Introductory Mathematical Methods for Biologists
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Lecture – 24
Cell Symmetry : Use of Polar Coordinates

Hi, welcome to this lecture on Mathematical Measures for Biologists. We have been discussing vectors and how one can use vectors to represent positions and displacements and so on.

Today you will learn something very specific to motion in the cell, what do we mean by motion in the cell. So, the topic today is cell symmetry cells have a specific symmetry, because of that we can use something called polar coordinates.

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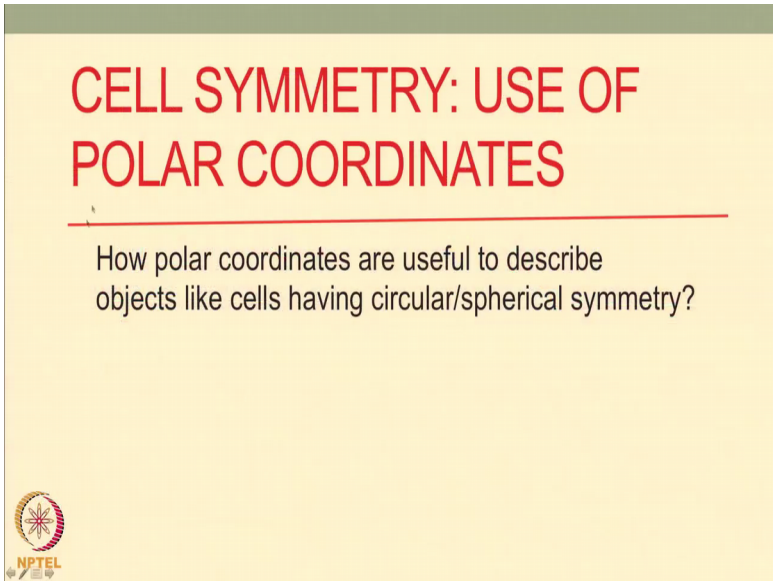

So, cell symmetry use of polar coordinate and I will explain what does this mean and how polar coordinates are useful to describe objects like cells this biological cells, having circular or spherical symmetry. So, this is the question that you will try and answer in this lecture.

So, now let us think about cells which are objects of our interest in biology very often. So, cells have a very specific symmetry right. So, cells typically have some kind of like some circular symmetry.

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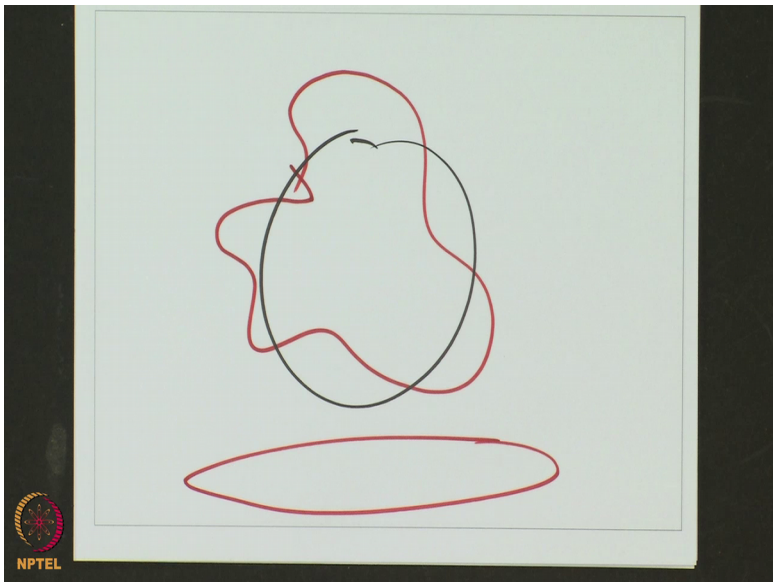

Need not be always, but you could think of many cells having some kind of circular symmetry. Even if the cells are not nearly circular or spherical, so in 3D it will have a spherical symmetry even if it is like this, even if it is like this it is in overall spherical symmetry is not like this I mean there could be some cells which have some different symmetry which is like a cylinder or, there are various objects typically sphere and cylinders they have some very specific nice symmetry and we can use that and use slightly different things from the plane xyz coordinates that we learned, we can use the different set of coordinates to understand these spherical objects.

