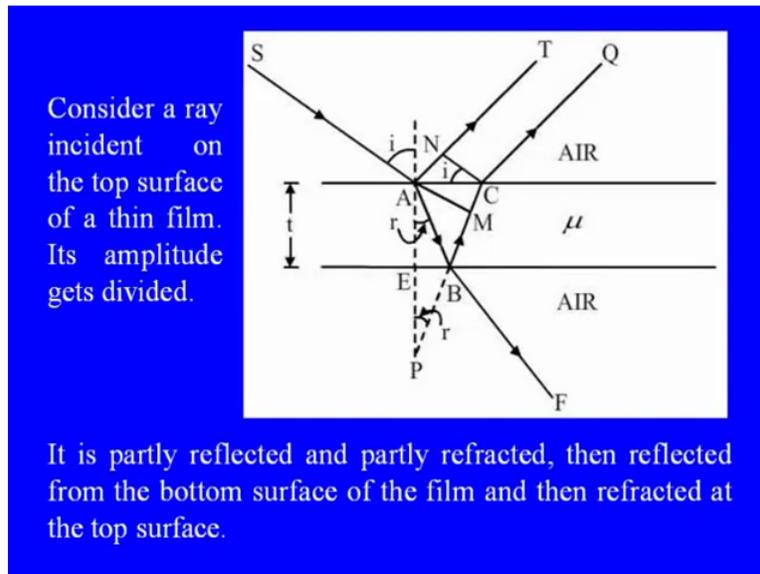


Engineering Physics 1
Dr. M. K. Srivastava
Department of Physics
Indian Institute of Technology-Roorkee

Module-03
Lecture-05
Interference by Division of Amplitude

Interference by Division of Amplitude by M K Srivastav, Department of Physics, Indian Institute of Technology, Roorkee. Uttarkhand. In this lecture, we shall take up interference by division of amplitude. Let us now go back to the setups which employ this procedure, for the creation of a pair of coherent sources. That is the basic requirement anyway.

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Consider a ray incident on the top surface of a thin film as shown in the figure. SA is that ray, its amplitude gets divided, it is partly reflected and partly refracted at the top surface and then reflected from the bottom surface of the film and then again reflected at the top surface. If AT is the top reflected beam, AB is refracted at the point B it is reflected BC and again comes out as CQ so AT and CQ are both in the reflected region. B after the beam which is transmitted goes through the film.

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Consider these two reflected beams in the reflected light. They have originated from the same beam and are therefore coherent. Their superposition gives a stationary interference pattern. The phase difference between them has been obtained earlier and is given by

$$\varphi = \frac{2\pi}{\lambda}(2\mu t \cos \theta_r) + \pi$$

Here θ_r is the angle of reflection and $2\mu t \cos \theta_r$ is the optical path difference.

Consider these two reflected beams in the reflected light. Remember, they have originated from the same beam and are therefore coherent. Their superposition gives a stationary interference pattern. The phase difference between them had been obtained earlier you know. And is given by 2π by λ times $2\mu t \cos \theta_r$, μ is the refractive index of the material of this film, t is the thickness of the film, and θ_r is the angle of reflection, this thing to $\mu t \cos \theta_r$ is called the optical path difference.

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The additional factor of π is due to the difference in the nature of reflection at the top and bottom surfaces. One is at the denser medium and the other at a rarer medium.

The conditions for maxima/minima in the reflected light are therefore

$$2\mu t \cos \theta_r = \left(m - \frac{1}{2}\right)\lambda \quad m = 1, 2, 3, \dots \quad (\text{max})$$

$$m\lambda \quad (\text{min})$$

Now, this additional factor of π which is there is due to the difference in the nature of reflection at the top and bottom surfaces. The top reflection is at a denser medium. We are assuming that μ is greater than 1. And the reflection at the bottom surface medium to air, that is at a rarer

medium. So, that is why this additional factor of Pi is there. The conditions for maxima and minima in the reflected light are therefore, this path difference is the optical path difference as we have said, is $m - \frac{1}{2} \lambda$, m can be take values 1, 2, 3.

This is for the maximum, remember, this additional half lambda is coming because of that factor of Pi which is there and if $2 \mu t \cos \theta$ is = $m \lambda$ and integer multiple of lambda, one leads to the minimum. The two beams reflected beams in this case are in opposite phase.

