Physics I: Oscillations and Waves.
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Lecture - 25
Beats

In the past few lectures, we have been studying the phenomena of interference and diffraction. In both of these phenomena, we had light; we had waves of the same frequency and same frequency and wave number being superposed. And we were interested in studying, these special intensity variations that arose, when we superpose such waves and these intensity patterns arose because the sub times at certain points the two waves or the various waves that will be superposed, where all in the same phase.

So, they added up to increase the intensity at some other points, the waves were such that the phases of the waves, was such that they cancelled out. And you had a very low intensity this gives rise to the pattern of fringes etcetera. Now, in today's lecture, we are going to discuss a slightly different situation, we are going to discuss a situation, where we have the super position of two waves of different frequencies. For two waves of different frequencies we are going to study that variation of the intensity with time.

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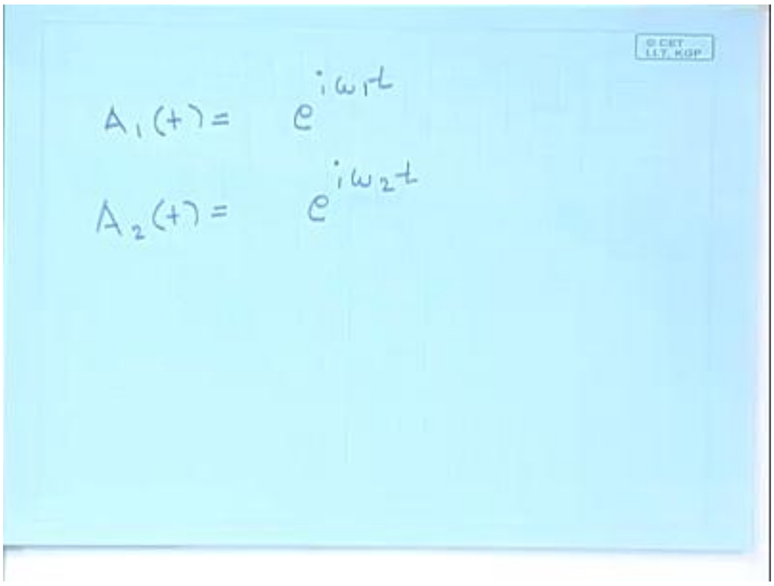

So, let us consider two waves one of frequency omega 1 e to the power i omega 1 t and A 2. So, we have two waves of two different frequencies, we are interested in studying the time evolution at fixed points. So, I have not shown the x dependence, but the x dependence is there are sinusoidal plain waves. So, they usual x dependence minus k x is going to be there, I have not shown, it the wave numbers are also different. So, we have two waves of different frequencies and we wish to study the super position of these two waves. So, these are complex so, this is complex notations.

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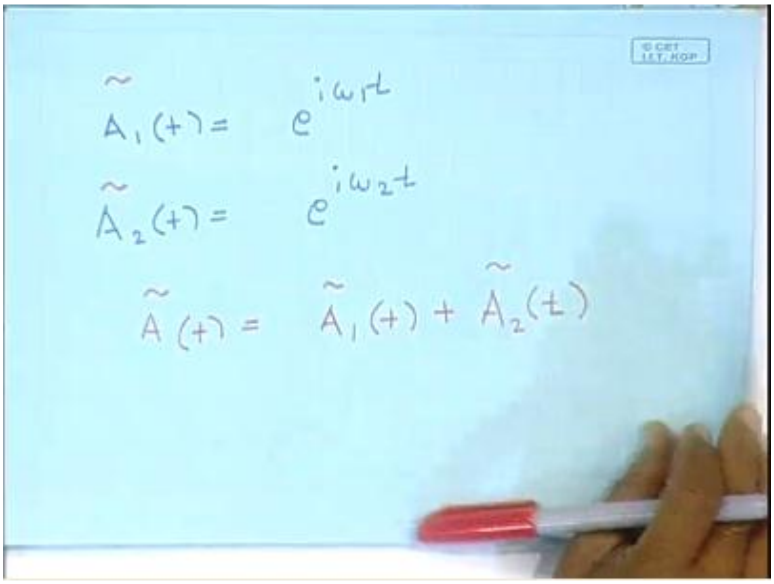

So, we have in this situation considered the amplitudes of two waves to be identical and we have superposed them. And this can be simplified to some extent.

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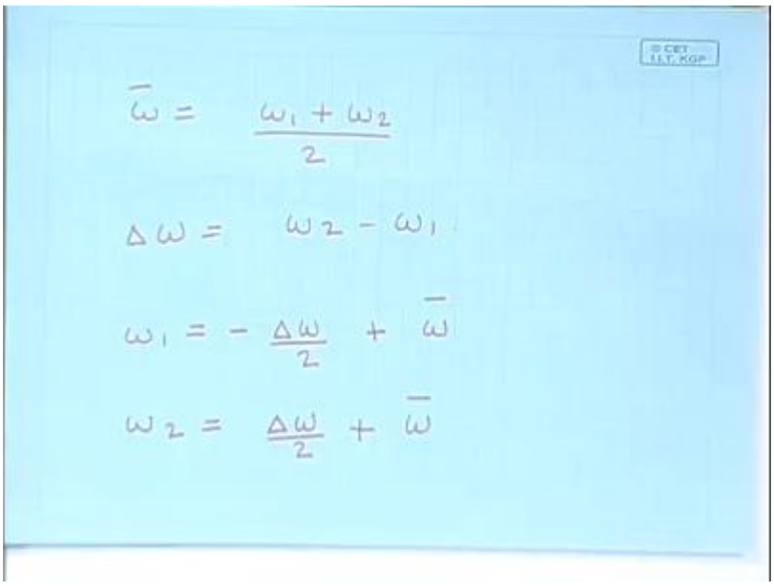

So, if we define the mean, frequency omega bar to the omega 1 plus omega 2 by 2 delta and the difference to the omega 2 minus omega 1. Then we can write omega 1 has delta omega by 2 with a minus sign plus omega bar and omega 2 is delta omega by 2 plus omega bar.

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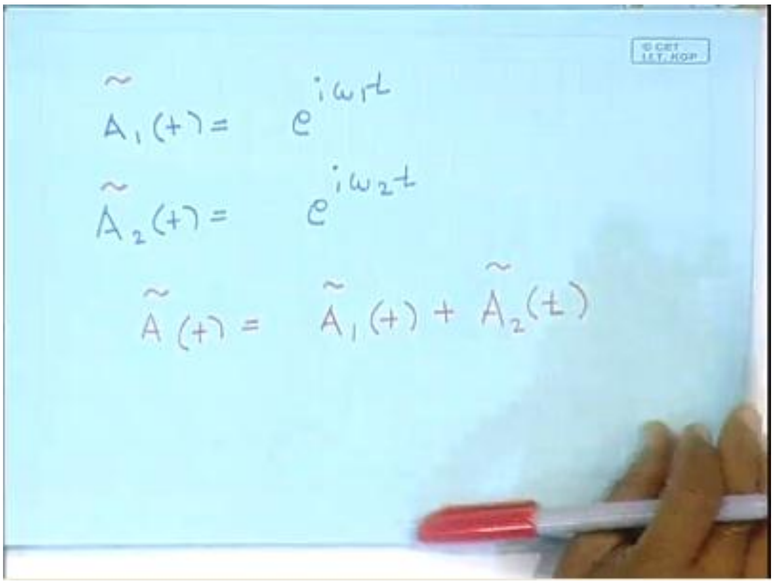

So, with this new notation of omega bar and delta omega the super position of these two waves are, let me write down these two waves themselves first. So, the first wave written in terms of these variables written in this wave the first wave becomes A 1.

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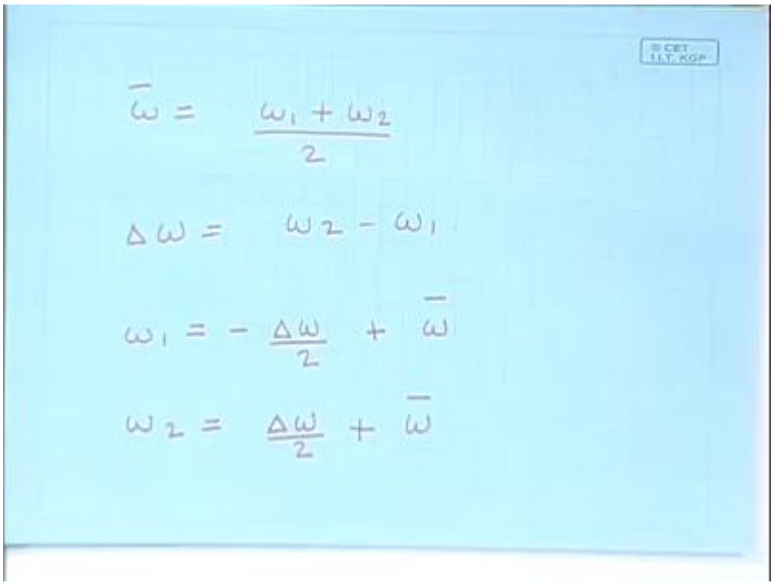

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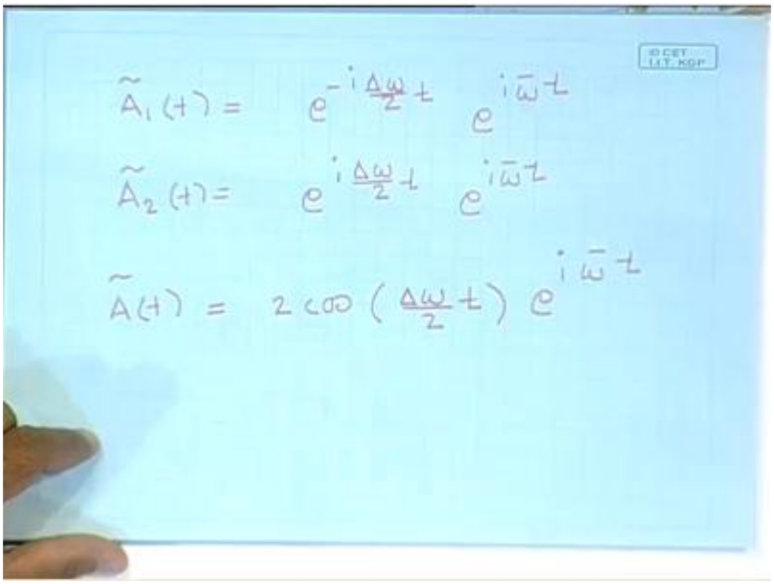

This is the first wave with frequency omega 1, this the second wave with frequency in omega 2 when we superpose these 2 we have A t is the sum of. So, this part can be taken common the sum of these 2 is going to be 2 cos. So, these is going to be 2 cos delta omega by 2 t into e to the power i omega bar t. And we are interested in a situation, where omega 1 and omega 2 are very close, if omega 1 and omega 2 are very close.