So, let us think about in 2D circular objects first and think how to use the ideas that we learned about vectors in the context of a circle and that would give us some idea about how one can describe motion inside a cell. We want to understand how for example. We started early with a question that how we can tell something is moving to the exterior of the cell something is moving towards the interior. So, we wanted to tell the direction which is the exterior of the cell interior of the cell towards the periphery, how do we say this towards the periphery using vector notation. That is the how do we communicate this mathematically, that new language is what we are learning now and for that we would consider a circle and learn how we would describe something which is pointed towards the periphery of the circle, towards the circumference of the circle. So, let us consider a circle.

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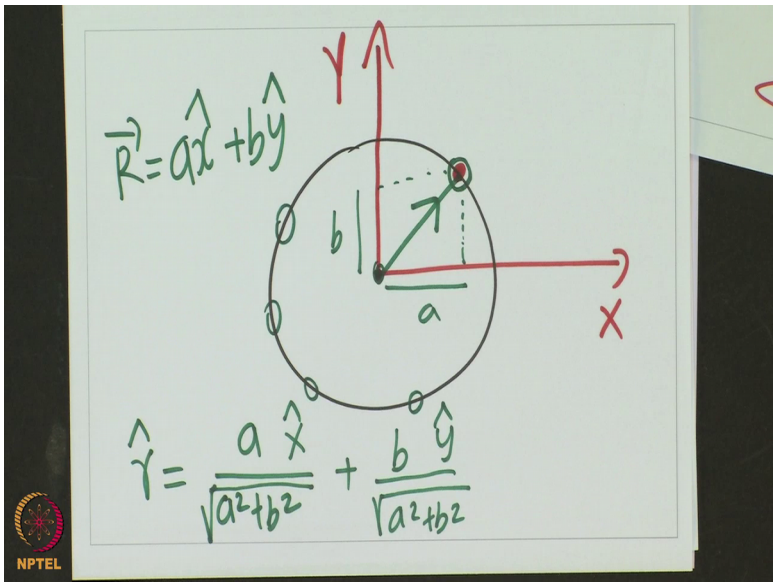

So, we have a circle and in the middle of the circle I will put an xyz axis. So, let us have a x axis and y axis here. So, this is my x and a y and this point in the cell. So, let us say we have a some point of interest in the cell and I can draw a vector from here to here. And we already learned that this vector if I call this R vector and then I could call this as my distance a which would be 3 unit, 4 units whatever and this distance as b which is again some number a and b are some two numbers. I could say that this R vector that is this position to reach this position from this origin I could go a along the x axis. So, if I go a along the x axis and b along the y axis b in the y direction. So, if I go a along the x direction b along the y direction, I will reach this point and that is what is represented by R is equal to a x cap plus b y cap right.

So, now this we also said that we can draw write a unit vector which is r unit vector which I could write as a by root of a square plus b square plus this is along the x direction and b by root of a square plus b square along the y direction this is a unit vector which would represent this direction. So, this is something that we learned.

Now, if you want to represent this, something on the circle like any point here since this is a circle any point on the circle will be the distance a square plus b square away from this because a square plus b square is the radius of the circle. So, we know that if we have a circle its radius is going to be the magnitude of this R vector.

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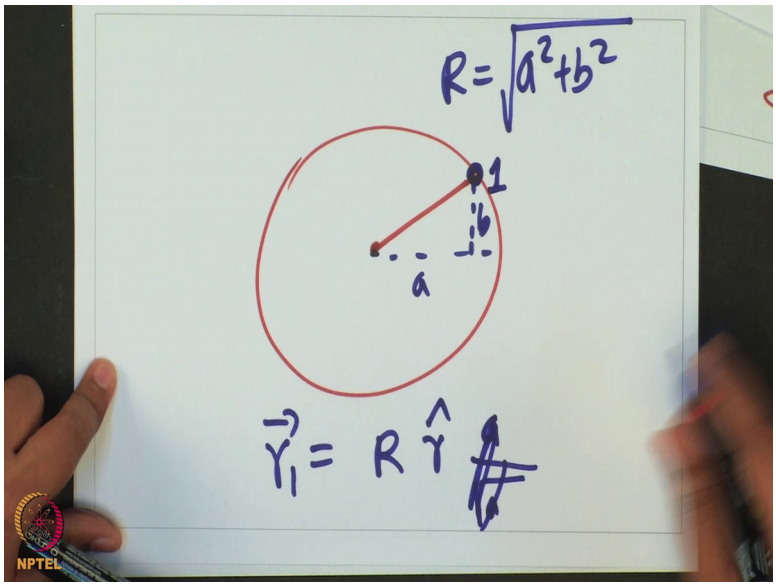

