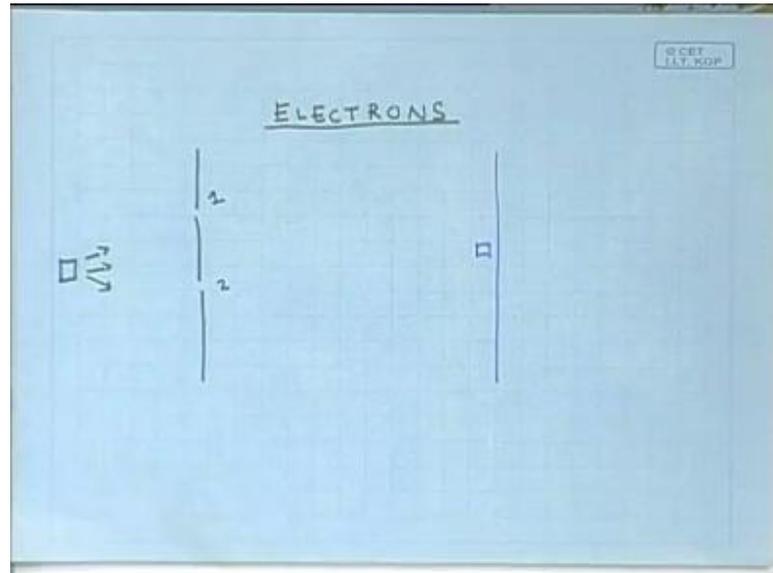


**Physics I : Oscillations and Waves**  
**Prof. S. Bharadwaj**  
**Department of Physics and Meteorology**  
**Indian Institute of Technology, Kharagpur**

**Lecture - 35**  
**Probability Amplitude**

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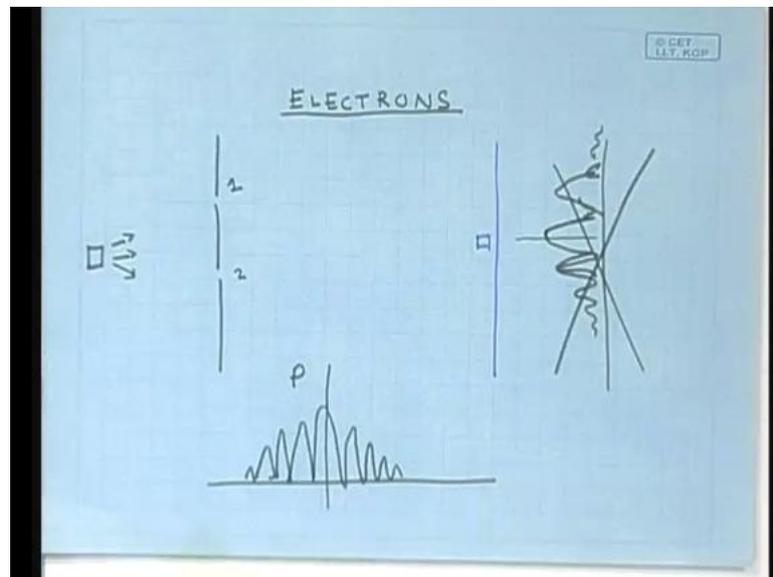
We have been discussing a situation where we have 2 slits and we have a source, which sends out electrons, these electrons are incident on the 2 slits. We had labeled the slits 1 and 2 the electrons, which pass through the slits are incident on a screen. And we have a detector, which can be moved to different points on the screen and what we find when if this, if we do this experiment, where electrons are shot here from the source over here. So, you have an electron gun, which shoots out electrons and you have a detector here on the screen, which can count the electrons arriving at any particular point. So, what happens? When you have done this experiment is that the detector will detect electrons. So, it will always find that either there are no electrons incidents or there will be single electrons.

So, you never receive half an electron, you always receive either a full electron, coming or no electron at all. So, the detector, if you switch on the whole experiment keeps on shooting electrons. And you put the detector here, for a time period let us say 5 minutes and wait; you will find that there will be certain counts, recorded by the certain number

of electrons recorded by the detector. This is going to be an integer, because you are always find that, there is going to be either 1 electron coming out nothing at all.

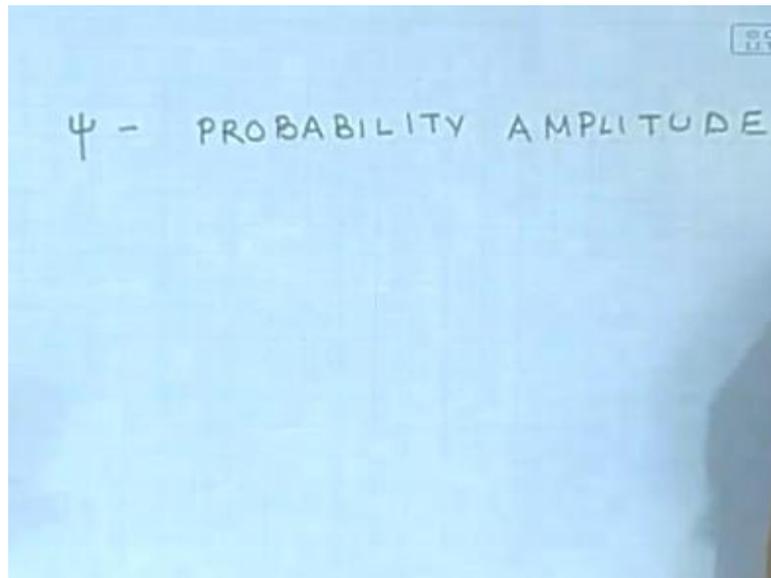
So, you will get some count may be 0 1 2 3 100 1000 something and then you can repeat the experiment placing the detector at different positions. And then, you if you know the number of electrons that are short out in that 5 minute duration, then you can divide the counts that, you receive here by the total number of electrons that, were short out in the same time interval. And by dividing these 2 we get the probability of an electron coming at any point on the screen and if you do this experiment, you will find that the probability distribution looks something like this.

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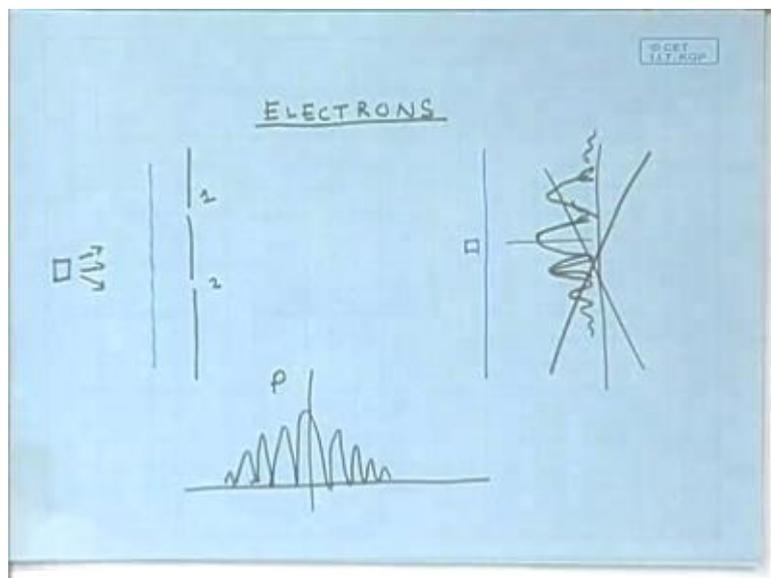
It will have a peak and then, there will be a pattern of maximas minimas, let me draw this again for you this is a picture has got has not come out well. So, if you plot the probability as a function of position on the screen, you will get something, which looks like this. So, you will get this is the probability of finding the electron on some point on the screen, this is the center of the screen. And you will find that, you will get a probability pattern, which looks just like the interference pattern. When we, which you expect to get, when we had discussed the Young's double slit experiment. I also told you that, the only way you can explain this, probability distribution on the screen is by invoking by associating a wave.

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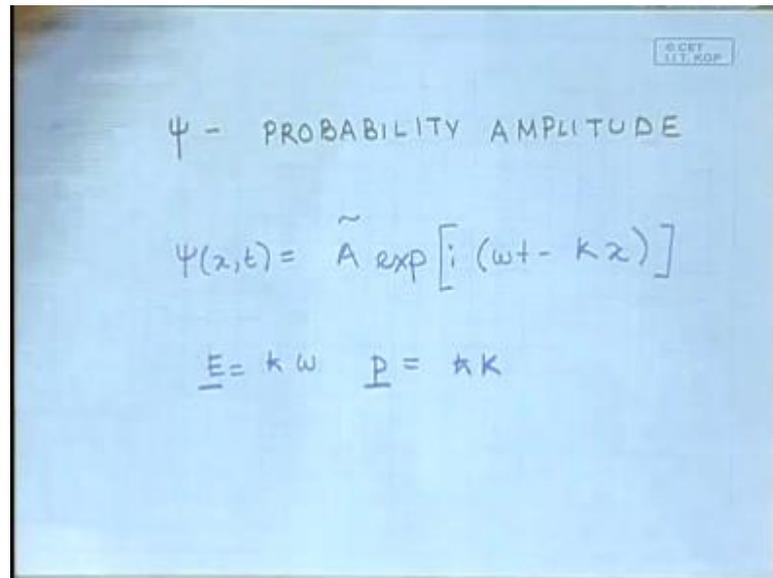
Psi with every electron, so, this wave psi is associating with every electron and this wave; I am referring to as the probability amplitude. So, complex number, which is defined everywhere.

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So, let us, just imagine 1 electron. So, every electron has a wave associated with it. So, when we have, when the wave associated with 1 electron. So, let we draw the wave front, this is the wave front 1 of the wave fronts are the wave associated with 1 electron.

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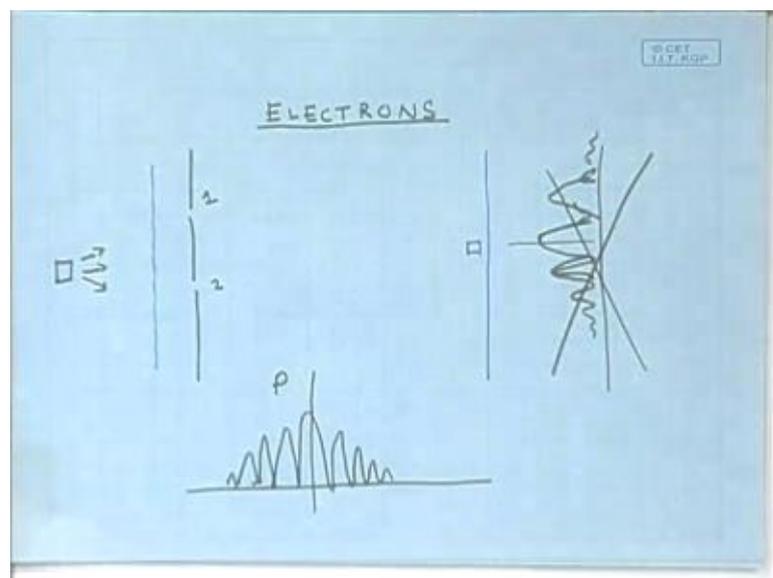


$\psi$  - PROBABILITY AMPLITUDE

$$\psi(x,t) = \tilde{A} \exp[i(\omega t - kx)]$$
$$\underline{E} = \hbar \omega \quad \underline{p} = \hbar k$$

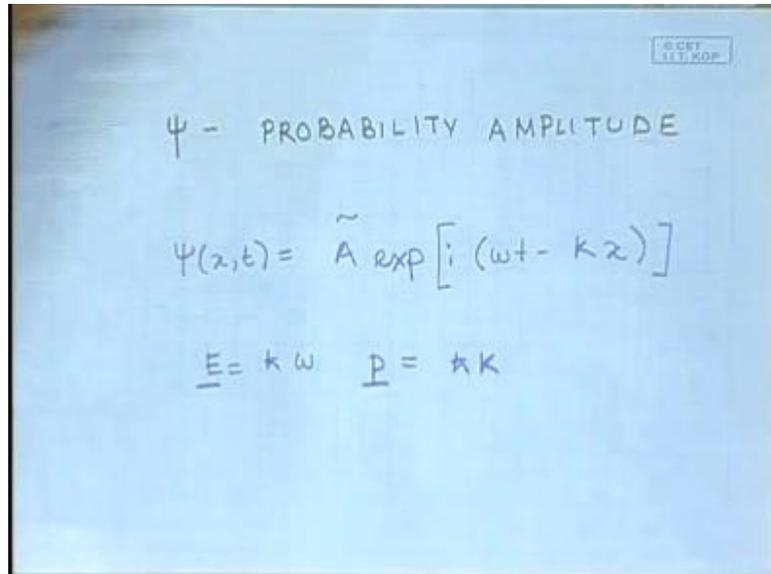
Let us, take a possible situation, where this looks like this I have also told you, that this  $\omega$  is related to the energy of the electron, which is  $\hbar \omega$  and this  $k$  is related to the momentum of the electron  $\hbar k$ . So, let us, just visualize this wave, this wave is a plane wave and associated with the electron with an energy  $E$  and momentum  $p$ . Where  $E$  is  $\hbar \omega$   $p$  along the  $x$  axis is  $\hbar k$  associated with an electron with this energy and this momentum, we write down a wave which looks like this.

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So, this wave the wave front corresponding to this wave, this propagating forward that is a forward propagating wave, which I have written down here..

