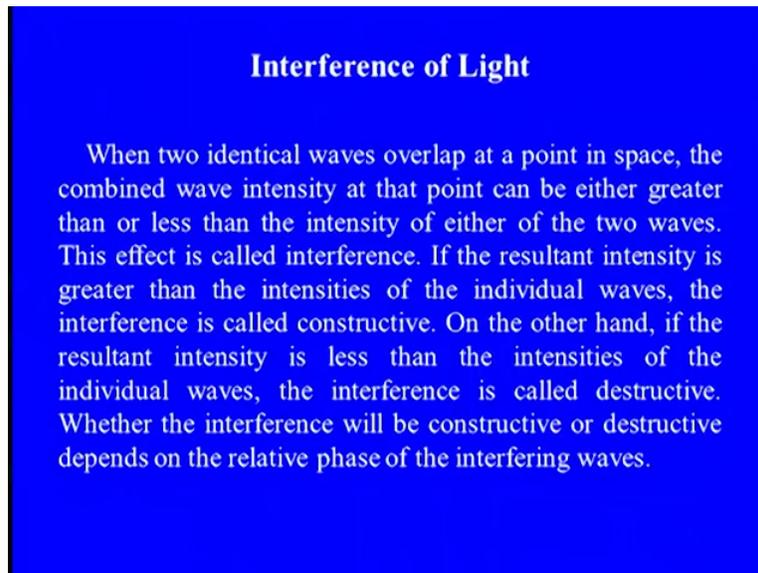


Engineering Physics 1
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Department of Physics
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Module-03
Lecture-01
Interference of Light – Part 01

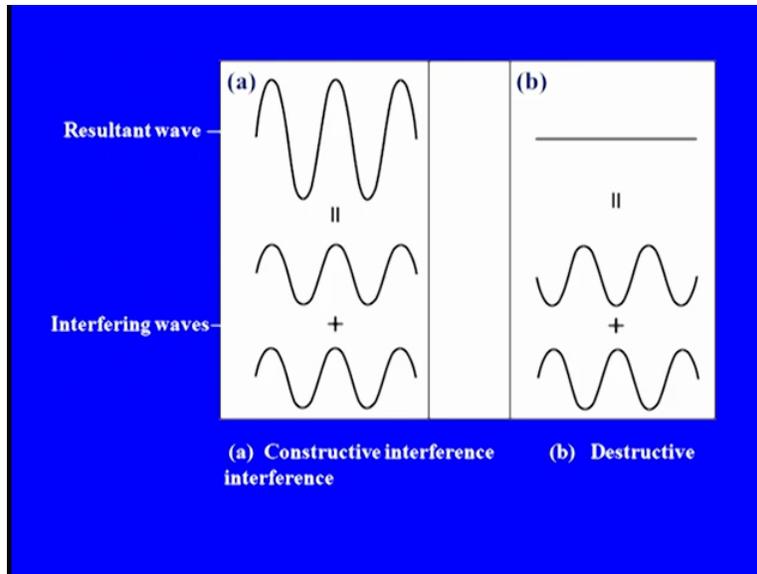
Myself, Dr. JD Varma, Associate Professor in Department of Physics. In this lecture I will discuss the basic theory of interference of light where I will discuss the conditions to get interference light in the laboratory. And I will also discuss some of the famous experiments which are used in the laboratory to study the interference phenomena. First I will discuss what is Interference?

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So, when two identical waves overlap at a point in space, the combined wave intensity at that point can be either greater than or less than the intensity of either of the two waves. This effect is called Interference. If the resultant intensity is greater than the intensities of the individual waves the interference is called constructive. On the other hand, if the resultant intensity is less than the intensities of the individual wave, the interference is called destructive. Whether the interference will be constructive or destructive depends on the relative phase of the interfering waves.

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If two identical waves arrive in phase at the same point in space, that is they line up crest to crest and trough to trough then, the amplitude of the resultant wave would be twice the amplitude of the component waves, as shown in the left panel of this figure. This is the condition for constructive interference. Now, if the waves at the point of consideration line up crest to trough and trough to crest, the amplitude of the resultant wave will be zero.

As it is shown in the right panel of this figure, this is the condition for destructive interference in which two waves cancel one another's effect at a point.

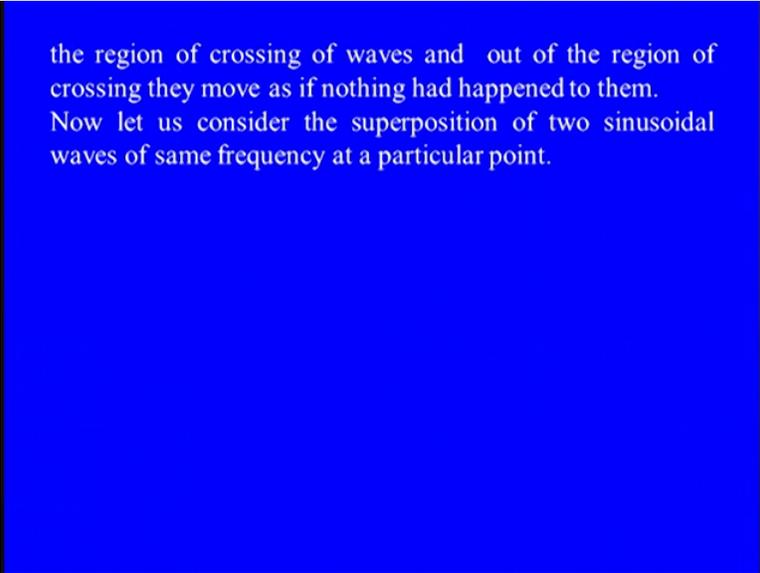
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Although, I have considered here the interference of two waves, but in principle any number of waves can interfere. The amplitude (intensity) of resultant wave can be determined by using 'superposition principle', first enunciated by Thomas Young. According to this principle if a medium is disturbed simultaneously by any number of waves, the instantaneous resultant displacement of the medium at every point at any instant is the algebraic sum of the displacements of the medium due to individual waves in the absence of the others. Here, it should be noted that modification in intensity takes place only in

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Here it should be noted that modification in intensity takes place only in the region of crossing of waves.

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the region of crossing of waves and out of the region of crossing they move as if nothing had happened to them. Now let us consider the superposition of two sinusoidal waves of same frequency at a particular point.

And out of the region of crossing they move as if nothing had happened to them. Now, let us consider the superposition of two sinusoidal waves of same frequency at a particular point.

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Suppose the displacements produced by each of the disturbance, separately at the point, are given by

$$x_1(t) = a_1 \cos(\omega t + \theta_1) \text{ and } x_2(t) = a_2 \cos(\omega t + \theta_2)$$

here, a_1 and a_2 are amplitudes of the waves, and displacements $x_1(t)$ and $x_2(t)$ are in the same direction. θ_1 and θ_2 are their initial phases. Now, according to the superposition principle the resultant displacement $x(t)$ would be given by

$$\begin{aligned} x(t) &= x_1(t) + x_2(t) \\ &= a_1 \cos(\omega t + \theta_1) + a_2 \cos(\omega t + \theta_2) \\ x(t) &= a \cos(\omega t + \theta) \end{aligned}$$

Suppose the displacement produced by each of the disturbance separately at the point are given by $x_1(t) = a_1 \cos(\omega t + \theta_1)$ and $x_2(t) = a_2 \cos(\omega t + \theta_2)$ here a_1 and a_2 are the amplitudes of the waves and displacement x_1 and x_2 are in the same direction, θ_1 and θ_2 are their initial phases. Now, according to the superposition principle the resultant displacement x would be given by $x_1 + x_2$.