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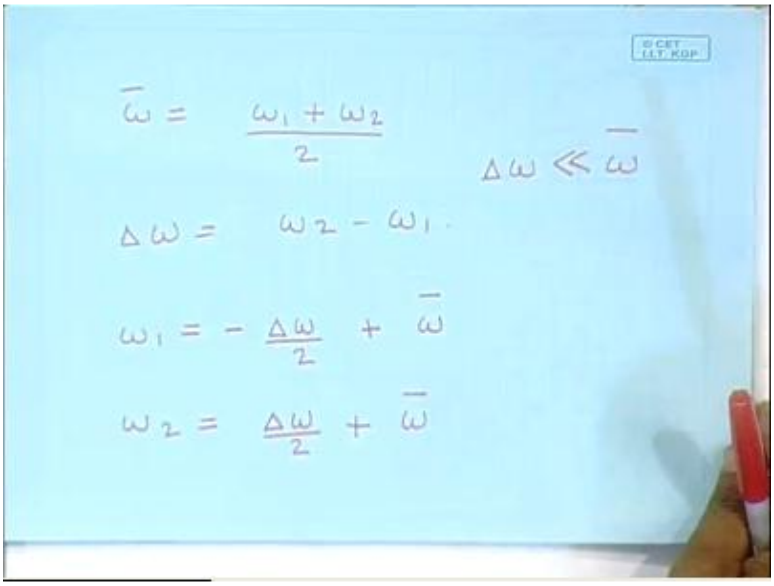

Then delta omega the difference in the frequency is much smaller than the average frequency. So, that is the situation, we are interested in, we are very to very close frequencies the super position of two waves of very close frequencies.

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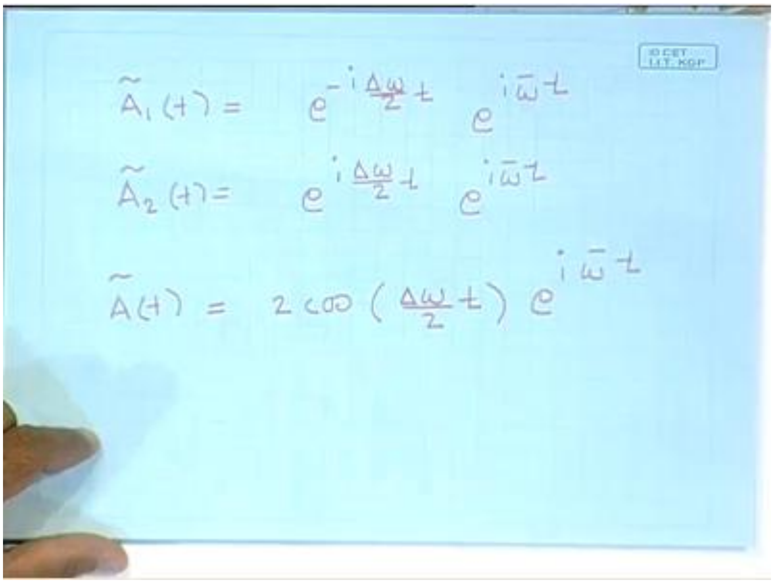

And the resultant the superpose of the super position of these 2 is of this is looks like this. So, this can be interpreted as follows, we have a fast oscillation at the average frequency of the two waves the amplitude of this fast oscillation itself oscillates slowly at the difference at the frequency which is the difference of the 2 frequencies. So, we see that if it superposes two waves, which are very close in frequency the resultant is a fast oscillation at the mean frequency. And the amplitude of the fast oscillation at the mean frequency itself varies slowly at the difference frequency, the difference at frequency, which is a difference of the 2 frequencies.

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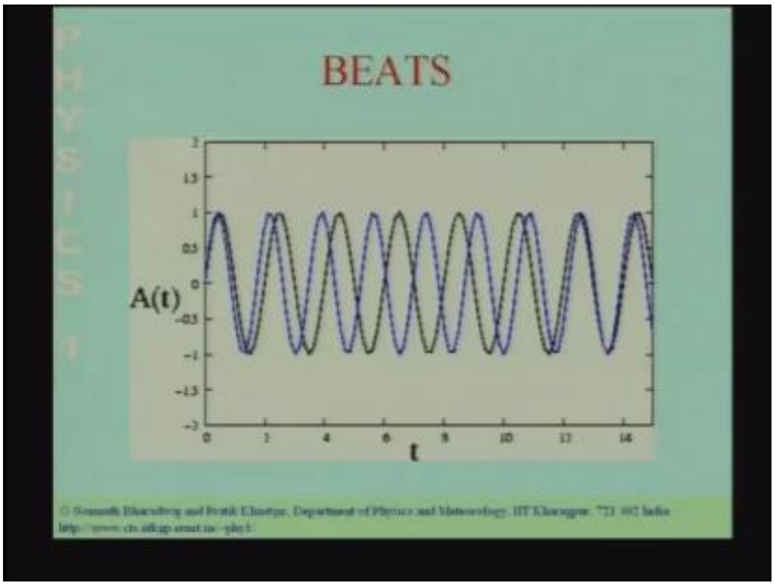

This as a we can interpret this physically, this is what I am going to do for you now. So, this picture shows, you the two waves and the two wave the oscillations of the two waves. They have two slightly different frequencies. So, omega 1 t and omega 2 t, there is a slight difference between these to start with difference 0 as t increases the difference keeps on increasing. And then this difference between omega 1 and t and omega 2 t, so, there is a difference between omega 1 t and omega 2 t and this difference keeps on increasing. And this is the difference, this difference keeps on increasing as time involves and so, the oscillation also differs as time involves.

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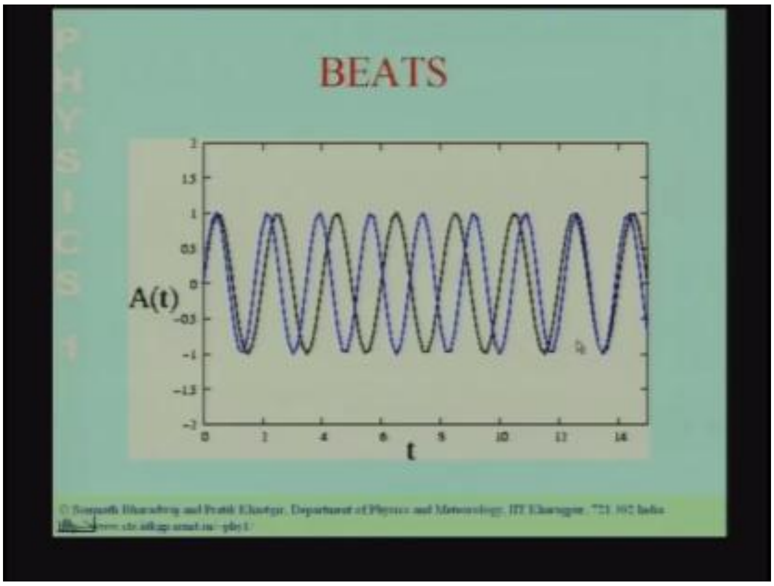

And the difference again vanishes, when this becomes 2 pi.

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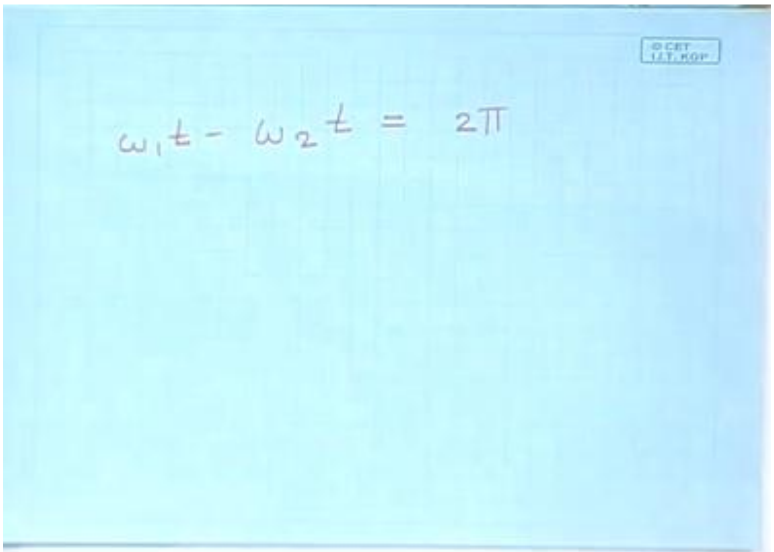

You see the the difference between the 2 oscillations keeps on increasing and then again, it goes down. And finally, the 2 oscillation again match, when the difference between omega 1 t and omega 2 t is 2 pi.

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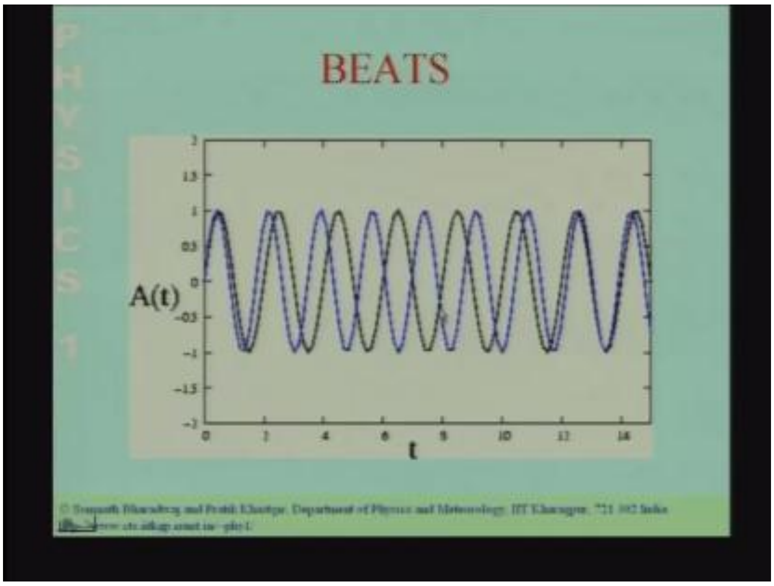

And the difference between the 2 oscillation, become the 2 oscillation becomes, exactly opposite, when the differences is exactly pi, which is what you see here. So, the 2 oscillations the 2 oscillations start out exactly the same. But there is a small difference, which keeps on increasing with time and then this small the difference, when it becomes pi the 2 oscillations are exactly out of phase which is, what you see over here and then they again become 2 pi. Here the differences become 2 pi. So, they are again in phase and again they will go exactly out of phase again, when it becomes 3 pi and again, it will be exactly in phase when a it is 4 pi etcetera.

So, super position of these two, is going to be the combination of these two is going to add up when you are somewhere in this region and then, it is going to cancel out the super position of these two waves is going to cancel out over here. And then again, it is going to add up over here, so, the so, what you have is essentially, that the two waves are going add up in these regions. They are going to cancel out these regions are again, they are going to add up in these regions. So, the fact whether the two waves are going to add up or cancel out is changing slowly, much slower than individual oscillations. And fact whether they add up or cancel out the super position of the two waves depends on this difference of the two.