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In the transmitted light, the conditions are complementary.

$$2 \mu t \cos \theta_t = \left(m - \frac{1}{2} \right) \lambda \quad m = 1, 2, 3, \dots \quad (\text{max})$$

$$m \lambda \quad (\text{min})$$

It should naturally be like this as there is no loss of energy. There is only redistribution.

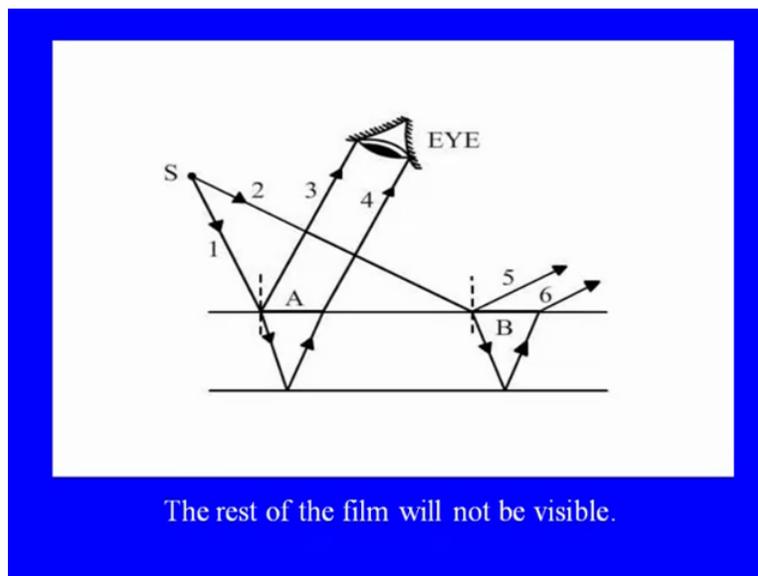
In the transmitted light, the conditions are complimentary to the conditions in the reflected light. So here, that additional factor of Pi is not there and that is why we have the conditions the $2 \mu t \cos \theta$ is = $m \lambda$ for a minimum and the top condition that is for a maximum. It should naturally be like this as there is no loss of energy. You see, there is only redistribution. Interference only causes a redistribution of energy and not a loss of energy.

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One thing should be noted. If the light is coming from a point source, the reflected light will be able to reach the eye only from a small portion of the film.

One thing should be noted if the light is coming from a point source; the reflected light will be able to reach the eye only from a small portion of the film. Let us see that. This figure shows:

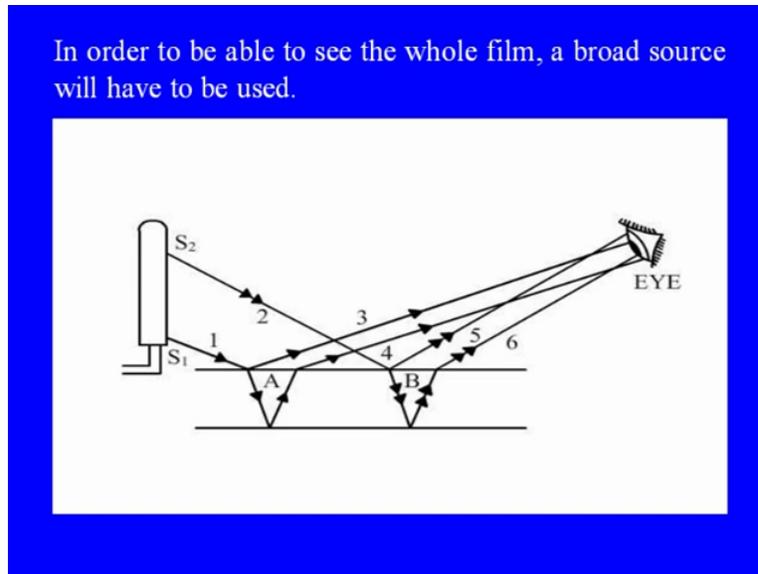
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This S is a point source, let us consider the ray 1, top reflected, giving you the ray 3, bottom reflected giving you the ray 4 and suppose they reach the eye. So, the eye will be able to see will the dispersion of the film, the portion A. Consider another ray coming from the point source ray 2, top reflected ray 5 goes through bottom reflected and gives you ray 6, 5 and 6 are also coherent because they cause interference but the light from there does not reach the eye.

