**Introductory Mathematical Methods for Biologists**
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**Lecture – 25**
**Gradient, Forces and Flows : Part I**

Hi. Welcome to this lecture on Mathematical Methods for Biologists. We have been learning about vectors; how to represent quantities having certain direction. So now, we will learn more things about vectors and how to apply this for some context in biology. So, in the topic of today's lecture is: Gradient, Forces and Flows.

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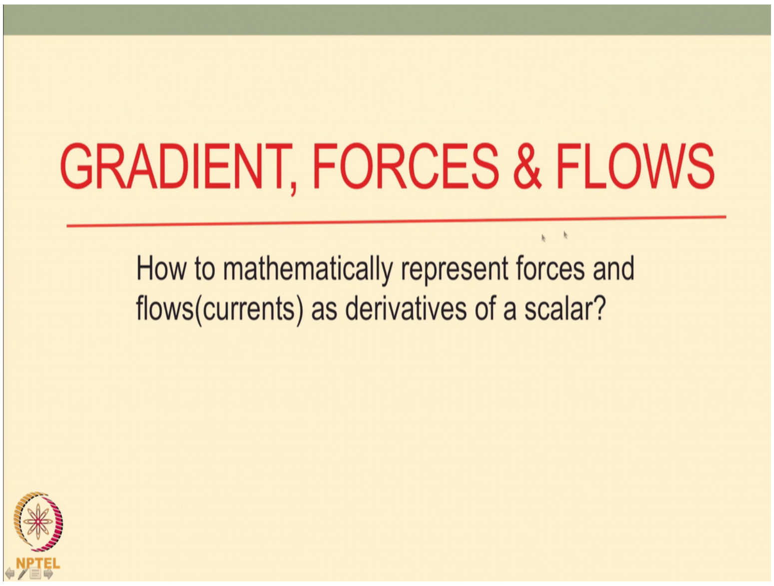


Gradient, forces, and flows: how to mathematically represent forces and flows that is currents as derivatives of a scalar. So, this is the question that we will answer in this lecture; how to mathematically represent forces and flows. Flows, by flows I mean currents which I will explain as derivatives of a scalar. Typically, these forces and flows are derivatives of a scalar and we will see how this comes.

So, the first thing we would know; of course we know the scalar is a quantity which is some magnitude and no particular direction and the examples of scalars are.

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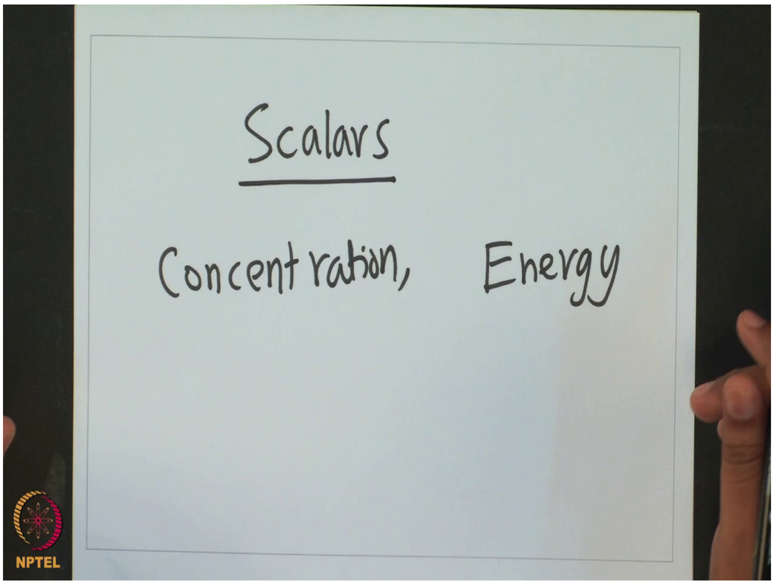


So, something that we know, scalars are like concentration, and energy; these are some examples of scalars for example two, examples of scalars are concentration and energy.

Now, if you think of concentration in a cell what would come to one's mind? So, let us think of something concentration in a cell or let us take first a very thin 1-dimensional cell.