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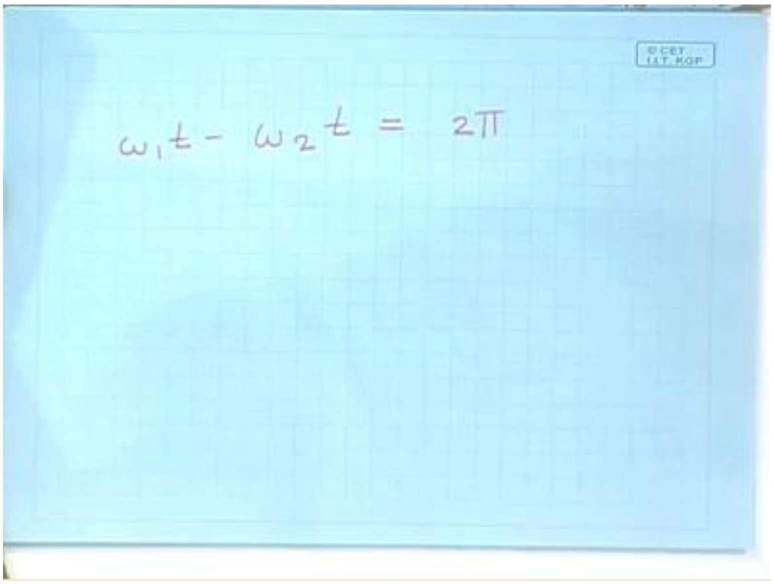

Angular frequency omega 1 and omega 2 and the resultant looks like this, so, you have the fast oscillation.

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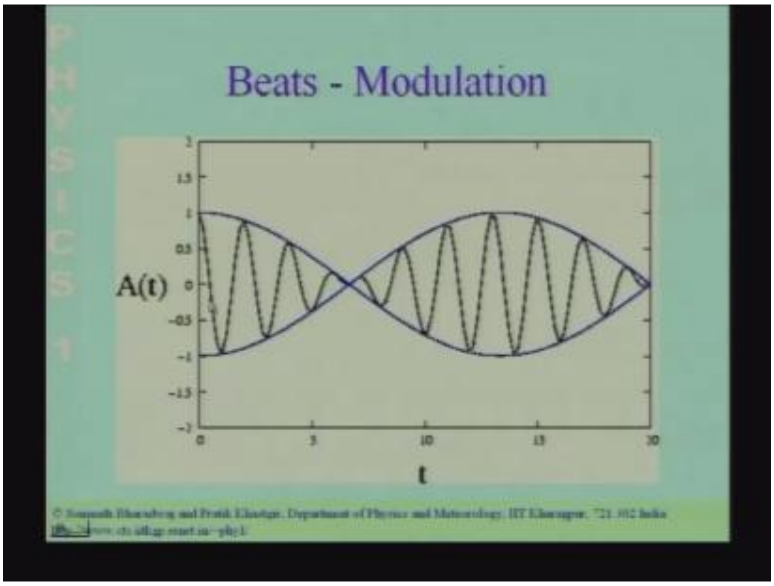

And the amplitude of the fast oscillation changes, it becomes 0 when the 2 oscillations are exactly pi out of phase.

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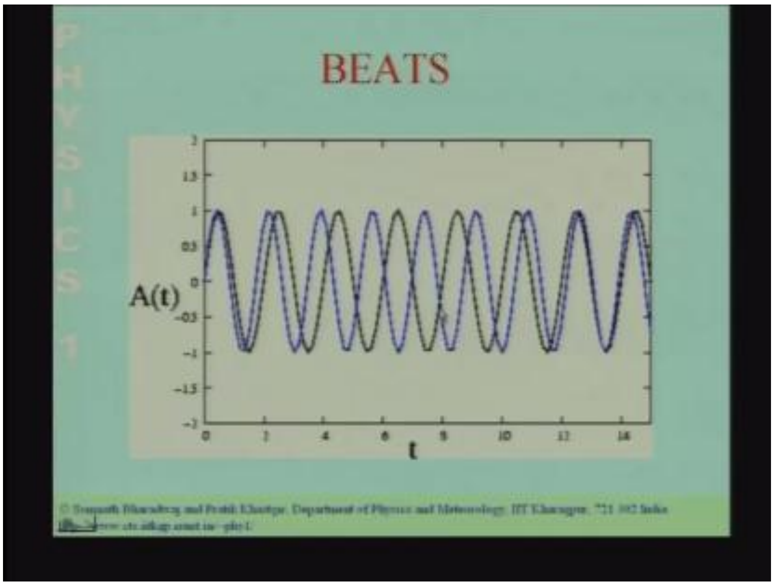

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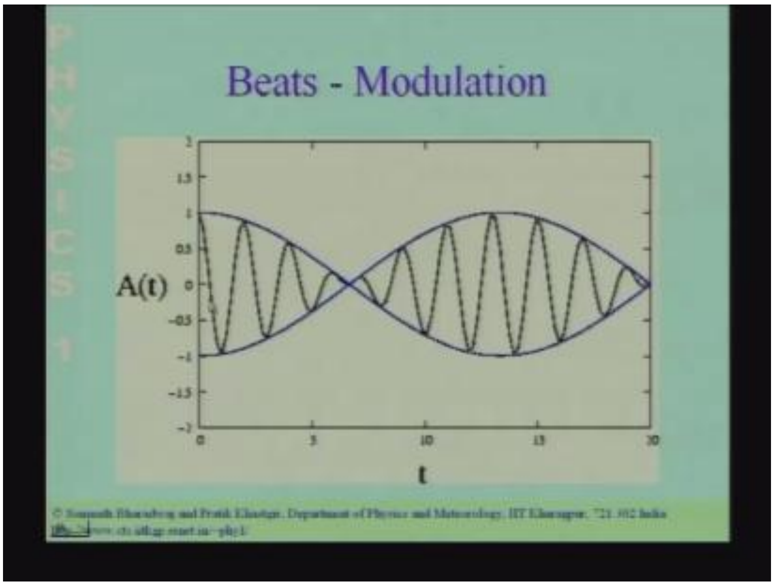

And then, it adds up when they are 2 pi out of phase again, it becomes 0, when they are 3 pi out of phase and so, forth. So, you have this slow modulation, slow change of the amplitude, which is refer to as a modulation of the amplitude. Now, let us what happens now? Let us discusses the situation where. So, this situation occurs in many contexts, so, for example, I have 2 strings, which produce nearly the same frequency. But not exactly the same frequency for example, I tuning a guitar and I have not exactly tuned it. So, I plug the 2 different strings, they produce very close sounds. But not exactly the same the question is what do I hear when I plug 2 strings, which produce very close frequencies, but not exactly the same frequencies. So, let me go through this, repeat this analysis which I had just performed.

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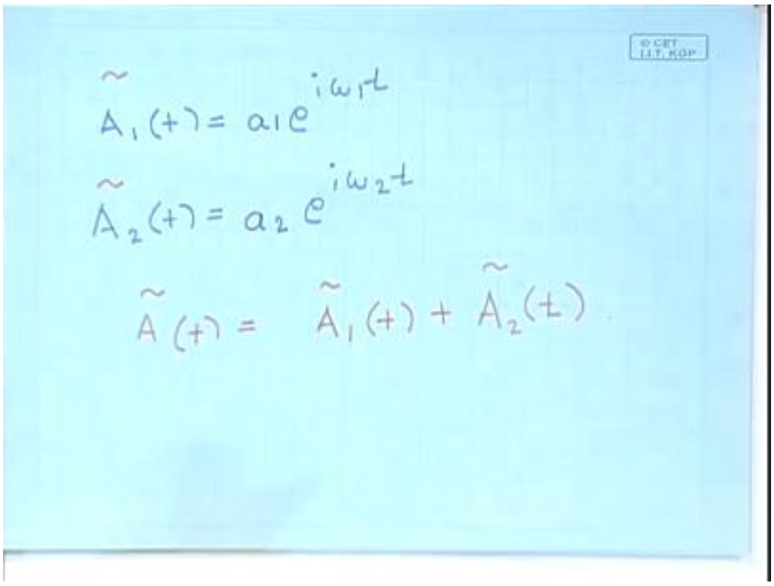

So, we have now super position of two waves of slightly different frequencies the amplitudes could also be difference. So, let me put different amplitudes here A 1 A 2 we are going to superpose these two.

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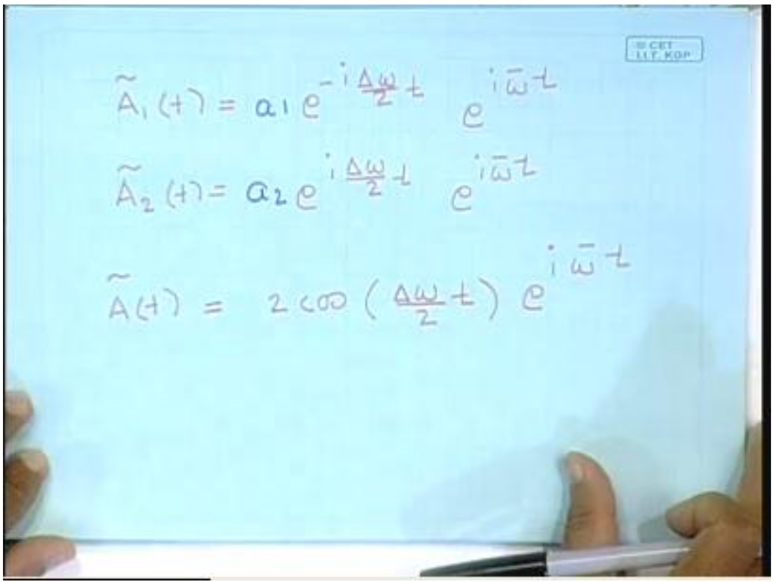

We have gone through, this exercises and I showed you this, could be written. It terms of delta omega and omega bar now, there will be factor of A 1 here and a factor A 2 here.

So, when I superpose these 2 let me write down, what we expect to get the super position of these two is going to give us.

