Introductory Mathematical Methods for Biologists

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Lecture – 05
Graphs: Logarithmic and Other Functions

Hi. Welcome to this lecture, we will continue to learn more functions. We have been learning trigonometric functions, so we will learn about cos function.

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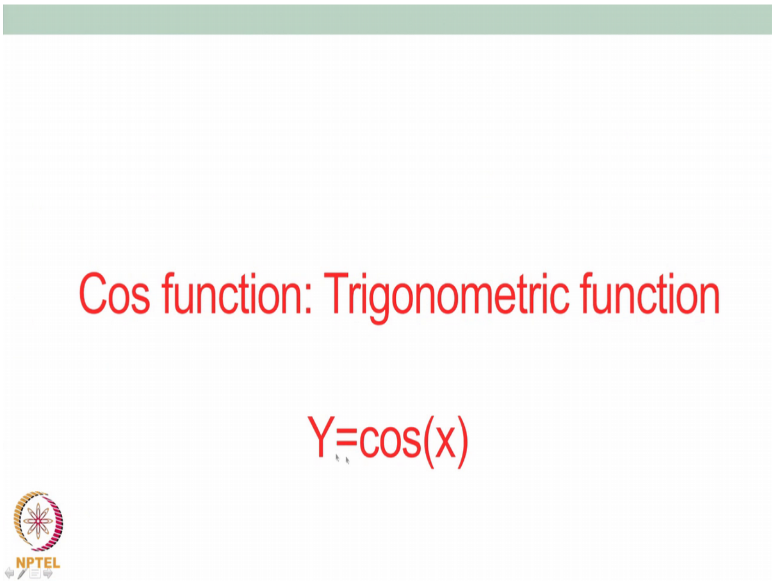

So, Y is equal to cos x; so another trigonometric function.

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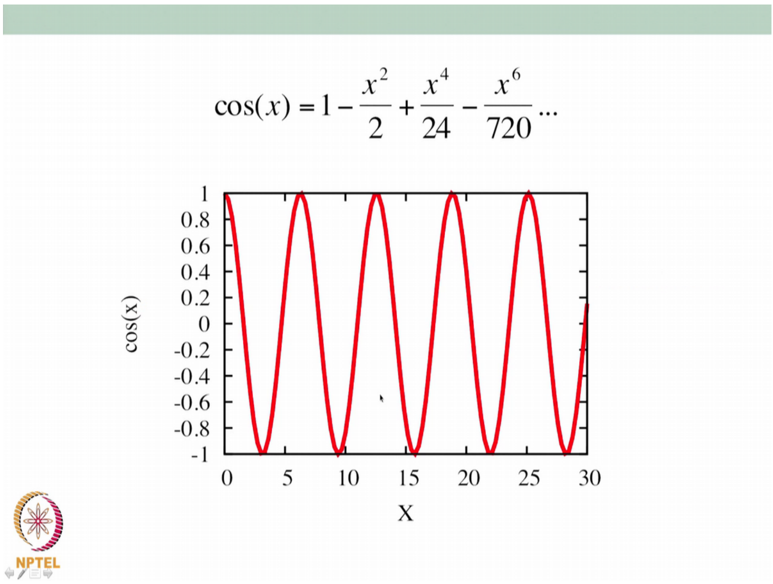

So, just like sin cos also can be written as some combinations of x power some powers of x. So here cos x is 1 minus x square divided by 2 plus x power 4 divided by 24; minus x power 6 divided by 720 dot, dot, dot, dot. So, this is again infinite series you have many more terms I have only written four terms here.

So, if you take an x value and substitute in this infinite series, you will get cos x or we can use the calculator and plot it or use any software and plot it. You can see that when x is 0 this term will be 0, this term will be 0; this term will be 0 all are further terms which we x powers of x will be 0. So, when x is 0 cos of x is just 1; so you can see that when x is 0 cos x is 1 and as x increases this will decrease, then further increase. So, this is also a oscillatory was oscillating function oscillate; again plus 1 and minus 1 for various values of x.

So, the difference with sin x; x is equal to 0 was 0 here, when x is equal to 0 cos x is 1; now as we said for sin x or cos x the x value there.

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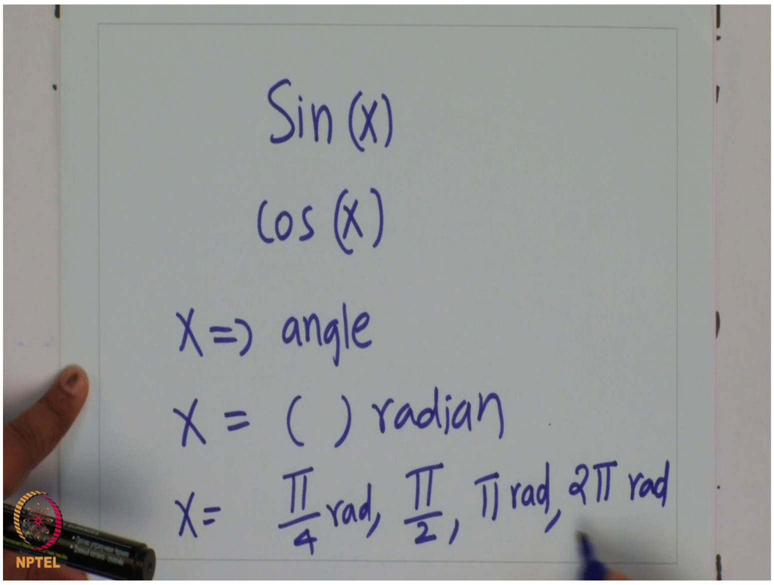

So, this is sin x or cos x x is typically in angles need not be angle, it could be any dimensionless quantity; again can be written as can be use as x. So, here you can see again that 1 minus x square x square divided by 2; so x again will not have any dimension.

So, angle is typically for example, angle is arc by radius; so this is the ratio of 2 lags. So, it will be a dimensionless quantity, so x typically is written as some radian. So, let us say some number radian, so as we set for sin; it could be pi by 4 radian pi by 2 radian pi radian 2 pi radian and so on and so forth. So, various values over various values of x; you can get the corresponding cos x or sin x values.

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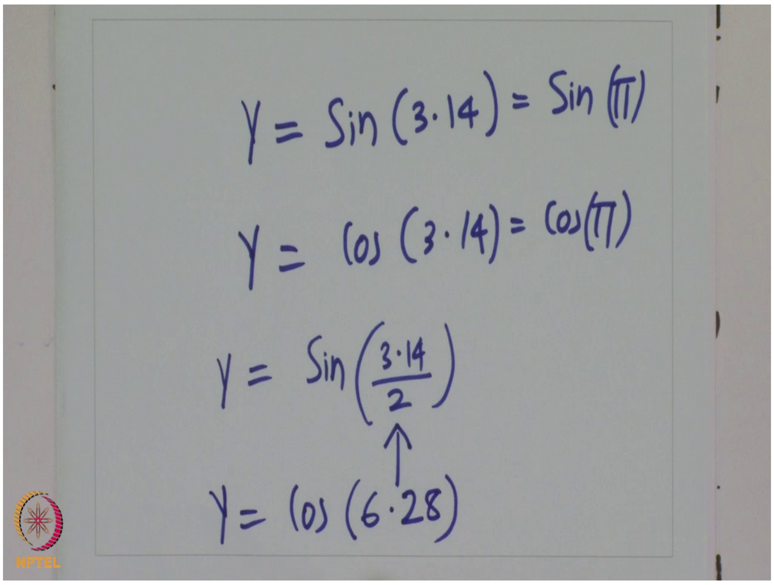

So, as we know you can have Y sin 3.14 dot, dot, dot, so this is the approximate value of pi and one can give this in a calculator and you will get the corresponding value. Similarly Y is equal to cos 3.14; which is same as sin pi cos pi, so this is a radian. Similarly, you can have pi by 2; which is Y is equal to sin 3.14 approximately; divided by 2. Similarly cos or 2 pi; so whatever be the value of the argument here, so this is often called an argument, you will get the corresponding Y value.

So, you could Y is equal to cos 6.28 is approximately 2 pi; so you have to remember, this is a numbers; this is also written in degrees.