So, the region B will not be visible to the eye. That is an important thing here, if S is a point source.

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Now so, in order to be able to see the whole film a broad source will have to be used. Let us see this figure. Now, S_1 , S_2 is a broad source light coming from S_1 top reflected then again the bottom reflected the rays 3 and 4. The ETI, I is able to see this portion of the film the region A, 3 and 4, they are coherent origin both have originated from ray 1. Consider the ray 2 coming from another point of the source the source is a broad source. ray 2, top reflected ray 4.

And then no, not ray 4 ray 5 and the other one which is 360, bottom reflected 5 and 6 are also coherent; both have originated from the ray 2. So, the I will be able seeing the region B of the film as well. Now, remember ray 1 and 2 are not coherent but they need not be coherent, the interference between 3 and 4, they are coherent between 5 and 6, they are coherent.

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This is thus a big difference between the division of wavefront and division of amplitude set-ups.

In the case of white light, films will show colours depending on the value of thickness t and θ_r .

So, thus we find this is the big difference between the Division of Wavefront and Division of Amplitude, set ups. Here a broad source is a must. If you have a point source okay then, the set up really does not work. You cannot see the whole, the whole pattern in the case of white light film, naturally, will show colors depending on the value of the thickness t of the film and the angle of reflection θ_r .

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If the angle θ_r does not vary much and the film is wedge shaped, straight line fringes parallel to the edge are formed. The fringe width is given by

$$\beta = \lambda/2 \tan \alpha \sim \lambda/2\alpha .$$

Here α is the angle of the wedge.

Now suppose if the θ_r does not vary much, suppose the light is almost parallel, parallel beam. When the film is wedge shaped then naturally, the Straight line fringes parallel to the edge

are formed. The fringe width in this case is given by, λ divided by $2 \tan \alpha$. α is the angle of the wedge and if this angle is small $\mu_{10} \alpha$ can be approximated by α . So, the fringe width is λ upon 2α . We are assuming that the film is an air film.

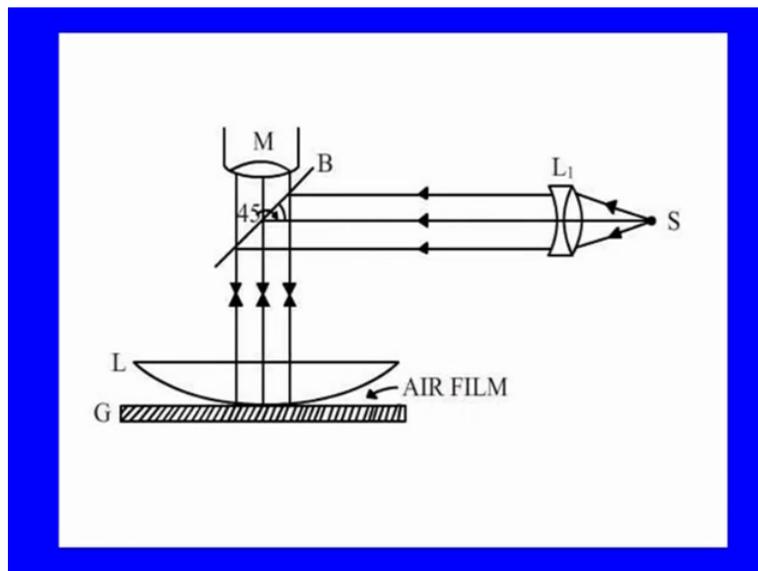
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Newton's Rings-Setup

Newton's rings arrangement provides a simple laboratory set-up for measuring the wavelength of light. A plano-convex lens is placed on a plane glass surface. A thin film is thus formed between the curved surface of the lens and the plane glass plate. The thickness of the film is zero at the point of contact and increases as one moves away.

Let us consider Newton's rings setup. This Newton's rings arrangement provides a simple laboratory setup for measuring wavelength of light. A plano convex lens is placed on a plain glass surface, a thin film is thus formed between the curved surface of the lens, curved surface of the plano convex lens and the plain glass plate. The thickness of the film is 0 at the point of contact and naturally it increases as one moves away. You see, this is the setup.

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G is the glass plate L is the Plano convex lens, air film is formed between the two surfaces, S is the source, θ is the angle, so, where the beam is coming a broad beam remember, we cannot have a point source here with a broad beam almost parallel is falling. B is a plate, glass plate which reflects the beam down. So, the light falls so vertically in normal incidence on the film. Remember the angle θ will be 0.