So, what we have is a circle and the radius of the circle R and if this is my a and this is my b if I have an xyz coordinates in the xy coordinate system whatever be that coordinate system wherever be it if this is a and this is b a square plus b square root will be my radius and therefore, the vector I represent this quantity will be equal to the radius of it and this point here will have will be R distance away from the origin. So, this particular point let me call this 1 the r 1 would be R distance away from the origin and what is the direction of this the direction is r cap direction which is x plus y. What is r cap? r cap is a by a square. So, we wrote R cap in the, this is R, what is R cap. So, R cap is a by a square plus b square x cap plus b by a square plus b square y cap where a and b are this distances from here to here, so this is my this.

So, if I have this r cap I could write r, r cap as the as this vector and do this yourself you will see that if I substitute r which is the magnitude r is root of a square plus b square and if I substitute this I would get essentially this vector we can see that let us see this ourselves. So, what are we saying? We are saying that any point on this vector if I want to represent a point on this vector on this circle.

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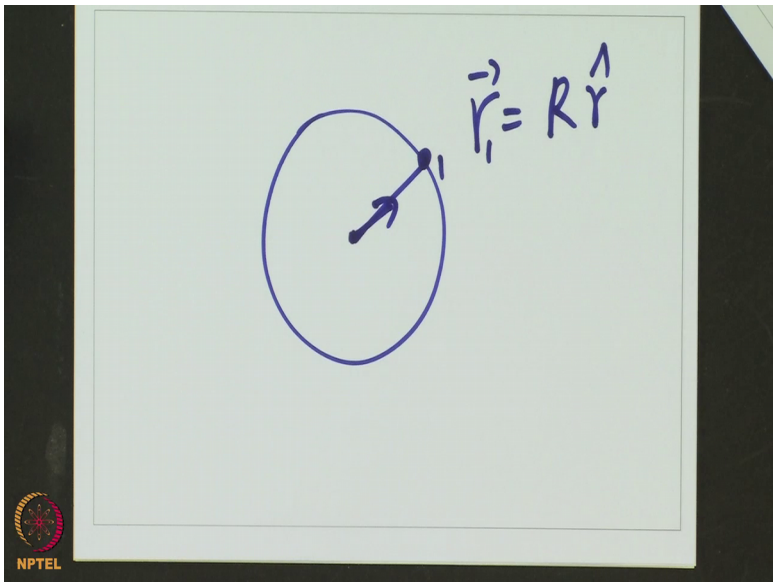

So, I have a point on the circle which is which is R this is away from the center I would represent r 1 as R r cap, this is this would give you me this direction and this distance away from this direction this is tell me something about this, but does this can be everything. So, let us think about slightly differently.

So, if I take a circle or if I take any point on this 2D plane, so I have this 2D plane and I have a circle in this plane.

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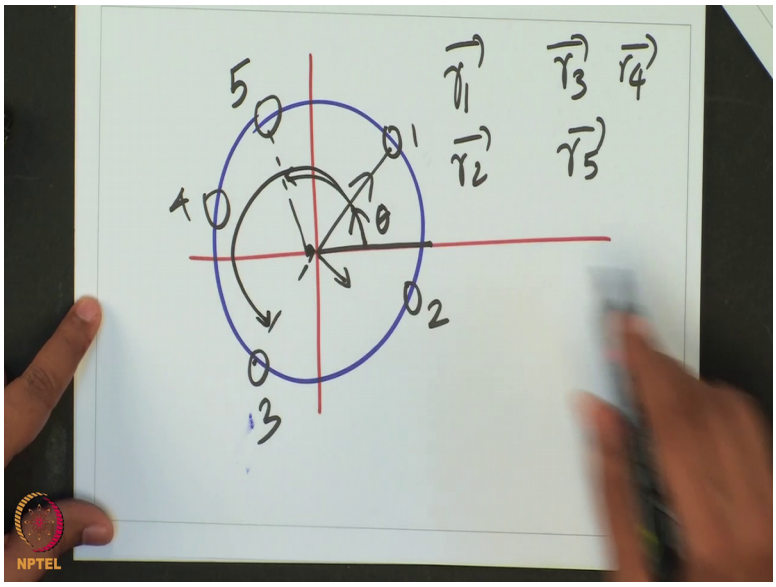

So, if I want to represent any point let this is a point, this is the point, this is a point, this is a point I have to specify. So, this is if I call this 1 2 3 4 5, I can write vectors r 1, r 2, r 3, r 4 and r 5, 5 vectors if I write I would get this point. But any point to get what two things I know is that which direction to go should I go this way or should I go this way and how much to go if I know this one would get that is one way. Another way of knowing is that from here how much angle should I go right if I just specify the angle if I this angle theta if I specify, if I want to go here from x axis if I go this much angle I would reach here, if I go if I want to reach here from the x axis if I go clockwise this much angle I would reach here.