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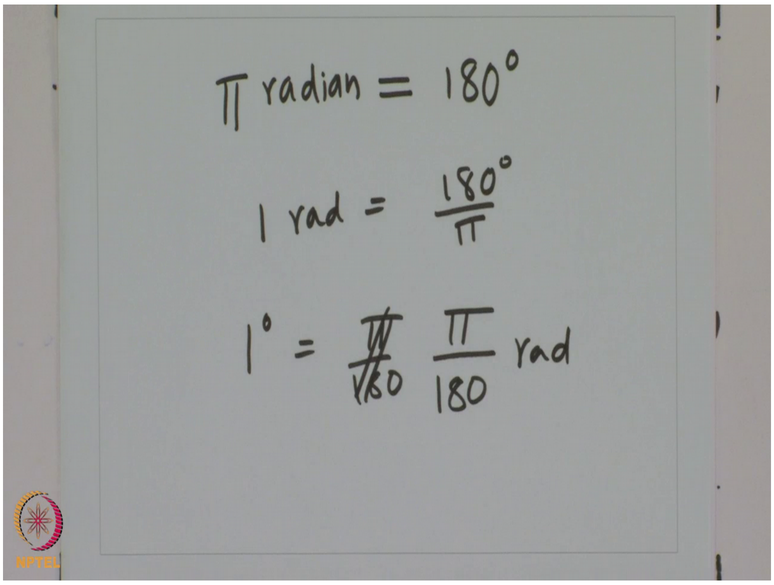

So, pi radian as we know is 180 degree; so this is I want all of you to remember. You of course, you would know this, but just for the; so pi radian is 180 degree. So, you can convert like; so which would mean that 1 radian is 180 by pi degree or 1 degree is pi by 180 radian pi divided by 1 degrees pi divided by 180 radian.

So, one can convert appropriately and write this things in either radian or in degree. So, this is something we should remember when we plot trigonometric functions.

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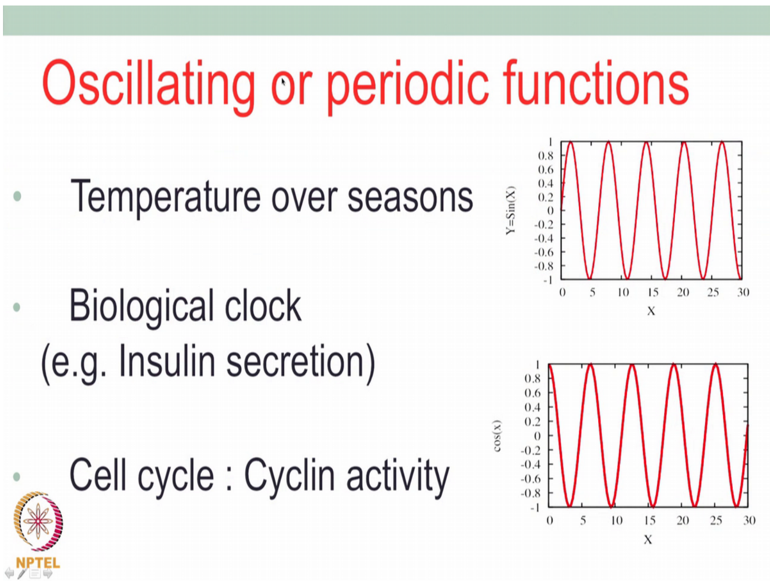

And as we said will this sin and cos are some oscillating periodic functions, we can use this appropriately to plot temperature over seasons, biological clock, cell cycle. These are the starting points, it does not mean that temperature can be written as sin x or cos x; which it may not be possible, but this is a starting point. Once we have a function, which is oscillating; we can manipulate this function to get this to plot these quantities; that is the train that you will learn. How to use these functions to plot the experimental data that we have which would be this, but you can see is see some correspondence are immediately that this is oscillating; similarly these functions are also oscillating.

So, we can use these functions to represent some of these quantities. Now you will have some other functions which has a very different shape for example, very often we would plot some of the free energy in thermodynamics.

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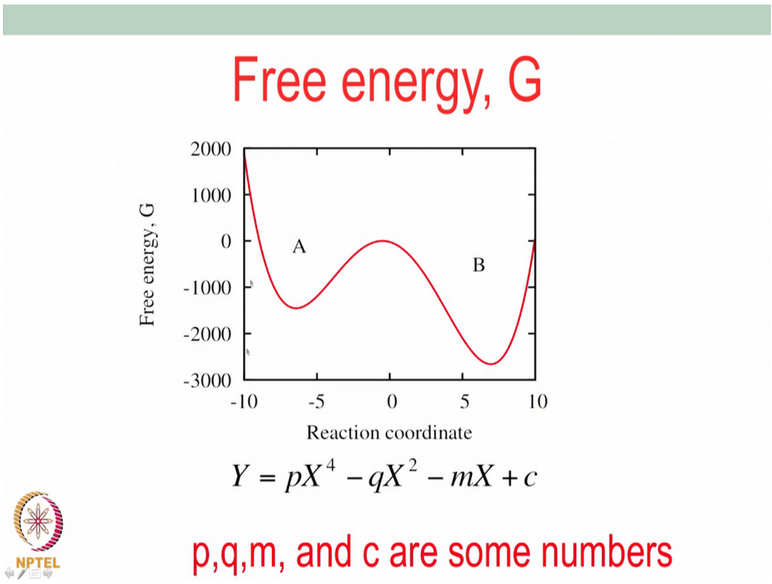

So, if we plot free energy as a function of reaction coordinated by their states, we can have two states A and B and all that. So, we can have some free energy versus x reaction coordinate. So, if you have a equation which is of the form pX power 4 minus qX square minus mX plus c, where p, q, m and c are some numbers by putting appropriate number.

I can get this curve, I have plotted this curve by substituting appropriate numbers for p, q, m and c. If you substitute some numbers, you will get this kind of a curve; so, you can plot this kind of curve again as a combination of powers of X. So, that is what I wanted to convey here.

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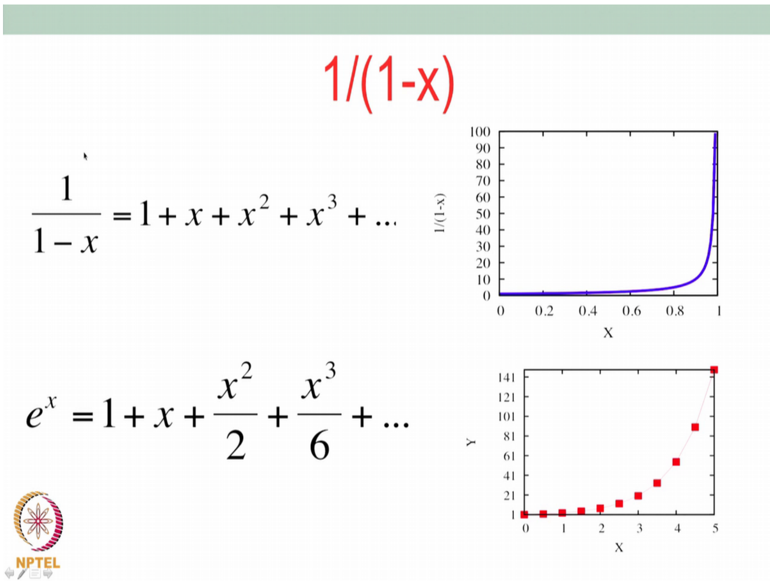

Another thing I just want you to familiarize with when we saw all this complicated curves like 1 plus x by x divided by some number plus x square divided by some number. Immediately, it would have come to your mind why not I simply add 1 plus x plus x square plus x cube.

So, if I just add simply add 1 plus x plus x square plus x cube plus x power 4 plus x power 5 plus infinite series like this one would get 1 divided by 1 minus x. So, if this function; if you expand it turns out that, you will get 1 plus x plus x power 4 plus so on and so forth, you should realize that one is nothing, but x power 0.

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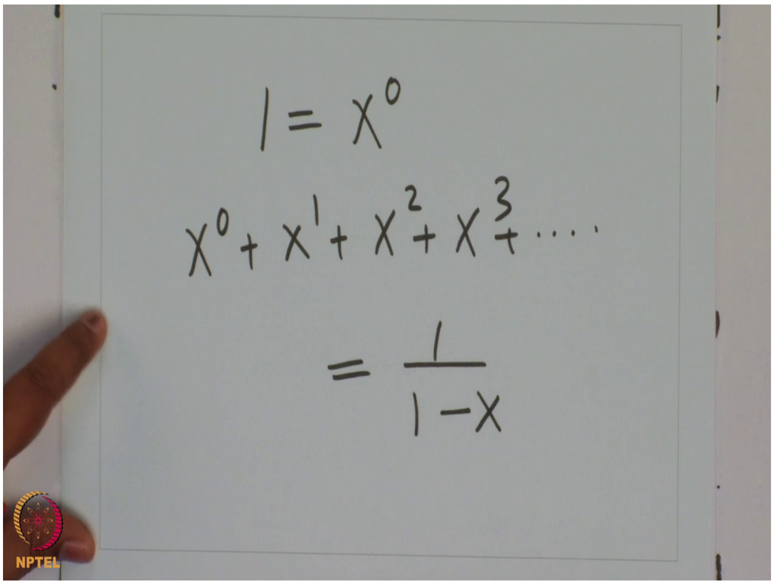

So, in all this cases I hope you have realized that 1 is X power 0; so if you have X power 0 plus X power 1 plus X power 2 plus X power 3 plus dot, dot, dot infinite series like this; this is 1 by 1 minus X.

