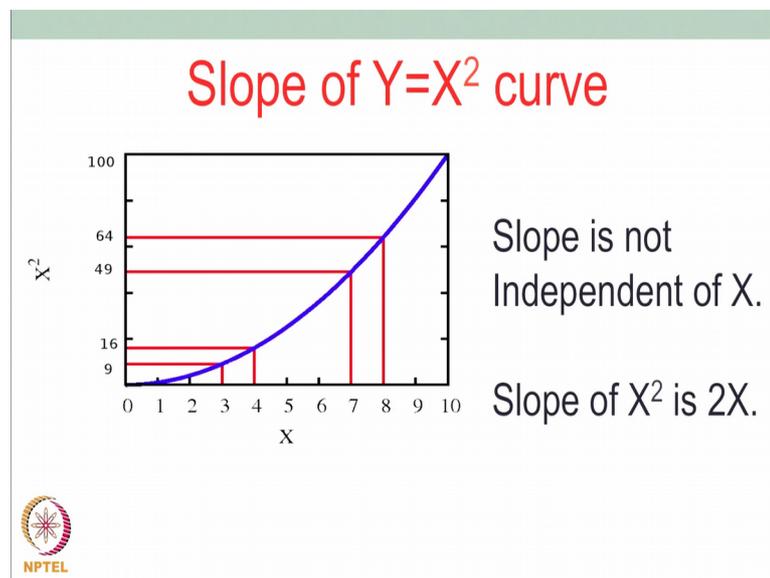


So, think of a curve like this; so I have here 0, 5, 10 and this will be in between 2.5; I would think and this is 7.5 and this is minus 5, minus 2.5, minus 10, 7.5 and the correspondingly let me draw some important values here. So, this is 0 of course, this is 10, 20, 30, 40, 60, 80 and so on and so forth.

So, if I want to calculate the derivative of this; I can calculate the derivatives at different points. So, let us say; I take this as my delta X 2.5; so these two points as my delta X and corresponding to this, I can get the corresponding Y value here and I can get the corresponding X value here. So, for this I can get the corresponding Y value; which is here. So, if I just take these 2 X value; which is my delta x, this is the corresponding Y value, this is my delta y. So, calculate this value and find this difference and divide; that would be this slope.

So, that is one way of this does; a way of calculating the slope. So, let us do that look at here.

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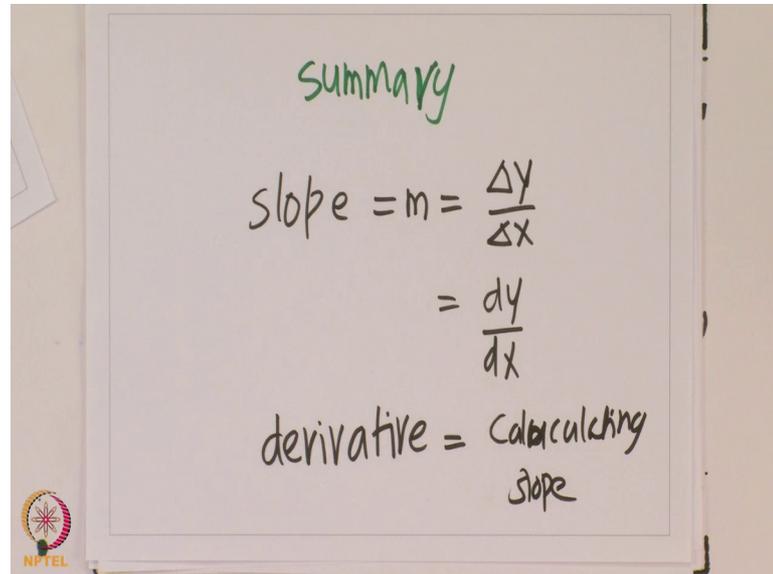


So, if you just take Y is equal to X square X square curve if I take 3 and 4 as my X values the corresponding Y values are nine and 16, so I will get 7. On the other hand; if I take 7 and 8, where delta X; the change in X is 1, the corresponding Y values will be 64 and 49 and this difference is very different from this difference.

So, wherever if I calculate that slope here and if I calculate the slope here; it appears very different; on the other hand in this earlier curve here; if I calculate the slope here or slope here is same. So, in the straight line curve slope was independent of X, but in this case slope is dependent of X and the slope for this is 2 X; which we will come back and learn about it. But at the moment, it is enough to understand that different places you calculate

slope; you will get different numbers and we will come back and learn about this in detail.

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So, to summarize what we just did so far; we talked about slope, which is m we call, which is ΔY by ΔX ; change in Y divided by change in X ; which is also we call it dy by dx and that is at the moment, they are the same and we calculate the slope for a straight line. And some curve we said that this is calculating; slope is calculating derivative is same as calculating slope; calculating slope is same as calculating derivative. So, this is what we learned in this lecture; we learn about this finding derivatives more in detail in the coming lecture; for this moment, we will stop by.