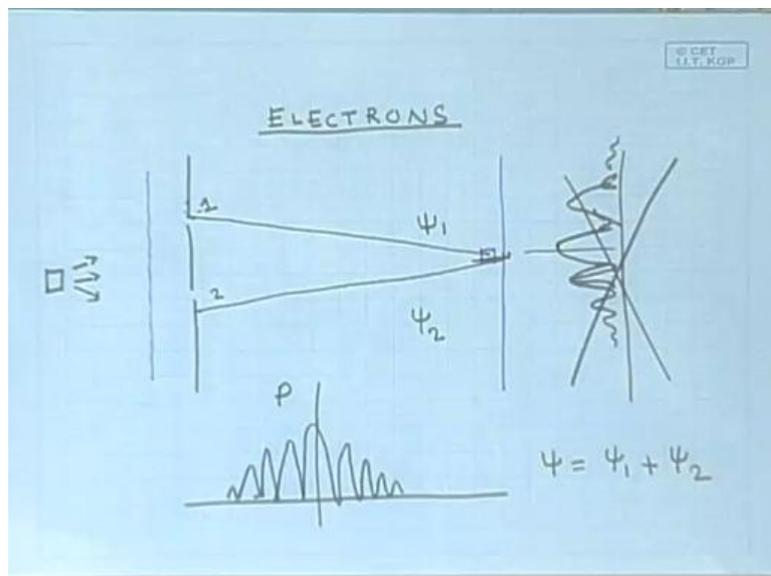
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$\psi$  - PROBABILITY AMPLITUDE

$$\psi(x,t) = \tilde{A} \exp[i(\omega t - kx)]$$
$$\underline{E} = \hbar \omega \quad \underline{p} = \hbar k$$

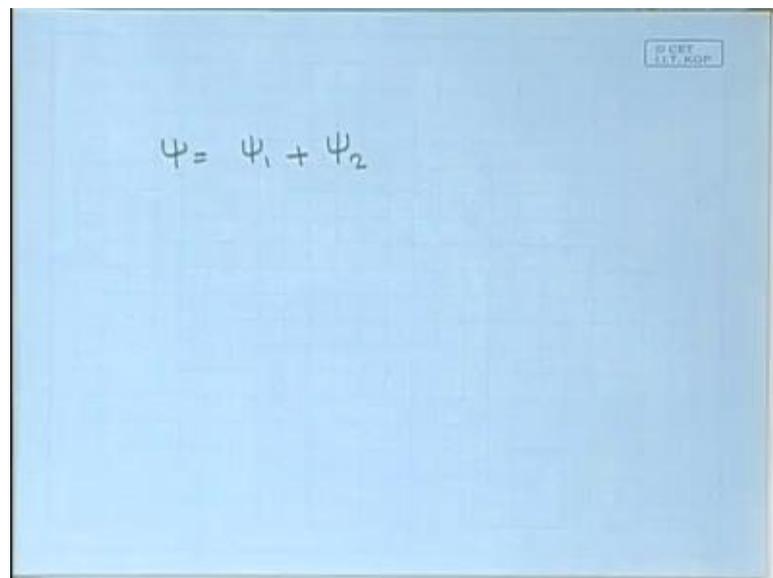
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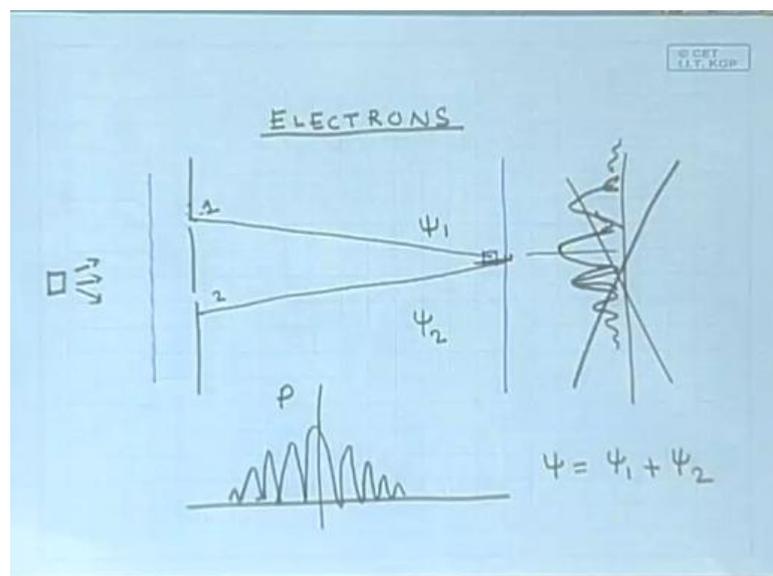
Now, this wave is incident on these 2 slits now, recollect what happens? When a wave is incident on 2 slits each slit now, acts like a source for a secondary wave. So, you can think of each secondary wave coming out from here. And if I wish to calculate the resultant wave  $\psi$  at this point, it will have 2 contributions 1 contribution will come from the first slit. The second contribution will come from the second slit, so, the way; we can

explain this kind of a probability distribution is by associating a wave with every electron. And the wave corresponding to an electron, when it is incident on the 2 slits it each of the slit now acts like a source for a secondary wave. If I wish to calculate the wave  $\psi$  at this point the probability amplitude  $\psi$  at this point on the screen, I have to add up the contribution from this secondary source. And this contribution, we called  $\psi_1$ , we have to also add to the contribution from this secondary source  $\psi_2$  slit 2, which is  $\psi_2$ . And the resultant at this point is going to be  $\psi$  is equal to  $\psi_1$  plus  $\psi_2$ .

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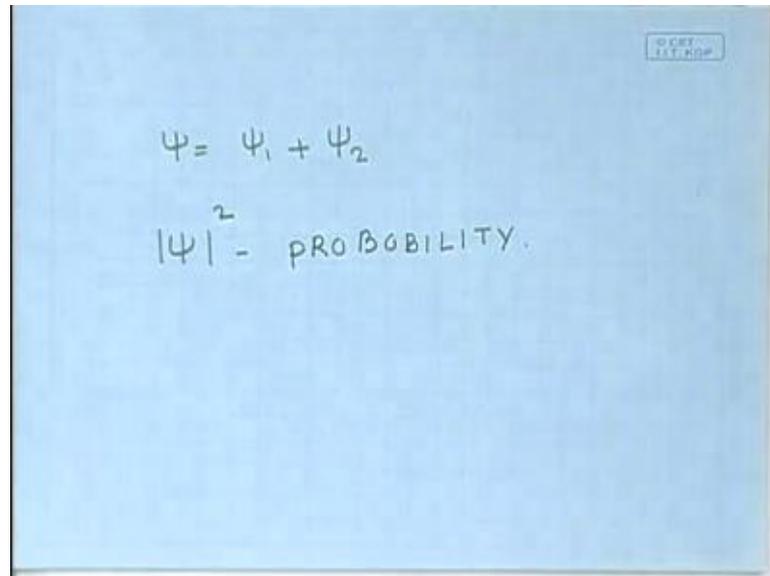

$$\psi = \psi_1 + \psi_2$$

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And to find the probability of of the electron arriving at this point, we have to add up...

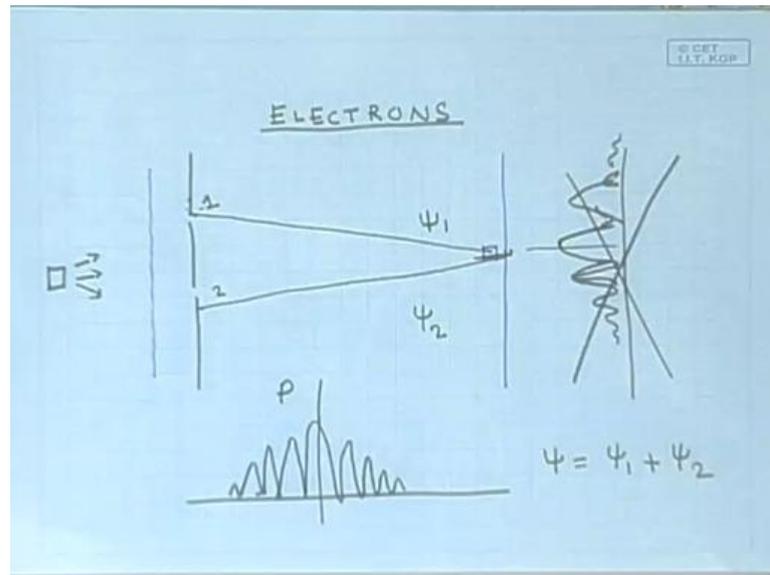
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The image shows a blue background with handwritten text. The first line is the equation  $\psi = \psi_1 + \psi_2$ . The second line is  $|\psi|^2 - \text{PROBABILITY.}$

We have to...The probability is calculated from the probability, amplitude by taking its mod and then squaring it. So, the, it is just like the ways, you calculate the intensity of an electromagnetic wave or a sound wave, you take the modulus and square it of take the amplitude. Find its modulus and square it that will give, you the intensity for an electromagnetic wave or sound wave for this probability amplitude; you take the modulus and square it. It gives you the probability of finding the electron at that particular point on the screen. So, if you calculate the psi the probability amplitude at that point on the screen and then take its modulus and squares it. It gives you the probability of finding the electron at that point. So, the let we make the key point once more if you think of.

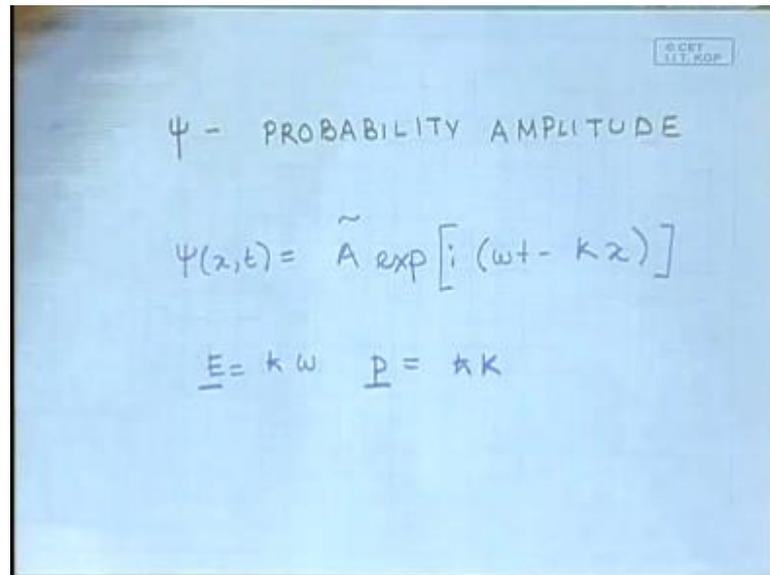
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The electron as a particle, you would associate a trajectory with the electron and if I were to tell you that the electron left here. And was detected at this point classically, you would think of there being 2 possibilities either 1 trajectory going through, this slit and then coming here. There would also be another possibility, where you had a trajectory where the electron went through this slit and arrived at over here. So, there are two possibilities by, which the electron can arrive from somewhere on that side to these two to this point it could be either through this slit or through this slit. Now, if you adobe this picture then the probability of the electron arriving here, would be the probability that the electron came through, this plus the probability that electron came to through this.

And these probabilities, being a positive number would always add up. So, the resultant probability of the electron coming here would be the sum of the probability, that it came through this or it came through this. And 1 could determine the probability of the electron coming to a point through this by closing this and opening this, one could determine the probability of electron coming here through this by closing this and opening this. And then you would add up the probabilities to calculate the probability of the electron coming here, when both are open. But if you do the experiment, you find that this is not true the instead of the probability increasing, when I open both the slits in there are places where the probability goes down. So, the way around this as, we have discussed is to introduce a wave associated with electron, whose and this wave is what we referred to as the probability amplitude.