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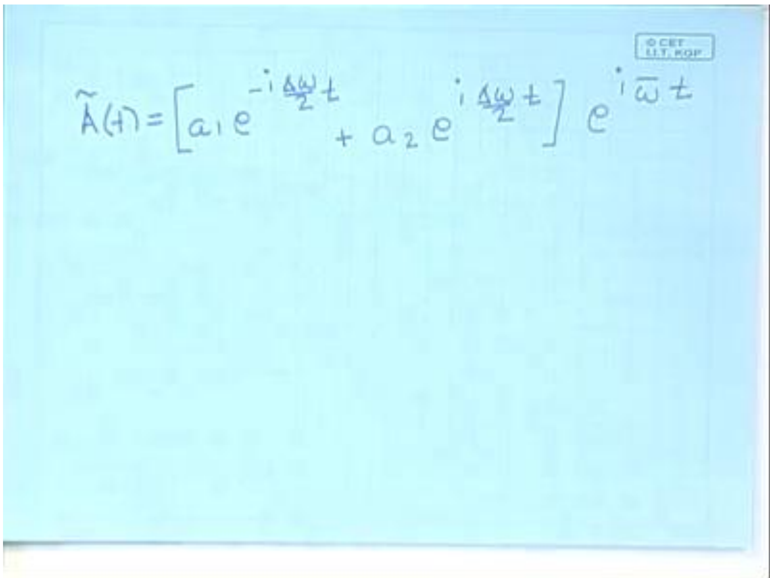

A t is equal to A 1 e to the power minus i delta omega y 2 t plus A 2 into the power i delta omega by 2 t into e to the power i omega bar t. Now, what we actually measure, what hears the sound? That we hear actually, measures is the intensity. So, let me calculate the intensity the intensity is the time average of what you measure is the time average actually, so, you see there is…

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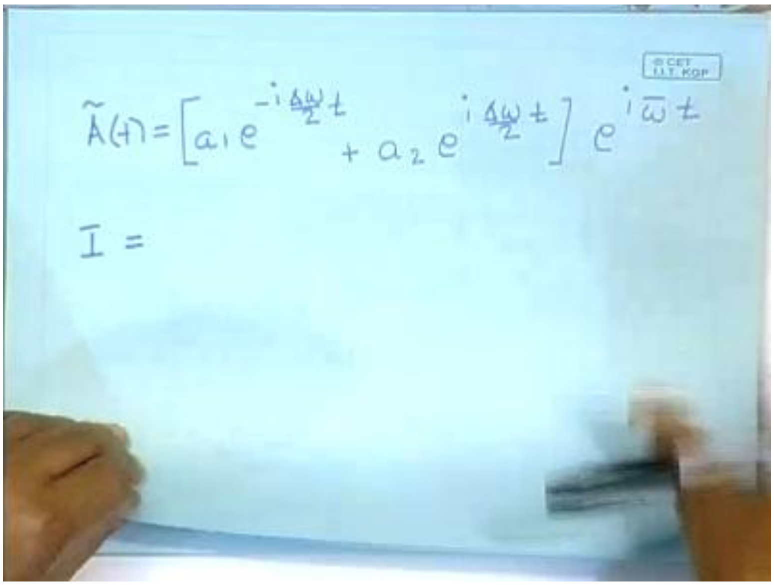

Let me measure; let me calculate the intensity and things will be clear then. So, the intensity is going to be before going into the question of the intensity, let me just come back to this.

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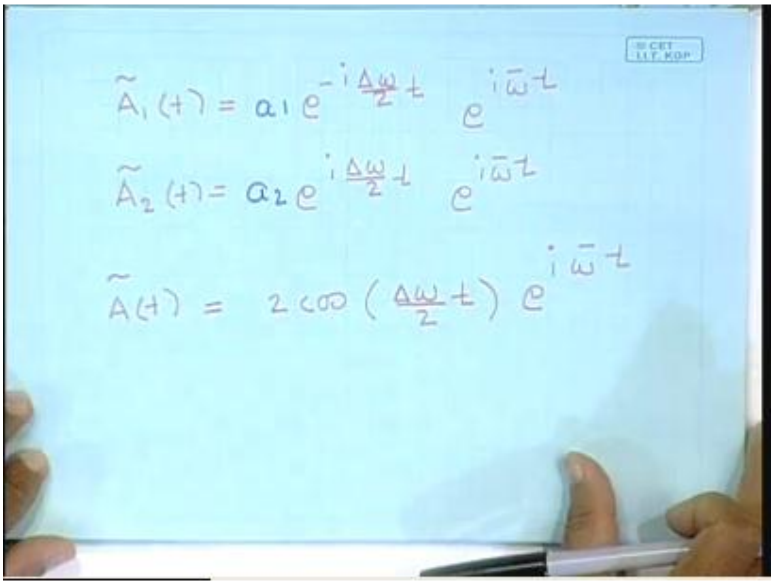

Suppose I had exactly the same, I had exactly the same amplitude, then this would be the result of the super position. And the question is now? How to calculate the intensity? Now, our ear cannot record the, when I have a sound wave the sound the air goes back and forth, that is what happens and our ear cannot record these individual movements. We do not record these individual movements are ear drum goes back and forth at the high speed at, which the sound is sound wave is making it oscillate. What we record is the time average, which intensity for example, if I have a sound wave the sound that 500 hertz's the ear drum back and 40 every 500 second, what we record is the time average intensity. So, the time average, that we record is over much larger is not over is not that is this, intensity is average intensity average to our sufficiently large types. So, there are many oscillations in between, now, suppose I have a situation where I have one sound and 500 Hertz's and another sound at 501 hertz's.

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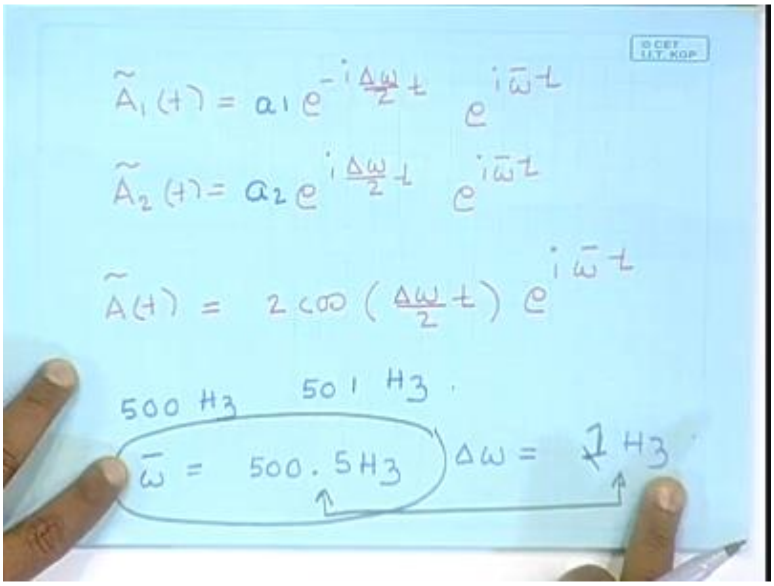

We have 2 where super position of these two is a fast vibration is a fast oscillation the fast oscillation is going to be at the average omega bar, which is 500.5 hertz's. And the amplitude of this fast oscillations is going to get modulated at delta omega, which is the difference, which is the delta omega is 1 hertz's. And the amplitudes is going to mod get modulated at point 5 hertz's. Now, the point, which you should, we should recognize here is that we will not experience. So, when we experience this sound will we not experience this, very fast variation that 5 100 hertz's.

We do not, we wrote 500.5 hertz's, what we record is a time average of this, but we will experience this radiation at 1 hertz's, this we will at 0.5 hertz's, this will be experienced. So, the point to note here is that we will the year does an time average on a times scale, which is inter mediate, somewhere in between the this and this. So, this thing gets averaged out, but this part does not. We can record this variation, because it will have a time period of the order of a second and our ear can record this variation. So, if you ask the question? What is the depths we are going to hear, we should take the time consider that time average over this radiation. But not over this variation, this is where we have to take a look and the instrument, which is doing the measurement and apply the suitable time average. So, let me do this in the fully real notation here.

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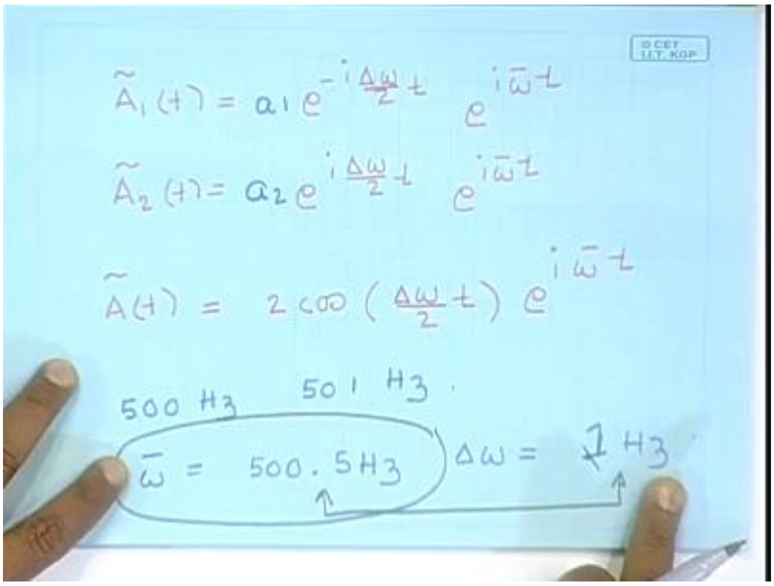

So, the way to calculate the intensity is to take the real the quantity, that you really measure. That is, the real quantity is the real part of this expression.

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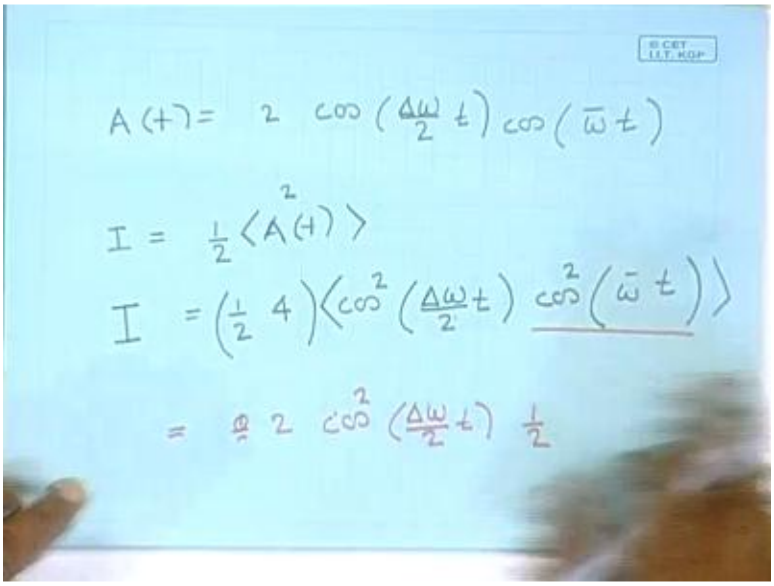

The real part of this expression is A t which is 2 cos delta omega by 2 t into cos omega bar t, this is the actual physical vibration and to calculate the intensity, I have to square this and put a factor of half then take the time average. This gives us half factor of 4 here, which is an overall normalization anywhere not of great interest. And then I have cos square, now, our ear the intensity that, we able to record without ear is going to be a time average. So, the question is over this time average of this, the question is should be average the product of this or over what time scale is the averaging.

So, as we have already discussed, we should average over a time scale, which is intermediate between this and this. So, this gets average out, but this will not get averaged out. And this time average is going to give us 2 this is going to still remain cos square delta omega by 2 t, this time average is going to give us factor of half. We are not going to be able to record the fast variation over here. So, what we will record? What how what we going to experience is slow modulation the slow change in the amplitude.