And if you substitute the value of x_1 and x_2 we get $a_1 \cos(\omega t + \theta_1) + a_2 \cos(\omega t + \theta_2)$. After simplification we will get $x = a \cos(\omega t + \theta)$.

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From this equation it is clear that the resultant disturbance is also simple harmonic in character having the same frequency but different amplitude and different initial phase. In other words the sum of two simple harmonic motions of the same frequency and along same line is also a simple harmonic motion of the same frequency.

We can show that the amplitude 'a' of the resultant wave is given by

$$a = [(a_1^2 + a_2^2 + 2a_1a_2\cos(\theta_1 - \theta_2))]^{1/2}$$

and the initial phase of the resultant wave θ is given by

$$\theta = \tan^{-1}[(a_1 \sin \theta_1 + a_2 \sin \theta_2) / (a_1 \cos \theta_1 + a_2 \cos \theta_2)]$$

From this equation, it is clear that the resultant disturbance is also simple harmonic in character having the same frequency but different amplitude and different initial phases. In other words, the sum of two simple harmonic motions of the same frequency and along same line is also a simple harmonic motion of the same frequency. We can show that the amplitude a of the resultant wave is given by root of $a_1^2 + a_2^2 + 2 a_1 a_2 \cos \theta_1 - \theta_2$.

And the initial phase of the resultant wave θ is given by $\tan^{-1} \frac{a_1 \sin \theta_1 + a_2 \sin \theta_2}{a_1 \cos \theta_1 + a_2 \cos \theta_2}$.

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Thus, the resultant amplitude a depends upon the amplitudes a_1 and a_2 of the component waves and their difference of phases $(\theta_1 - \theta_2)$.

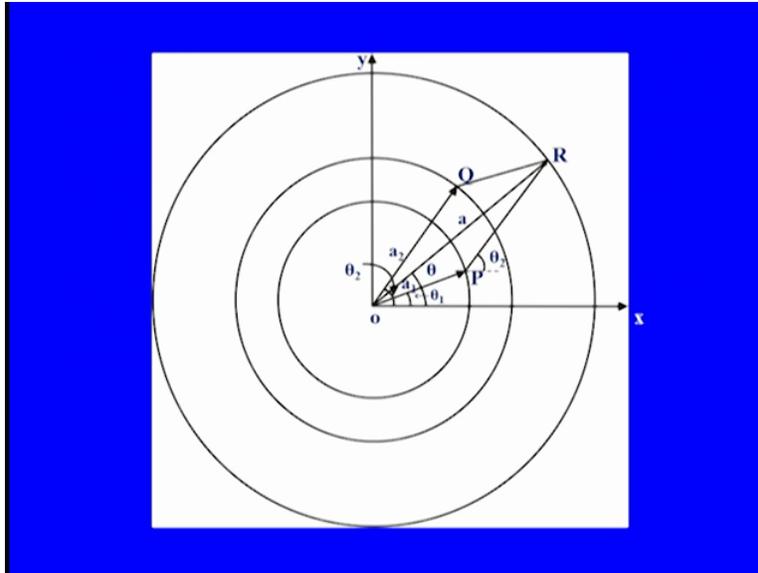
Above we have used algebraic method to find the resultant of two harmonic (sinusoidal) waves. But the resultant can also be obtained quickly by graphical method. This method is more useful when we have a large number of superposing waves as we will see while discussing the phenomenon of diffraction.

To obtain the resultant of two disturbances: $x_1(t) = a_1 \cos(\omega t + \theta_1)$ and $x_2(t) = a_2 \cos(\omega t + \theta_2)$ by

Thus the resultant amplitude a , depends upon the amplitudes a_1 and a_2 of the component waves and their difference of phases $\theta_1 - \theta_2$. Here, we have used algebraic method to find the resultant of two harmonic waves but the resultant can also be obtained quickly by graphical method. This method is more useful when we use, we have a large number of superposing waves as we will see while discussing the phenomena of diffraction.

So, to obtain the resultant of two disturbance $x_1(t) = a_1 \cos \Omega t + \theta_1$ and $x_2(t) = a_2 \cos \Omega t + \theta_2$.

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By the graphical method we draw vectors OP and OQ of magnitude a_1 and a_2 respectively such that OP and OQ makes an angle θ_1 and θ_2 with the x axis respectively as shown in this figure. Here, if we assume the directors OP and OQ rotate in clockwise direction with angular frequency Ω then, the x coordinate of OP will be $a_1 \cos \Omega t + \theta_1$ and that of OQ will be $a_2 \cos \Omega t + \theta_2$. At $t = 0$ the rotating vectors are at point P and Q.

We use the law of Hologram to find the resultant OR of the vectors OP and OQ. The length of the vector OR will represent

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resultant wave and the angle θ that OR makes with the x-axis is the initial phase of the resultant wave. Further, as the vectors OP and OQ rotate on the circumference of the circles of radii a_1 and a_2 , the vector OR rotates on the circumference of the circle of radius OR with the same frequency.

From the triangle formed by a_1 , a_2 and a , using the law of cosines we can find

$$a = [(a_1^2 + a_2^2 + 2a_1a_2\cos(\theta_1 - \theta_2))]^{1/2}$$

From the figure we have

$$a \cos \theta = a_1 \cos \theta_1 + a_2 \cos \theta_2$$

and $a \sin \theta = a_1 \sin \theta_1 + a_2 \sin \theta_2$

The amplitude of the resultant wave and the angle θ that OR makes with the x axis is the initial phase of the resultant wave. Further while the vector OP and OQ rotates, on the circumference of the circles of radii a_1 and a_2 , the vector OR rotates on the circumference of the circle of radius OR with the same frequency. From the triangle formed by a_1 , a_2 and a , and using the law of cosines, we can find the amplitude of the resultant wave a is $=\sqrt{a_1^2 + a_2^2 + 2 a_1 a_2 \cos(\theta_1 - \theta_2)}$.

From the figure, we have $a \cos \theta = a_1 \cos \theta_1 + a_2 \cos \theta_2$ and $a \sin \theta = a_1 \sin \theta_1 + a_2 \sin \theta_2$.

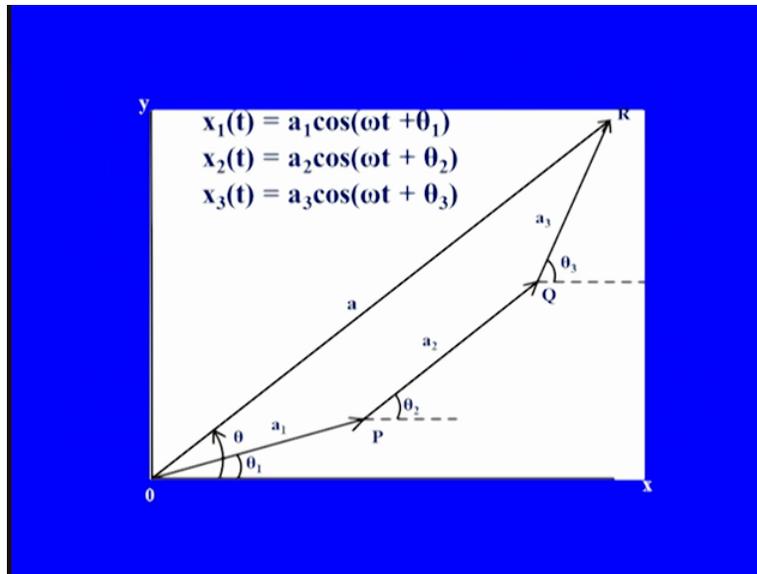
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With the help of these expressions we can obtain the expression for the initial phase of resultant wave. Here we will get the same results as obtained earlier, using algebraic method.