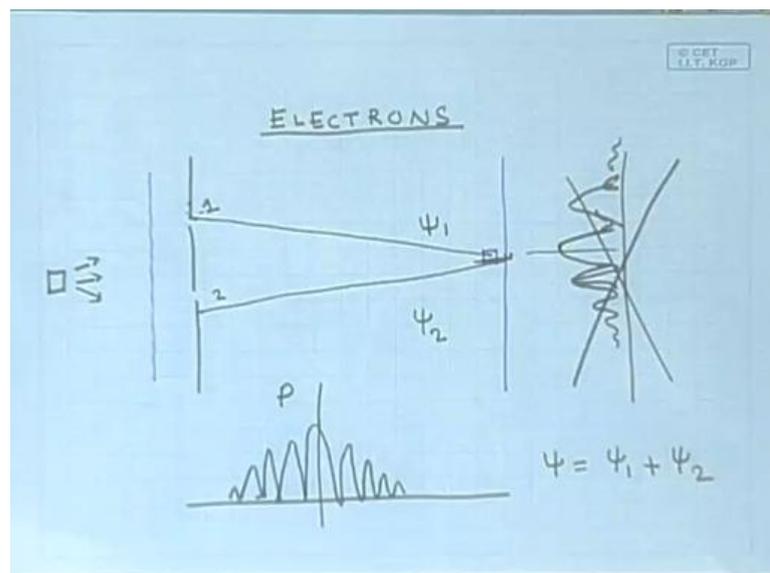
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$\psi$  - PROBABILITY AMPLITUDE

$$\psi(x,t) = \tilde{A} \exp[i(\omega t - kx)]$$
$$\underline{E} = \hbar \omega \quad \underline{p} = \hbar k$$

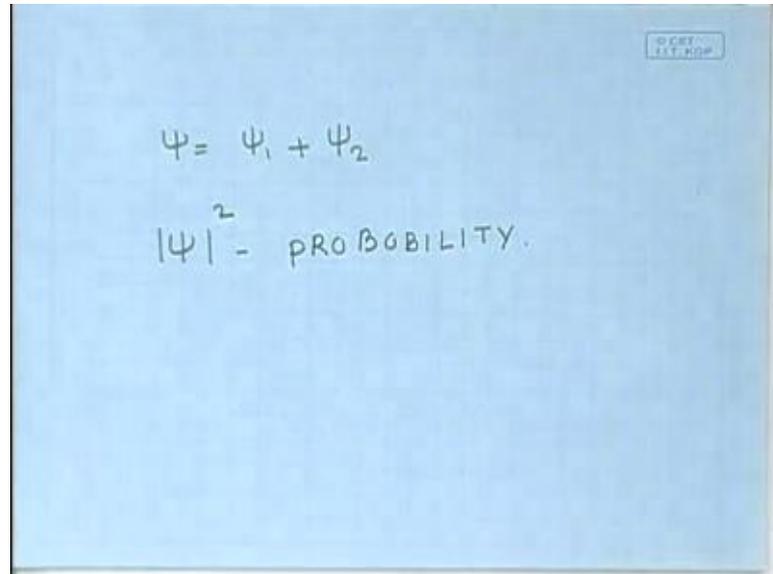
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So, now, when there are two possibilities the electron can go either through this or through this, you have to add up the probability amplitude contribution for this possibility. Where the electron goes through, slit 1 with the probability, amplitude contribution from slit 2. So, you have to add up the waves that contribution to the wave at this point. Or contribution to the probability amplitude of the electron at this point to calculate this, you have to add up the contribution from possibility, where the electron goes through slit 1 which is  $\psi_1$ , if to this you have to add up the contribution from the possibility, that the electron goes through slit 2, which is  $\psi_2$ . So, both of these the

probability and amplitude contributions from both of these have to be added to determine the total probability amplitude at this point.

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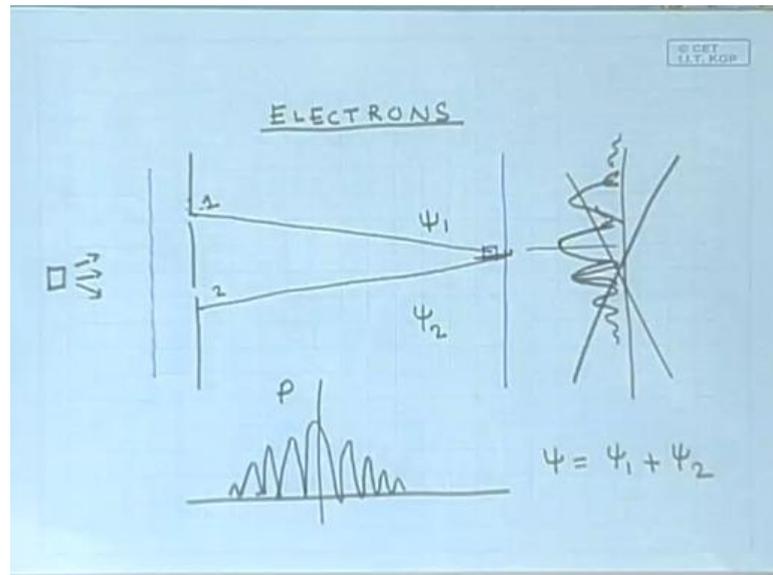


A photograph of a blue surface with handwritten text in black ink. The text consists of two lines: the first line is the equation  $\psi = \psi_1 + \psi_2$ , and the second line is  $|\psi|^2$  - PROBABILITY. The word "PROBABILITY" is written in all caps. There is a small, faint rectangular stamp in the top right corner of the blue area.

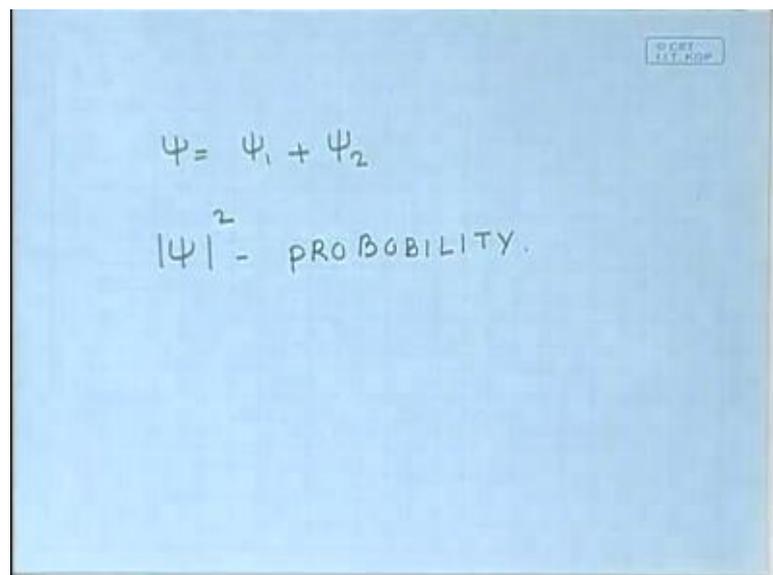
$$\psi = \psi_1 + \psi_2$$
$$|\psi|^2 \text{ - PROBABILITY.}$$

And the square of the probability amplitude gives, you the probability of the electron coming here and since this probability amplitude at that point on the screen at this point of the screen is the sum of these two probability amplitudes.

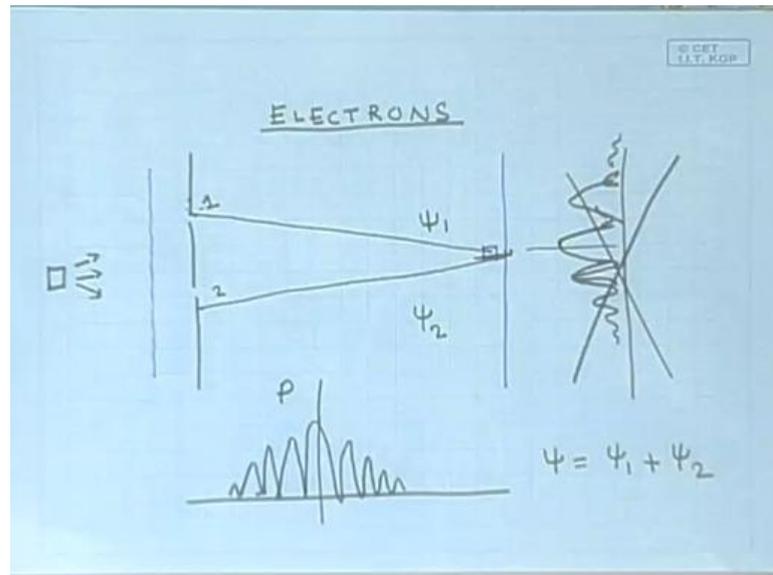
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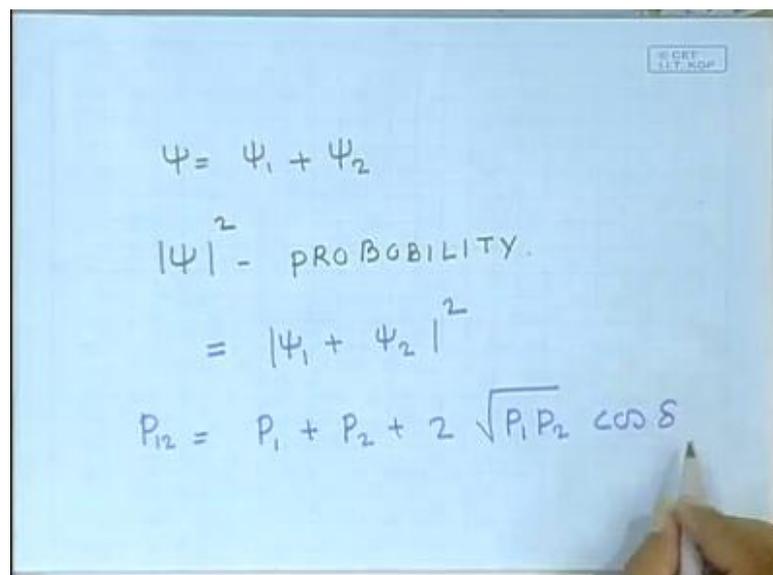
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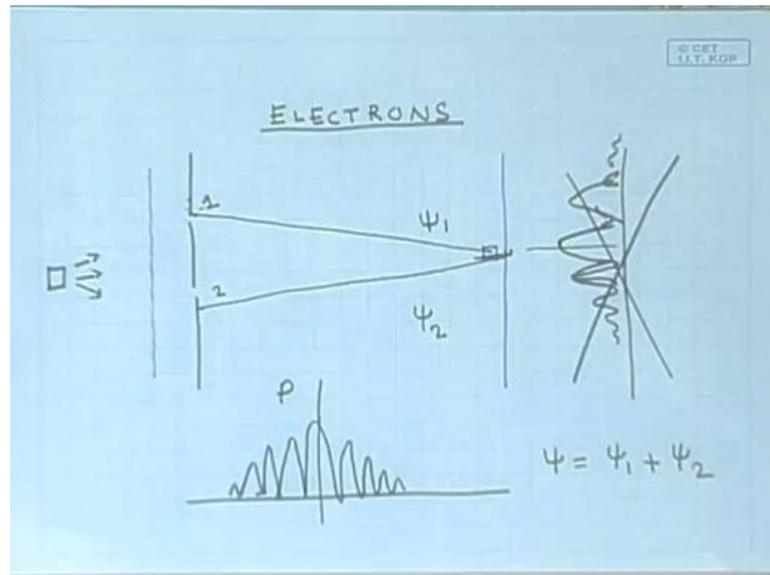


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When I take the modulus and square, it this is going to be equal to psi 1 plus psi 2 square and these probability amplitudes are necessarily complex quantities. So, when you calculate the probability of the electron arriving at any point, when both slits are open it is going to be the probability, when 1 slit is open plus the probability that, the next slit is open plus 2 time the square root of P 1 P 2 cos delta where delta is the phase difference between these 2 probabilities amplitudes, so, it is the phase difference.

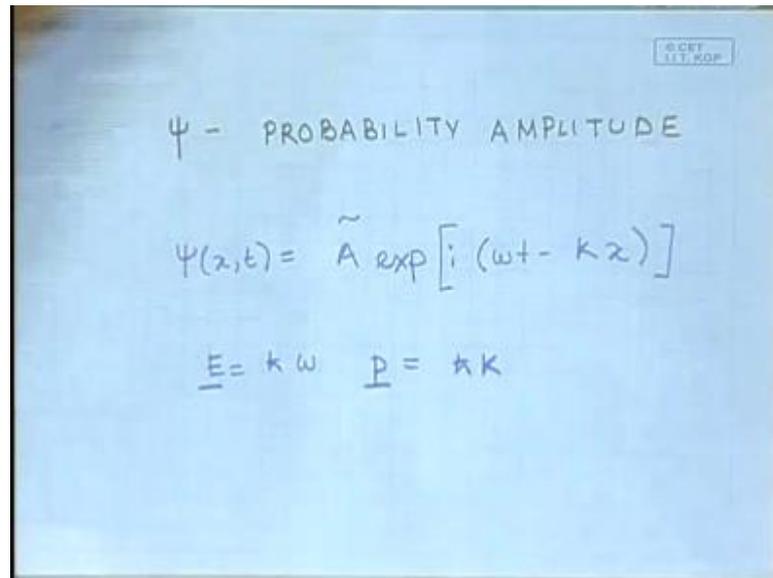
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Between this, contribution from this possibility plus the contribution from this possibility. So, you see the main thing is that the probabilities do not add up. So, when you have 2 alternatives, that the electron can take to reach this particular point on the screen this particular point, then you do not have to you cannot add up the probabilities. To get the resultant probability of the electron arriving there, you have to add up the probability amplitudes. And the probability amplitudes are necessarily complex and you have the phenomena of interference, because the probability amplitudes can be in phase they can be out of phase. If the probability to, if the probability amplitude contributions from the two possibilities are in phase the probability increases, if you are at such a position, where the probability amplitude contributions are out of phase.

The probability actually goes down, it goes below the probability, when only 1 slit is open or only the slit 2 is open. So, you have the phenomena of interference. Now, let us ask the question first question that we can ask is what happens to our old picture where the electron arrives at this point, either through, this or through this? The moment you think of the electron as a wave, you see you cannot think now, you can no longer think of the electron as a particle, because the particle has a well defined trajectory. We now, have to abandon that picture, when we want to understand this kind of a situation and we have to now, think of the electron as a wave. So, we have a single electron the wave corresponding to that single electron is there everywhere.