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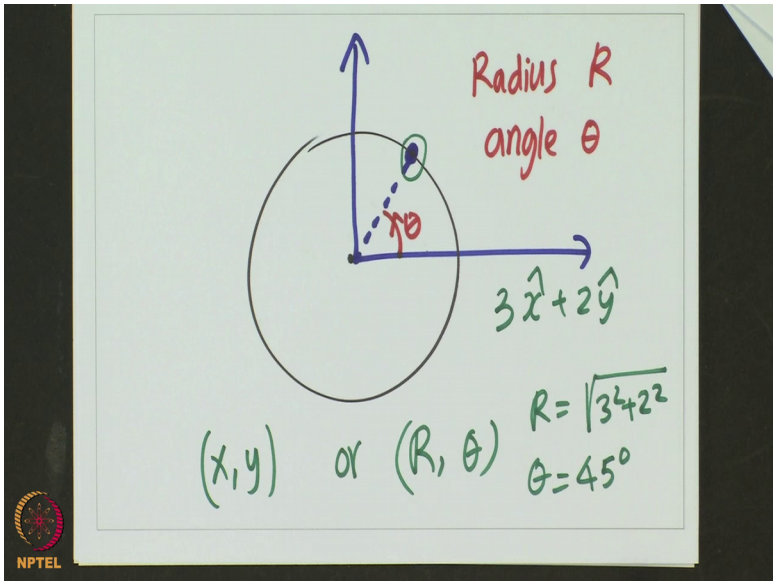

So, by specifying angle you could get every points on a circle. So, what we are saying is that if we have a circle if you want to get any point on the circle this point in the circle if I specify this angle from the x axis and the radius. So, if I know the radius which is this distance and this angle theta in the counterclockwise direction. So, in this direction theta in this direction and this radius which is along the radial direction if I know these two quantities R and theta I can get this point right.

So, in general any point in this 2D plane can also be obtained if I know an angle from the x axis and also distance along a radial axis. So, this is up. So, either to represent a point either one can use – x y to specify x x and y coordinates of this which was a and b in the notation or I could specify R and theta. These two are equivalent method. So, I could just

say this is 3 x plus 2 y or I could specify the radius and theta if I specify that that also will specify this particular point if somebody tells me if radius is 3, something right which is some number root of three square plus 2 square is going to be my radius and theta is 45 degree then I would go three square plus root of 3 square plus 2 square which is root 13 along the radius and then go an angle along the x axis and go an angle 45 degree I would reach this point.

So, this method of specifying a point by the radius and the angle is basically known as plane polar coordinates. So, what does it the statement is the precise mainly statement is following.

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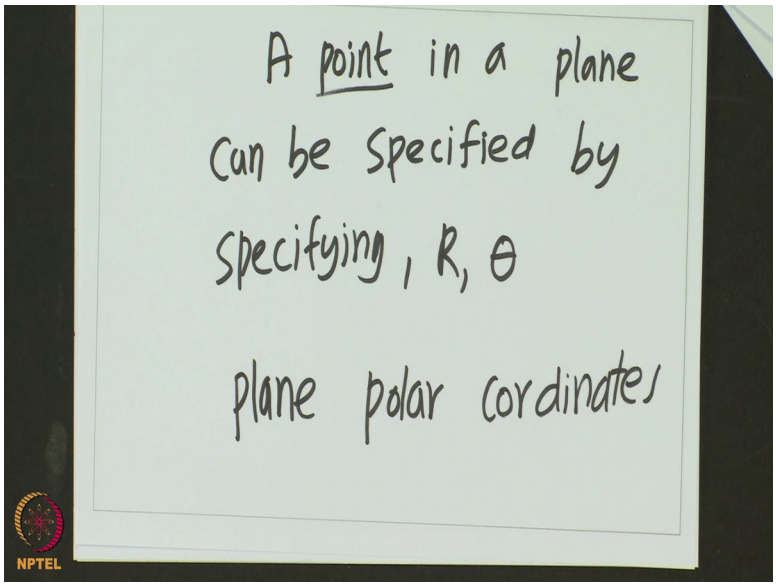

A point in the in a plane a point in a plane can be specified by specifying some radial distance R and an angle theta with respect to x axis in the counterclockwise direction anticlockwise direction. If I specify R and theta I can specify the location of a point this is the position of a point a location of a point, a point, is a point location, in a plane can be specified. So, this is called plane polar coordinates.