Thus, if we want to find the resultant of the two displacements, then we must first draw a vector (OP) of length a_1 making an angle θ_1 with the x-axis; from the tip of this vector we must draw another vector (PQ) of length a_2 making an angle θ_2 with the x-axis as shown in this figure. The length of the vector OQ will represent the resultant amplitude and the angle that it makes with the x-axis will represent the initial phase of the resultant displacement. Now, if we have third

With the help of these expressions, we can obtain the expression for the initial phase of the resultant wave. Here, we will get the same result as we have obtained earlier using the algebraic method.

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Thus if we want to find the resultant of two displacement then, we must first draw vector OP of length a_1 making an angle θ_1 with the x axis. From the tip of this vector we must draw another vector PQ of length a_2 making an angle θ_2 with the x axis as shown in this figure. The length of the vector OQ will represent the resulting amplitude and the angle that it makes with the x axis will represent the initial phase of the resultant displacement.

Now, if we have third displacement $x_3 = a_3 \cos \Omega t + \theta_3$ then from the point Q we draw a vector QR of length a_3 making an angle θ_3 with the x axis. The vector OR will now represent the amplitude of the wave resulting due to superposition of x_1 , x_2 and x_3 . The angle θ made by OR with the x axis will be the initial phase of the resultant wave. Now, I will discuss the conditions for constructive and destructive interference.

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Conditions for constructive and destructive Interference

As we have seen, the amplitude of the resultant wave in terms of amplitudes and initial phases of the interfering waves is given by

$$a = [(a_1^2 + a_2^2 + 2a_1a_2\cos(\theta_1 - \theta_2))]^{1/2}$$

Now let us represent phase difference $(\theta_1 - \theta_2)$ by δ . So from this equation it is clear that if the phase difference between the waves

As we have seen the amplitude of the resultant wave in terms of amplitude and the initial phases of the interfering wave is given by root of $a_1^2 + a_2^2 + 2 a_1 a_2 \cos \theta_1 - \theta_2$. Now, let us represent the phase difference $\theta_1 - \theta_2$ by Δ . So, from this equation it is clear that if the phase difference between the waves Δ is $=0, 2\pi, 4\pi$ R in general $2n\pi$ where n is $=0, 1, 2, 3$ like that then $a = a_1 + a_2$.

From wave theory we know that phase difference of $0, 2\pi, 4\pi$ that is $2n\pi$ means, the waves are in phase. Thus if the two displacement are in phase then the resultant amplitude is the sum of the amplitudes of the interfering waves. This is known as constructive interference and this will be the, will be obtained when Δ is $=2n\pi$ where n is $=0, 1, 2, 3$ etcetera.

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On the other hand, if the phase difference

$\delta = \pi, 3\pi, 5\pi, \dots$, i.e., $(2n+1)\pi$ ($n=0, 1, 2, 3, \dots$), then $a = (a_1 - a_2)$, i.e. if two displacements are out of phase then the resultant amplitude is the difference of the two amplitudes. This is known as destructive interference and the condition for this is $\delta = (2n+1)\pi$ ($n=0, 1, 2, \dots$).

Since phase difference δ is equal to $(2\pi/\lambda) \times$ Path difference, we can also write the above conditions for constructive and destructive interference in terms of path difference.

Suppose two waves initially in phase meet at a point after travelling different paths. At this point there would be constructive interference if Path difference = $n\lambda$ and destructive interference if Path difference = $(n+1/2)\lambda$ (where $n=0, 1, 2, \dots$).

On the other hand, if the phase difference Δ is equal $2\pi, 3\pi, 5\pi$, that is in general, $\Delta = 2n + 1\pi$ where n is $=0, 1, 2, 3$, etcetera then, $a = a_1 - a_2$. That is if two displacements are out of phase then, the resultant amplitude is the difference of the two amplitudes. This is known as destructive interference and the condition for this is $\Delta = 2n + 1\pi$ where n is $=0, 1, 2, 3$, etcetera.

Since phase difference $\Delta = 2\pi$ by λ into path difference we can also write, the above conditions for constructive and destructive interference, in terms of path difference. Suppose two waves initially in phase meet at a point after traveling different paths. At this point there would be constructive interference, if the path difference is $= n$ times λ . And destructive interference, if path difference is $= n + \text{half}$ times λ where n is $= 0, 1, 2, 3$, like that. So, now I will discuss the variation of intensity of resultant wave as a function of phase difference.

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Intensity distribution

As we know the intensity of a wave is proportional to its amplitude and proportionality constant is taken as one. Therefore, the intensity of the resultant wave, I , is equal to the square of the amplitude a of the resultant wave.

$$I = a^2 = I_1 + I_2 + 2(I_1 I_2)^{1/2} \cos \delta$$

here, $I_1 (= a_1^2)$ and $I_2 (= a_2^2)$ are the intensities of the interfering waves and δ is the phase difference ($\theta_1 - \theta_2$) between them.

As we know the intensity of a wave is proportional to its amplitude and proportionality constant is taken as 1. Therefore the intensity of the resultant wave I is = the square of the amplitude a of the resultant wave. So, we can write intensity of the resultant wave $I = I_1 + I_2 + 2$ into root of I_1 into I_2 into $\cos \Delta$. Here I_1 is = a_1 square and I_2 is = a_2 square are the intensities of the interfering waves and Δ is the phase difference $\theta_1 - \theta_2$ between them.

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Since the maximum and minimum values of $\cos \delta$ are +1 and -1, respectively.

Therefore, the maximum intensity

$I_{\max} = (I_1^{1/2} + I_2^{1/2})^2$, when $\delta = 2n\pi$, $n = 0, 1, 2, \dots$, i.e. when the waves are in phase.

Since the maximum and minimum value of $\cos \Delta$ are + 1 and - 1 respectively. Therefore, the maximum intensity I_{\max} will be = root of I_1 + root of I_2 whole square, when Δ is = $2n\pi$.

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The minimum intensity $I_{\min} = (I_1^{1/2} - I_2^{1/2})^2$. This occurs when $\delta = (2n+1)\pi$, $n=0,1,2,\dots$, i.e. when waves are out of phase.

If the amplitudes of both waves are same, i.e. $a_1 = a_2 = a$, then $I_1 = I_2 = I_0$

In this case the resultant intensity

$$I = 4a^2 \cos^2(\delta/2) = 4I_0 \cos^2(\delta/2).$$

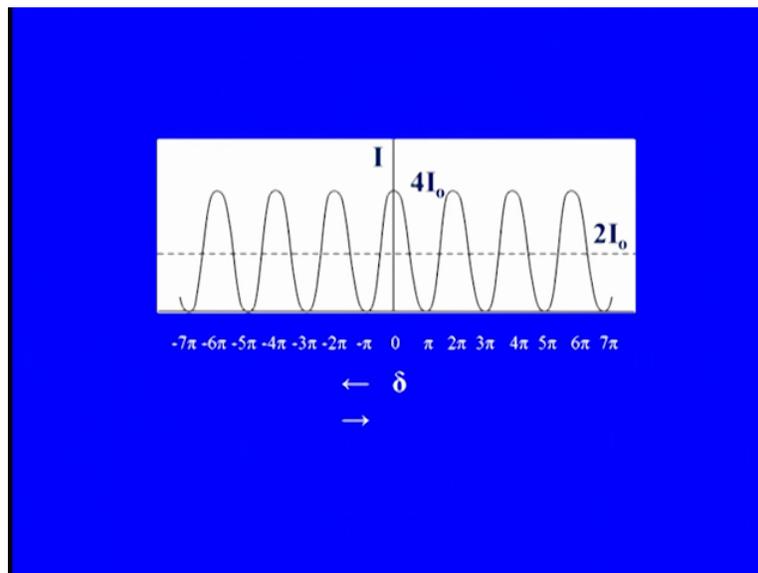
In this case

$$I_{\max} = 4I_0 \text{ and } I_{\min} = 0$$

This figure shows I vs δ plot.