And if we plot it what we get here is this curve which again looks like exponentially increasing curve, it increases very fast much for small change in X; it increases rapidly and e power X although increases rapidly. So, e power X is 1 plus x plus x square; this is divided by 2 x cube here; divided by 6. So, you are dividing by something here but here is just 1 plus x plus x square plus x cube plus so on and so forth.

So, these two curves look similar in some sense of course, there are various differences at a beyond X equal 1. This will have; this cannot be here you can see by at X equal 1's already infinity, so there are some differences; which here I want you to think about.

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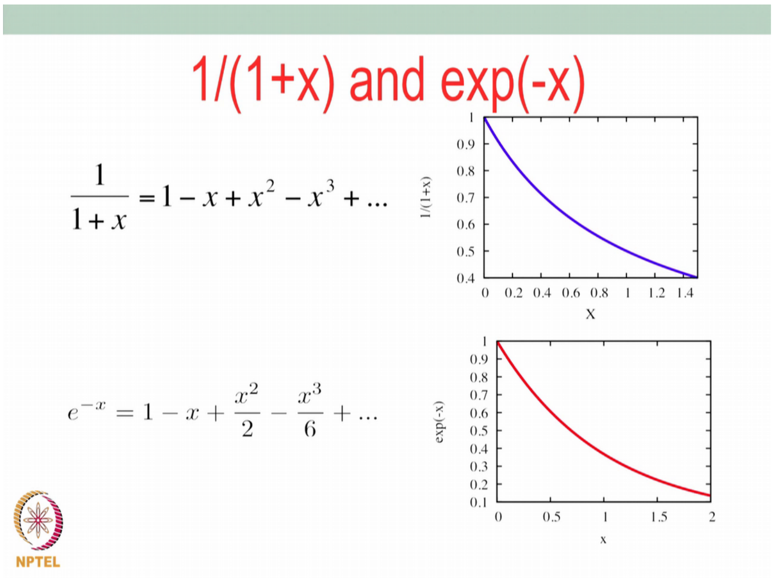

Now, similarly if you just take 1 by 1 plus x and compare with e power minus x e power minus x; we saw is 1 minus x plus x square by 2 minus x cube by 6 plus dot, dot, dot. If I do not divide by this numbers; if I just take 1 minus x plus x square minus x cube plus x power 4 minus x power 5 and so on and so forth. If I do this, I will get 1 by 1 plus x and if I plot 1 by 1; plus x, it is a decreasing function like this e power minus x is also a decreasing function is look somewhat similar.

So, this expansion looks similar; similarly this one corresponding functions also if we plot they will look similar. So, I want you to get familiarized with various functions here.

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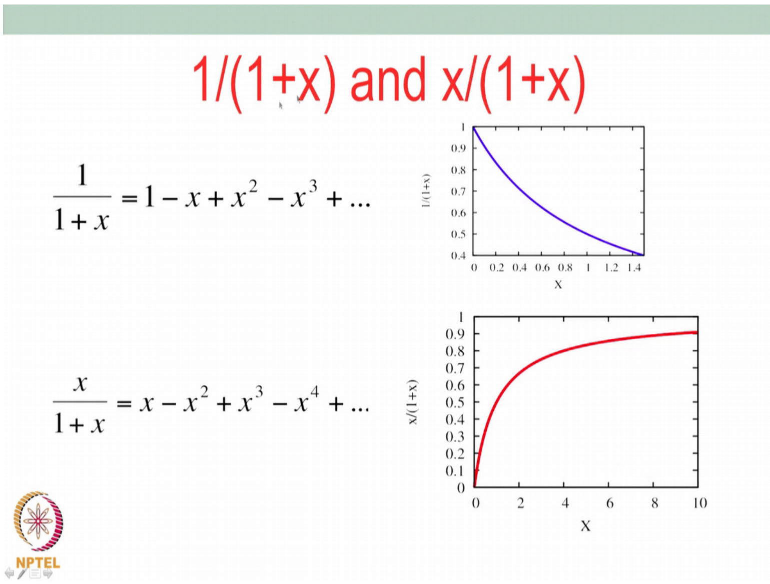

Another function that we saw 1 by 1 plus x; just now if we plot x by 1 plus x that is if I just multiply everywhere with x. So, I multiply this with x; it will be x minus x square plus x cube minus x power 4; plus dot, dot, dot. The behavior changes and you would get a function, which is like this which is a saturating function, so is x by 1 plus x as x goes very large.

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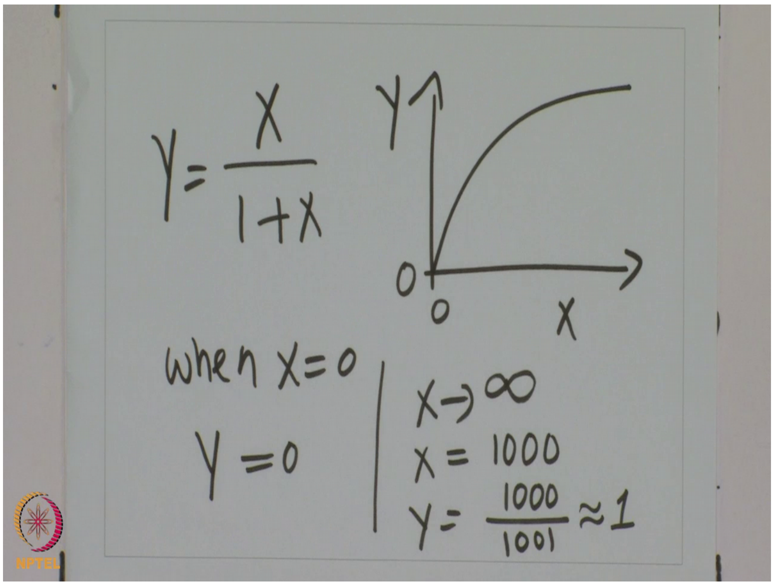

So, you can; I want you to familiarized again; with this function X by 1 plus X and we saw that this function, this looks like this.

So, this is Y; so this is Y and this is X; when X is 0; Y is 0 plus 0 divided by 1 plus 0 is 0 divided by 1 which is 0. So, this will be 0 for; 0 and it will increase when X is very large; X goes to infinity, X is very large. When X is this will become a constant; that is why saturating here. So, if you just put for example, X is equal to let us say 1000; then Y is 1000 divided by 1001; this is approximately 1.9999 something.

Now, if I just put X is 10000; it is 10000 divided by 10001. So, again that is also 1; that is why it is saturating as x goes this ratio will tend to 1; so that is what the curve that we have.

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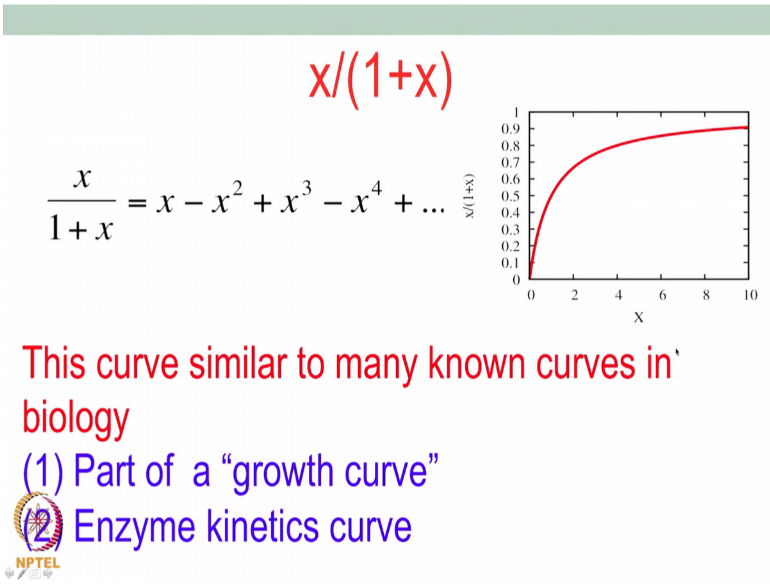

And I am saying this because; this curve is you might be very familiar with this curve in biology a part of a growth curve; might look like this or even enzyme kinetics curve might look like this. You might have various curve; which looks like this, which is a function, which is again a combination of x, x square, x cube, x power 4 etcetera.