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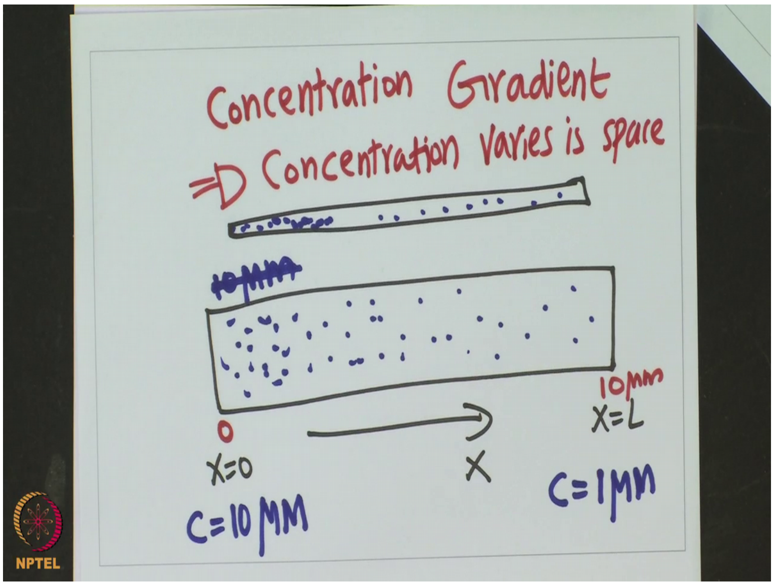


Like some a cell which would look like a very thin line like this; a thin line like this. So, essentially like a 1-dimension. Now if you think of concentration here, let us say there is very high concentration of something here and very small concentration of something here. I will draw this big for the purpose of description. So, for the purpose of description let me draw this big. So, we have essentially a 1-dimensional along the line which I would call x along x. So, this is x equal to 0 and this is x equal to L. So, along this there is a very long thin line.

Now if you think of concentration of a particular protein in this kind of a cell, let us assume that there is large concentration here and as you go along there is lesser and less code less concentration; like. This is I mean a concentration like this. So, high concentration here, so let us say this is 10 micromolar is a concentration here micromolar. So I should write here, here concentration is 10 micromolar. Here at this end let us say is like essentially 1 micromolar. And the length let us say here this is in microns. So this is 0 micron, and here let us say this is some particular amount of micron 10 micron micrometer. So, this is the length x equal to l which is 10 micrometer.

So, you have a 10 micro meter long thin tube which is like a my few imagine cell like a thin tube like this, and you have gradient of concentration which is from 10 micromolar to 1 micromolar. So, such things are called concentration gradients; concentration gradient. What does that mean? It means that concentration is changing in space; concentration varies in space concentration varies in space. So, this spatial variation of concentration that is high concentration here and as I go along the space I have low concentration here; so this spatial variation of concentration from one part of the cell to the other part is concentration gradient. And how do we mathematically represent this; this is the question that we will first discuss. So, to mathematically represent the concentration gradient, I have to first let us.

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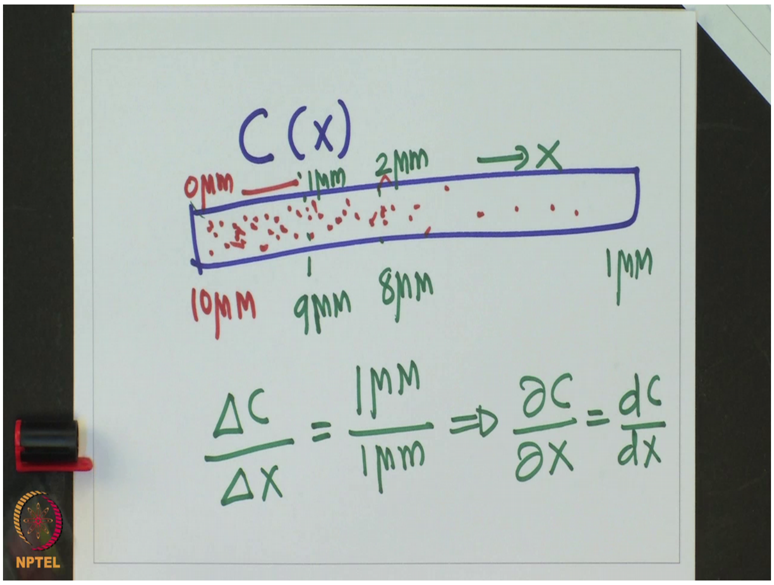


So, if c as a function of x is a concentration along this, there is a high concentration here and low concentration as we go along. So, this is the concentration gradient c of x. Now, as I go move this is 10 micromolar here and let us say I go by 1 micrometer and this is like 0 micrometer is the distance. As I go 1 micrometers, as I move here and the concentration is 9 micromolar. And I say go here, let say the concentration is 8 micromolar and so on and so forth. And it will become like 1 micromolar here.