And then on top we have got a microscope which is there to observe the fringes. The microscope is focused on the film.

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The loci of constant thickness are circles with point of contact as centre and therefore circular fringes are formed when this arrangement is illuminated at normal incidence.

Now, the loci of constant thickness or circles they are circles because of the circular symmetry here, with point of contact at the center and naturally therefore the fringes depend on the thickness and there are circular fringes are formed here. When this arrangement eliminated at normal incidence which is the case here; which is actually, always the case.

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The radii of dark rings are given by

$$r_m = \sqrt{m\lambda R}, \quad m = 0, 1, 2, 3, \dots$$

where R is the radius of curvature of the convex surface of the plano-convex lens. The radii of the rings thus vary as the square root of natural numbers. This expression also shows that the rings will become close to each other as the radius increases.

The radii of dark fringes are given by m at dark fringe is square root of m lambda capital R m can take values 0, 1, 2, 3, R is the radius of curvature of the convex surface of the Plano convex lens, the radii of the rings does vary as the square root of natural numbers. That is a very interesting result and because it is varying as the square root, this also shows that the rings will become close to each other, as the radius increases.

And another thing you like all to be pretty large if you want wide rings if r is small rings will be too narrow.

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The radii of the bright rings are given by

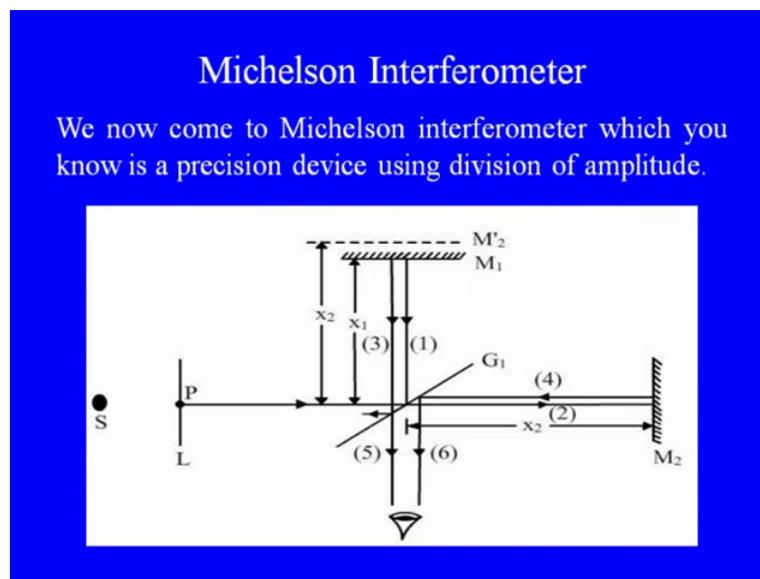
$$r_m = \sqrt{\left(m + \frac{1}{2}\right)\lambda R}.$$

To observe these rings the microscope or (or the eye) has to be focused on the upper surface of the film. These are *localized* fringes.

Now the radii of the bright rings are given by, r_m is = square root of $m + 1/2$ times λR . You see, this half is coming as I pointed out before because of that difference in the nature of reflection at the top of the film and the bottom of the film. To observe these rings, the microscope or the eye has to be focused on the upper surface of the film. You see, these films are localized fringes.

Naturally they have to be localized, they are formed where the bottom and top reflected waves they superimpose and lead to the interference pattern.