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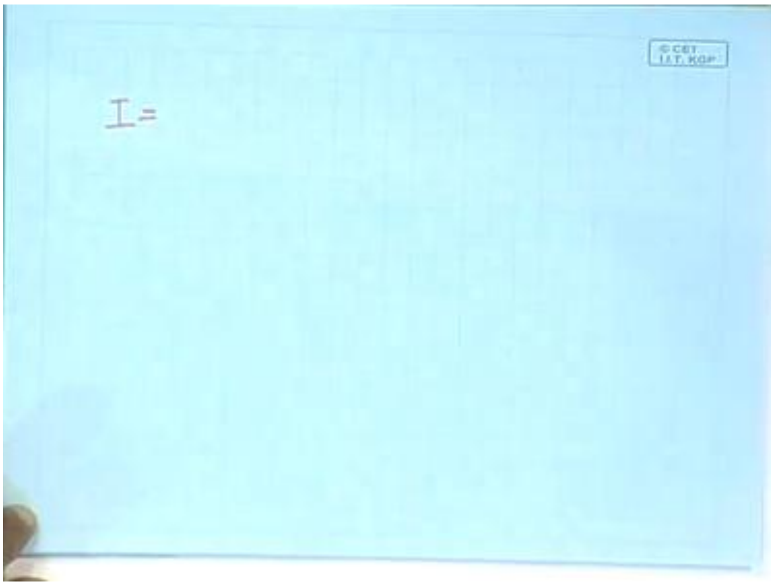

Because that is going to occur on the order of second and this cos square delta omega by 2 t can be written as with a factor of 2 outside can be written as, we know that cos square cos square x is 1 minus cos 1 minus cos to x. You can write, it in terms of cos 2 x and you see, that you will hear.

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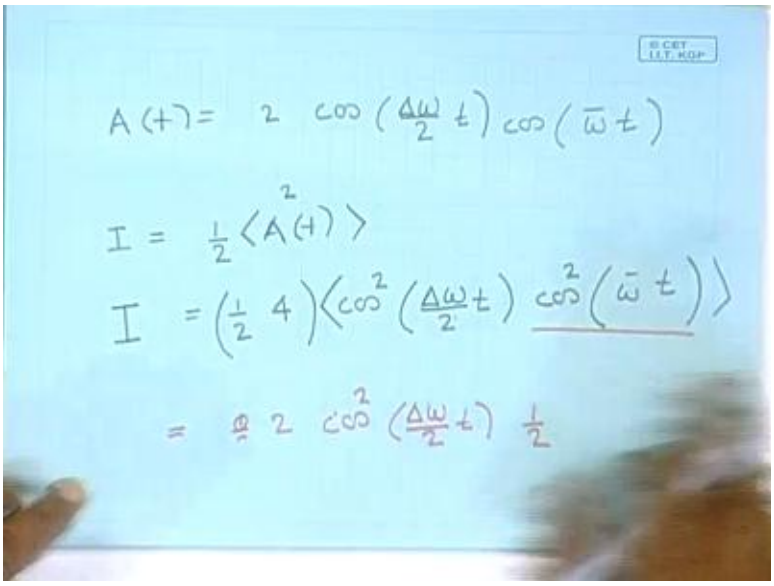

You essentially hear a modulation in the intensity at an angle of frequency the corresponding to the difference and this will occur at. So, this is going to occur at the time scale of 1 hertz's, which your ear can which our ear can make out. So, that is the point which I was trying to make here. So, let me go back to the calculation which we were doing. So, we have the super position.

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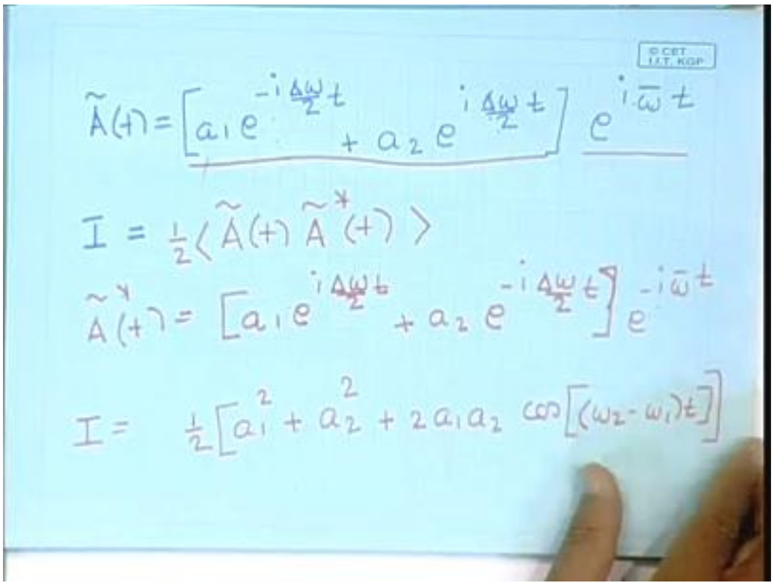

We have this kind of a wave, which is a product of a slow variation here and a fast variation over here and a question is that, what is going to be the intensity? So, time average intensity is the time average of the square of this, with a factor of half, which I can write in this fashion. And we should remember to do an average over, all other time scales, which are over a time scale, which is somewhere intermediate between this and this. So, if you take the complex conjugate of this time average, this complex conjugate of this what you get is, this into the power minus i omega bar t and we multiply these twos these two factors will cancel out, and this into this is going to give us A 1 square, this into this is going to give us A 2 square half of that factor of half. So, this is going to be the intensity of the first wave plus, intensity of the second wave plus.

So, the intensity is going to be half A 1 square plus A 2 square the product of this of and this is going to give us, is going to be a A 1 square plus A 2 square. And then we are going to have the cross terms the cross terms are going to give me a factor 2 A 1 A 2 cos omega 2 minus omega 1 t. So, what, we see is that the intensity of the super position of these two waves is going to vary slowly, with the difference in the two frequencies. So, the intensity is going to have a slow variation with the difference of the two frequencies. If A 1 A 2 are exactly, same the intensity is going to have a maximum value, which is 4 a square 4 times the individual intensity the minimum value is going to be 0.

If A 1 and A 2 are not identical the variation is not going to go between 0 and the maximum value, it will have a minimum and maximum value. The minimum being more than 0 and the maximum being a less than, what we have this particular case. So, this is the phenomena of beats. So, what you will hear, if I play 2 strings, which are whose frequency are very close, but not exactly the same the intensity is going to up and down slowly. It is as, if sum 1 were slowly turning the volume knob is going to go up and down slowly and it will be a descendible the closer the frequency is get the slower is going to be the variation. And finally, the variation going to stop, when the two frequency is are exactly the same, it is going to get very at large time, you going to have a very large time period variation, which is what I have shown you over here.

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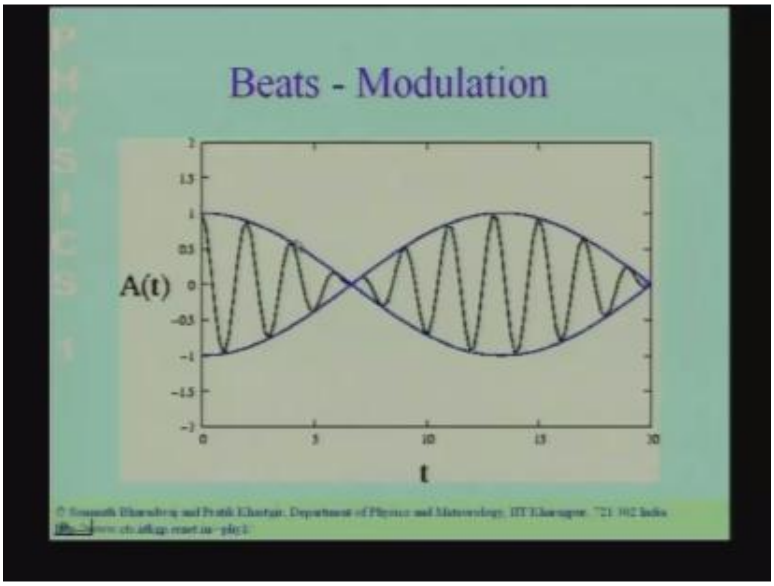

Also the amplitude essentially, goes up and down slowly and the time period of this variation of the amplitude is inversely proposal to the difference in the frequencies that you have. And this is also referring to as the modulation of the amplitude, this was the phenomena of beats.

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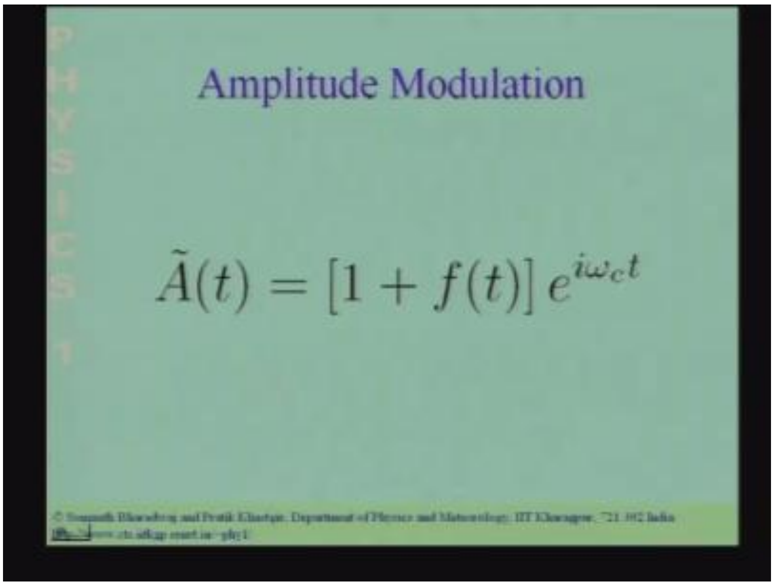

Let me go on to a different application of this discussion, this as to do with amplitude modulation and how signals are transmitted. So, let us ask the question, how does how are radios? How are signals transmitted in using radio waves? We all must have seen a radio receivers and transmitters the question is, how does this work? We have already, discussed 1 aspect of how it works? How the antennas we have already, discuss that there will be antennas. If you give an oscillating signal to the antenna, it is radiation. And then you have other antennas, which when there is radiation not incident on, it will produce an oscillating signal. But now we ask the question how you use this whole thing to transmit a signal. So, let us discuss that so, the way the whole thing works is as follows, in the radio station where the signal is being transmitted.

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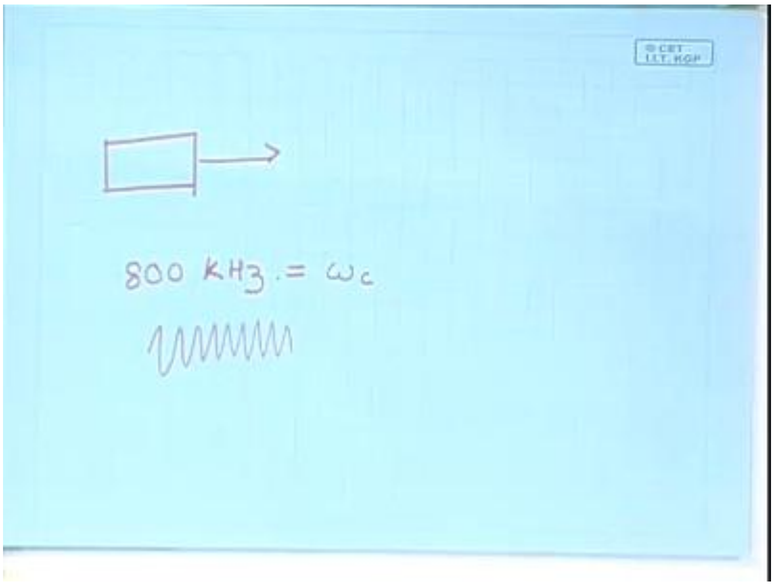

You have a signal generator, which produces what is called a carrier wave, so, this produces what is called a carrier wave. So, let us consider a situation, where the carrier wave as let us say 800 is at 800 kilohertz's frequency. So, the signal generator over here, this is going to be refer to as omega c the signal generator over here, produces a wave at 800 kilohertz's, it is wave like this sinusoidal wave like this itself as no information. Now, I would like to use this, wave to carry the voice signal. So, let us just say, that I would like to somehow encode the, my speech the voice, my voice on to that on to this radio wave.