So, the concentration is changing 1 micromolar every micrometer. So, this is 1 micron the distance here this is 0 micrometer position and here it is like 1 micrometer is the distance, and here it is 2 micrometer is the distance. And as I go along the concentration is changing, so delta c the concentration is changing as I go along the distance x. So, as I go along x the concentration is changing and this is let us say about 1 micromolar per every micro meter.

So, this is the change in concentration. And this change in concentration when I take very small delta x, I can write this as del c by del x. Or, since this is del here is a partial derivative which means the concentration is varying only with respect to x and not with respect to y z or t or anything. So, this is only with respect to x is what it meant. Here since we have known nothing else this could also be a can be written as dc by dx. So, I would use dc by dx here there is no I neglect time and other things and only assume that this changing along x and only variable is x. Therefore, this is dc by dx.

So, the concentration here is the change; dc by dx is the change in concentration and this is we know to mathematically represent the change in concentration as the derivative dc by dx, as I go along x the concentration changes. So, that is what this dc by dx means.

Now, it turns out that this change is along the x direction. If you take now this 2-dimensional thing like if you take x and y this change would have can happen and y or a in the along the x and along the y even in the one direction it could increase along the positive x or increase along the negative x. So, let us contrast two things. So, first thing is.

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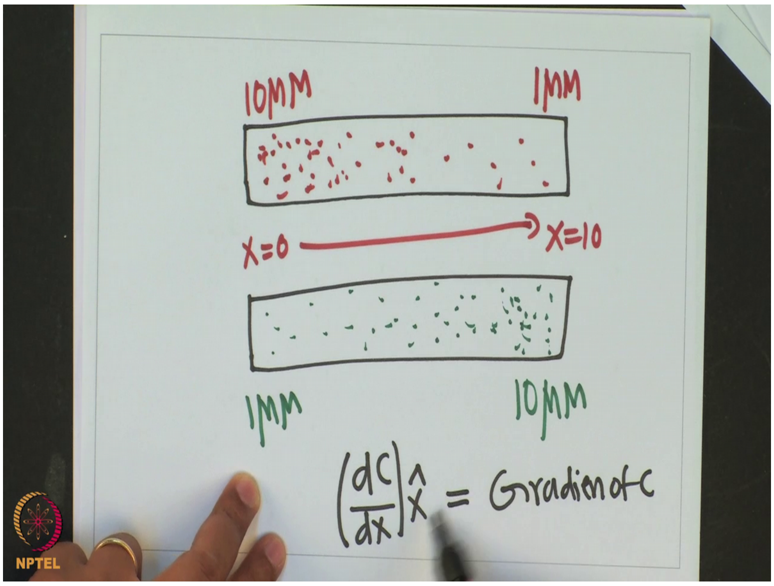


This, let us draw two boxes, and if you have high concentration here. So, here it is 10 micromolar here, 10 micromolar here and 1 micromolar here. And let us think of the opposite here, which is very high concentration here and as I go along little here. So, here this is 10 micromolar and 1 micromolar. Now, if these two scenarios this is my x axis x direction, so this is x equal to 0 and here x equal to 10 in units of micro meter. So, this is my increasing direction of x.

So, here as x increases the concentration increases, here as x increases the concentration decreases. So, as x increases here sorry; here as x increases the concentration decreases dc by dx and here as x increases the concentration would increase. So, dc by dx will have different sign. So now when I would say this, one can to represent this and this differently. One would want to have a vector sign; here the concentration is increasing this way while here the concentration is increasing this way. So, to represent this mathematically one could write dc by dx x cap, this would means that the concentration is changing along the x direction and the dc by dx sign would tell this is positive or negative. So, this symbol this x cap would tell us that the change is happening along the x axis.