The only thing is that I want to specify two vectors. So, let us know understand this plane polar coordinates little bit more. To do that let us go back to our example of point on a circle if you have a point on a circle this is the point and this is the vector this vector R vector can be written as this x and this y.

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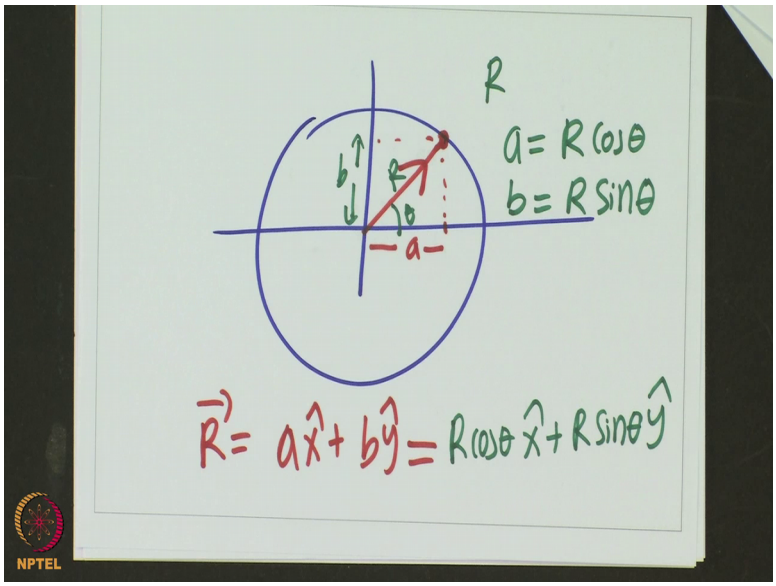

So, this distance is let us call it a. So, a along the x axis plus b along the y axis the same thing can be represented if I know this angle theta. So, I know this radius R which is this radius and this angle theta then this a is nothing but this R cos theta a will turn out to be R cos theta and the b which is this distance which is my b, b will turn out to be R sin theta right b will turn out to be R sin theta which is same as this distance.

So, you can see that a is equal to R cos theta and b is equal to R sin theta. So, I could write this as well as R cos theta x cap plus R sin theta y cap. So, this is the same thing now representing in terms of R and theta. I could specify R cos theta x cap plus R sin theta y cap and that would specify as something which is this particular point based on theta and R. I could specify this and I would get these two points. So, this is basically the plane polar coordinates where instead of specifying x and y, I could just specify R and theta. And the good part is this is very useful for a circle because R is same for every point. So, I have to only specify theta. So, if I know R and if I just vary theta I will get every point along the circle, but x and y would be different for different points in the circle. So, let us understand this why is it very useful for a circle.

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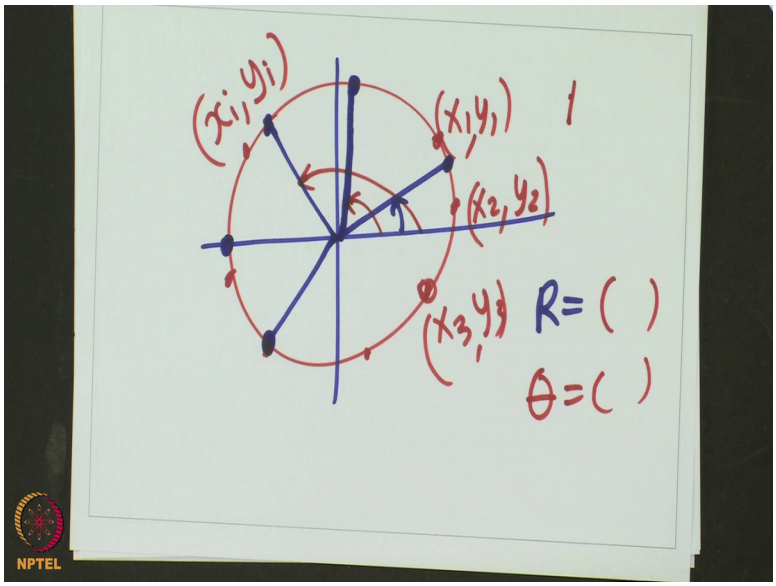