So, that is what I wanted to say that you might have seen such curves to represent enzyme kinetics growth curve. And all that you might remember Michaelis Menten equation and some such function will come there which might have this kind of shape which will have this kind of shape.

So, I want you to familiar; we got familiar with various functions. Now another function I want you to get familiarized is Y is equal to root X, so what is it? Y is equal to root X.

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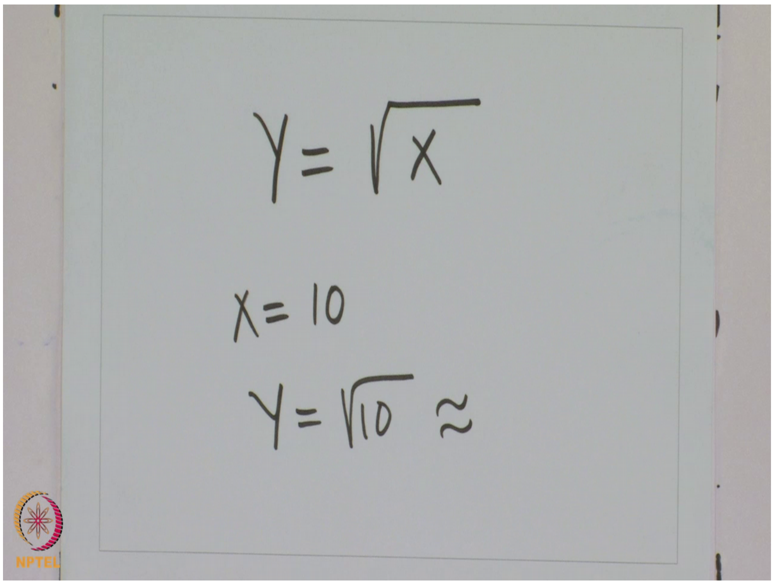

So, if I just take X is equal to 10; Y is root 10, which is some number above is 3; root 9 is 3 and root 4 is root 16 is 4, so some number between 3 and 4. So, I want you to familiar with this function Y is equal to root X.

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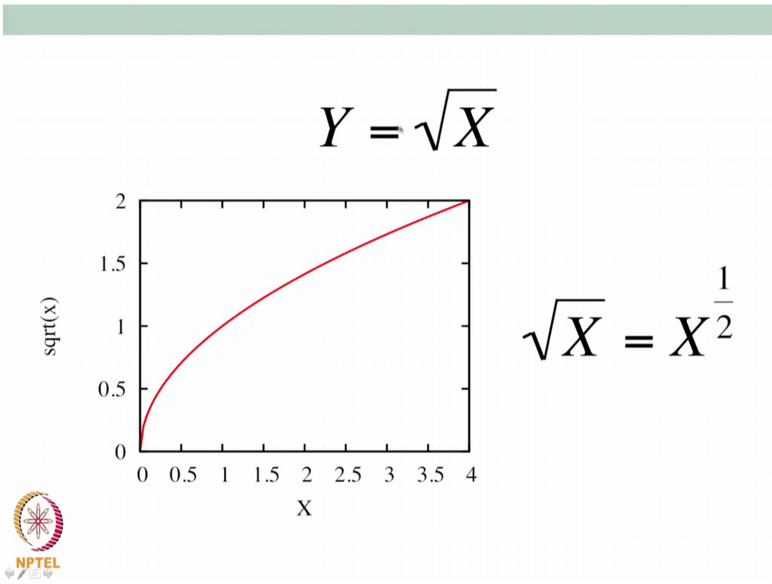

If we plot Y is equal to root X; it would look like this. So, this is X, this is square root of X square root of X, can also be written as x power half. So, this is Y is equal to X power half, this would look like this. So, as X becomes larger and larger this changes like; this is very specific way of changing.

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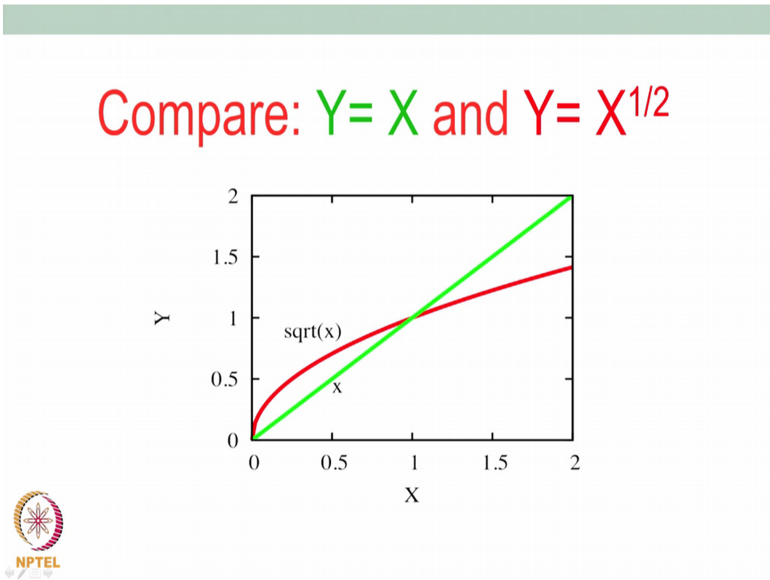

I want you to compare Y is equal to X and Y is equal to X power half. So, Y equal to X is a line which is a green, here Y is equal to root X or X power half; this the red curve at 0, they are the same when X is 1 root X and X are same below 1 root X is larger than X above 1 X is larger than root X.

So, I want you to think about this and know that the root X will have this shape; whenever you remember root X; remember this kind of a shape, which is increasing and slowly it is not really saturating per say; its increasing what very slower, it increases in a particular way which is slower than X.

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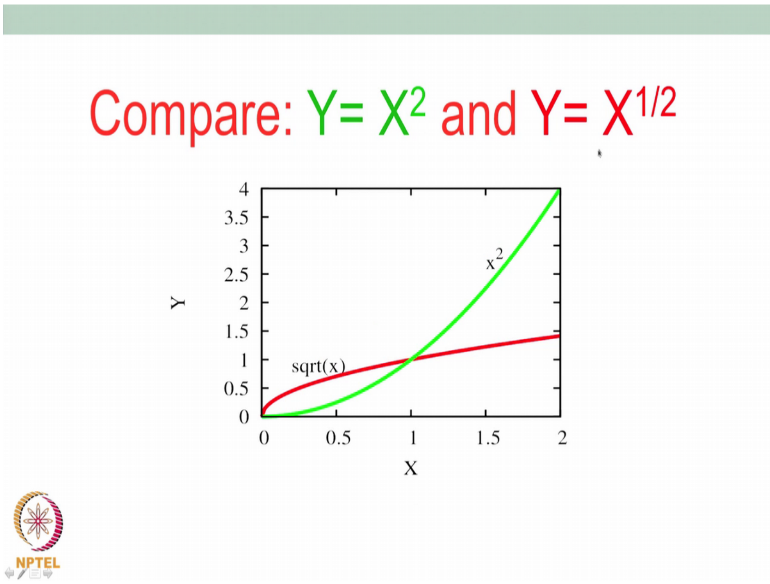

If you compare X square and root X, you will see that X square increases like this root X has this particular behavior.

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Functions that decrease with X

- Height of a falling object decreases with time
- Concentration of nutrient in bacterial culture
- Free energy vs Temperature

$$\Delta G = \Delta H - T\Delta S$$

So, some examples of functions that decrease with X; for example, height of a falling object of course, will decrease with time concentration of nutrient in bacterial culture as the bacterial grows. The nutrient concentration will decrease free energy versus temperature; the free energy will decrease; if these things are a constant as T increases delta G will decrease because of this minus sign.

So, there are some examples where the function that would decrease and you can think of many other examples. So, I want you to think about various examples where this will be function will be decreasing with the X.