Now, you could have a situation where the change is happening along the y axis, so to represent that we would do differently. So, this if I just write dc by dx x cap. So, this is gradient of c this is a vector, this has a direction it is changing in a particular direction along the x axis. Therefore, del c by del x x cap would mean that gradient of c and it as the direction along the x cap and it could be minus x cap or plus x cap depending on a sign if I put a negative sign it could have the opposite sign. So, this would represent that it is changing along the x axis I would use dc by dx x cap, and therefore this has a particular direction. Therefore, this gradient is like a vector.

Let us understand this a little bit more carefully by taking x and y. So, let us take x and y.

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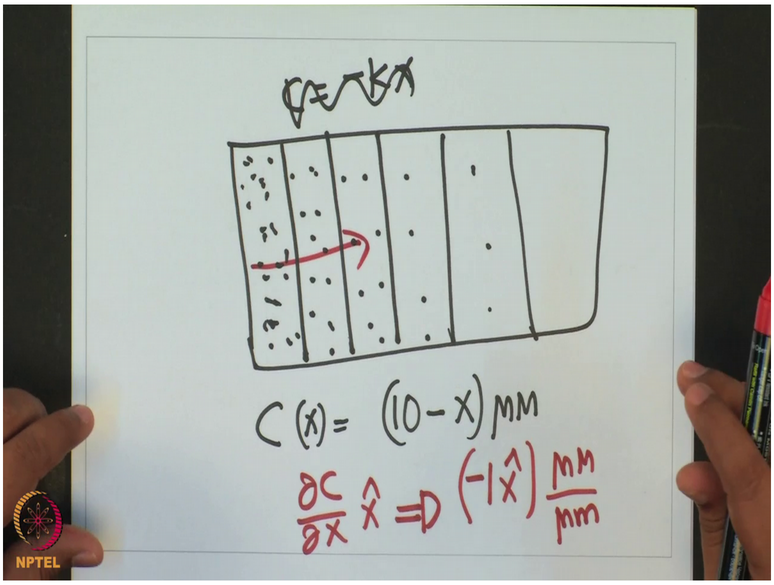


So, this is like more like an x. And y and let us say the concentration is changing only along the x axis. So, it is let us the same concentration everywhere here. And next trip the concentration is reduced a little bit, and the next trip the concentration is reduced further, and the next trip it is reduced further, here the concentration is further reduced, here the concentration is 0.

This would tell us that the concentration is kind of independent of y everywhere; you take or for all y value the concentration of same. So, let us say the concentration here is minus kx vary as the x increases the concentration decreases. So, let us say the concentration. Let me write an equation for concentration c of x is equal to 10 minus x in micromolar. This is the concentration; that means x is equal to 0 it is 10 micromolar, when x is equal to 1 it is 9 micromolar, when x is equal to 2 it is 8 micromolar, and so on and so forth.

So, this is the formula for the concentration along this I would write. Now, the fact that the change in concentration happens only along x, so would mean that the dc by; del c by del x x cap would means that the concentration varies only along x axis and not along y axis. Or, this represents the way change in concentration along x axis; del c by del x x cap would represent the change in concentration along y axis. So, this in this case would be derivative of 10 is 0 derivative of x is minus 1, so minus 1 x cap- the answer is going to be minus 1 x cap. And the unit del c by del x would be micromolar per let us say micro meter; micromolar per micrometer this is typically a unit like this would emerge.

So, this would means that the concentration is changing along the x axis minus 1; the change in concentration is minus 1 micromolar per micrometer as we go along the x axis like this. Now let us think of concentration changing both along the x axis and y axis. So, if you think of a real cell.

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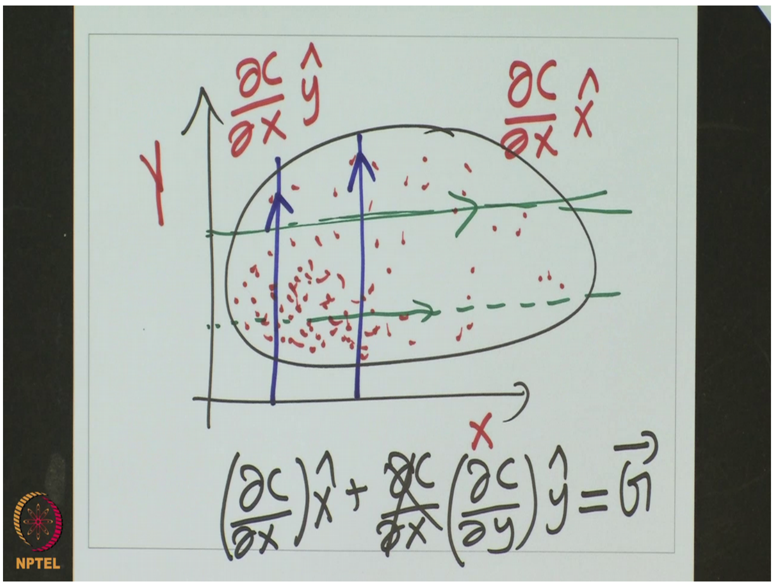