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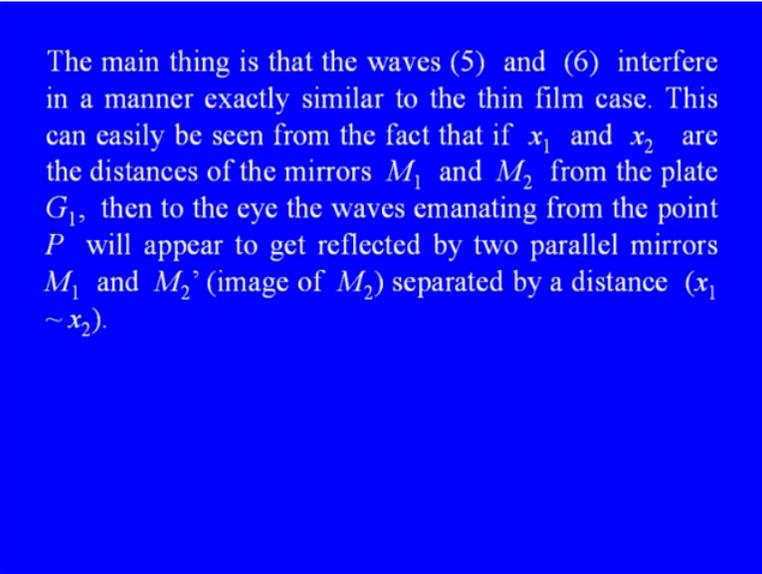
Michelson interferometer is another very interesting device. It is a precision device using division of amplitude as a resource. P could be a screen or sometimes the lens is used to make the beam parallel. But the main thing is ultimately we have a broad source. When this light G is a splitting plate, it disposes, half of the light gets reflected, which is the ray 1 half of it gets through which is the ray 2.

Ray 1 is being reflected by the mirror m_1 form the ray 3 which falls on displayed G, part of its reflected but then, it goes to becomes 5, goes to the observer. The ray tube which has gone through this splitting plate is reflected by the mirror m_2 . If therefore and when this ray of 4 falls back on the spitting plate G_1 gets reflected and we have the ray 6. Now, both 5 and 6 reaching the observer have originated from the same basic ray.

To begin with, therefore, forms a coherent pair and they will cause the interference pattern. You see, these mirrors are front polished mirrors. One thing you will observe here, we can think of an image of the mirror m_2 formed in the splitting plate m_2 prime which is there, same distance which is x_2 and that distance of x_2 here. We can imagine that the inference pattern is the thin film type interface pattern formed between the mirror m_1 and the image of the mirror m_2 that is m_2 Prime.

The distances here are x_1 and x_2 , so, the thickness of the film, the width of the film will be really the difference between the two. That is $x_2 - x_1$.

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The main thing is that the waves (5) and (6) interfere in a manner exactly similar to the thin film case. This can easily be seen from the fact that if x_1 and x_2 are the distances of the mirrors M_1 and M_2 from the plate G_1 , then to the eye the waves emanating from the point P will appear to get reflected by two parallel mirrors M_1 and M_2' (image of M_2) separated by a distance $(x_1 \sim x_2)$.

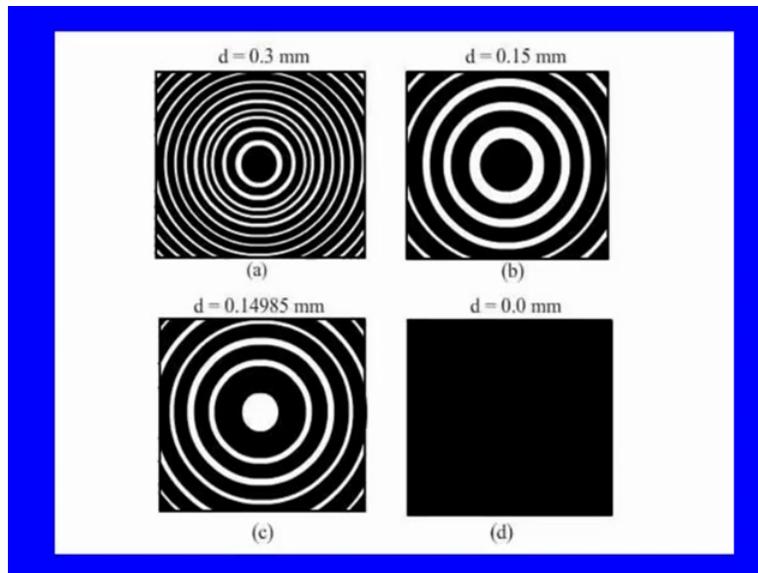
So, that is again rays 5, 5 and 6 to interfere. This can easily be seen from the fact that if x_1 and x_2 as I said, the distances of the pedals m_1 and m_2 from the plate G_1 and then to the eye. The waves emanating from the point P will appear to get reflected as I said by two parallel mirrors m_1 and m_2 , prime separated by a distance $x_1 - x_2$.