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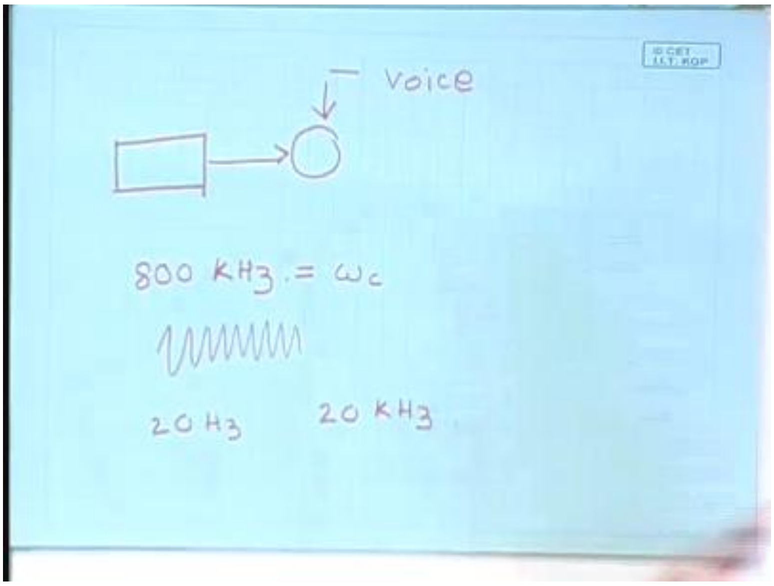

So, my voice spans the frequency range from let us say 20 hertz's to 20 kilohertz's in sound waves. So, by using a microphone, my voice could be converted into an electrical signal and what is done is that this is my voice, my voice comes over here, my voice this is my voice the signal which we wish to send transmit. And this is the carrier wave and the amplitude of the carrier wave is modulated. So, that it the chances in the amplitude are the signal, which I wish to send, which is what is have shown over here, so, this what is called amplitude modulation.

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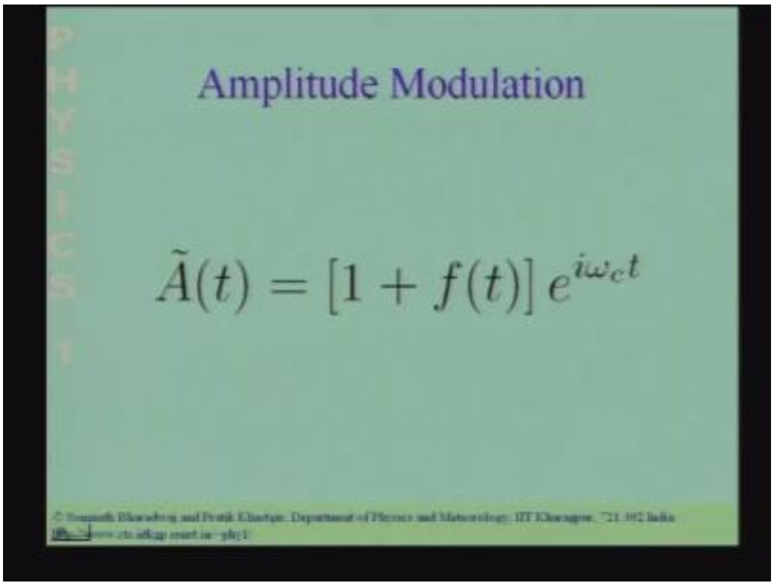

So, the amplitude of the wave of the carrier wave is modulated it is change. So, that the it has my voice signal, so, let me explain this pictorially.

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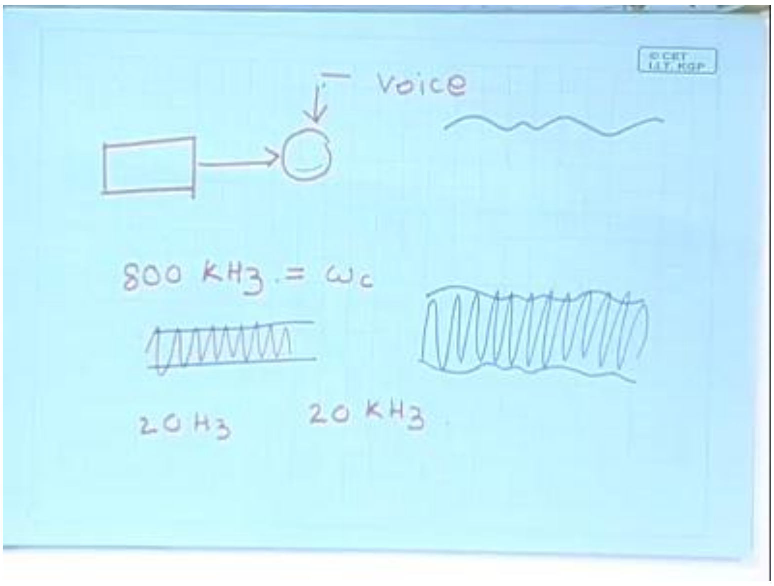

Suppose my voice signal looks something like this, so, I will now, change the amplitude to start with the amplitude of this signal is constant. I will now, change the amplitude of the signal. So, that it now looks like this and you have this, so, the carrier wave is still over here. So, if the amplitude of the carrier wave is change by means, of sum electrical devices the voice, which I wish to transmit is the amplitude of this carrier signal is modulated. So, that it has the signal which I wish to transmit and mathematically, we can express it like this.

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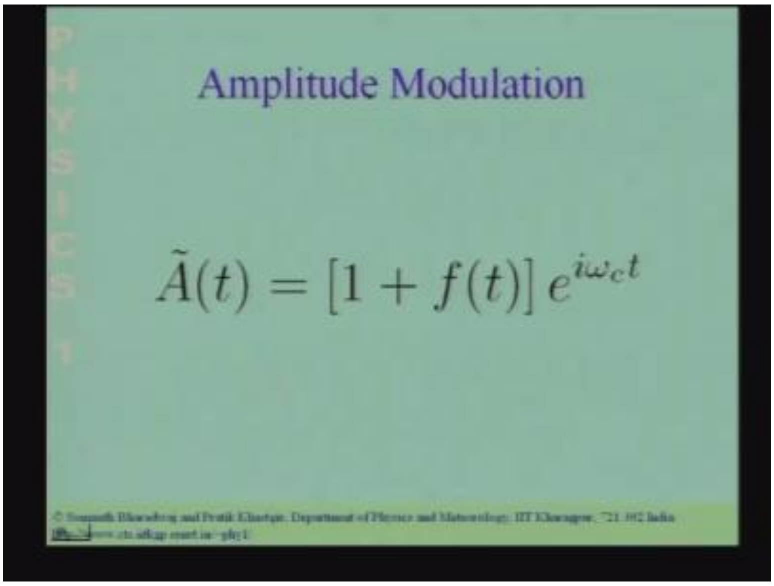

F t is the signal, that we wish to transmit and the 1 plus f t is the amplitude of the signal that is coming out and this signal is now, fed into the antenna, which transmitted. So, the signal generator produces a signal at the carrier frequency the amplitude of this is then modulated with the signal that we wish to transmit. And this is what is transmitted at the receiver end again, there are devices by through, which only the changes in the amplitude are recovered and from this, you can get back the voice which you wish to transmit.

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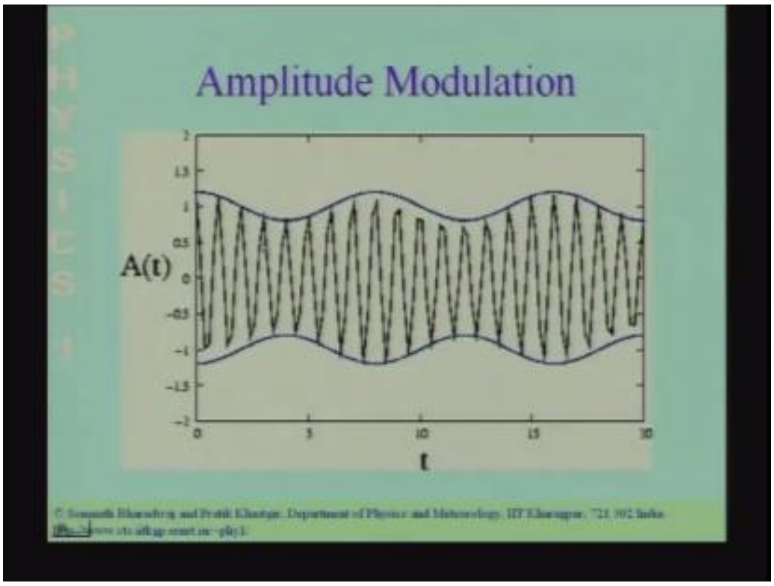

So, this is what is call amplitude modulation and this is shown over here pictorially, so, this the carrier signal, it oscillate at the carrier frequency and this the signal which you wish to transmit. So, this signal, that you wish to transmit is encode in it is put in to the modulation of the amplitudes. So, the amplitude of the signal is made to change as per the signal, which you wish to transmit. This is, what is, why is called amplitude modulation? It is the modulation in the amplitude. That carry the signal that you wish to transmit now, let us consider an example of amplitude modulation.