The minimum intensity I_{\min} will be $=(\sqrt{I_1} - \sqrt{I_2})^2$. This occurs when $\Delta = 2n + 1\pi$. That is when waves are out of phase. If the amplitude of the waves are same that is $a_1 = a_2 = a$ then, $I_1 = I_2 = I_0$. Say, in this case, the resultant intensity I will be $=4a^2 \cos^2(\Delta/2)$ or this will be $=4I_0 \cos^2(\Delta/2)$. In this case, the maximum intensity I_{\max} will be $=4I_0$ and the minimum intensity will be $=0$.

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This figure shows I versus Delta plot. From this figure, it is clear that when two beams of light arrived at a point exactly out of phase, they interfere destructively and the resultant intensity is 0. On the other hand, when they meet exactly in phase that is they interfere constructively, then the

resultant intensity is 4 times I_0 . Here it should be noted that there is no violation or conservation of energy in the phenomena of interference.

The energy which apparently disappears at the minima is actually still present at the Maxima where the intensity is greater than that would be produced by the two beams acting separately. In other words, the energy is not destroyed but merrily redistributed in the interference pattern.

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Now suppose there is no phase relationship between two interfering (superimposing) waves, i.e. they are produced by incoherent light sources, then the phase difference δ will remain constant for a very short duration (about 10^{-10} s) and δ would also vary with time in a random way. In this case $\cos^2(\delta/2)$ will vary randomly between 0 and 1, and average of $\cos^2(\delta/2)$ would be $\frac{1}{2}$. So we have $I = I_1 + I_2 = 2I_0$, which has been marked by dashed line in this I vs δ plot. Thus for two incoherent sources, the resultant intensity is the sum of the intensities produced by each one of the source independently and no interference pattern is observed.

Now suppose, there is no phase relationship between the two interfering waves that is they are produced by incoherent light sources, then, the phase difference Δ will remain constant for a very short duration of time, which is of the order of 10^{-10} second. And Δ would also vary with time in the random wave. In this case, $\cos^2 \Delta/2$ will vary randomly between 0 and 1 and the average of $\cos^2 \Delta/2$ would be half.

So, we have, resultant intensity $I = I_1 + I_2 = 2I_0$ which has been marked by dashed line in this I versus Δ plot. Thus for two incoherent sources the resultant intensity is the sum of the intensities produced by each one of the source independently and no interference pattern is observed. Now, I will discuss what are the various conditions that should be satisfied to get stable or stationary interference patterns?

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Conditions for interference

To observe the well defined interference pattern experimentally, the experimental set up must fulfill the following conditions:

1. The two interfering beams must originate from same source of light. This is essential for producing stationary maxima and minima. Since for stationary maxima and minima the phase difference δ should not vary with time, i.e. the initial phase difference between the interfering waves must be zero or constant. The sources producing this type of waves are said to be coherent. Two independent sources can never

So to observe the well defined interference pattern experimentally, the experimental setup must fulfill the following conditions: Number 1: The two interfering beams must originate from the same source of light. This is essential for producing stationary maxima and minima since for a stationary maxima and minima. The phase difference Δ should not vary with time.

That is the initial phase difference between the interfering waves must be zero or constant. The sources producing this type of waves are said to be coherent. Two independent sources can never be coherent.

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be coherent. This is because light from any one source is not an infinite train of waves and there are sudden changes in phase occurring in very short intervals of time ($\sim 10^{-10}$ s). However, two coherent sources can be accomplished experimentally from single source by either making one source image of the other by reflection or by dividing the light waves into two parts by reflection or by refraction or by partial reflection. In these circumstances any change of phase which the original light wave undergoes is shared by its two parts instantaneously, with the result that the phase difference between the waves remains constant and hence the positions of maxima and minima remain stationary.

This is because light from any one source is not an infinite train of waves and there are sudden changes in the phase occurring in every short interval of time of the order of 10^{-10} second. However, two coherent sources can be accomplished experimentally, from single source by either making one source image of the other, by reflection or by dividing the light waves into two part by reflection or by refraction or by partial reflection.

In these circumstances any change of phase which the original light wave undergoes is served by each two part instantaneously with the result that the phase differences between the waves remain constant and hence the position of maxima and minima remain stationary. The second condition which the sources must satisfy is that the waves must have the same period and wavelength.

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2. The waves must have the same period and wavelength. Also their amplitudes must be equal or very nearly equal.
3. The original source must emit light of single wavelength, i.e., monochromatic source.
4. The two interfering waves must propagate almost in the same direction or the two interfering wave fronts must intersect at a very small angle.

Also the amplitude must be equal or very nearly equal. Third condition is the original source must emit light of single wavelength that is monochromatic source. Fourth, the two interfering waves must propagate almost in the same direction are the two interfering wave front must intersect at a very small angle.

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In the interference experiments the conditions 1-3 will be satisfied if the interfering beams of light are obtained by dividing the light of a single source. This can be achieved either by division of wave front or by division of amplitude. Thus the interference experiments may be divided into two main categories: (i) experiments based on division of wave front and (ii) experiments based on division of amplitude. In the former class, the wave front is divided laterally into segments by mirrors or slits. In the later class the amplitude of the incoming wave of light is divided into two or more parts by partial

In the interference experiment, the conditions 1 to 3 will be satisfied if the interfering beams of the light are obtained by dividing the light of a single source. This can be achieved either by division of wave front or by division of amplitude. Thus the interference experiment may be divided into two main categories. Number 1: the experiment based on division of wave front and number 2: the experiments based on division of amplitude.

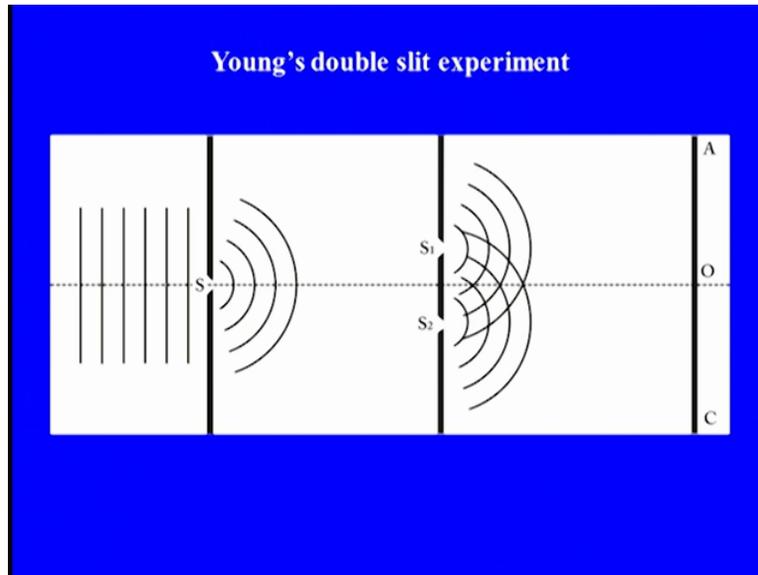
In the former class, the wave front is divided literally in to segments by mirrors or slits. In the later class, the amplitude of the incoming wave of light is divided into two or more parts by

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reflection and refraction resulting in two or more beams which are made to reunite to produce the interference effect.