If I just draw an x axis and a y axis and draw a real cell like this; some real cell like this and the concentration here could change both along x and y, there would be very high concentration below here and low concentration as we go up and as we go along the right. So, as I go along the x axis the concentration is decreasing, as I go along the y axis concentration is also decreasing. So, the change in concentration along the x axis can be written as del c by del x along x cap. And the change in concentration along y axis can be written as del c by del x along y cap.

So, these two things combined together I could write: del c by del x which is the concentration change along x cap plus concentration change along y cap which is del c by del y along y cap. This together if I do calculate this will be a number, if I calculate this will be another number together this there will be some vector which is called the gradient vector; let us we call let me call it G at the moment it is the gradient vector. So, the gradient vector G can be written as some derivative this would give us a number times x cap plus some other number times y cap. And this is the x change along the x axis and change along y axis for any given x. So, I take a y value and though that is my y value here and I study the change along x and this would keep the y constant and what is the change along x; that is what this will give you. Take another y value, for a given y value what is a change along x is given by this del c by del x. This is partial derivative. Therefore, y is a constant for a given y value the change along x how the concentration is changing along this line; how the concentration is changing along this line. That is what this gives. This gives for a given x value how does the concentration change along the y axis.

So here, this would give us for a given a x value how is the concentration changing as I go along this line. So keep this x value, I go along this line how is the concentration changing is del c by del y and the direction is y cap. Similarly take any other y x line. So, this is x equal to some constant, if as I go along the line how does the concentration changes that is del c by del y; there is a partial derivative that means x is a constant only change along y is measured this is what is the meaning of the virtual derivative and this would give us all the overall change in this 2-dimension.

So, that vector is the gradient vector. Now, there is a way to represent the gradient mathematically there is a symbol. So, the symbol typically is written as this way the gradient of concentration.

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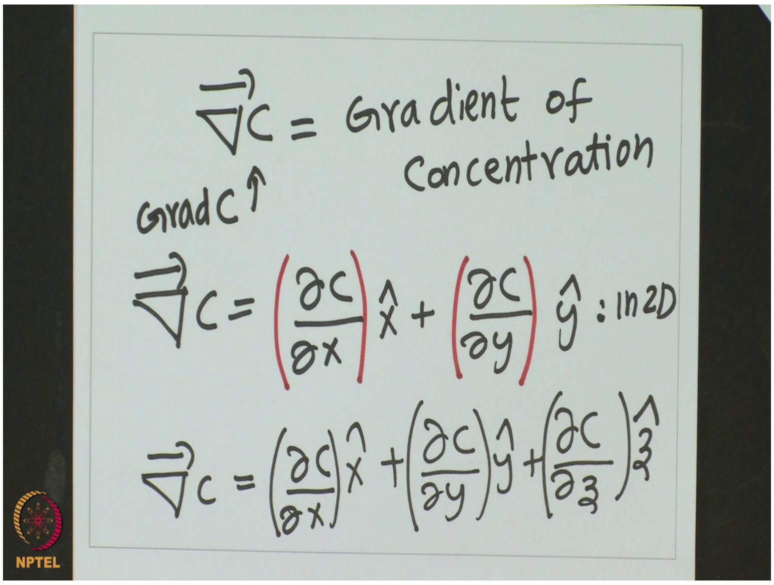


The gradient G is typically written by this symbol del c, which would mean gradient of concentration. What is the definition of del c? In 2-dimension del c is del c by del x along x cap plus del c by del y along y cap. So, this is the definition of this. As we know del c by del x will give us a number and this will also give us a number and along x this is the change along y this is the change and this is the gradient in 2-dimension.