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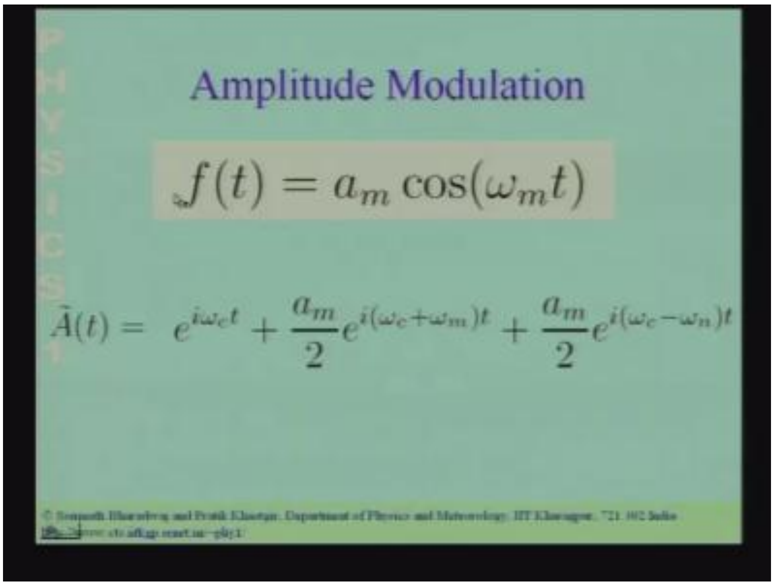

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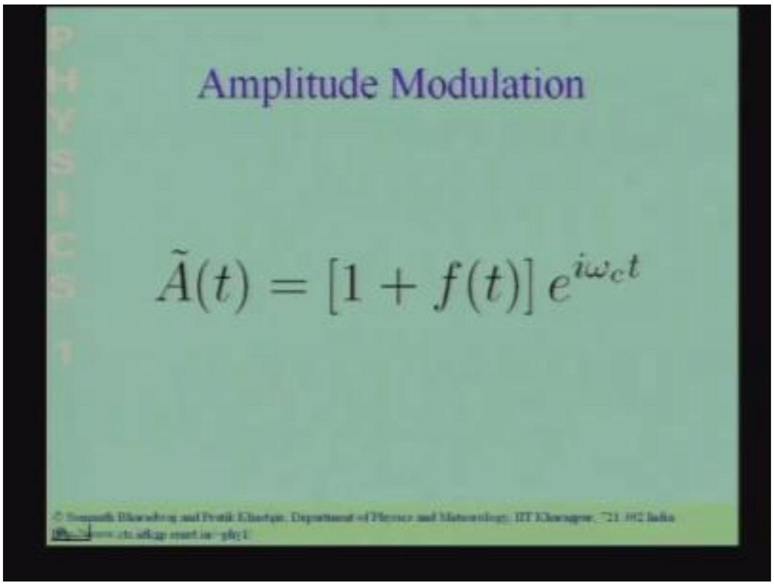

So, we will consider as specific situation, where the signal that we wish to transmit this is the signal that, we wish to transmit, we will consider a specific situation where the signal that we wish to transmit is a pure cosign a sinusoidal wave of a single frequency called omega m the modulation frequency.

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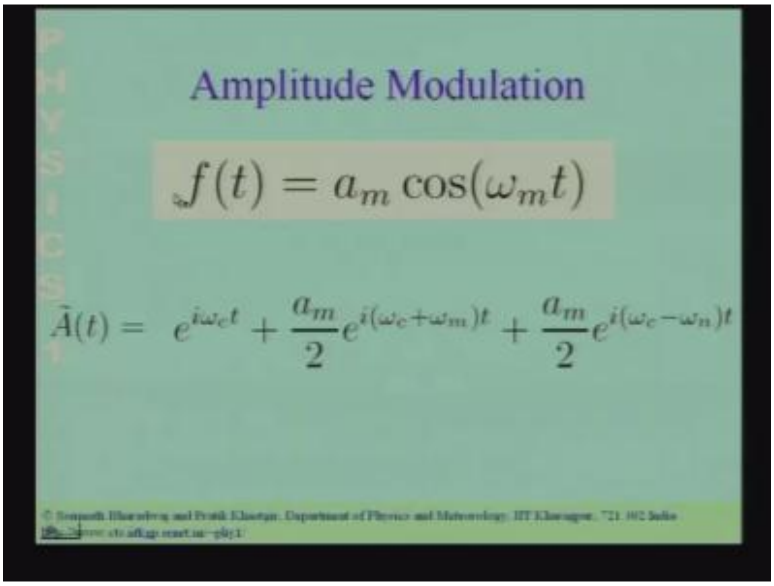

And it has amplitude am, so, this is the signal, which that we wish to transmit. So, this the signal that is modulated on to the amplitude.

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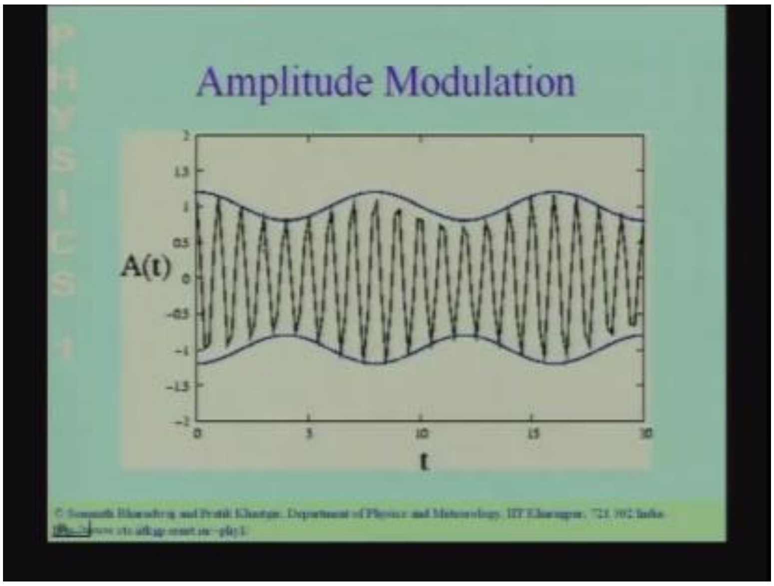

So, the amplitude modulations in this case are sinusoidal oscillation at a different frequency the modulation frequency.

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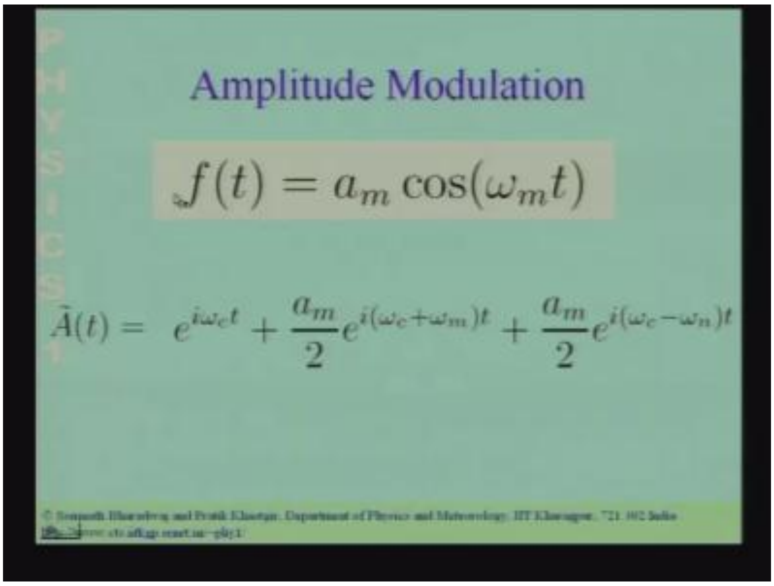

And we are going to assume that the modulation frequency is much smaller than the carrier frequency.

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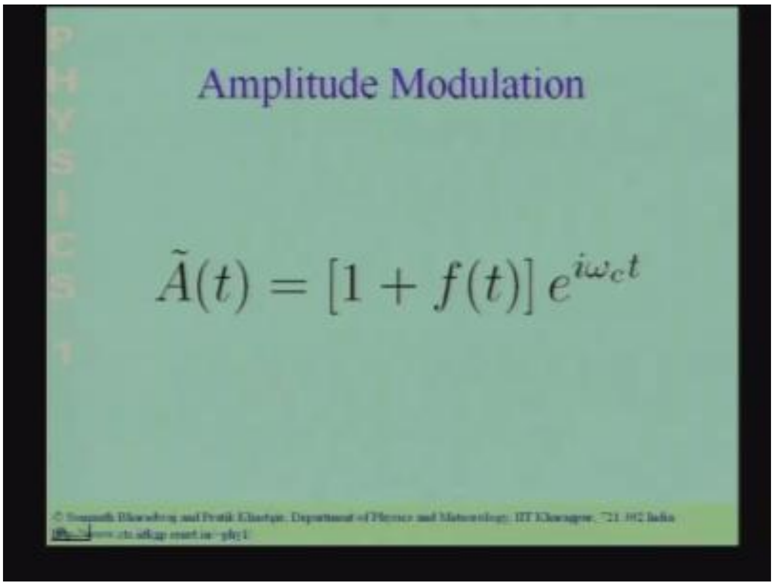

So, now, this is the… so, we are going to consider a spec specific situation, where the signal the signal that is being modulated. That is being the modulation is itself a pure sinusoidal at angular frequency omega m the modulation frequency.

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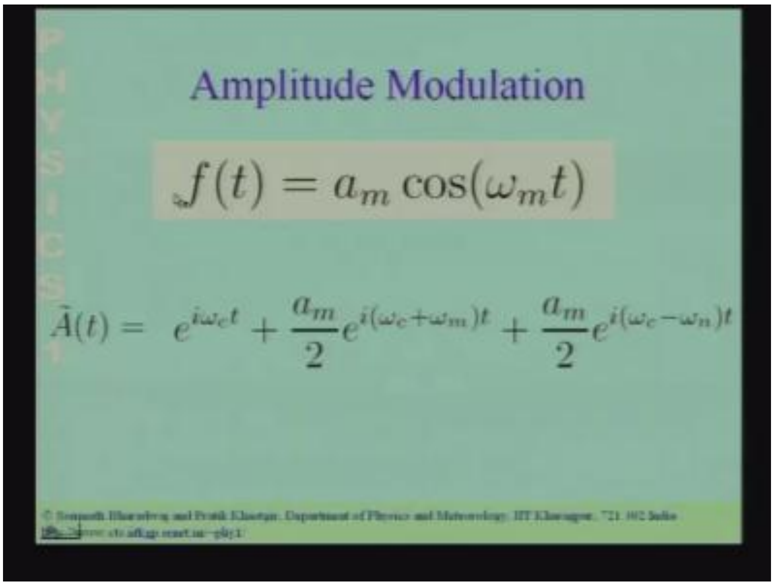

And it has an amplitude am, so, in that in such a situation the total signal that comes out of the radio transmitter can be written like this. So, the carrier frequency is still there, e to the power i omega c t, but you have these 2 extra terms and the 2 extra terms are at a different frequencies. And they occur at the, at frequency omega c plus the modulation frequency and omega c minus the modulation frequency. Now, if you ask the question? What is the frequency content of the signal that is actually, transmitted?