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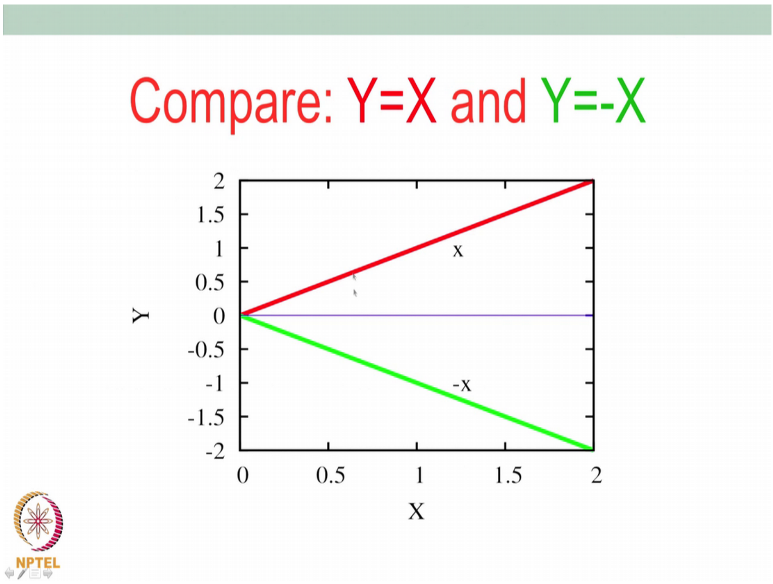

Now, we plotted X; I also want you to think about Y is equal to minus X. So, this is Y is equal to X; which will increase Y is equal minus X will decrease; so this is Y is equal to X and Y is equal to minus X.

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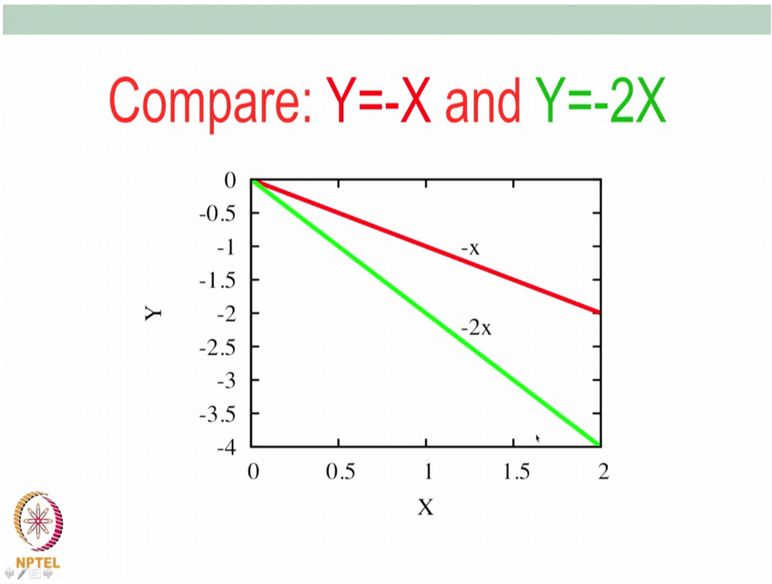

Similarly, I want you to think about Y is equal to minus X and Y is equal to minus 2X; compare those. So, we had then Y is equal to X and Y is equal to 2X; you can compare similarly Y is equal minus X and Y is equal minus 2X; if you compare these two you can see again that.

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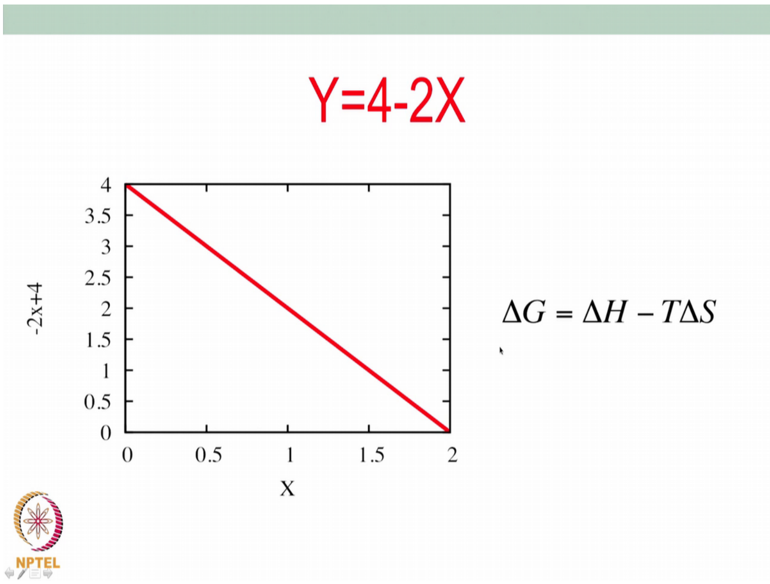

This is decreasing function, you can also think in general Y is equal to 4 minus 2X; which could be some example like this; you can this is general Y is equal to mX with some c plus c with some m having a minus sign here; its minus 2. So, you can then you can find some analogy between this equation and this equation, where delta H is 4 and X is T and delta S as 2; if you wish.

So, you can think of you can find some analogy; we can sign find some correspondence between these two equations. And you if you plot minus 2X plus 4; so in other words 4 minus 2X; if the Y axis is X, you will again get a decreasing function; another thing from whatever we learned. So, far if we compare Y is equal to X square and Y is equal to minus X square; these are the two things that we can compare.

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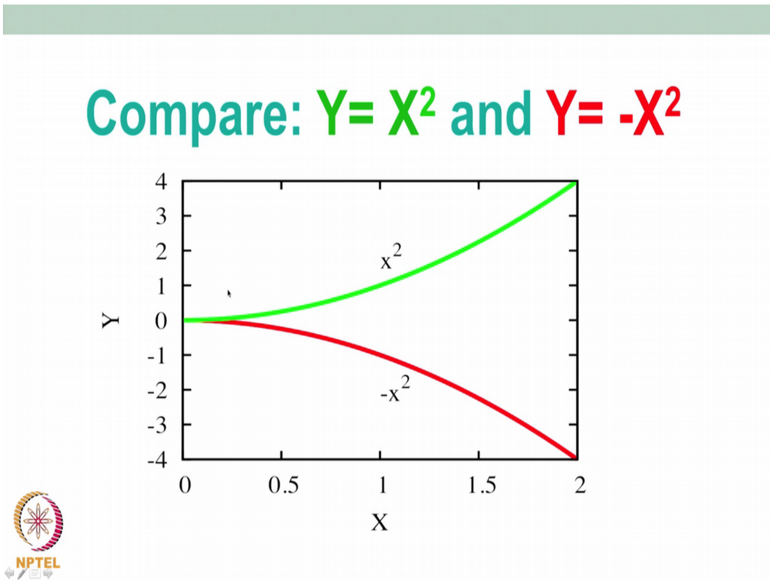

And Y is equal to X square is an increasing function; Y is equal to minus X square is a decreasing function. Again I want you to plot both sides of this, so I urge you to plot this on four quadrants and how it looks. You want to look at how this would appear, if you plot on either sides.

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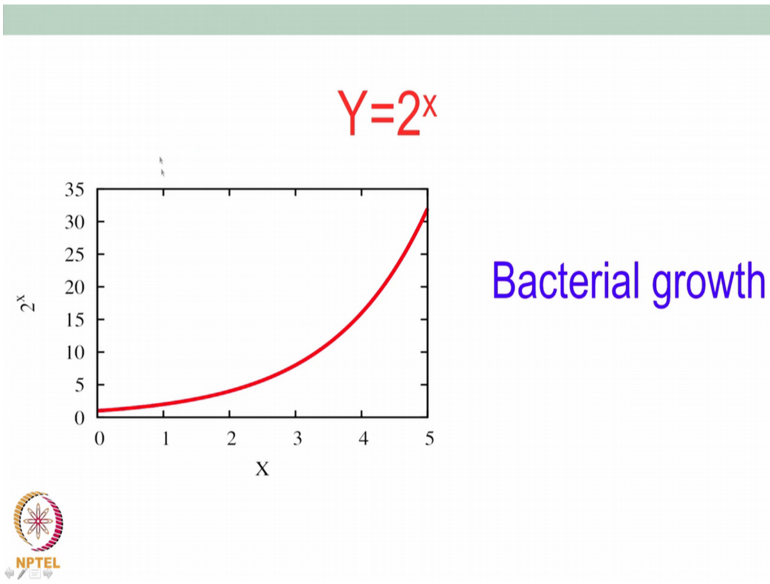

Now, another function we learned e power X; I want you to also think about 2 power X because at the end of the day; e is just a number; so, e.