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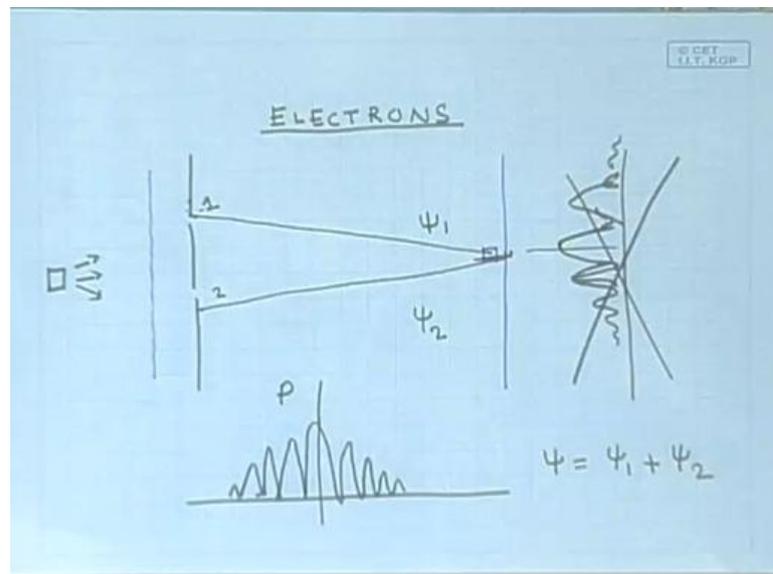


$\psi$  - PROBABILITY AMPLITUDE

$$\psi(x,t) = \tilde{A} \exp[i(\omega t - kx)]$$
$$\underline{E} = \hbar \omega \quad \underline{p} = \hbar k$$

If you write down a wave, which looks like this is defined all over space.

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So, the electron, which is left let out from here is represented by a wave the wave is defined all over this the wave front is propagating forward. When this is incident on the 2 slits some of the wave passes through this and reaches here, some of the wave passes through this and reaches here. So, now, you see that the wave, which is a distributed which is distributed all over space, can simultaneously pass through both the slits. And arrive at the detector and you have a value, which you get by superposing these two

contributions over here. So, you have to abandon the picture that the electron goes either through slit 1 or through slit 2, it the wave actually, goes through both. And arrive at arrives at this point and the resultant at this point is the superposition of both of these contributions. These 2 contributions may be in phase or out of phase and they may add up or cancel out giving rise to this kind of a interference pattern in the probability.

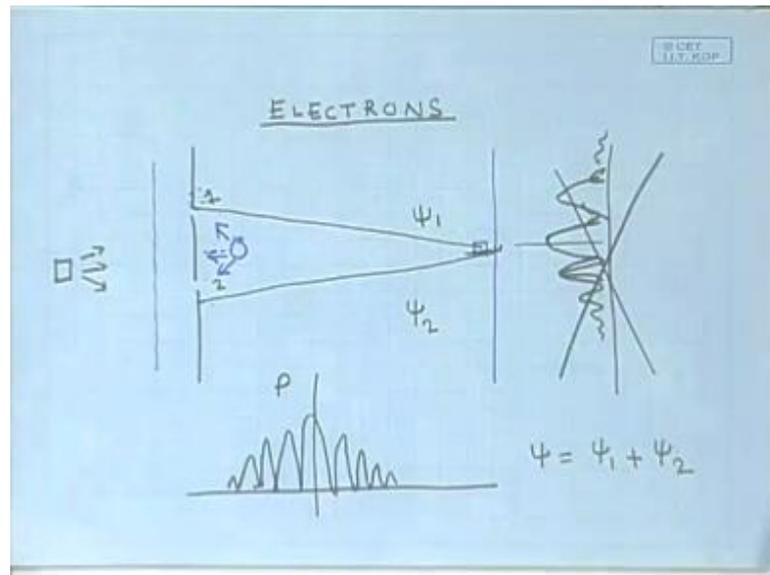
So, the electron is not going you have to abandon the picture, where the electron goes either through this or through, this the wave corresponding to the electron goes through both. So, it is as if the electron, were going actually, going through both. And you have to calculate the probability amplitude over here by superposing these two contributions. So, it is the electron is actually, distributed all over the space the wave corresponding to the electron is distributed all over the space. And in some places these the 2 contributions it at at any point here, it is a contribution of 2 sum of 2 contributions at some places a 2 contribution add up and some places they subtract. Now, so, we have this electron distributed all over this region the wave correspond to the electron at least is distributed all over the space.

Now, when you put a detector and detect the electron, you do not detect a fraction of the electron, because which you may think is what you would expect, because the electron as I have told you distributed all over this region. So, you would expect, you may think that if I were to put a detector here, I would detect the part of the electron, that is here no. This probability amplitude the wave corresponding to the electron, it tells you the probability of finding the electron somewhere, when you actually, detect the electron somewhere, you will always get a full electron. So, the electron, when you the moment you make detection the electron gets localized to that particular point. Until, you make the detection, you cannot say where the electron is the wave corresponding to the electron is there all over the moment; your detector gives you a click, which tells you that the electron is here.

The electron you know now, is at that instant when the electron is detected, you know that the electron is here and nowhere else. So, this is the kind of picture, we have to adopt, if we wish to explain, if we wish to understand this kind of probability distribution that is observed on the screen. And here, we have to allow for the possibility that the electron passes, through both the wave corresponding to the electron passes through, both the slits. Or another way of putting, it is that we cannot we do not cannot answer the

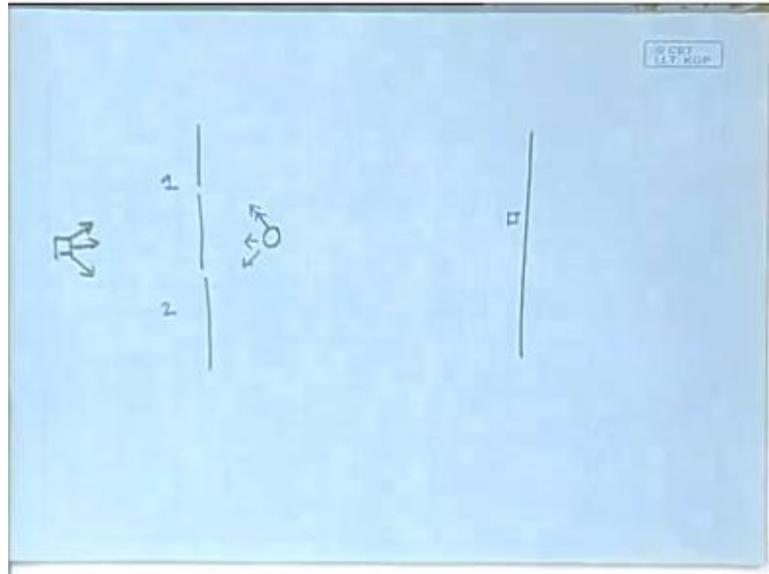
question through, which slit the electron actually passes. The wave corresponding to the electron passes through both the slits. So, you may say that the electron passes through, both the slits and arrives at the detector. You have to really, abandon this picture where electron either passes through this or through this, now, I could modify this experiment slightly.

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So, let me now, discuss that. So, I could what I could do is I could put a light source somewhere over here. So, suppose I were to put a strong light source, somewhere midway between the 2 slits such that this light source illuminates both the slits. Now, if an electron let me draw this picture again, so, we have the 2 slits.

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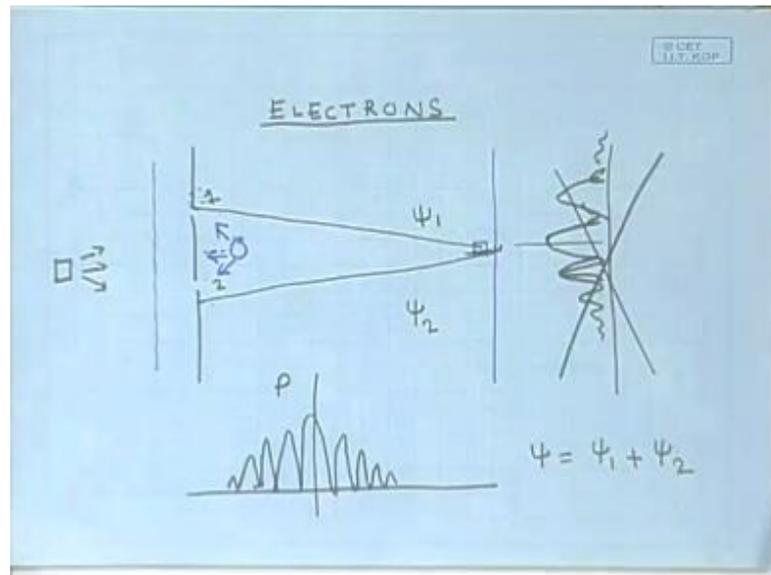
And we have the source, which shoots out electrons and we have a strong light source over here. We have the screen over here; we have the strong light source which illuminates both the screens quite strongly. So, now, if I have an electron, which goes through slit 1 and then arrives at the screen. The moment the electron appears at screen 1, it will scatter some of the light, which is being sent from this light source, we have learnt about this scattering.

So, the movement the electron is incident, so, the electrons are be shot out from here, if the electron passes through slit 1. It will scatter some of the light incident on slit 1 and you will see that there will be a sudden scintillation. It will suddenly get brightened over here; because the scattered light you could you will be able to make out the scattered light. And then, you may find a click at some point at a detector at some position. So, you will now, know that for sure the electron has gone through slit 1 and arrived over here, if you wait for some more time.

Then you will you will find that the there is a sudden brightening over here the electron has scattered a photon over here. And then, you will record a click at some detector on the screen and you will know that the electron has come through slit 2 and then arrived at the screen. So, by such an experiment by such an experiment, you will be able to make out through, which slit the electron has gone and hit the screen over here. So, by such means, you can now, determine through, which slit the electron is going. So, now, we

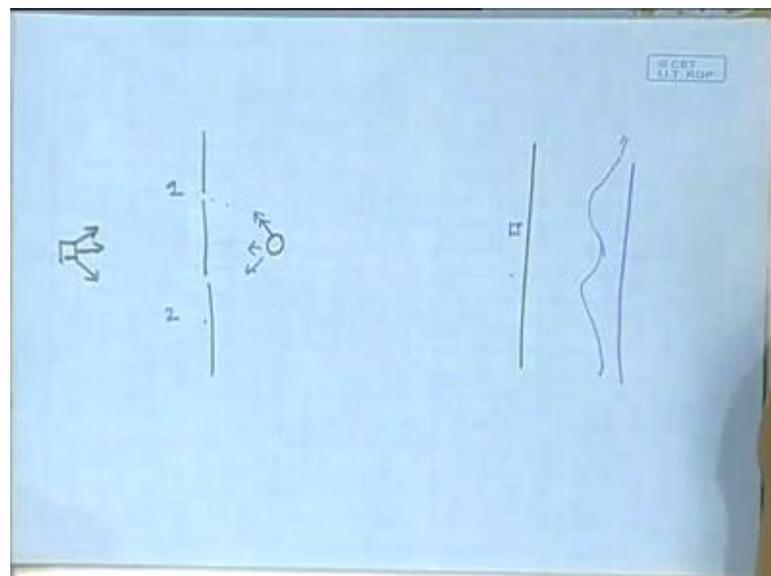
have a situation where you can tell through, which slit the electron goes and then hits the detector. You're longer in the situation, which you had earlier where you could not tell through, which slit the electron goes and it is. So, happens that the movement, you put a light source here which can detect the 2 electrons the electron the whether the electron goes through this slit or through this slit.

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It, so, happens that the interference pattern which you had earlier, when you were not able to tell through, which slit the electron went is now washed away.

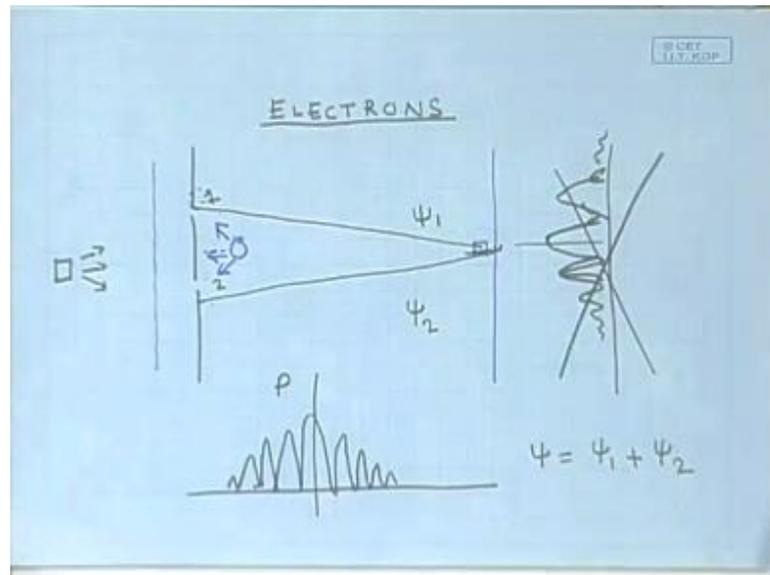
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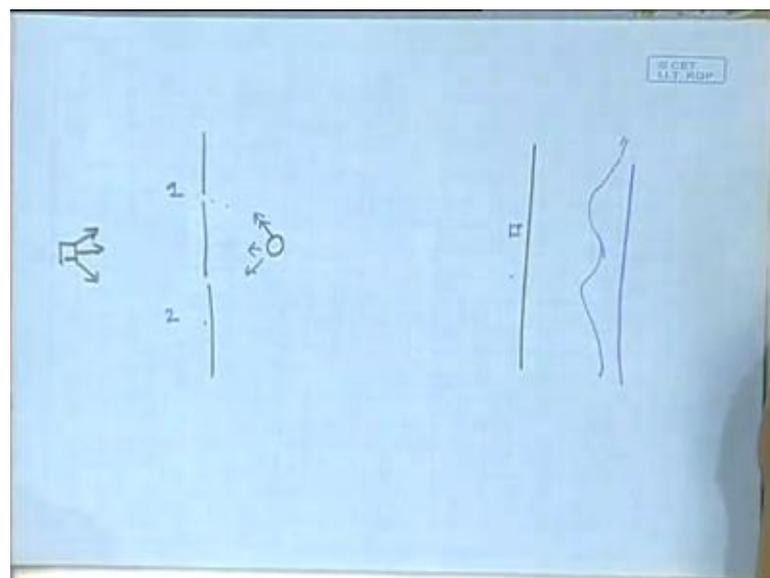
What you now, get will look like the sum of the 2 probability distribution. So, it will have 1 peak somewhere here and then there will be one more peak somewhere here, it will look like. It is the sum of the 2 probability distributions one of the electron going through this, slit and then arriving at the screen. The second being of the probability of the electron arrive going through, this slit and arriving on the screen. Now, you see why does this happen? We can also understand that the moment, you see the light emitted by the source; we have already learnt that the light you could that you should think of the light in terms of photons. And the photons carry energy and momentum, so, when an electron scatters a photon there will be an exchange in the electrons change in the electrons energy and momentum.