, partial reflection and refraction, resulting in two or more beams which are made to reunite to produce the interference effect. I will describe some famous experiments of both classes. So, first I will discuss some of the experiment which is based on division of wave front.

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So, first experiment which I will discuss is a famous Young's double slit experiment. In this experimental setup, monochromatic light of wavelength λ is allowed to pass through a slit s and then, through two slits S_1 and s_2 , which are equidistance from s as shown in this figure. The slit s , S_1 and S_2 are perpendicular to diagram and parallel to each other.

The two sets of cylindrical wave front from the slits S_1 and S_2 interfere with each other to form interference pattern on the screen AC placed at some distance in front of the plane containing two slits. If it comes in the light on this screen, we see, in any space bright and dark bands which are known as interference fringes. Since the path $s S_1 = s s_2$, the wavelets arrive at slits S_1 and S_2 at the same instant.

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Therefore, secondary wavelets diverge to the right from slits S_1 and S_2 which have precisely equal phases at the start. Furthermore, their amplitude, wavelength and velocity are also equal. Now let us determine the positions of maxima and minima on the screen. On the screen point O is equidistant from S_1 and S_2 . Let us consider a point P on the screen at a distance x from O. If the point P is such that $S_2P - S_1P = n\lambda$, where $n = 0, 1, 2, \text{ etc.}$, then the disturbances reaching the point P from the two sources will be in phase and hence the interference will be constructive leading to maximum intensity.

Therefore, secondary wavelets diverge to the right from slits S_1 and S_2 which have precisely equal phases at the start. Furthermore, their amplitude, wavelength and velocity are also equal. Now let us determine the positions of maxima and minima on this screen. On this screen, point O is equidistant from S_1 and S_2 . Let us consider a point P on this screen at a distance x from O. If the point P is such that $S_2P - S_1P = n\lambda$ where $n = 0, 1, 2, 3, \text{ etc.}$

Then, the disturbance is reaching the point P from two sources will be in phase. And hence, the interference will be constructive leading to maximum intensity.

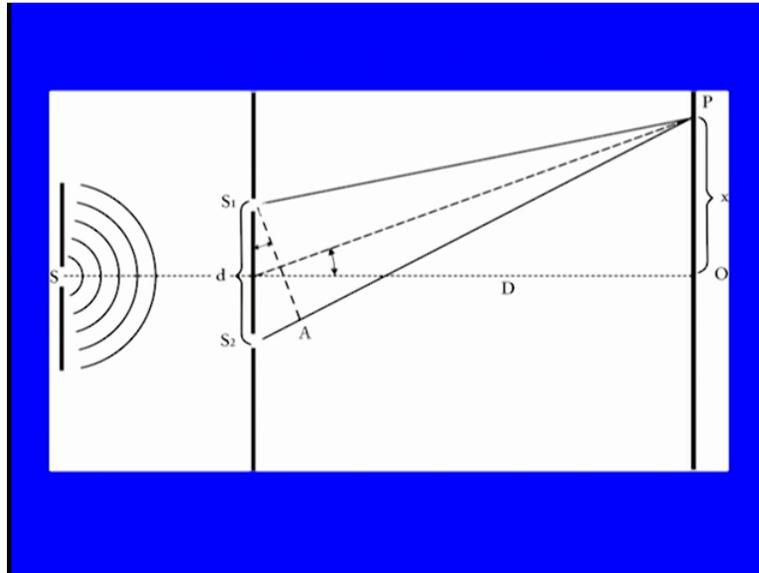
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On the other hand, if the point P is such that $S_2P - S_1P = (n+1/2)\lambda$, $n = 0, 1, 2, \text{ etc.}$, then the disturbances reaching the point P from the two sources will be out of phase and therefore the interference will be destructive and the intensity will be minimum.

Since for the point O, we have $S_1O = S_2O$, both waves will arrive at O in phase. Therefore, if point P coincides with O the intensity will be maximum. As the point P moves up and down on the screen starting from point O, the magnitude of the path difference $|S_2P - S_1P|$ increases and hence the conditions for maxima and minima will be satisfied alternatively.

On the other hand, if the point P is such that $S_2 P - S_1 P = n + \text{half lambda}$ again n is =0, 1, 2, 3, etcetera then, the disturbance reaching the point P from the two sources will be out of phase and therefore, the interference will be destructive and the intensity will be minimum.

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Since the point O we have $S_1 O = S_2 O$, both waves will arrive at O, in phase. Therefore, a point P coincide with O the intensity will be maximum and the point P moves up and down on the screen starting from point O. The magnitude of the power difference has to P - $S_1 P$ increases and hence the condition for maxima and minima will be satisfied alternatively.

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If we calculate the phase difference δ at any point we can find the intensity at that point using the equation $I = 4a^2 \cos^2(\delta/2)$, where a is the amplitude of the interfering waves.

So, now it remains to evaluate the phase difference in terms of the distance x of the point P on the screen from the central point O, the separation d of the two slits, and the distance D from the slits to the screen. This can be obtained with the help of this figure. If the point A on S_2P is such that $S_1P = AP$, then $S_2P - S_1P = S_2A$.

If we calculate the phase difference Δ at any point we can find the intensity at that point using the equation $I = 4a^2 \cos^2 \frac{\Delta}{2}$ where a , is the amplitude of the interfering waves. So, now it remains to evaluate the phase difference in terms of the distance x of the point P on the screen from the center point O , the separation D of the two slits and the distance D capital D from the slits to the screen. This can be obtained with the help of this figure. If the point A on S_2P is such that $S_1P = AP$ then $S_2P - S_1P$ will be $=S_2A$.

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In the experimental set up $D \gg d$ and $D \gg x$. Hence angles θ and θ' are very small and practically equal. Under these conditions, ΔS_1AS_2 may be regarded as a right triangle, and the path difference becomes $d \sin \theta' = d \sin \theta$. If θ is small, we have $\sin \theta \approx \tan \theta$.

So, path difference $\Delta = d \sin \theta = dx/D$

and phase difference $\delta = (2\pi/\lambda) \Delta$.

Now, for intensity to be maximum ($I = 4a^2$), δ should be an integral multiple of 2π , which will occur when path difference is an integral multiple of λ . Hence for maximum intensity

$$xd/D = 0, \lambda, 2\lambda, \dots = n\lambda \quad n=0,1,2,\dots$$

In the experimental setup D capital D is much greater than small d and capital T is much greater than X . Hence angle θ and θ' are very small and practically equal. Under these conditions, triangle S_1AS_2 may be regarded as a right angled triangle and the path difference becomes $d \sin \theta' = d \sin \theta$. If θ is small, we have $\sin \theta \approx \tan \theta$. So, path difference Δ will be $=d \sin \theta$ will be $=d x$ divided by capital D .

And the phase difference Δ will be $=2\pi$ by λ into path difference. Now, for intensity to be maximum Δ should be an integral multiple of 2π which will occur when the path difference is an integral multiple of λ . Hence for maximum intensity x into small d divided by capital D should be $= 0, \lambda, 2\lambda$ are in general, $= n$ times λ . So, the position of Maxima x_n will be $= n$ times λ into capital D divided by small d where n is $=0, 1, 2, 3,$ etcetera.

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So the positions of maxima $x_n = n\lambda D/d$, where $n=0, 1, 2, 3$, etc.

The minimum value of the intensity will occur at those points where $\delta = \pi, 3\pi, 5\pi, \dots$. For these points the path difference

$$x\lambda/D = \lambda/2, 3\lambda/2, \dots = (n + 1/2)\lambda, n=0, 1, 2, \dots$$

So the positions of minima $x_n = (n + 1/2)\lambda D/d$, $n=0, 1, 2, 3$, etc.