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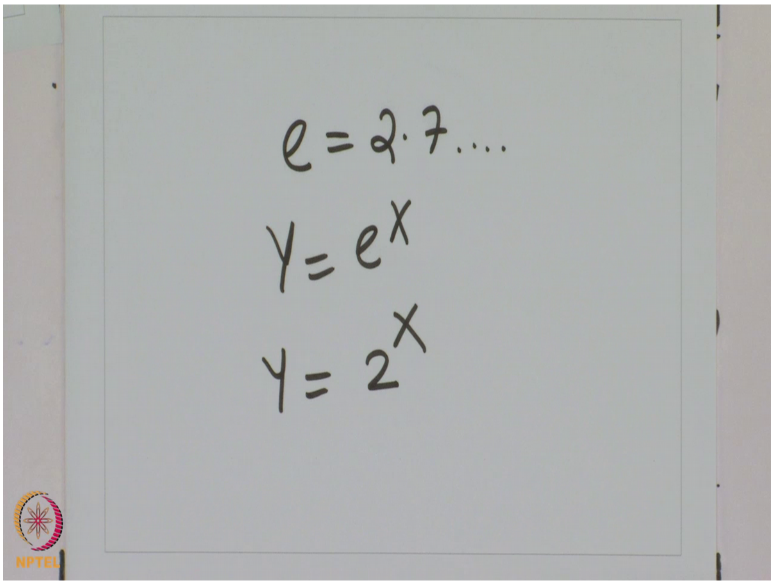

As you would know e is a rational number which is 2.7; some number like this. So, instead of plotting Y is equal to e power X; we can plot Y is equal to 2 power X and this is relevance for bacterial growth. As you would see in a minute and the curve would look like this. So, if you plot two power x; the curve would look somewhat similar 3 power X; it would be increasing with X and this is a lot of relevance for bacterial growth; how is relevant? Let us see this.

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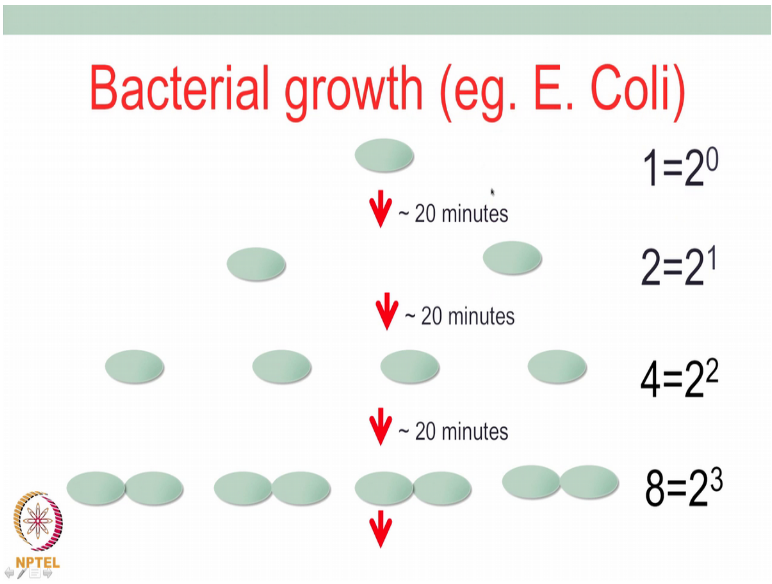

So, think about E. Coli growth for example, you have you start with 1 cell in about 20 minutes, it will divide and become 2 cells; this would divide and this would give you 2 cells; separate this would this 2 cells would become 4 cells; these 4 cells in the next generation would become 8 cells.

So, if we look at t equals 0 at; 0 generation you have 2 power 0, 1 cell in the first generation you have 2 power 1; which is 2 cells in the second generation; which is again after 20 minutes. So, there are you have 2 power 2 cells; which is 4 in the third generation.

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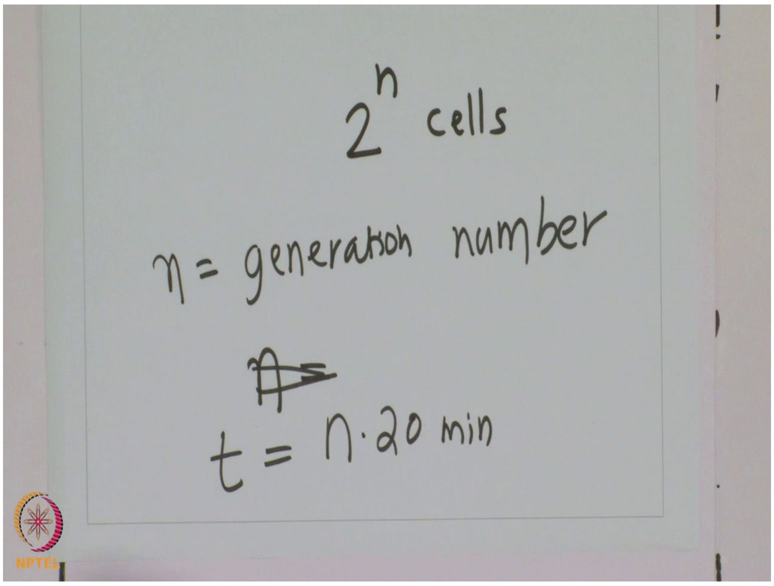

You have 2 power 3; 8 cells; so in general in nth generation, you will have 2 power n cells. So, what you would get in a bacterial growth is after n generation; you will have 2 power n cells, where n is generation number. Now, if you assume each generation is on an a average 20 minutes; n is each generation is 20 minutes. So the time after n generation will be n times 20 minutes; approximately on an average, the time will be 20 n minutes.

So, what would this imply? This would imply that 2 power n; I can write n as t by 20 from this.

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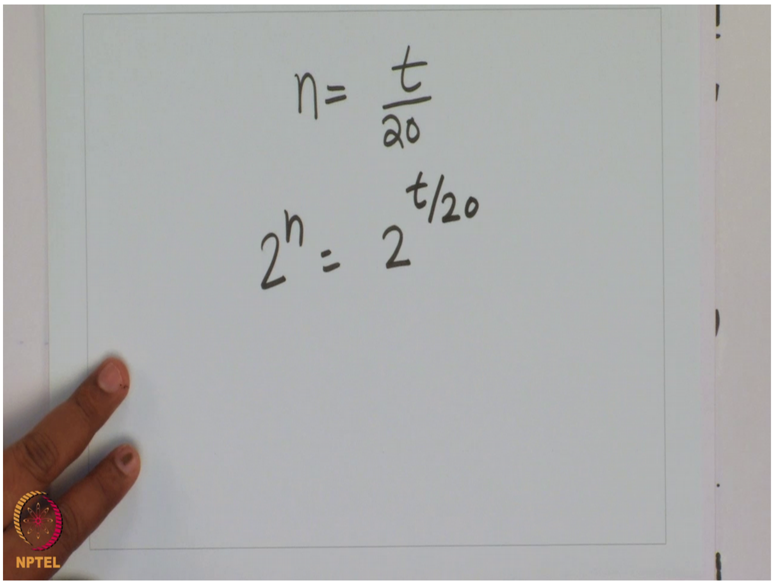

So, if I just take 20 on the other side; again right n has t by 20, if n t is in minutes; those in minutes, this is for E Coli and what one would get is that 2 power n can be written as 2 power t by 20.

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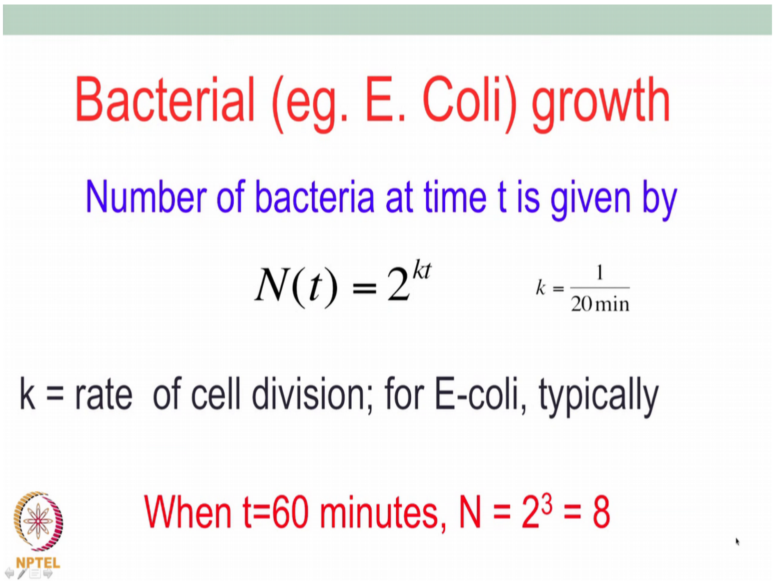

What would this imply is that I can write 2 power k t; this can be written as the number of bacteria at a time t can be written as 2 power k t, where k is 1 by 20 minutes. So, k will have a; k is like a rate which is the dimension of time inverse here, it is minute inverse or it could be express as minute inverse or second inverse.