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Here also an extended source is used. With that source no definite interference pattern will be obtained on a photographic plate placed at the position of the eye. Instead if we have a camera focused for infinity, then on the focal plane we will obtain circular fringes. These will look like

Here also an extended source is used. With that source, no definite interference pattern will be formed on a photographic plate placed at the position of the eye. Instead if we have a camera focused for infinity then, on the focal plane, we will obtain circular fringes and these fringes will look like;

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So, they had been shown here for different values of the difference of the distance. That is the thickness of the film. For the thickness 0.3 millimeters, the fringes are very fine as this thickness decreases fringes become wider and wider, difficult to find 0.15 millimeters. It is still slightly less 0.149 and naturally when D is zero the film will be completely dark. Remember, not completely bright, because of the difference in the nature of reflection.

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Now if the beam splitter G_1 is just a simple glass plate, the beam reflected from the mirror M_2 will undergo an abrupt phase change of π (when getting reflected by the beam splitter) and since the extra path that one of the beams will traverse will be $2(x_1 \sim x_2)$, the conditions for destructive and constructive interference will be

$$2(x_1 \sim x_2) \cos\theta = m\lambda \quad \text{or} \quad m(\lambda+1/2)$$

respectively.

Now, a practical point if the beam splitter G_1 is just a simple glass plate, the beam will reflected from the mirror m_2 , will undergo an abrupt phase change of π , as I pointed out earlier, when getting reflected by the beam splitter. And since the extra path that one of the beams will traverse will be twice of $x_1 - x_2$, the conditions for destructive and constructive interference will be twice the difference $x_1 - x_2$, $\cos\theta = m\lambda$ for destructive interference because of additional factor of π as I pointed out earlier, or m into $\lambda + \text{half}$ for constructive interference.

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As $d = (x_1 \sim x_2)$ decreases the fringe pattern tends to collapse towards the center. Conversely, if d is increased the fringe pattern will expand.

If N fringes collapse to the center as the mirror M_1 moves by a distance d_0 , then we must have

$$2d = m\lambda$$
$$2(d - d_0) = (m - N)\lambda$$

As d decreases and that is why the distances x_1 and x_2 approach each other, the fringe pattern tends to collapse towards the center and naturally, conversely, if d is increased, the fringe pattern

will expand. As d is being increased different it will appear to originate from the center. That is the idea. Now, if n fringes collapse, to the center held the mirror M_1 moved by a distance d naught then, we must have relation $2d = m \lambda$ as before.

Now, that distance has changed by d naught, so $d - d$ naught and fringes have disappeared collapses so $m = N \lambda$.

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θ has been put equal to zero here as we are looking at the central fringe. Thus

$$\lambda = 2 d_0 / N$$

This provides us a method for the measurement of the wavelength.

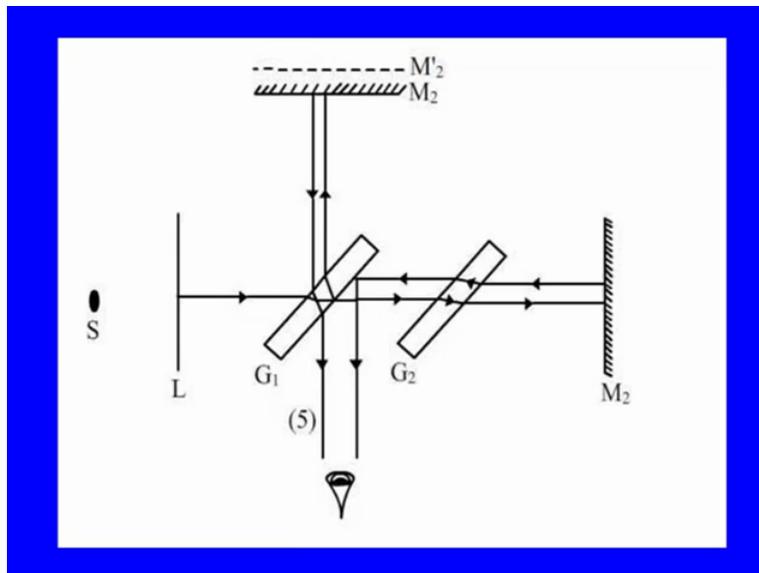
Here θ has been put = 0, because we are looking at the central fringe, where the light is incident normally, so, $\cos \theta$ is 1. And thus from these relations we find that, $\lambda = 2 d$ naught divided by N . And this provides us a method for the measurement of the fringes. Look at the circular fringes, move the mirror, count the number of fringes which have collapsed and you get a capital N measure d naught you can find out the wavelength.