The whole number n , which characterizes a particular bright fringe, is called the order of interference. Thus, the fringes with $n = 0, 1, 2, \dots$ are called the zero, first, second, etc order.

The minimum value of the intensity will occur at those points where Δ is $=\pi, 3\pi, 5\pi$, so on. For this point, the path difference x into small d divided by capital D will be $=\lambda/2, 3\lambda/2$, like that; that is, in general $n + \text{half } \lambda$. So, the position of minima x_n will be $=n + \text{half } \lambda$ into capital D divided by small d . The whole number n which characterizes a particular bright fringe is called the order of interference. Thus the fringes with n is $=0, 1, 2, 3$, are called the 0, first, second, etcetera order.

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We can calculate the distance on the screen between two successive bright fringes and between two successive dark fringes by evaluating $(x_{n+1} - x_n)$ using the expressions of x corresponding to bright and dark fringes. This distance is known as fringe width w . In both cases we get

$$\text{Fringe width } w = x_{n+1} - x_n = \lambda D/d$$

Since the value of fringe width w is constant, on the screen we observe equally spaced dark and bright fringes. Thus, fringe width varies directly with slit screen separation D , inversely with the separation of slits d and directly with the wavelength λ of light employed.

We can calculate the distance on the screen between two successive bright fringes and between two successive dark fringes by evaluating $x_{n+1} - x_n$, using the expressions of x and x

corresponding to bright and dark fringes. This distance is known as fringe width w . In both cases we get fringe width w is $=x_{n+1} - x_n = \lambda D / d$.

Since the value of fringe width w is constant on the screen, we observe equally spaced dark and bright fringes. Thus fringe width varies directly with slit screen separation D inversely with the separation of slits is d and directly with the wavelength λ of light used. Now we will discuss some other operators which are based on division of wave front.

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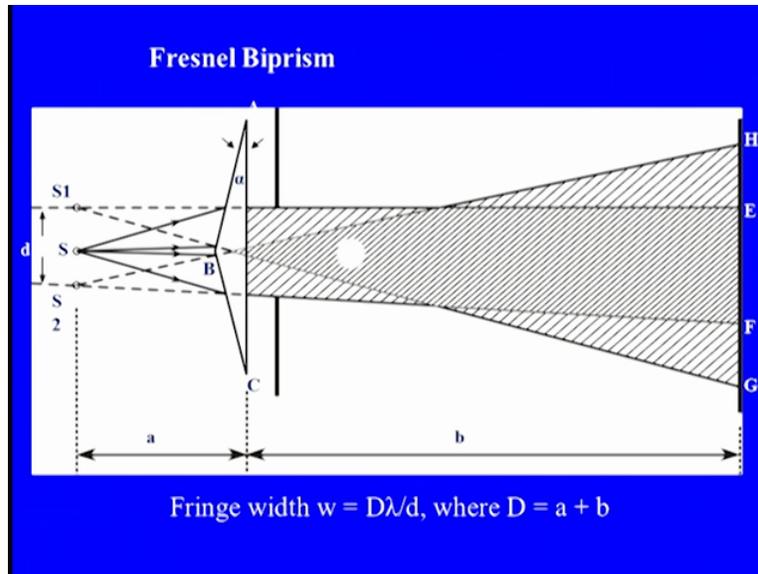
Other apparatus based on division of the wave front

Soon after the double-slits experiment was performed by Young, the objection was raised that the bright fringes he observed were probably due to some complicated modification of the light by the edges of the slits and not to true interference. Later, Fresnel brought forward several new experiments in which the interference of two beams of light was proved in a manner not open to the above objection. In the following I will discuss some experiments in which two interfering beams are obtained by refraction and reflection of light.

Now, soon after the double slit experiment was performed by Young, the objection was raised that the bright fringes he observed, were probably due to some complicated modification of the light by the edges of the slit and not to true interference. Later, Fresnel brought forward several new experiments in which, the interference of two beams of light was proved in a manner not open to above of objection.

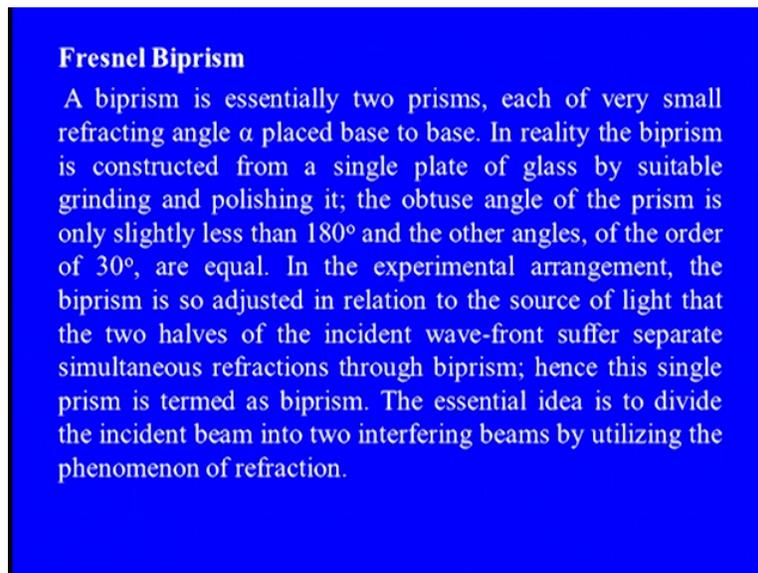
In the following I will discuss some experiments in which two interfering beams are obtained by refraction and reflection of light. So now, I will discuss the Fresnel Biprism experiment.

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A Biprism is essentially two beams, each of very small refracting angle α placed base to base. In reality, the Biprism is constructed from a single plate of glass by suitable grinding and polishing it. The obtuse angle of the plates is only slightly less than 180 degree and other angles of the order of 30 degree are equal. In the experimental arrangement the Biprism is so adjusted in relation to the source of light that the two halves of the incident wave front suffer separate simultaneous refraction through Biprism. Hence, this single prism is Tom Reyes Biprism.

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The essential idea is to divide the incident beam into two interfering beams by utilizing the phenomena of refraction.

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In this experiment light from a narrow-slit S, illuminated with monochromatic light of wavelength λ , is allowed to fall symmetrically on a biprism. The intersection of the two inclined faces forming the obtuse angle must be adjusted accurately parallel to the length of the slit. In this figure, the slit S is perpendicular to the plane of the diagram. Under this condition, the edge B divides the incident wave front into two parts. Firstly the one which in passing through the upper half ABD of the prism is deviated through a small angle towards the lower half of the diagram and appears to diverge from the virtual image S_1 . Secondly the one which in passing through the lower half CBD is deviated

In this experiment, light from a narrow slit S illuminated with monochromatic light of wavelength λ is allowed to fall symmetrically on a Biprism. The intersection of two inclined faces forming the obtuse angle must be adjusted accurately parallel to the length of the slit. In this figure, the slit S is perpendicular to the plane of the diagram. Under this condition, the edge B divides the incident wave front into two parts.

Firstly, the one which in passing through the upper half ABD of the prism, is deviated through a small angle towards the lower half of the diagram and appears to diverge from the virtual image S_1 . Secondly, the one which is passing through the lower half CBD is deviated through a small angle towards the upper half of the diagram and appears to diverge from the virtual image S_2 .

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through a small angle towards the upper half of the diagram and appears to diverge from the virtual image S_2 . The two emergent wave fronts, which intersect at a small angle, are derived from the same wave front and hence the fundamental condition of interference is satisfied. The virtual images S_1 and S_2 , being the images of the slit S , function as coherent sources in this experiment. As a consequence, interference fringes are observed on the screen in the overlapping region EF of the two emergent beams of light.