So, the number of bacteria at time t can be written as 2 power k t, where k is the rate of cell division for E Coli. Typically it is like 1 by 20 minutes and when t is 60 minutes.

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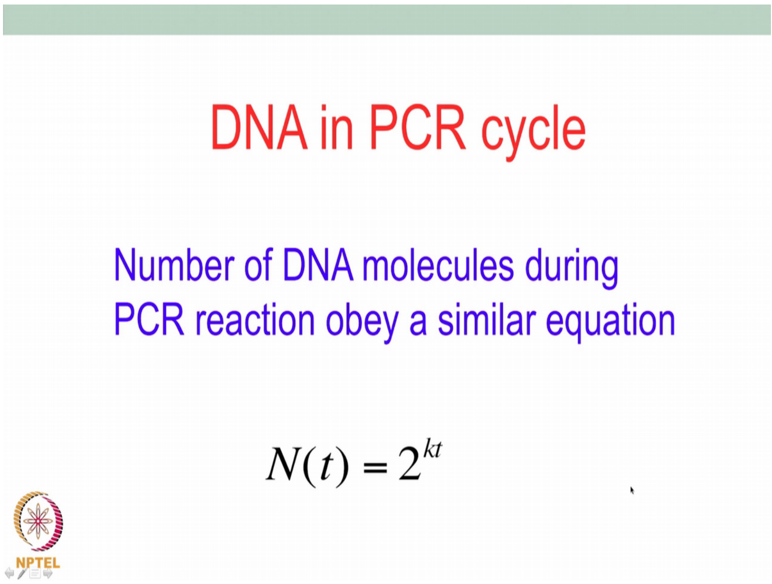

So, 2 power 3 is 8; another example for this would be DNA in PCR cycle. So, you will have number of DNA molecules during PCR reaction; will obey a similar equation N of t is 2 power k t. So, this is another example where again you will have divided this in 2.

Now, you can also write this as e power something and we will learn about this. So, I urge you to figure out; how to write this as e power something, how to write 2 power k t as exponential some number, if something that I want you to learn let me try to do that quickly here.

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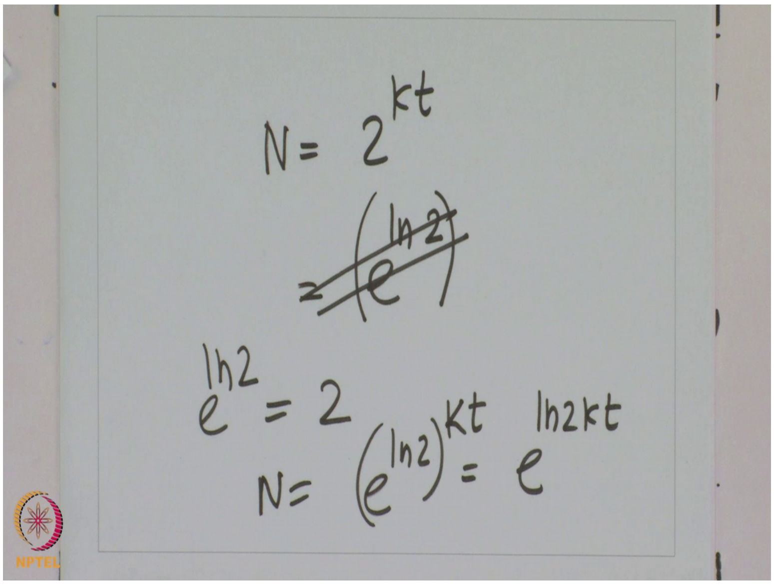

So, here we have n is 2 power k t; you can write this is e power log 2 is nothing, but 2. So, e power log 2; so let me write e power log 2; lon 2 is nothing, but 2.

So, since this is 2 power k t rate can be written as e power lon 2. So, 2 power k t can be written as e power lon 2 power k t; which is nothing, but e power lon 2 k t. So, one can write 2 power k t, this N as e power lon 2 k t. So, one can write this is an exponential function also and I want you to learn this.

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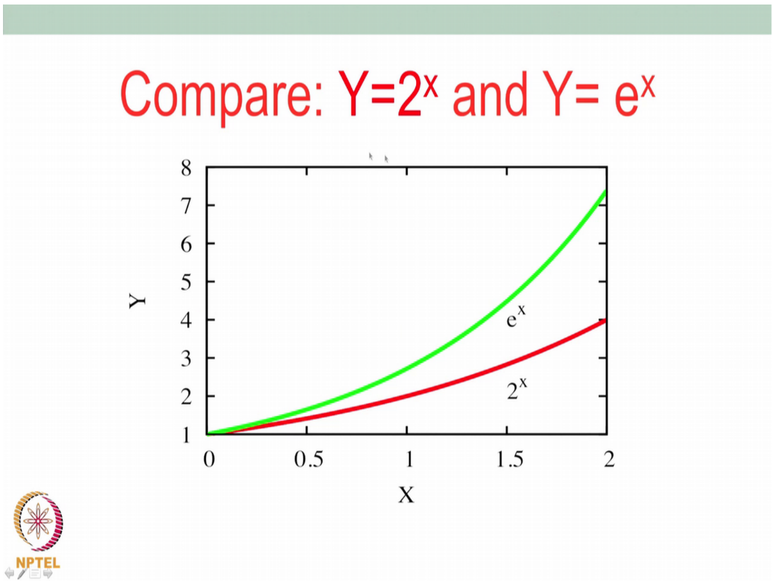

So, if you compare 2 power x and e power x; they have similar shape, but e power x since e being a slightly larger number e power x will be about 2 power x.

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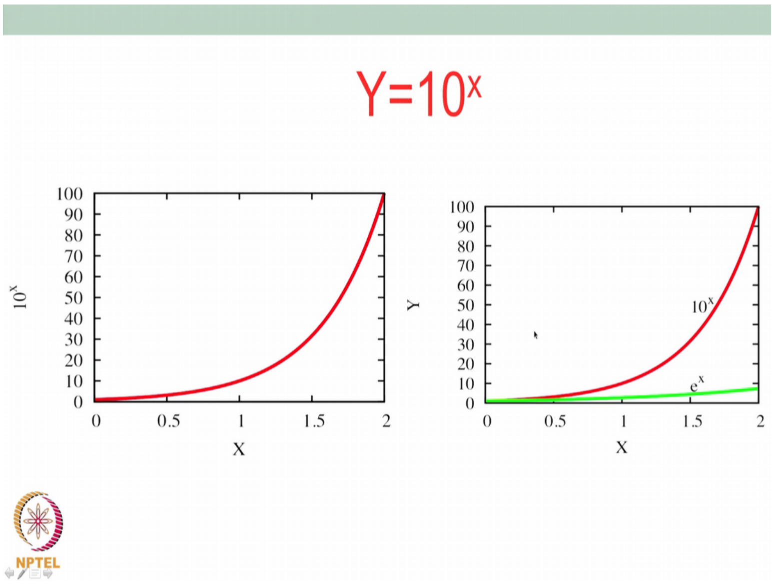

Similarly, we can have 10 power x; so I will first quickly go through few functions. So, you can take 10 power x and plot it; so this is a 10 power x and e power x is 10 is larger than e.