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In an actual Michelson interferometer, the beam splitter G_1 consists of a plate (which may be about $\frac{1}{2}$ cm thick), the back surface of which is partially silvered and the reflections occur at the back surface.

Now, I need a problem, simple problem though in this setup, in an actual interferometer, the beam splitter G_1 , it is not a line, it consists of a plate naturally, which may be about half a centimeter thick. The back surface of which is partially silvered such that 50% light goes through 50% is reflected and these reflections occur at the back surface. This is the picture.

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In an actual Michelson interferometer, the beam splitter G_1 is a plate of about half a centimeter thick. Now you see, the light which is coming, it is partially reflected at the back surface, half of it going towards the mirror m_2 straight and half of it going to the other mirror. And then reflected, reflected from there. But the main thing to be seen is the distance traveled in the plate G_1 is not same for both the beams.

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We find that the beam (5) traverses the glass plate thrice and in order to compensate for this additional path, one introduces a 'compensating plate' G_2 which is exactly of the same thickness as G_1 .

We find that the beam 5, traverses glass plate in three times. And in order to compensate for this additional path, one introduces a compensating plate G_2 which is exactly of the same thickness.

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The compensating plate is not really necessary for a monochromatic source because the additional path $2(n - 1)t$ introduced by G_1 can be compensated by simply moving the mirror M_1 by a distance $(n - 1)t$, where n is the refractive index of the material of the glass plate G_1 .

The compensating plate is not really necessary for a monochromatic source because the additional path twice $n - 1$ into t and is the refractive index of the material of this compensating plate. This is the additional path introduced by G_1 it can be easily compensated by simply moving the mirror, no problem by this, much of distance and $1 - n - 1$ into t . The difficulty is for the white light source it is not possible to simultaneously satisfy the 0 path interference condition for all wavelengths since the refractive index depends on wavelength.

Note this point, we want to compensate for this additional path difference of the light through the splitting plate and this should be there satisfied for all the wavelength components present in the white light.

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However, for a white light source it is not possible to simultaneously satisfy the zero path-difference condition for all wavelengths since the refractive index depends on wavelength.

But this by itself is not possible because the optical path difference depending on the refractive index depends on the wavelength. It cannot be compensated on all the wavelengths at the same time. Let us see them.

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For example, for $\lambda = 6560 \text{ \AA}$ and 4861 \AA , the refractive index of crown glass is 1.5244 and 1.5330 respectively. Now if we are using a 0.5 cm thick crown glass plate as G_1 , then mirror M_1 should be moved by 0.2622 cm for $\lambda = 6560 \text{ \AA}$ and 0.2665 cm for $\lambda = 4861 \text{ \AA}$. The difference between the two positions corresponds to over hundred wavelengths.

For a wavelength of 6560 angstroms and another at 4861 angstroms, the refractive index of crown glass for example is 1.5244 and 1.5330 respectively. Now, if we are using a 0.5 centimeter thick crown glass plate as the beam splitter G1 then the mirror M1 should be moved by 0.2622 centimeters for the wavelength 6560 angstroms. And it should be moved by 0.665 centimeters for wavelength 4861 angstroms.

They are not same. Not only that, the difference between the two positions corresponds to over a 100 limits.

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Thus, if we have a continuous range of wavelengths from 4861 Å to 6560 Å, the path difference between any pair of interfering rays will vary so rapidly with wavelength that we would observe only a uniform white light illumination. In the presence of compensating plate G_2 one would be able to observe a few coloured fringes around the point corresponding to zero path difference.

Thus if we have a continuous range of wavelengths from 4861 angstroms, 6560 angstroms, as we would have in a white light cells the path difference between any pair of interfering rays will vary so rapidly mid wavelength that we will observe only uniform white light illumination, no pattern. In the presence of a compensating plate G2, see now all this problem gets eliminated right away;

Whatever happens in G1 it gets compensated by G2 for every wavelength component present in the white light. One would be able to observe a few colored fringes around the point corresponding to the 0 path difference.