Similarly, we could have the gradient of course in 3-dimension if in reality everything will be 3-dimensions: del c in 3-dimension is del c by del x along x along x cap plus del c by del y along y cap plus del c by del z along z cap. So, this symbol means gradient; so grad also written as grad gradient. So, gradient of concentration grad c this can also be written as grad c, this is also has a meaning grad c. So, gradient of concentration grad c is equal to del c by del x x cap plus del c by del y y cap in 2D; 2-dimension.

In 3-dimension, del c is del c by lx around x cap del c by del y along y cap del c by del z along the z cap which is the z axis. So, along x axis this is the component, along y axis this is the component, along z axis this is the component. So this is three components, I could wrote this is a number, this is another number, this is another number; these three components together will form this vector del c.

Now of course, as we said this as the dimension of concentration by in a space right; in in per length; so like will change in concentration per unit length. So, that is the dimension of this quantity. Now it turns out that in biology proteins diffuse one part to the other. And the fundamental idea of diffusion is that one would naively understand diffusion as flow of proteins from higher concentration to lower concentration. So, there is a current or a flow of concentration or protein molecules then they will flow from higher concentration region to lower concentration gradient.

Now it turns out that this how much is this flow; if you want to quantify this flow one needs to understand this idea of gradient. So, the question we have is that; the question that isn't biologically one need to pause and ask is that.

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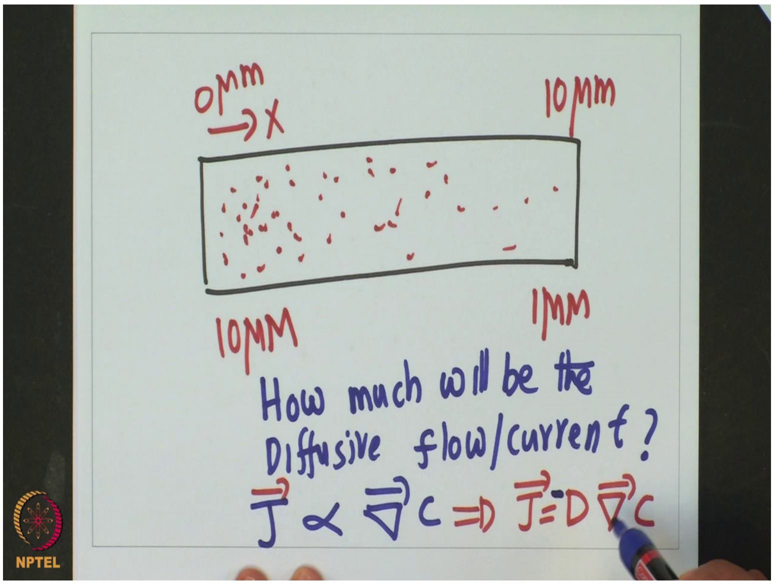


Imagine that you have a concentration gradient like this where you have as we were discussing high concentration here and lower constant low concentration here. So, here it is very high concentration, let us say 10 micro molar to 1 micro molar and this is the distance is x which is 0 micrometer to 10 micrometer.

So, as I go along the x the concentration changes. And it turns out that the flow how much the question we have to ask is how much will be the flow or diffusive flow; how much will be the diffusive flow in this case, diffusive flow or a current. This is the question that we want to answer. How much will be the diffusive flow given that there is 10 micro molar here and 1 micro molar here along this gradient of 10 micrometer. It turns out that this answer to this question is that the flow is J is proportional to the gradient of concentration. J is proportional to gradient of concentration and it turns out that it can be written that J is equal to some constant called the diffusion constant times the gradient of c.

So the J is a vector, the vector is proportional to del c which is the which something we learned. And this is exactly equal to some constant D which depends on the property of the medium and the particles size of the protein and so on and so forth, which we will learn what is this D. But, once we know the D which is the property of the system with the property of the medium molecule and all that, the flow is given by J is equal to D del c.

So, we will understand this a little bit more in detail. And it turns out that this is also true for forces. And we will come and learn this little bit more carefully that, how the forces also is like a gradient and how one can understand this force is a gradient.