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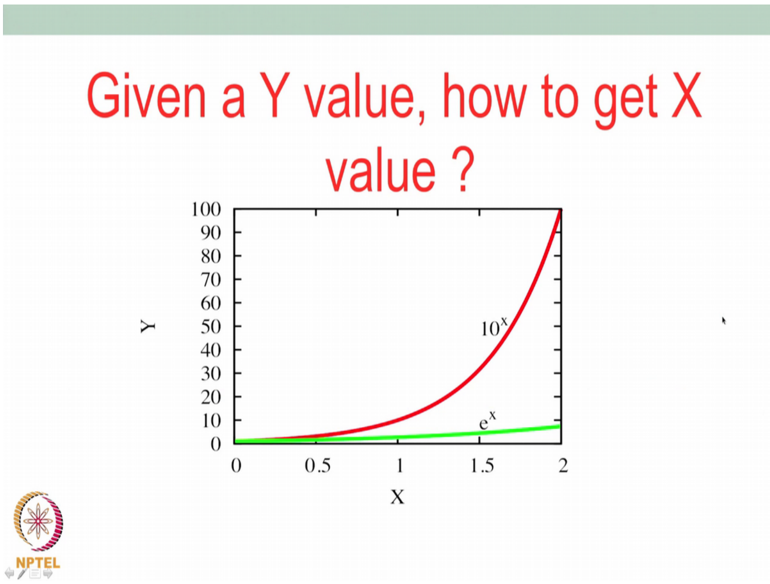

This will go above this; now you can also think of given a Y value; how to get an X value? If Y is 100 and Y is 10 power X; the corresponding X value can be found out.

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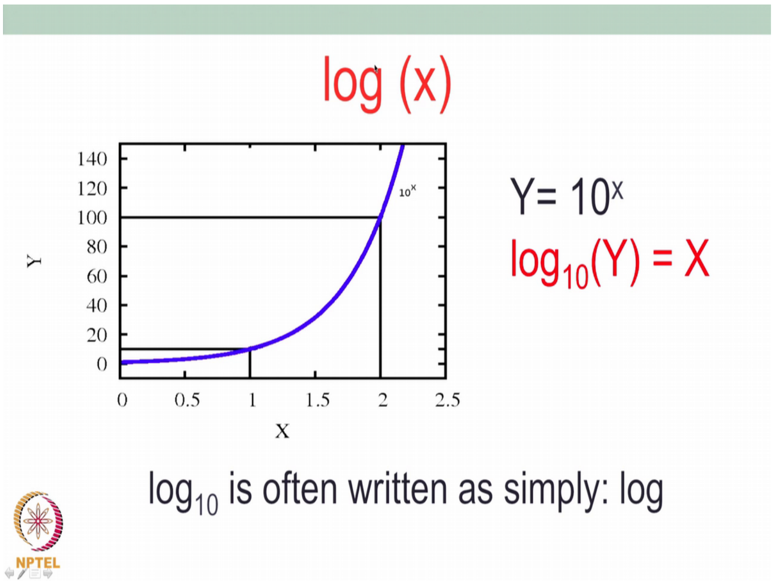

And that function is log; so when you say log X; what does it mean? If Y is 100 and you take a 10 power X line and look at the corresponding X value this will be 2. So, I just start from 100 and draw a line to the 10 power X curve and come down this will be 2. So, this is nothing, but log to the 10, 100. We know that log to the 10, 100; so what I am saying?

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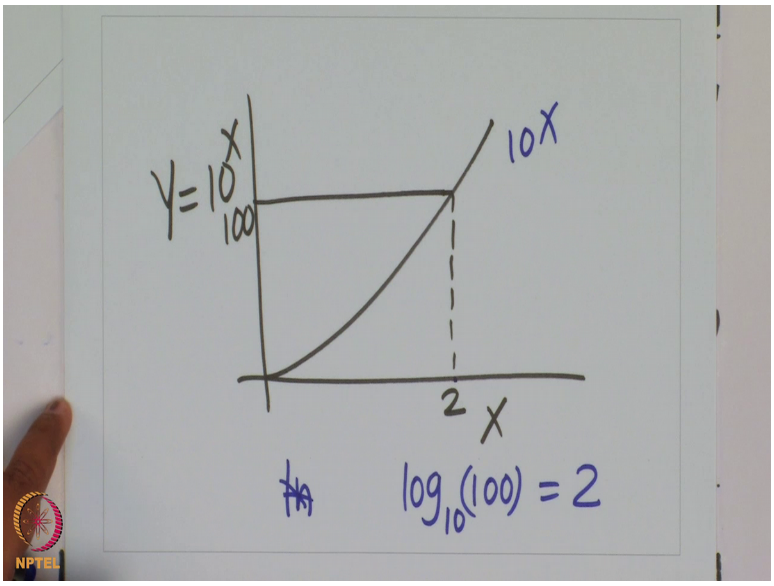

I am saying that; if we have a curve which is Y is 10 power X and this is X and this is 100; this will be 2, what does it imply? What does it mean? It means is that 2.

So, log to the base 10, 100 is 2; so, I start from 100 and I go to this curve; I take this curve 10 power X and I draw a line and come back; whatever the value in the X axis will be log of 100. So, similarly log of 10 is 1; similarly if I instead of this 10 power X; I take the curve e power X and I take a Y value and find the corresponding X value, then that will be log to the base e. So I want all of you to familiarize; as the law of function and if you plot the law of function.

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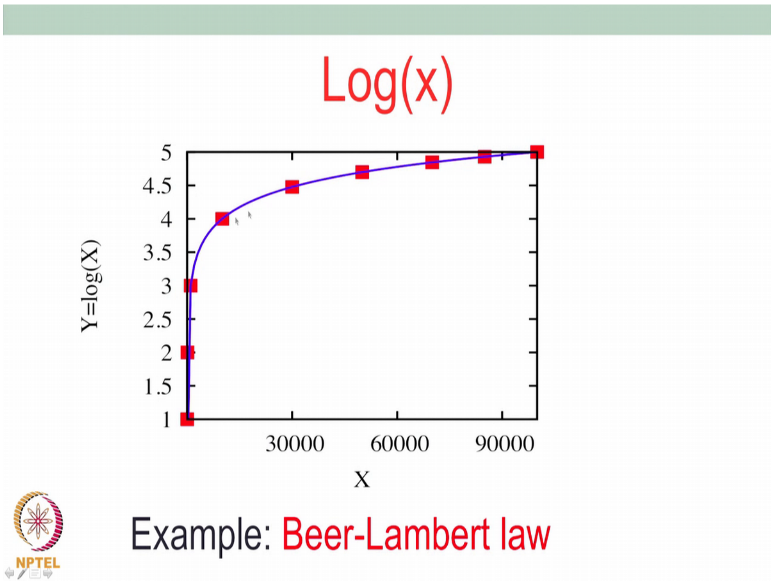

This will look like this; so this is the law of function; this is again with your various examples Beer-Lambert law the various examples, where you will have law of function.

So, we also learned few other functions; so I want to summarize all the functions that we learned so far.

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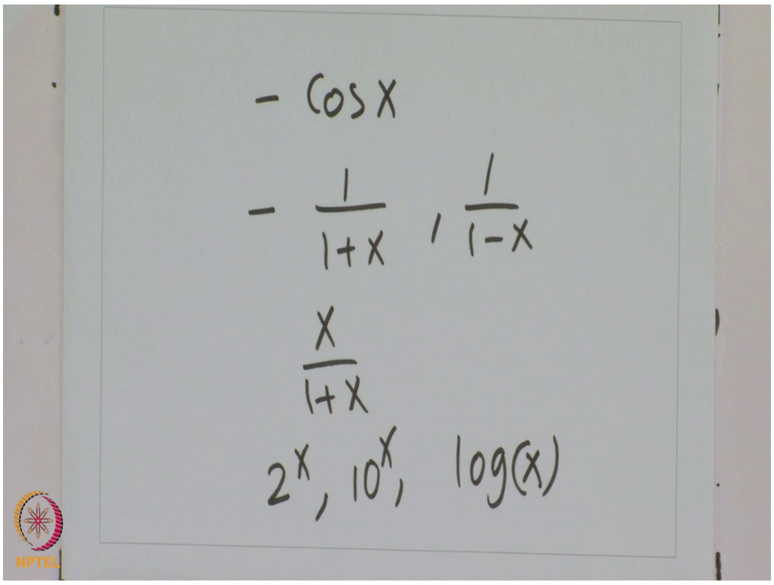

So, we learned cos X, we learnt free energy and various other things and we learned 1 by 1 plus X; 1 by 1 minus X; X by 1 plus X and all that. And finally, we learned 2 power X 10 power X; log X, lon X and so on and so forth.

So, I want you to familiarize with these functions plot various functions and think about how this function will behave for various values of X. And familiar as yourself with all kinds of functions because these functions are the words in this language of mathematics; only if you have a very good vocabulary, you can speak a language.

Similarly, if you want to use the language of mathematics; we should be knowing all kinds of functions. So, I want you to familiarize with all kinds of functions; you may not understand this, but just plot it; use the software, use a calculator and plot the function and see how it changes with X and that will tell you a lot. And we will see as we go ahead, how we can use these functions to describe various biological phenomena. With this, we will stop today's class and continue in an in the next class, bye.