If I take a circle every point has different x and y, the x y values this will have x 1 y 1, this will have x 2 y 2, this point is x 3 y 3. So, this point in general could be like some x n, y n, xi yi. So, every point along the circle will have a different x value and y value, but the R the radius is same for everywhere the R is same. So, R is same, but the only ways specifying theta if theta is 10 degree this would be the point, if theta is close to 90 degree let us say 80 degree or 85 degree this would be the point if the theta angle is more than 90 degrees let us say 110 degree or something then this could be the point. So, by if the theta is 180 degree this would be the point if the angle is more than 180 somewhere here could be the point and depending on various values of this angle. So, this angle is where different for this is different for this. So, depending on this angle from the x axis I could specify different points on the circle.

So, specifying R and theta is very useful for a circle because R is constant only theta is varying, and just by playing around the theta one would get the positions if I want to say that something is moving along the circle I had to only talk about changing theta right. If I just say that something is moving along the periphery of the circle along the circumference I have to only say how does the theta change right.

So, if you want to motion along the circle, if I want to discuss motion along the circle, so I want to say something is moving from here to here to here and so on and so forth.

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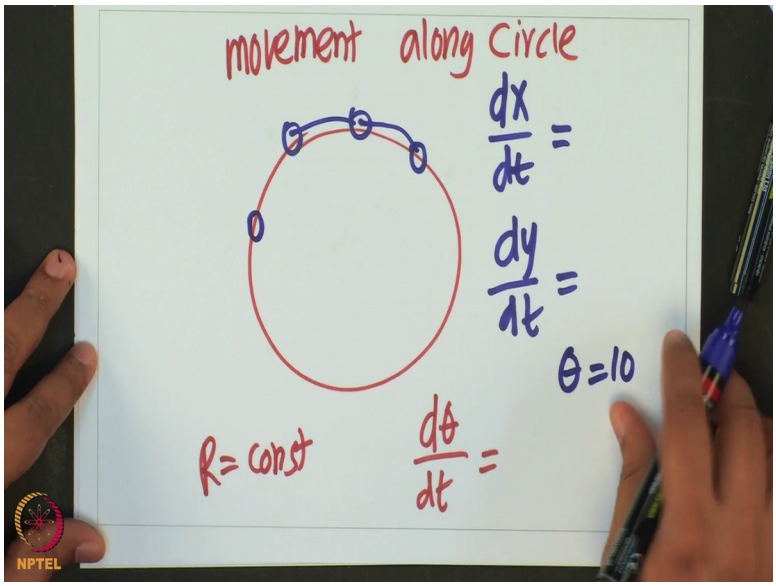

Either I could say dx by dt and dy by dt I have to say two things, I have to specify two things then only I would say how this is moving. But instead of this I could just say d theta by dt this alone, it is just one variable would describe the motion along the circle why if I use xy coordinates I need two cordinates, two numbers to specify two equations to specify the movement along the circle.

So, movement along the circle it is much easier to use polar coordinates like theta where R is a constant, since R is a constant one could just by specifying theta is varying from 0, 90 and so on and so forth I can tell that the particle is moving from here to here right. So, just by specifying the angle as a function of time I can specify the position of this particle along the circle if I want to say that particle if theta is 10, 20. So, if I just would say that let us say we have a table which is time here theta here, at t equals 0 the theta 0 which is which means that the particle is here after 1 minute the particle move 10 degrees. So, the particle has reached here.

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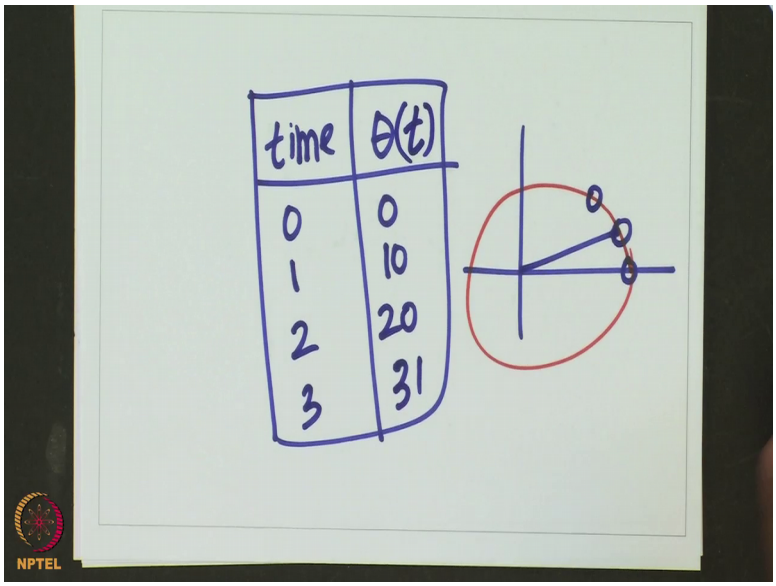