So, the wave, which was coming, the wave associated with the electron, is now, going to get disturbed and this fact that, you have disturbed the electron will erase will wipe out the interference pattern that you had that you would expect, if you were not to disturb the electron. So, the disturbance the momentum important to the electron will kick it, give it a kick and that you cannot predict, where the electron is going to go as a consequence of that kick. So, the, and this disturbance also wipes out the interference pattern, which you would have seen, if you were not to put in this disturbance. So, the act of determining through, which slit the electron passes disturbs the wave associated with the electron and this interference pattern interference pattern is washed away.

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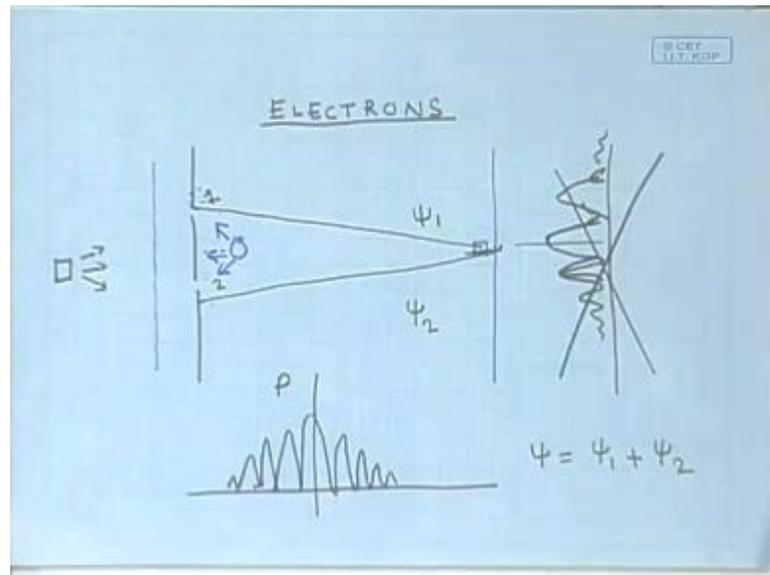
And this is what you get now; you could try and try to overcome this restriction. So, 1 possible thing, that you might you could possibly think of doing a possibility, which you may consider you would you might consider try and overcome this. So, that you could detect, both suppose you wish to try and detect both, that is both the interference pattern as well as determine through, which whole the electron goes this, would be something which would contradict everything right. The interference pattern, we see arises, because of the possibility that the wave corresponding to the electron can go through, both of these. But if you knew through, which whole the electron slit the electron went then,

there would be no need for considering the wave going through both ends. If you were to still detect the interference pattern then, we know that they would be some kind of a contradiction and we would be in deep trouble. So, the question is does this thing arise or does it arise through some means, by which it never arises. So, you may try to make such a situation arise by reducing the intensity of the light now. What will happen if you reduce the intensity of the light is as follows.

If you reduce the intensity of light, you will find that there are some photons, there are some electrons, which do not scatter a photon over here, but reach the detector on the screen. Because you will find that, there will be clicks on detectors located at the screen, but you will find there will be no corresponding brightening suddenly at near 1 of these 2 slits. So, you will not know, which through which of the 2 slits the electron has gone, but you will find that the electron has reached here. This will sometimes, you will find that the electron produces a sudden brightening near slit 1 sometimes, it produces a slight sudden brightening near slit 2 followed by a click on the detector on the screen. So, when you reduce the intensity of the light, you do not reduce the energy or the momentum of the photons, what you do is you reduce the number of photons.

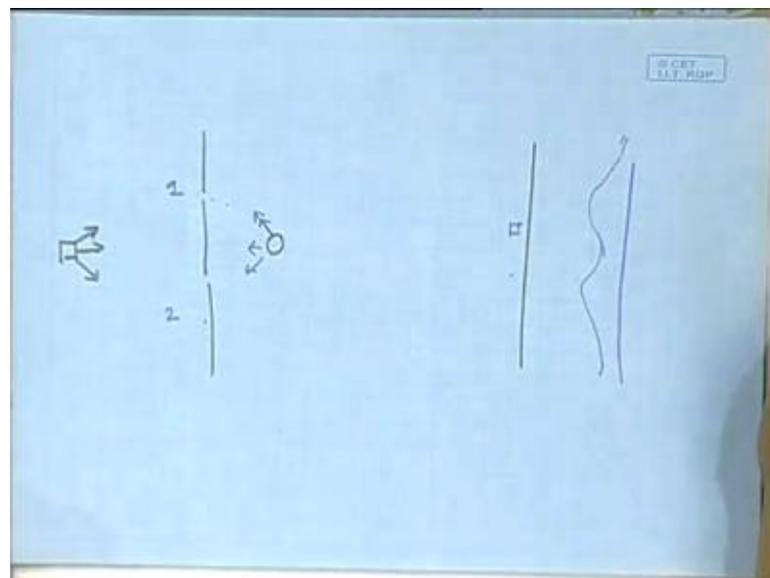
And if you reduce the number of photons then there will be some electrons, which will get through the slits without being able to scatter a photon and such electrons, will not produce any kind of brightening either here or there they will just directly reach the screen. Some other electrons will scatter photons and for those electrons, you will know that, it has gone through, either this or through this. Now, in such a situation, if you record the electrons which produce scattering here, so, these are the electrons, which you can tell that this electron went through slit 1 this electron went through slit 2 this electron went through slit 2 that went through slit 1. So, for these electrons, you will find that the probability distribution looks, just like the same good old thing that, you would expect, if you did the experiment with bullets. The probability distribution would be the sum of the 2 probabilities one of the electron going through, this the other for the electron going through this. Whereas for the electrons for, which there was no scattering, you will get if you if you plot the probability of those electrons arriving on the screen.

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For those electrons, you will get the interference pattern, where which you would have got if this light source were not there.

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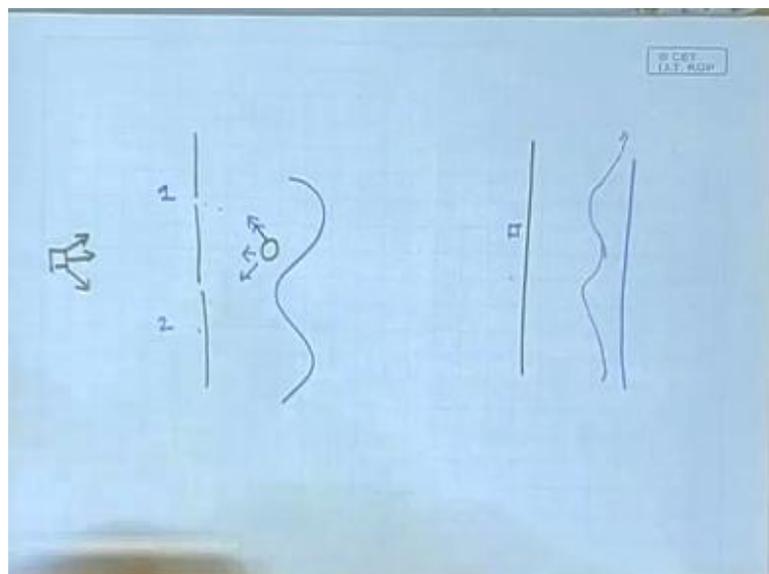


So, you see for the electrons for, which you can tell through, which slit it has gone the interference pattern is washed away for the electrons for which you cannot tell through, which slit, it goes the interference pattern remains intact. Now, you could try another thing, you could try to reduce the wave length of the light that is coming out of the source. When we discussed Compton Effect, we learnt that, the energy and the

momentum of the photon are proportional to the frequency or inversely proportional to the wavelength of the light. So, if you use light of a very large wavelength, you may be able to detect scatter the electron. So, you will be able to detect the scattered photon, so, so, if you get scattered photon from here, then you will know that the electron has gone through this. If you get scattered photons from here, you will know the electron has gone through this. So, if you can reduce the wavelength, so, make it. So, small that the momentum the, momentum of the photon is very small.

So, you can then scattered the electron without disturbing the electron too much and may be, you will still get the interference pattern. So, let me make the point again, you can try modifying this experiment by reducing the frequency or increasing the wavelength of the photon with the hope, that it now, carries a very small momentum. The photon now, carries a very small momentum. So, even if, it scatters of the electron, it will not disturb the electron too much. And I will still see the interference pattern, because the disturbance is not too much, but the photon would have got scattered and I will be able to say through, which slit the electron a has gone. But unfortunately, in this situation, what will happen? Is as follows we have learnt that the resolution of any optical instrument anything that depends on waves is the resolving power to resolve. The angular resolution is proportional to the wavelength that is the larger the wavelength. The poorer the resolution in the sense, that I will not be able to resolve very fine angles. I will only be able to resolve very large angles.

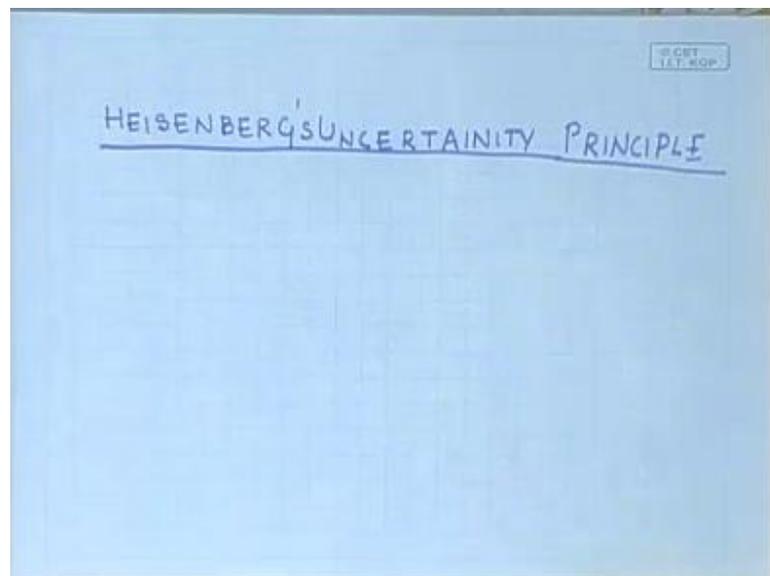
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So, if you increase the wavelength and do this experiment it, so, happens that if you choose a wavelength, which is sufficiently large that the momentum transferred to the electron is not in is is sufficiently low. So, that the wave is not disturbed and you do get the interference pattern it. So, happens that in all such situations the resolution is so poor that, you are not able to say through, which slit the electron passed. Let we make this point again, if you increase the wavelength, if you make your wavelength very large, it is true that the momentum transfer to the electron is going to be small.

You will still get the interference pattern after scattering, but you will not be able to say the wavelength being so large you will not be able to say whether the electron passed through this or this. The resolution is going to be larger than, this and it is going to be you are going to get a very course picture. So, you will know the electron has passed, because you will see the scattering passed through one of these, but you will not be able to tell through which 1 it has passed. So, you will the interference pattern, but you will not know, through which slit the electron has gone. So, there is a underlying principle, which is on which the whole of quantum mechanics is based.

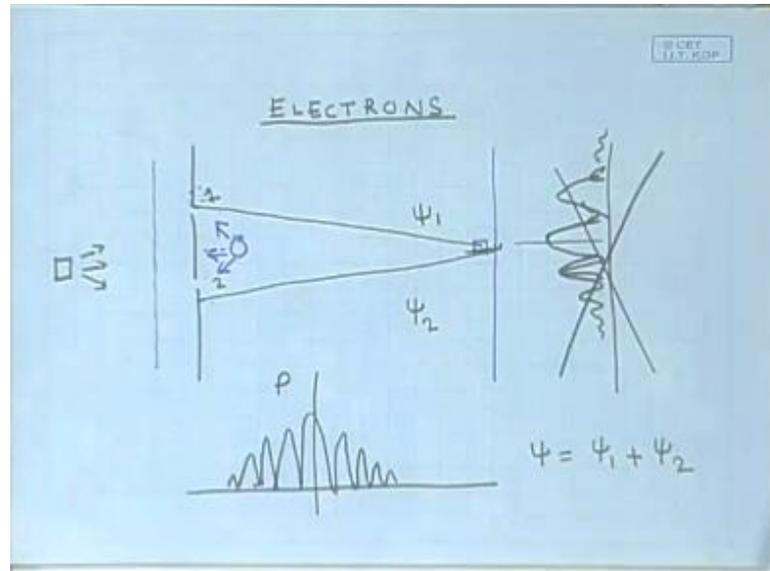
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The underlying principle is the Heisenberg's Heisenberg, which was originally proposed by a German scientist Heisenberg uncertainty, principle Heisenberg's uncertainty principle. So, in most general terms the Heisenberg's uncertainty, principle states that if

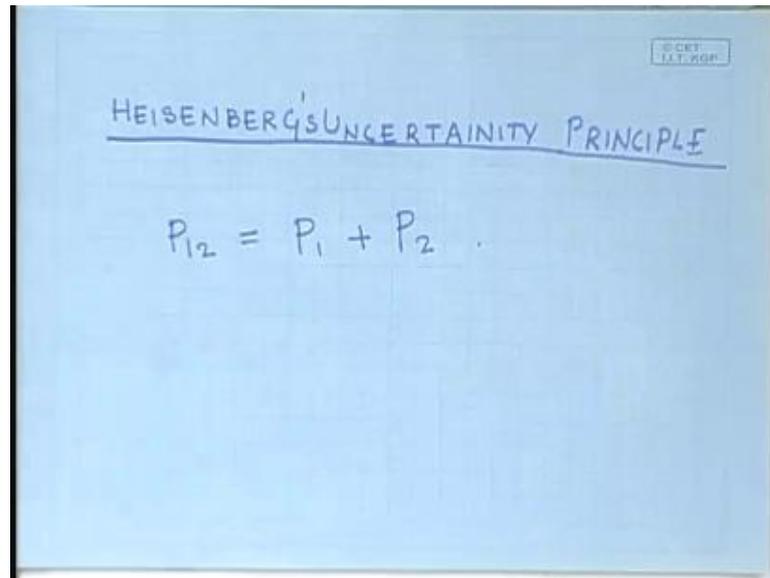
there are two different alternatives, which the electron or the particle could take for example, in the situation which we have been considering.