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If $S_1S_2 = d$ and D is the distance between the plane of S_1S_2 and screen, then like double slit experiment we can obtain the expression of fringe width $W = D\lambda/d$.

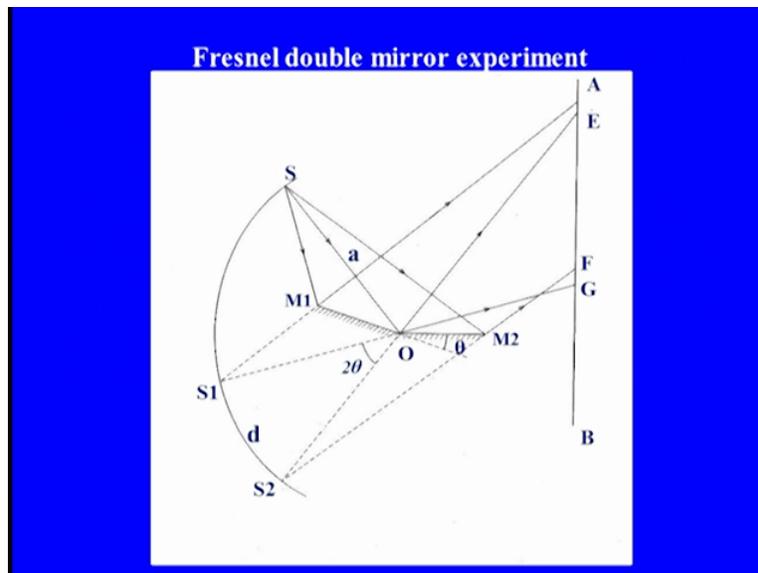
Fresnel Double Mirror

In this experiment of Fresnel, to obtain the sustained interference effects the incident wave front is divided into two coherent interfering wave fronts by utilizing the phenomenon of reflection. In this experimental set up two optically plane mirrors OM_1 and OM_2 are mounted vertically inclined to each other at a small angle θ and

If S_1, S_2 is $=D$ and capital D is the distance between the plane of S_1, S_2 and screen then like double slit experiment, we can obtain the expression for fringe width w is $=\text{capital } D \text{ lambda divided by small } d$. Now, I will discuss another important experimental set up which is known as a Fresnel Double mirror experiment. In this experiment of Fresnel, to obtain the sustained

interference effects, the incident wave front is divided into two coherent interfering wave fronts by utilizing the phenomena of reflection.

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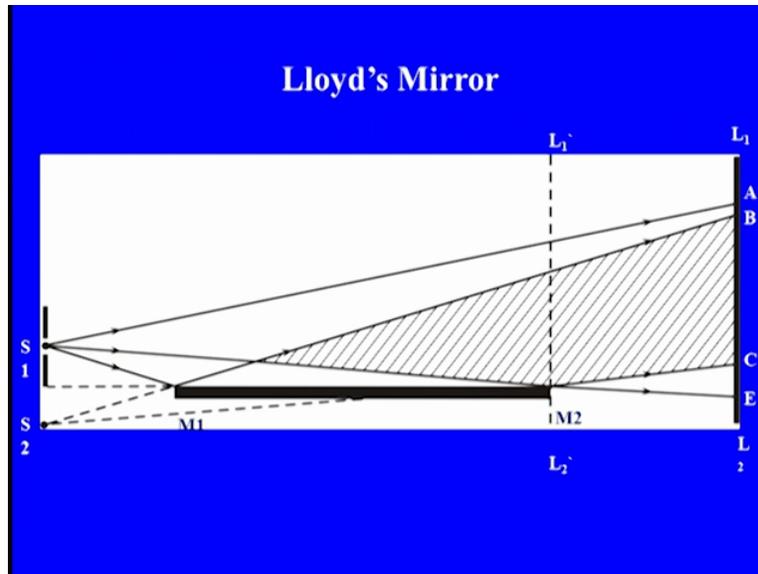
In this experimental setup, two optically plane mirrors OM_1 and OM_2 are mounted vertically, inclined to each other at a small angle θ and touching at the point O . The light from a narrow slit, illuminated with monochromatic light of wavelength λ , is allowed to fall on the mirrors. The line of intersection through O must be adjusted parallel to the length of the vertical slit, as one half of the incident wave front is reflected from OM_1 and appears to diverge from the point S_1 , which is consequently the virtual image of the source slit S in OM_1 .

The other half of the incident wave front is reflected from OM_2 , giving rise to an image S_2 of the source slit S . Obviously, S_1 and S_2 are the virtual coherent sources in this experiment since both are the images of the same source slit S . Thus the fundamental condition of interference is again satisfied here. Furthermore, since the angle θ between the mirrors is very small, the separation D between the coherent sources S_1 and S_2 given by $d = 2a\theta$, where $OS = a$, is also very small.

Thus the condition for the observation of widely spaced interference fringes is also satisfied. The light on the screen AV appears to come from two virtual coherent sources S_1 and S_2 and in the

region EF where the two beams overlap interference fringes are observed parallel to the slit. Now, let us discuss another important experiment which is known as the Lloyd's mirror arrangement.

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In this arrangement, light from a slit S_1 is allowed to fall on a plane mirror and grazing incidence. The light directly coming from this slit S_1 interferes with the light reflected from the mirror forming an interference pattern, in the region BC of this screen. In this experiment, the central fringes at O cannot be observed on the screen L_1, L_2 unless, it is moved to the position L_1 prime L_2 prime where it touches the end M_2 of the mirror, because here light is received only from S_1 .

If the central fringe is absorbed with white light, it is found to be dark. This implies that the reflected beam undergoes a certain phase change of π on reflection. Consequently, when the point P on the screen is such that $S_2P - S_1P = n\lambda$ where n is $=0, 1, 2, 3$, etcetera. We get minimum that is destructive interference. On the other hand, if $S_2P - S_1P = n + \frac{1}{2}\lambda$ where n is $=0, 1, 2, 3$, etcetera we get maximum.

Thus, this experiment provides an experimental confirmation of the fact that a sudden phase change of π occurs on reflection in a rarer medium. That is from the surface backed by a denser medium. The mirror employed here should have slurring on the front surface to avoid multiple

internal reflections and its surface should be adjusted so as to make it exactly parallel to the vertical source slit S_1 . So now, I will discuss the interference with white light.

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Interference with white light

In the Young's double slit, Fresnel biprism and double mirror experiments we have considered monochromatic light of single wavelength to obtain large number of bright and dark fringes. Let us now study the changes produced in the fringes when monochromatic light source is replaced by white light in these experiments. White light consists of wavelengths varying from 4000 to 7000 Å, i.e. from the violet to the red end of white light spectrum. In the experimental set up at any point P_o , situated on the perpendicular bisector of coherent

So, earlier in the Young's double-slit tunnel Biprism and double mirror experiments we have considered monochromatic light of single wavelength to obtain large number of bright and dark fringes. Let us now study the changes produced in the fringes when the monochromatic light source is replaced by white light in these experiments. White light consists of wave lengths varying from 4000 to 7000 Angstrom. That is from the violet to the red end of the white light spectrum.

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Sources S_1 and S_2 , the geometrical path difference is zero for the interfering light waves of all wavelengths present in the white light. Here we get the light of every colour in exactly the same proportions as it exists in white light. As a consequence, the resultant illumination at P_o , due to superposition of zero order (or central) bright fringes of all the wavelengths (colours), is white. The spacing between the consecutive bright or dark fringes, $W = D\lambda/d$ is a function of the wavelength. Obviously, the smaller the wavelength, the closer will be corresponding fringes.