So, if I have a nice circle which is not nicely describe shown here, but if I have a circle if I want to move along this I puts after 2 minutes it became 20 degree, 3 minutes 31 degree and so on and so forth and this would tell you the movement along this I just need to only specify this theta variable which is a function which is the angle. This is the use of polar coordinates.

So, since if the object has a circular symmetry it is always enough to use plane polar coordinates. Same thing is true for a sphere, if I have a sphere you have to use three coordinates one R which is the radius of the sphere and then you have to have two angles to specify. So, in the case of a sphere it is going to be something called spherical polar coordinates. I urge you to read a little bit about in understand it spherical polar coordinates.

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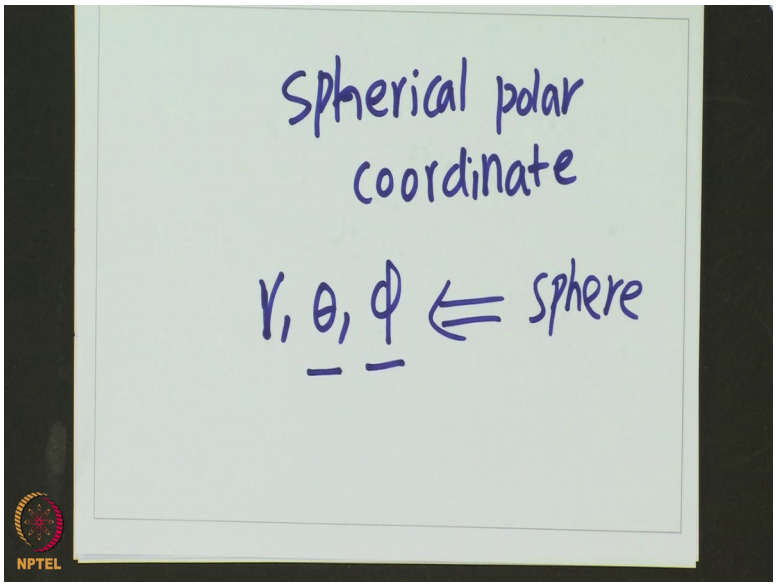

So, here there we can specify r one, angle, theta which is the radius theta and phi another angle phi. So, two extra angle would be needed for a sphere. If the radius if something is moving along the sphere r is constant only theta and phi are changing. So, just by specifying theta and phi as a function of time I can specify the movement of a particle anything that is moving in on the on the sphere.

Now, let us come back to our question that how do I know specify, some say the thing is moving towards the interior or to the exterior of this cell.

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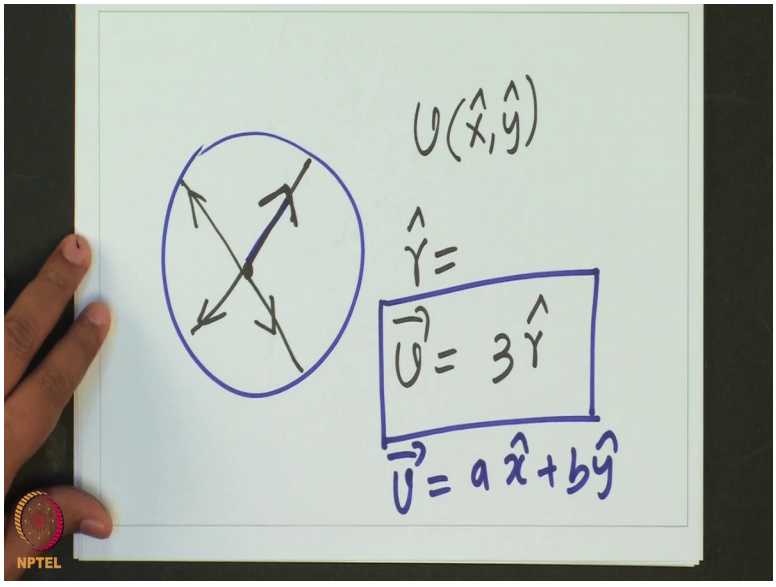