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The two alternatives are, the electron could go through, this slit or this slit then, what the Heisenberg's uncertainty, principles states is that if the act of measurement. If we try to measure through, which of these 2 alternatives the particle chooses, if we try to measure in this particular case through, which of the slits. The electron goes then the act of measurement will disturb the electron. So, much if you can determine through, which of these 2 slits it goes or if more generally if there are 2 alternatives. If we actually, determine through which of these 2 alternatives, it chooses. Then the act of measurement will disturb the particle in this case the electron so much that the interference will be wiped away. And if the interference is wiped away, then you have to just add up the probabilities for the 2 alternatives.

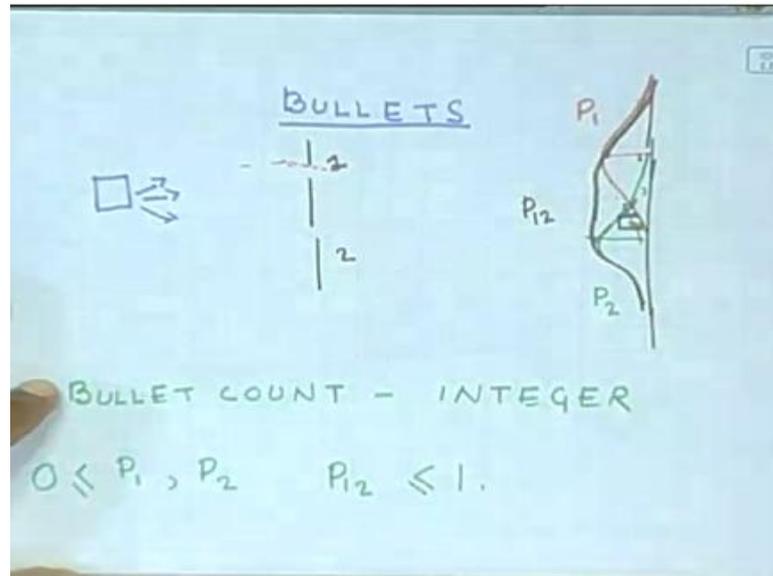
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So, in such a situation the probabilities  $P_{12}$  is going to be  $P_1$  plus  $P_2$ . So, if you measure the act of measurement, itself is going to disturb the system. So, much that you will not be able to you will wipe out the interference pattern. So, if you make a measurement, which can tell you if there are 2 alternatives for the system. And if you do not know, which of these 2 alternatives the system is going to go through, then you have to add up the probability amplitudes and you will get interference. Whereas, if you make a measurement and determine, which of these 2 alternatives the particle chooses the act of making the measurement is going to disturb the system so much. That the act of measurement, itself at this disturbance introduced by the act of measurement is going to wipe out the interference pattern.

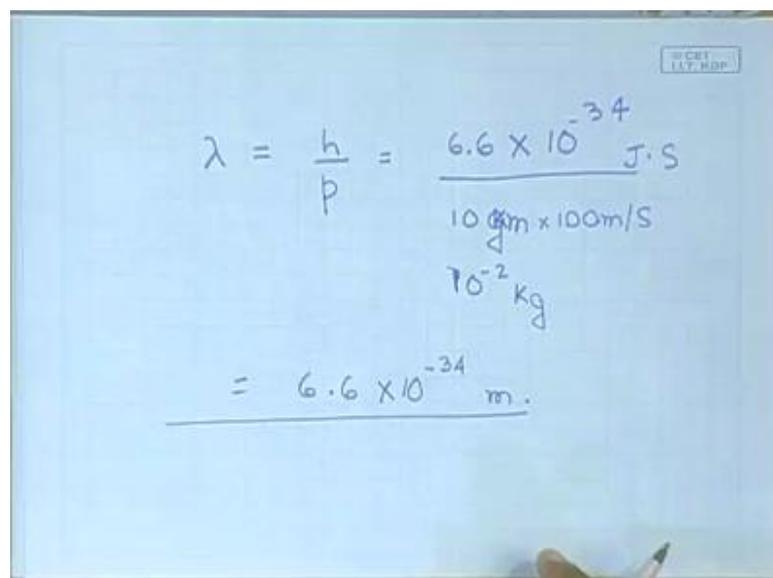
So, let me repeat again, what the summary the gist of what we have been? What I am trying to get across and what I am trying to get across is a is a different picture, which has to be invoked. When we are dealing with the microscopic world, when thing with things happening in the microscopic world, you may ask the question, why do not we see this kind of an interference pattern? In the probability distribution or in the distribution of bullets for example in the distribution of bullets, so, if I shoot 2 bullets, that is the experiment, that is the situation which we have discussed in the last lecture.

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If we shoot bullets on to 2 slit and observe the probability distribution of the bullets on a screen placed behind the 2 holes or the 2 slits. Then the probability distribution, when both slits are open is the sum of the probability distribution, when only 1 is open that is only 1 slit 1 is open which is what is shown in red. And only when slit 2 is open, what is shown in green if I open both of them, I will get the sum of both of these probabilities, which is what is shown in black. The question is why do we not see, the interference pattern corresponding to the bullets. So, let me remind you that the wavelength what.

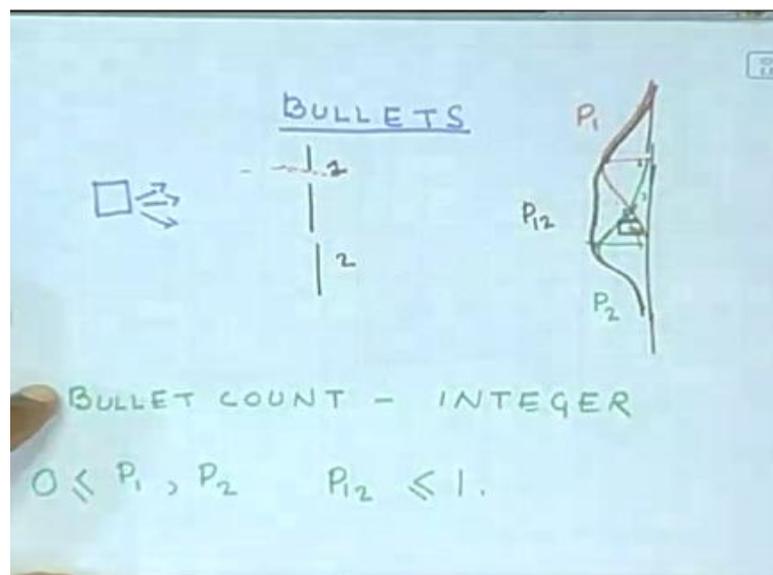
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Corresponding to this probability amplitude wave, remember that this probability amplitude wave is the same thing that, which we have been talking about today. And in the last lecture, is the same thing that de Broglie had hypothesized, so, what we have been discussing is basically, the interpretation of the wave originally hypothesized by de Broglie and remembers that the wavelength is expected to be  $h$  by the momentum  $p$ . And the value of  $h$   $6.6 \times 10^{-34}$  joules second I am not writing the units. We are working in SI units and let us, also estimate the typical momentum of a bullet. So, of some classical particle need not be a bullet.

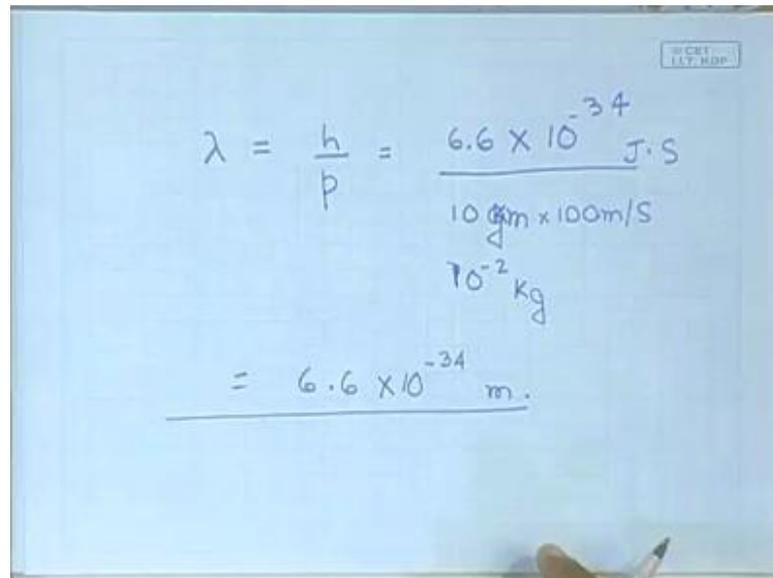
So, let us, take the mass to be say 10 grams some typical some number, which I am putting in a bullet need not be really 10 grams, but this is just to get an idea. And let us, take a speed of let us, say 1 kilometer or 100 meters per second. So, 100 meters per second, so, 10 grams 100 meters per second so, 10 grams would be  $10 \times 100^2$  Kg, and if I multiply it with 100 meters per second. So, I will get a factor of  $10^4$  in the denominator and the wavelength, this is joule second let me put the units. So, the wavelength is going to come out to be of the order of  $6.6 \times 10^{-34}$  this is  $10^{-34}$  this is  $10^{-34}$  meters. So, this exactly, cancels out in the denominator and this is of the order of  $10^{-34}$  meters.

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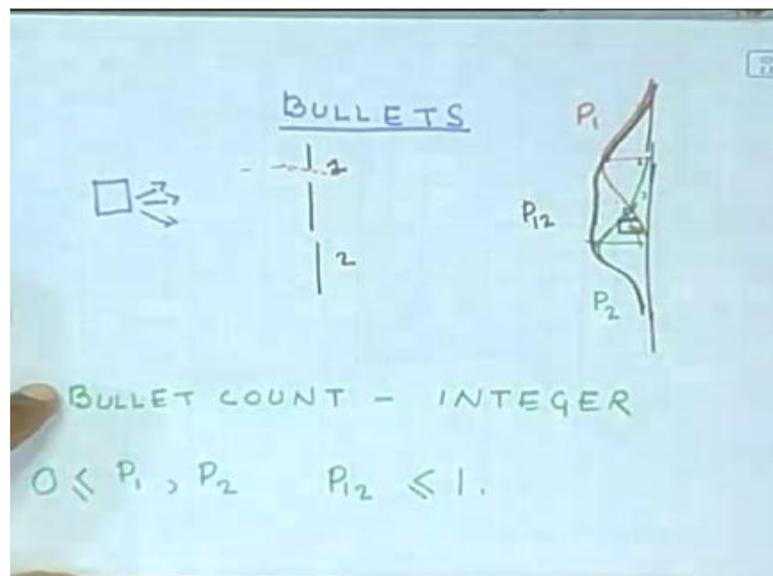
And so, the wavelength, if I were to associate a wave with the bullet the wavelength would be of the order of  $10^{-34}$  meters.

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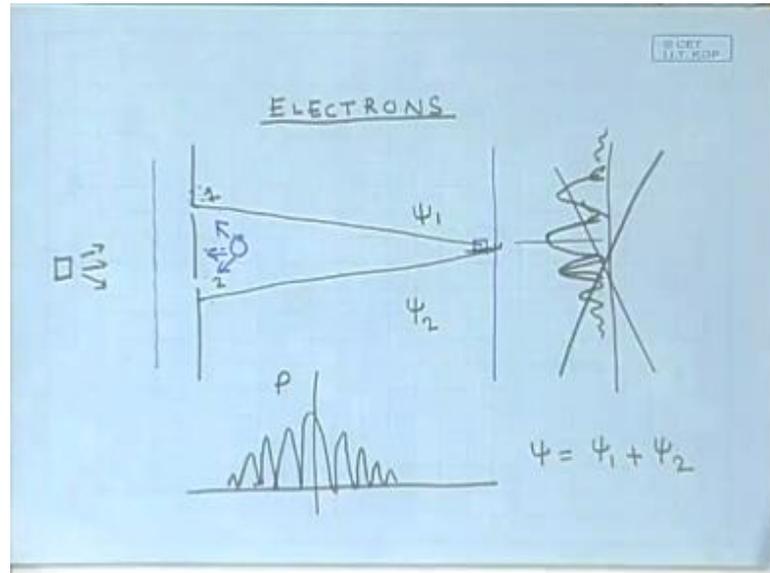
A handwritten calculation on a whiteboard showing the de Broglie wavelength formula. The equation is  $\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34} \text{ J}\cdot\text{s}}{10^{-2} \text{ kg} \times 100 \text{ m/s}}$ . The final result is  $\lambda = 6.6 \times 10^{-34} \text{ m}$ .