In the experimental setup, at any point P naught, situated on the perpendicular bisector of the coherent sources S1 and S2, the symmetrical path difference is 0 for the interfering light waves of all wavelengths present in the white light. Here, we get the light of every color in exactly the same proportions as it exists in white light. As a consequence, the resultant illumination had P naught due to superposition of 0 order or central bright fringes of all the wavelengths, is white.

The spacing between the constructive bright or dark fringes W is $= \text{capital } D \text{ lambda divided by small } d$ is a function of wavelength. Obviously, they smaller the wavelength the closure will be corresponding fringes.

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Since λ of violet end of visible spectrum is least, therefore, the condition of constructive interference, path difference = λ , will be satisfied for the violet colour and then for other colours, in the spectral order, as we move away from the central fringe. As a consequence, the bright fringe nearer to the central fringe shall have strong violet tinge, followed by other bright fringes having a strong tinge of colours in the spectral order. In reality, no bright fringe is of saturated spectral colour. After 8 or 10 fringes, the

Since λ of violet end of visible spectrum is least, therefore, the condition of constructive interference, that is path difference is $= \lambda$, will be satisfied for the violet color and then for other colors in the spectral order. As you move away from the central fringe, as a consequence, the bright fringe nearer to the center fringe shall have a strong violet tinge followed by other bright fringes, having a strong tinge of colors in the spectral order. In reality no bright fringe is of saturated spectral color.

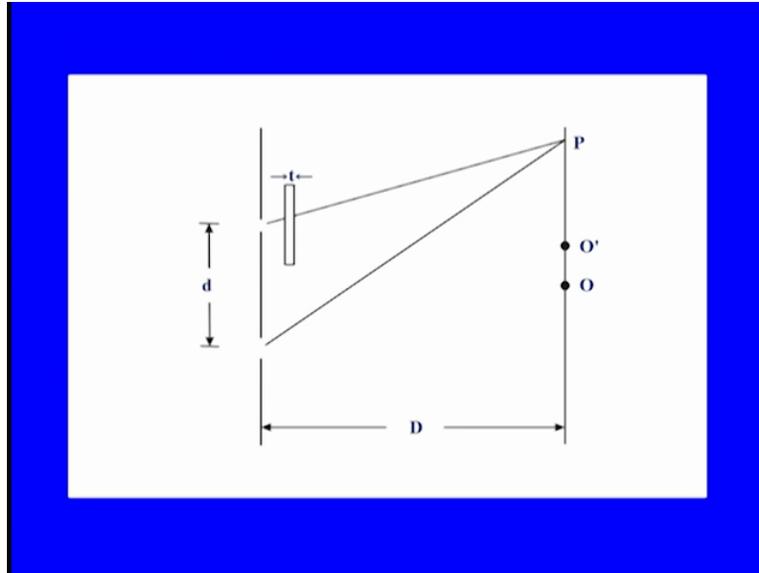
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path difference becomes so large that the condition for constructive interference, path difference = $n\lambda$, may be simultaneously satisfied for a number of wavelengths, for example, we have the relation: path difference = $5\lambda_1 = 7\lambda_2 = 9\lambda_3$ and also at the same point the condition for destructive interference, path difference = $(n + \frac{1}{2})\lambda$, may be simultaneously satisfied for many colours. As a consequence, for large optical path difference, the dark fringes of some wavelengths are completely masked by the bright fringes of other wavelengths. Thus, with white light we get a white central fringe at the point of zero path difference along with a few coloured fringes on both the sides, the colour soon fading off to white.

After 8 or 10 fringes, the path difference becomes so large that the condition for constructive interference part of path difference is = $n\lambda$, may be simultaneously satisfied for a number of wavelengths. For example, we have the relation path difference is = $5\lambda_1 = 7\lambda_2 = 9\lambda_3$ and also at the same point, the condition for destructive interference path difference is = $n + \frac{1}{2}\lambda$ may be simultaneously satisfied for many colors.

As a consequence, for large optical path difference, the dark fringes of some wavelengths are completely masked by the bright fringes of other wavelength. Thus in the white light we get a white central fringe at the point of 0 path difference along with a few color fringes on both the sides, the color soon fading off to white. Now, I will discuss what will happen if we put a glass plate in the path of one of the interfering beam.

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So, let us take the example of the Young's double slit experiment. So suppose, t is the thickness of the transparent plate and it is introduced in the path of one of the interfering beam as shown in this figure. From the figure, it is clear that a light wave traveling from S_1 to P has to traverse a distance t in the plate while the rest $S_1P - t$ it travels in air. Thus, the time required for light wave to reach from S_1 to the point P is given by,

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Displacement of fringes

Let us now discuss the effect of introducing a thin transparent plate of thickness ' t ' in the path of one of the interfering beam as shown in this figure on the interference pattern. From the figure it is clear that a light wave traveling from S_1 to P has to traverse a distance t in the plate while the rest $(S_1P - t)$ it traverses in air. Thus the time required for the light wave to reach from S_1 to the point P is given by

$T = (S_1P - t)/c + t/v$, where c and v are the velocities of light in air and plate, respectively.

$t = S_1P - t$ divided by $C + t$ divided by B where C and V are the velocities applied in air and plate respectively. So, capital t time taken by the beam capital t will be $=S_1 P + \mu - 1$ into t divided by C , where μ is $=C$ by V is the refractive index of plate.

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$$T = [S_1P + (\mu-1)t]/c$$

where, $\mu (= c/v)$ is the refractive index of plate. The physical interpretation of this equation is that due to introduction of the plate effective path from S_1P to P becomes $[S_1P + (\mu-1)t]$ in air. Similarly, the effective path from S_1 to O , the point equidistant from S_1 and S_2 , becomes $[S_1O + (\mu-1)t]$ in air. Thus, when the plate is introduced, the central fringe corresponding to equal path from S_1 and S_2 is formed at the point O' , where $S_1O' + (\mu-1)t = S_2O'$. From geometry we can show that $S_2O' - S_1O' = dOO'/D$. So, $(\mu-1)t = dx/D$, where $OO' = x$.

The physical interpretation of this equation is that, due to introduction of the plate effective path from S_1P to P becomes $S_1P + \mu t - t$ in air. Similarly, the effective path from S_1 to O , the point equidistant from S_1 and S_2 becomes $S_1O + \mu t - t$ in air. Thus when the plate is introduced the central fringe corresponding to equal path from S_1 and S_2 is formed at the point O' where $S_1O' + \mu t - t = S_2O'$. From the symmetry, we can show that $S_2O' - S_1O' = d \cdot OO' / D$. So, $(\mu - 1)t = dx / D$ where $O, O' = x$.

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Thus, the fringe pattern shift by a distance x equal to $D(\mu-1)t/d$. By using white light we can measure the displacement x in the central fringe on introduction of thin transparent plate and with this measured value of x we can determine the thickness of the plate by using the above expression for x .

Thus in the fringe pattern shift by a distance $x = D(\mu - 1)t / d$. By using white light, we can measure the displacement x in the central fringe, on

introduction of thin transparent plate. And with this measured value of x , we can determine the thickness of the plate by using the above expression for x .

Now, let us, let me summarize what I have discussed in this lectures. So, in this lecture, I have discussed what is interference and what is the basic conditions that must be fulfilled to get the stationary interference pattern in the laboratory. And we have discussed, how to get the two coherent sources to get the interference pattern, that is, which is, that is a division of wavefront and the division of amplitude.

And we have also discussed some of the famous experimental set up which are used in the laboratory to get the stationary interference pattern based on division of wave front. In the next lecture, I will discuss some of the experimental setup which are based on division of amplitudes.