So, I have this, I want you to just to say that the cargo are moving in this particular direction right in this, is the I want to say that these are moving along this direction. So, the velocity that we have for the speed can be written as in the x y direction, velocity can be written as in the x y direction, it could also be written as in the R theta direction so there is you can directly unit vector along the radius. So, if I just say that R is equal to R cap which is a unit vector along this radius. So, I could just say that this is going. So, if R if I just say that v is equal to 3 r cap this would mean that it is going in the direction along the radial direction with some speed of 3 unit per second whatever the unit of 3 is, if I specify v is equal to 3 r cap this would mean that this is moving in this particular direction of r cap and 3 is the magnitude.

So, this is something that we can specify and using this coordinate system which is also equivalent of specifying v is equal to some a x cap plus b y cap. These are equivalent specifications instead of ax cap and b y cap this simplify things a lot and therefore, one can specify just by 3 r cap and one can just specify this as just x cap plus y cap also. Both are descriptions of specifying movement along this direction.

Now, if something is moving this the angle is not changing only the position is changing right. So, I could also, if I want to think of a molecular motor and a cargo, so you have microtubule on which a cargo is moving the angle is not going to change let us say, only the radial direction is going to change the angle is same. So, this is my x and this is my sorry this is my y and this is my x y this angle is same, but it is going to move along the r radial direction. So, just by specifying the position of this the position of this cargo the position of this cargo rc let me write, the position of this cargo can be specified as a r cap.

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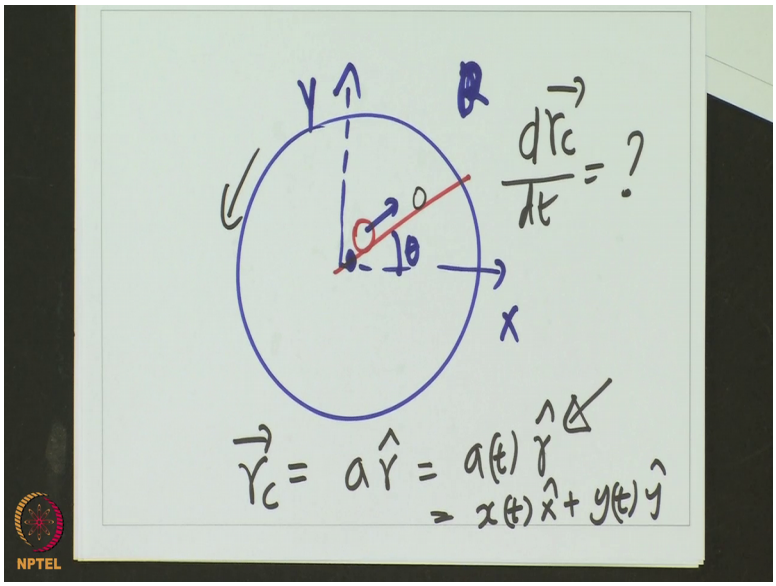

So, along this particular direction r cap the magnitude is this. So, if I just specify this it would just specify the position of this cargo. So, as the time changes a could change a could be a function of time, but it is going in this direction of r cap and which is same as specifying x y as a function of time, this is there, there is an equivalence between these two if we understand r cap as we discussed earlier one can also specify this just by specifying r cap. So, this is the thing and the change in this motion drc by dt what is this, this is something we will try and understand and this will help us many things to help us a lot in thinking about the velocity if how does this question change that would tell us what is the velocity.

So, the idea and the bottom line is that if there is spherical and circular symmetry we can use special coordinate system not the x y coordinate system that we learned so far, but a R theta coordinate system where we take an angle and radius. This would simplify things a lot. Just like we saw that if I moving along microtubule track I do not have to specify I have to specify only one coordinate system, I have to specify only how far it is from the center that is all I have to specify, just by specifying this how far it is from the center in a particular direction I can specify the position of the cargo. Similarly if something is moving along the periphery I have to only specify the angle.

So, this would simplify things a lot, just by specifying position from the center or position along the periphery angle along periphery I could specify the position of objects

this is very useful. So, this is the use of polar coordinate system and the symmetry helps us and therefore, very often in biology one would make use of this spherical and circular symmetries to describe things using this special coordinate system which is plane polar or spherical polar coordinate system.

With this I will stop this lecture. Bye.