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And remember the Young's Double Slit experiment the interference pattern in terms of angle.

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If the in terms of angle subtended over here in terms of that the spacing between the maximas is of the order of lambda by D. Where lambda is the wavelength of the wave D is this separation. So, if I take the separation to be something like 1 micrometer, so, 1 micrometer lambda by D. So, this is the value of lambda.

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$$\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34} \text{ J}\cdot\text{s}}{10^{-2} \text{ kg} \times 100 \text{ m/s}}$$

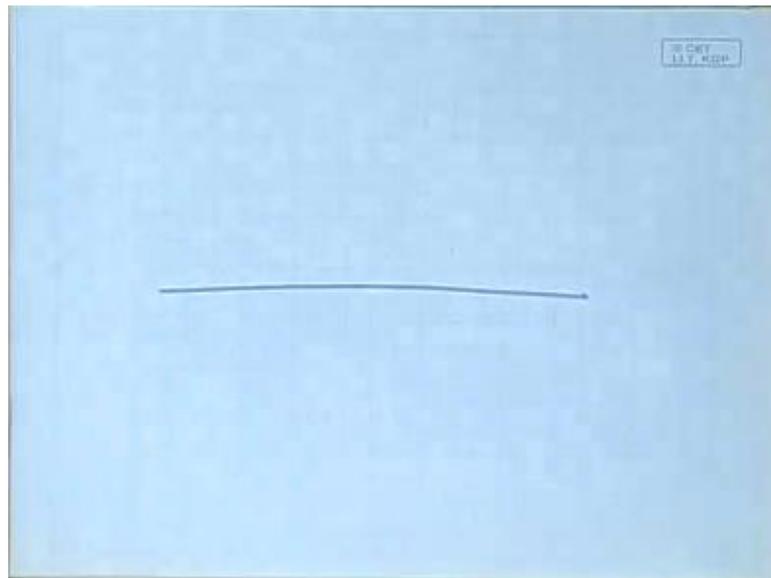
$$= 6.6 \times 10^{-28} \text{ m.}$$

$d \sim 1 \mu\text{m} \quad \frac{\lambda}{D} \sim 6 \times 10^{-28} \text{ radians}$

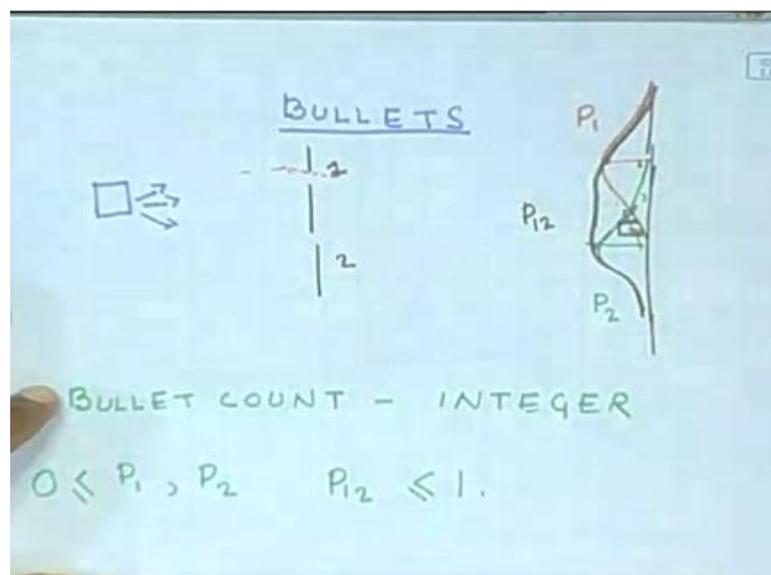
And the spacing between the 2 slits let me take to be 1 micrometer. So, lambda by D is going to be of the order of 6 into 10 to the power minus twenty eight meters. Right I am dividing 6 point 6 3 6 point 6 10 to the power minus 34 divided by 10 to the power

minus 6. So, I get 10 to the power minus 28 meters. So, this is now, this sorry this is  $\lambda$  by  $D$  it is an angle in radians. And if I put the screen 1 meter away, then I am going to get this is going to be the spacing between the maximas, if I if my screen is 1 kilometer away, then I am this is going to get multiplied by 1000. And still it is very small number.

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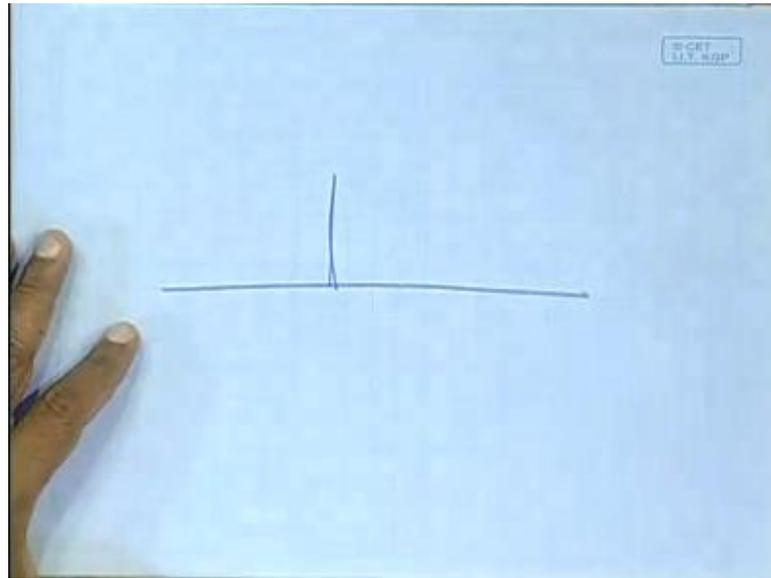


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So, the intensity pattern that I am going to see on the screen over here if the screen is 1 kilometer away from the 2 slits there are going to be...

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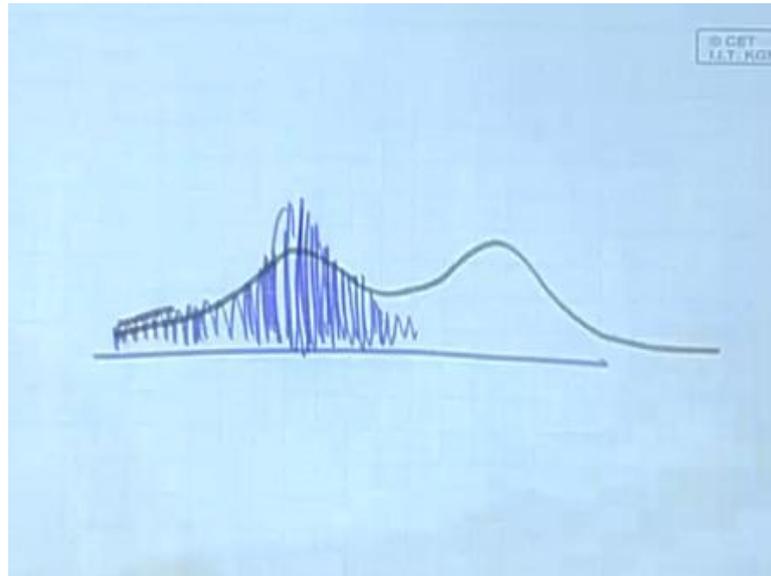
These intensity variations, but the spacing between the maximas is going to be extremely.

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$$\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34} \text{ J}\cdot\text{s}}{10^{-2} \text{ kg} \times 100 \text{ m/s}}$$
$$= 6.6 \times 10^{-34} \text{ m.}$$
$$d \sim 1 \text{ pm} \quad \frac{\lambda}{D} \sim 6 \times 10^{-28} \text{ radians}$$
The image shows handwritten calculations on a blue background. The first equation is  $\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34} \text{ J}\cdot\text{s}}{10^{-2} \text{ kg} \times 100 \text{ m/s}}$ . The second equation is  $= 6.6 \times 10^{-34} \text{ m.}$ . The third equation is  $d \sim 1 \text{ pm} \quad \frac{\lambda}{D} \sim 6 \times 10^{-28} \text{ radians}$ . A hand is visible on the left side of the image, pointing towards the calculations.

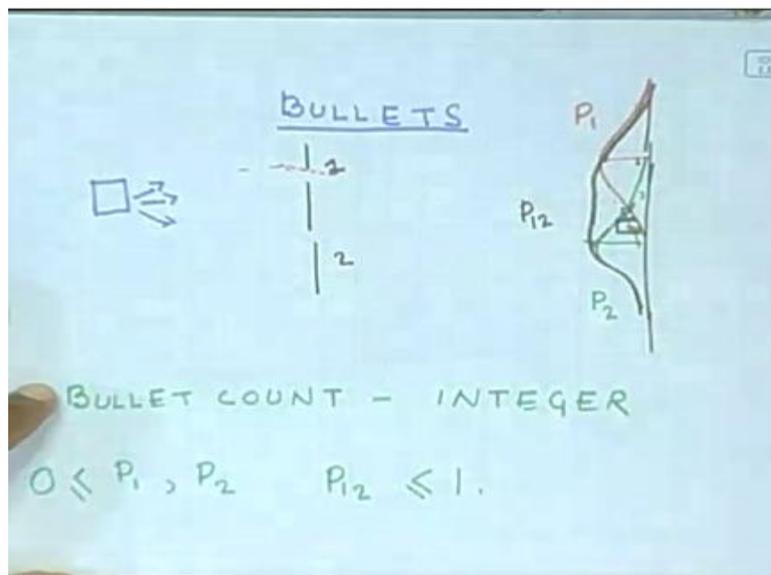
Small for 1 kilometer, it is going to 10 power minus 25 meters, because this is the angle multiplied by the distance, I will get the separation between the fringes.

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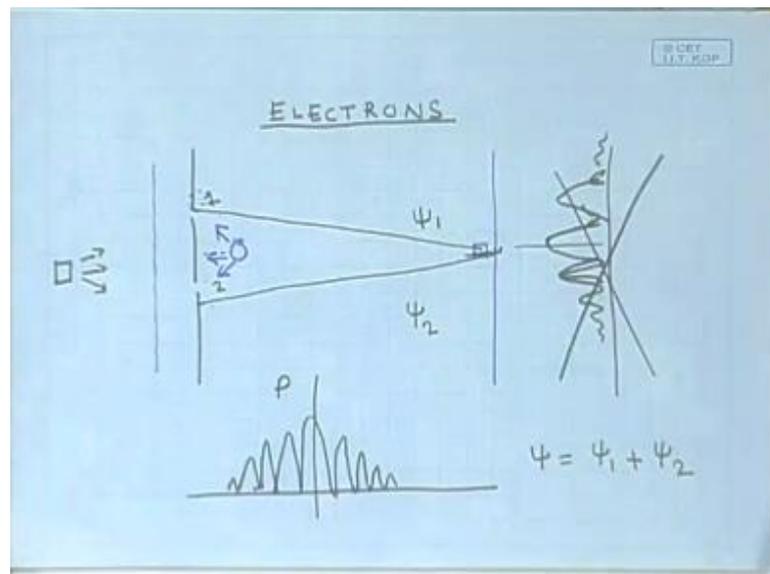
So, this spacing is going to be the order of  $10^{-25}$  meters, but it is going to be very close. So, finally, what we are going to get is a set of maximas which are very very close to each other the spacing being  $10^{-25}$  meters and these maximas are going to. So, closely spaced, that what we are going to see actually, what we are going to be able to make out is going to be sum the average of this, which is going to look something like this the probability distribution, which is going to be the sum of the probability distribution, when corresponding to the 2 slits.

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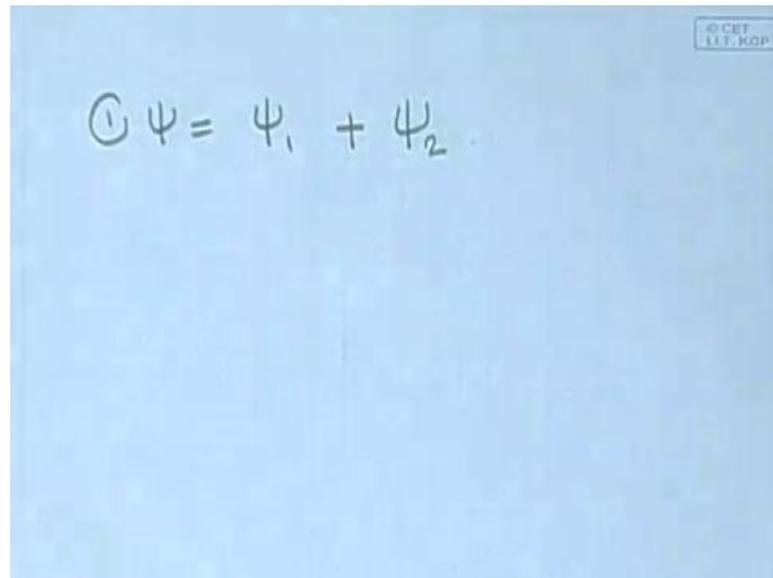
So, the point is that the interference pattern varies the distance between the maxima is small, that you cannot make it out. And what you see looks like the classical expectation, which is why you do not see this for microscopic objects so, let me now, again come back to the point, which I was making. So, in the situation, where we have let we recapitulate the key points, which I am trying to make, so, in the situation where we have an electron.