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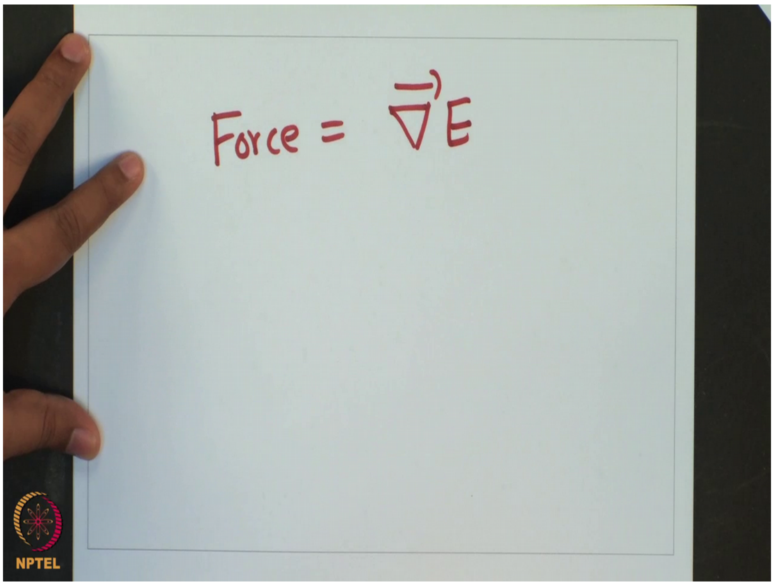


So, let us just to state this that the force can also be written as gradient of energy. So, there is one thing which; break, I do know there is the mistake I will not repeat. So, J is proportional to minus del c and J is proportional to del c and this it turns out that this is minus D del c; J is equal to minus D del c. So, this actual flow or current is D del c with a negative sign.

So, we will learn about this in detail soon, we will understand this how why what is the meaning of this. But, the answer to this question that how much will be the diffusive flow or current it turns out that it is J minus D del c. In other words, let me rewrite this a little bit more carefully.

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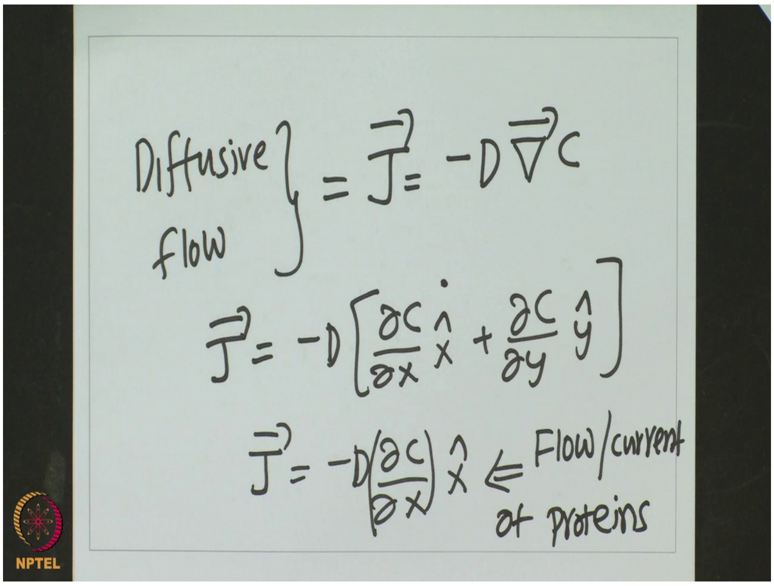


Diffusive flow is J which is a vector, which is minus D del c how much will it flow given a concentration gradient this is the amount it will flow where D depends on the property of the medium and the material proteins that we study. And we will calculate D D was famously calculated by Einstein in 1905 and we will discuss about this in the coming lectures. This, if I expand I would get minus D times del c by del x along x axis plus del c by del y along y cap and so on and so forth.

Let us write in 2D. In 1D it is just minus D del c by del x x cap. So, that is the change in the slope; the change in concentration along the x axis minus D times x cap. This is the flow; this will be the exact amount of flow or current of proteins. We will come back and understand this little bit more in detail soon.

With this, we will stop this lecture by summarizing that we learned that the gradient is change in concentration and which is essentially is we can gradient can be written as del c, grad c, and this gradient is useful in computing for example diffusive flow and many other things including forces which we will learn in the coming lecture.

With this we will stop this lecture. Bye.