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Measurement of difference between two close wavelengths

Michelson interferometer can also be used to measure difference in wavelengths if the light used has two close wavelengths λ_1 and λ_2 (with $\lambda_1 \sim \lambda_2 \ll \lambda_1, \lambda_2$). If the difference d in the distances of the two Michelson mirrors is such that

$$2d = m\lambda_1 = (m + \frac{1}{2})\lambda_2, \quad \lambda_2 < \lambda_1$$

Okay and the interesting measurement can be carried out this is the measurement of difference between the two closed wavelengths. And also be used to measure difference in wavelength if the light use has two closed wavelength, lambda 1 and lambda 2 with the difference being very, very small compared to the individual wavelengths. If the difference d, in the distances of the two Michelson's mirror is such that 2d is = an integer times lambda 1 but is = m + half times lambda 2.

See I have assumed lambda 2 being smaller than lambda 1. So, for lambda 1 is an integer multiple for lambda 2 is an integer + half multiple. Then what will happen?

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Then the bright fringes of λ_2 will fall on dark fringes of λ_1 and the total fringe pattern will disappear.

The wave length difference is given by

$$\lambda_1 - \lambda_2 = \lambda_1 \lambda_2 / 4d = \lambda^2 / 4d$$

where λ is the mean wavelength.

The bright fringes of λ_2 will fall on dark fringes of λ_1 and the total fringe pattern will disappear. The wavelength difference can be calculated and is given by $\lambda_1 - \lambda_2$ is = $\lambda_1 \lambda_2$ divided by 4 times d , $\lambda_1 \lambda_2$ can be written as λ^2 square between wavelengths. So, the result is $\lambda_1 - \lambda_2$ is = λ^2 square upon $4d$.

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Newton's ring arrangement can also be very easily used to study/demonstrate this and measure the difference ($\lambda_1 - \lambda_2$). We know that the path difference here is related to the radii of the rings, $2d = r^2/R$. The above relation therefore becomes

$$\lambda_1 - \lambda_2 = \lambda^2 R / 2r^2$$

or

$$r^2 = \lambda^2 R / 2(\lambda_1 - \lambda_2)$$

Newton's rings arrangement can also be very easily used to study and demonstrate this and measure the difference $\lambda_1 - \lambda_2$. You see, we know the path difference here is related to the radii of the rings $2d$ is = small r square upon capital R or small R is the radius of the Ring, capital R is the radius of curvature of the plano convex lens used in the arrangement.

So, the above relation therefore becomes $\lambda_1 - \lambda_2$ is = λ^2 square times capital R divided by $2r$ square or in terms of r square is = λ^2 square capital R divided by twice $\lambda_1 - \lambda_2$.

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This is the radius of the ring where the fringe pattern vanishes.

Another method could be to lift the plano-convex lens, so that the path difference at the point of contact becomes such that this point now corresponds to maximum for one wavelength and minimum for the other. If d is the distance through which the lens has been lifted, then

$$d = \lambda^2/4(\lambda_1 - \lambda_2).$$

So this relation gives you a radius of the ring where different pattern vanishes and the method could be to lift the Plano convex lens. Remember, you have a Plano convex lens placed on a glass plate. Now, we are lifting it. So, the path difference at the point of contact, you see at the point of contact, the thickness is 0; now we are increasing that thickness.

The path difference at the point of contact becomes such at this point now corresponds to maximum for one wavelength and minimum for the other. So if d is the distance through which the lens has been lifted then this d will be = λ^2 divided by 4 times $\lambda_1 - \lambda_2$.

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With Michelson interferometer, one can keep moving one of the mirrors to obtain the distance required to be moved for two consecutive vanishing of the fringes. If this distance is d' , then

$$d' = \lambda^2/2(\lambda_1 - \lambda_2)$$

With Michelson interferometer one can have a different method; one can keep moving one of the mirrors, to obtain the distance required to be moved for two consecutive vanishing of the fringes. Let us understand this. Suppose, for a certain distance, the fringe pattern has vanished. This means maximum of one wavelength falls over the minimum of the other. You keep moving the fringe, one of the mirrors, situation will come, when the fringes appear again continue to move the fringe the mirror;

Again a situation will come when maximum of one wavelength falls over the minimum of the other and fringe pattern vanishes. This is what we are talking here. One can keep moving one of the mirrors to obtain the distance required to be moved for two consecutive vanishing of the fringes. If this distance is d' then the d' is = as before, $\frac{\lambda_1^2 - \lambda_2^2}{2\lambda_1 \lambda_2}$.

So this method, so this method can be used to find the difference between two closed wavelengths present in the light coming from the source, ok. I think this we have come to the end of this lecture, thank you.