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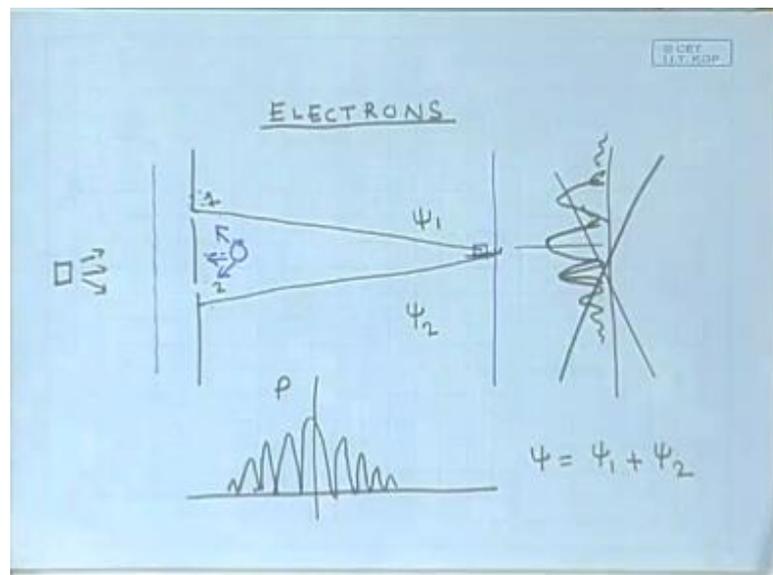
For example, going through these two slits or where there are various alternatives that the electron can take or where any microscopic particle can take there are various alternatives. So, here there are two alternatives by which the electron can reach here it, can either go through this slit or it can go through this slit then we have to add up.

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$$\textcircled{1} \psi = \psi_1 + \psi_2$$

The probability amplitude corresponding to each of these alternatives, so, if there were three alternatives the electron could have taken to arrive at the screen. Then we would have had to add up the contribution for the probability amplitudes for three such alternatives.

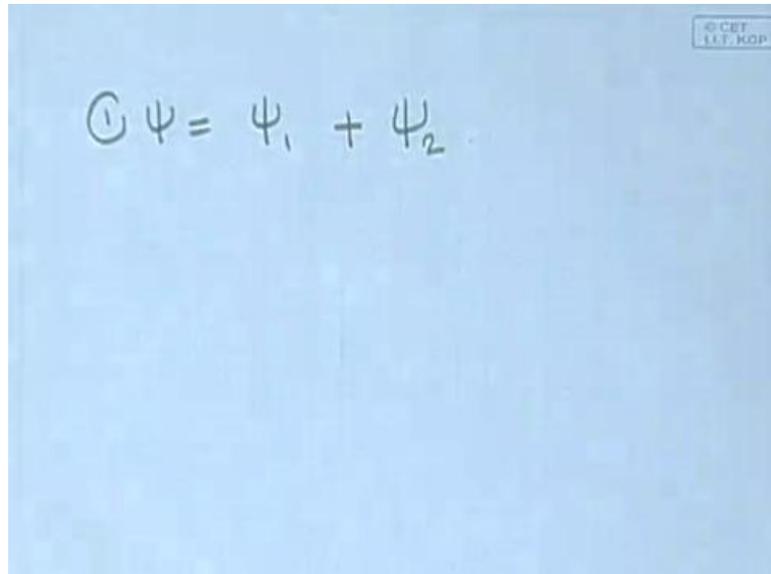
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So, if there were 3 slits then, I would have added had to add up the probability amplitude contribution from 3 such slits and there would be Psi 1 plus psi 2 plus psi 3 and these

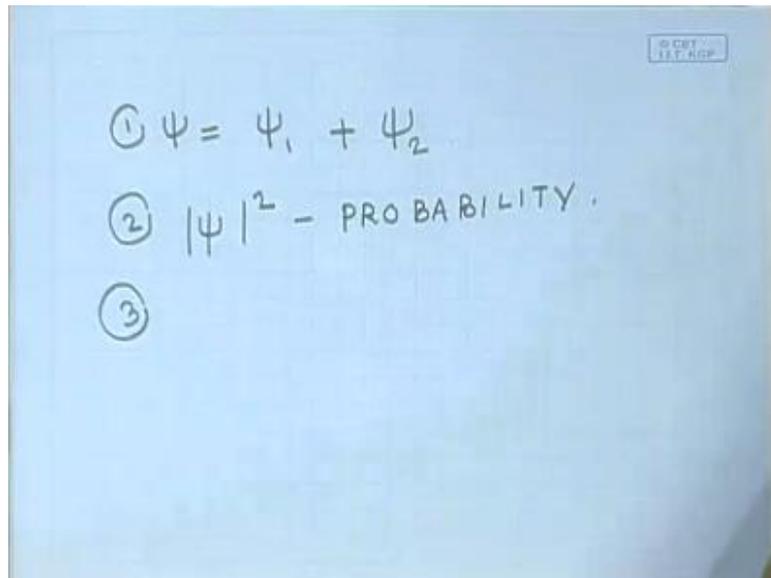
probabilities, amplitudes are necessarily complex quantities these are waves defined all over.

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$$\textcircled{1} \psi = \psi_1 + \psi_2$$

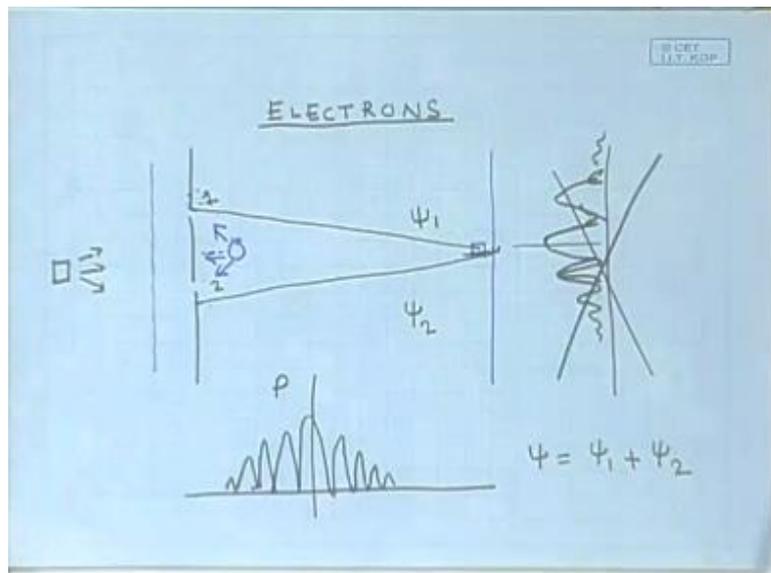
Now, when we want to calculate the when we actually do a measurement and the quantity, which you can calculate predict for the measurement is only the probability you cannot calculate trajectories for electrons. And such macroscopic, microscopic particles; you have to think of them in terms of waves the probability amplitude wave. Now, when you actually go and put a detector and make a measurement although, the wave is distributed all over space, you will always find that the electron is there at only 1 point.

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And what you can predict is the probability of finding that electron at different points and that probability is to be calculated by taking the modulus square of the probability amplitude this gives the probability. And the third the third thing is that, if we do a measurement which can tell us through which of the alternatives is being chosen.

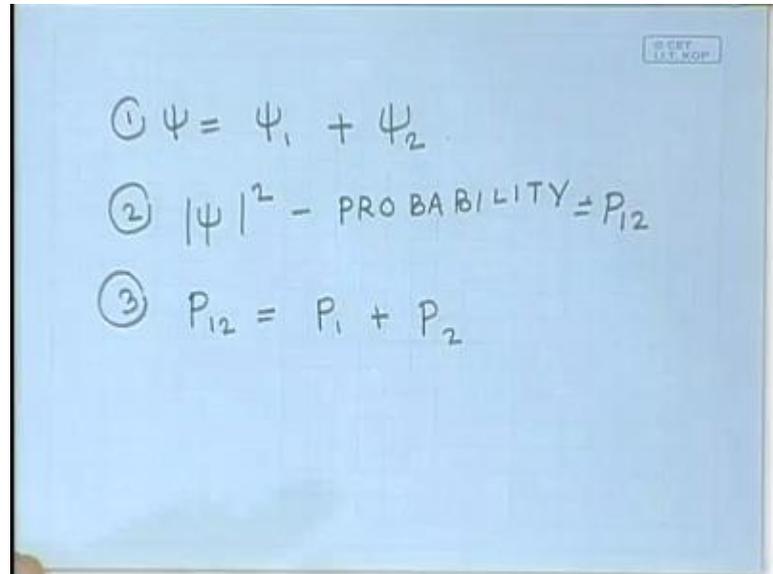
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By the particle, so, in this case if we do a measurement, which can tell us through, which of these 2 slits the electron is actually, going then the interference is washed out and the resultant is just the sum of the probabilities. That the electron goes through, this or the

electron goes through this or it is just the sum of the probabilities that the particle, chooses either alternative 1 or alternative 2.

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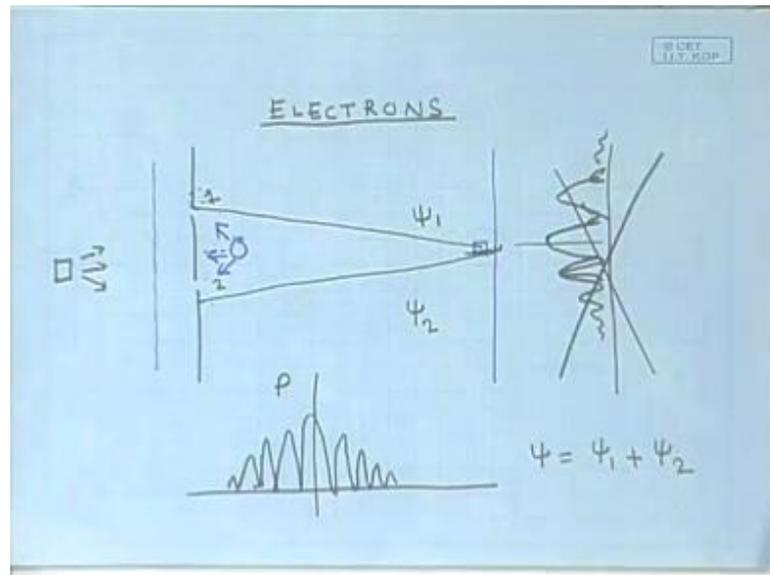
①  $\psi = \psi_1 + \psi_2$

②  $|\psi|^2 - \text{PROBABILITY} = P_{12}$

③  $P_{12} = P_1 + P_2$

So, that is the third thing, if you make a measurement then P this is going to be P 1 2 when you do not make a measurement the act of measurement disturbs. The wave and as a consequence the probability is now, the sum of the probability corresponding to the two alternatives. So, these are the three main points, which I have been trying to make we have to abandon the particle picture over here the, if you wish to understand. If you wish to understand and model the propagation of the electron through these two slits. It has to be thought of in terms of waves, we have to abandon the particle picture, where the electron has a definite position and momentum, you know for sure where you know for sure through which slit the electron goes. This has all to be abundant we have to think of this whole thing of the electron.

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Incident on these slits, we have to think of it in terms of a wave incident on the 2 slits this wave splits into 2, it goes through both the slits and then arrives at the detector. Now, a final point, which I should make before I close, we have been discussing a situation, where there is only 1 electron. And so, a single electron, wave goes through both and the single electron is detected at 1 point on the screen. So, every time an electron goes through, this you will get 1 click over here, from just sending 1 electron. You will never be able to get you will not get, this full probability distribution the same experiment has to be repeated many times. So, you have to actually, send many electrons through this when you send many electrons, you will get a variation in number counts. So, if you put your detector here and send many electrons you will get a certain number of counts which you can convert to a probability, put your detector slightly different position again, send many electrons count how many of them arrive here.

And then that will tell you the probability of the electron reaching here. The experiment has to be repeated many times with many electrons to finally, get this full probability distribution. 1 electron will only give you, 1 count at 1 position from, there you will not be able to tell the probability distribution, it has to be repeated many times. Finally I should also I should also tell, you that you do not have to necessarily consider a situation, where you are sending the electrons 1 by 1. But the point that, you have to remember is that the waves corresponding to two different electrons do not interfere; it is the interference occurs, between only between the wave only for the wave of a particular

electron it interferes, only with itself. It does not interfere with the wave corresponding to a different electron. It is as if these two waves are incoherent, so, you if you have many electrons being sent out, we do not have to bother about the possibility of 1 electron's wave interfering with the other electron's wave that need not be considered.

It is only the wave of a single electron, which passes through both of these and then produces the interference pattern over here. We can never simultaneously record the entire interference pattern, what we can do is we record the electron at a single position and by repeating this many times. We can construct the full probability distribution, which is the mod square equivalent of the intensity pattern that you get for light or sound. Here it is the probability distribution, we have to we can get that by sending electrons 1 after the other or by sending many electrons and looking at their distribution. So, this brings to a close today's lecture in the consecutive lecture. The forth coming lectures, we are going to develop on the mechanics on the way to manipulate this, probability amplitude, which we are also going to referred to as the wave function. And that takes us into the realm of wave mechanics or quantum mechanics.