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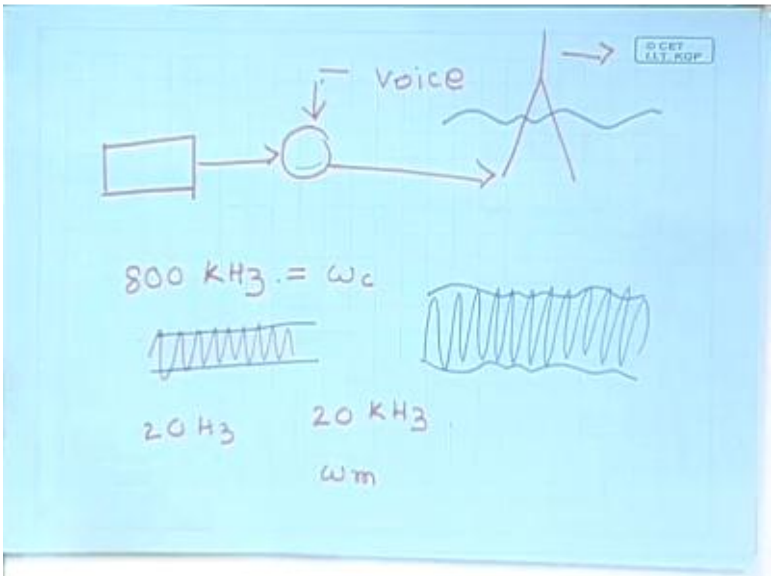

So, this signal that comes out over here is now, set to your antenna, this is your antenna, they will some tower on which the antenna is placed. So, the question so, let me go through the steps again, you have a signal generator that produces the carrier frequency, which in this case is eight 100 kilohertz's. The amplitude of the carrier frequency is modulated in this case the modulation; itself is again other sinusoidal at omega m. And we ask the so, the amplitude of it is modulated and this is now, fed to the transmitter and the question being asked is what are the frequencies which are present in the signal that is transmitted and we have the exercise over here.

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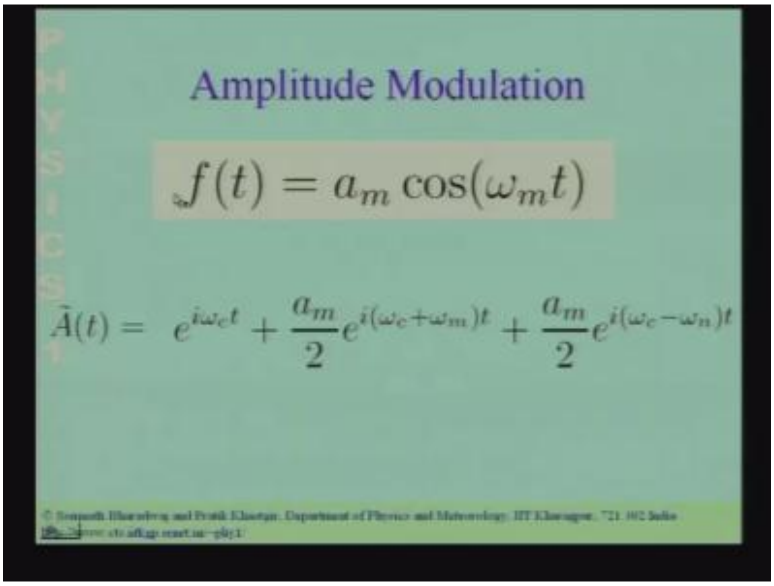

We find that they are two new frequencies, that are coming.

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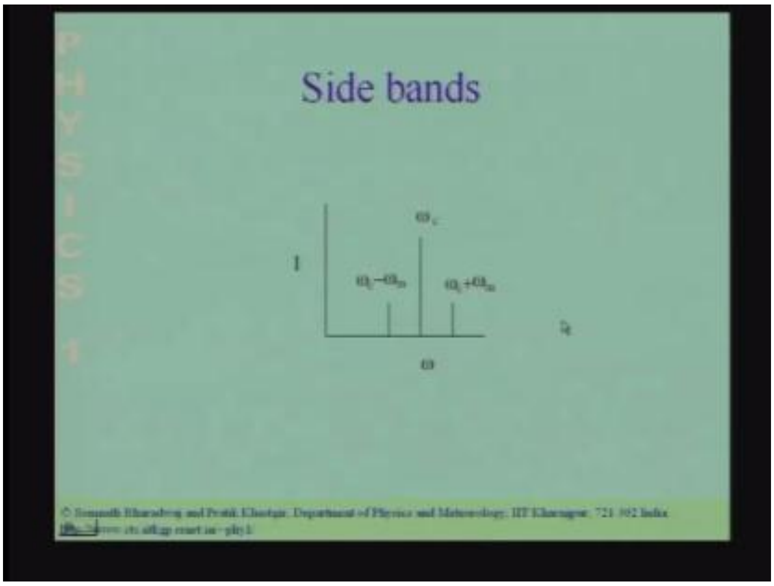

So, you do a send the signal that comes out in to a spectrometer, you will find that there are three different frequencies, which are present the carrier frequency is present no doubt. But there are two new frequencies 1 is the carrier frequency, plus the modulation frequency, and another is the carrier frequency minus the modulation frequency. So, there are two new frequencies that are present so, what we have seen over here is.

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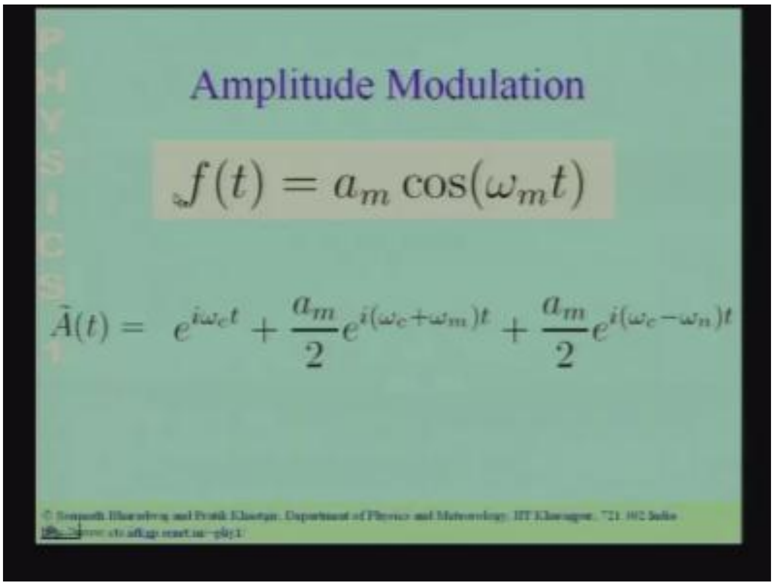

Something very important, what we have done? Is that we have modulated the amplitude of the carrier wave and the modulation. Itself is a pure sinusoidal of a fixed frequency the modulation frequency, and we see that as a consequence of this modulation. We introduce two new frequencies these two new frequencies that introduce.

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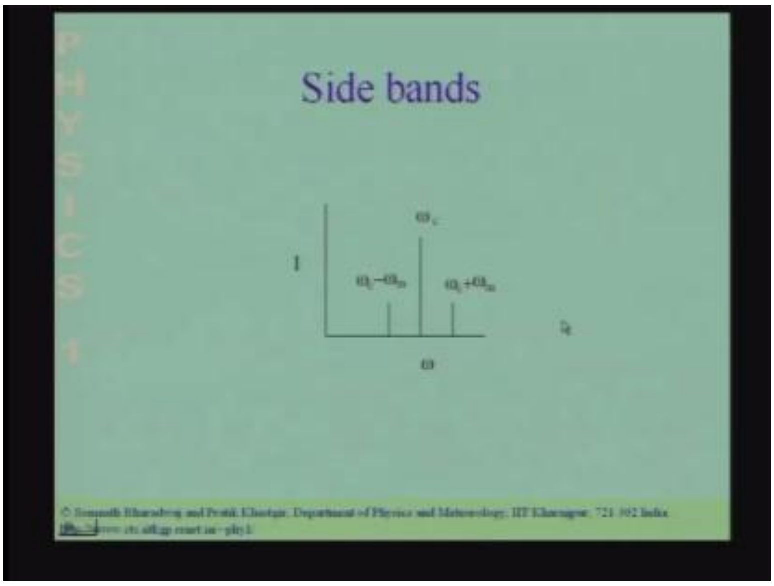

Are called side bands, so, there are two new side bands two side bands, which have been produced 1 at the sum of the carrier are the modulation frequency 1. And the difference of the carrier and the modulation frequency, now, here we have considered a specific time with modulation, which at a unique at a, at a single frequency.

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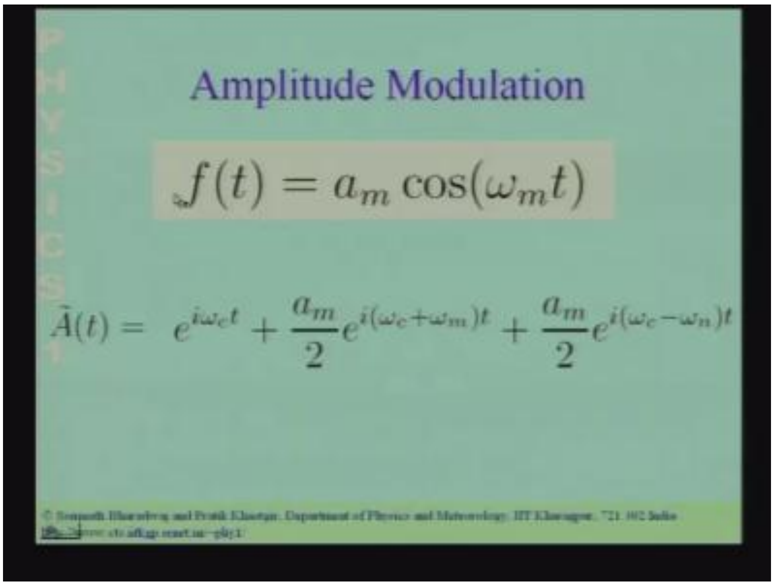

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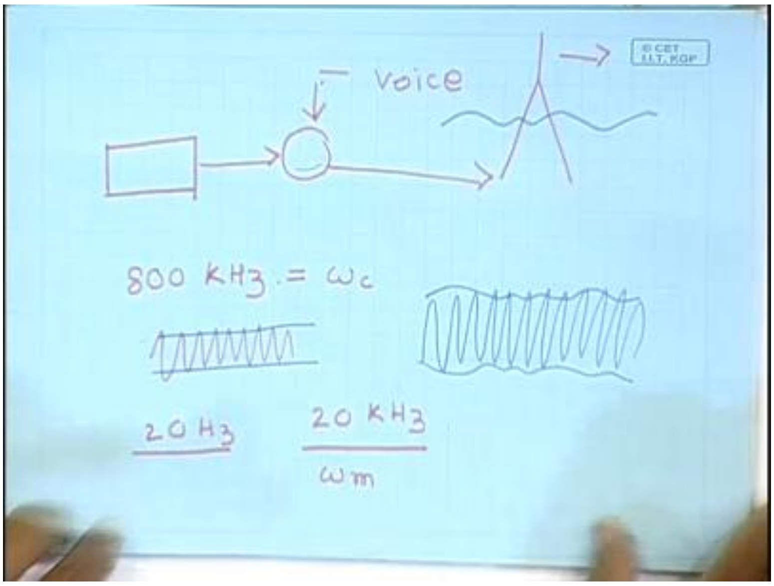

Now. if I modulate, if you modulate my voice for example, on to the carrier frequency, my voice spans the entire frequency range from 20 hertz's to 20 kilohertz's. So, what is going to happen when you modulate this 800 kilohertz's carrier wave with my voice? The difference that is going to occur is as follows.

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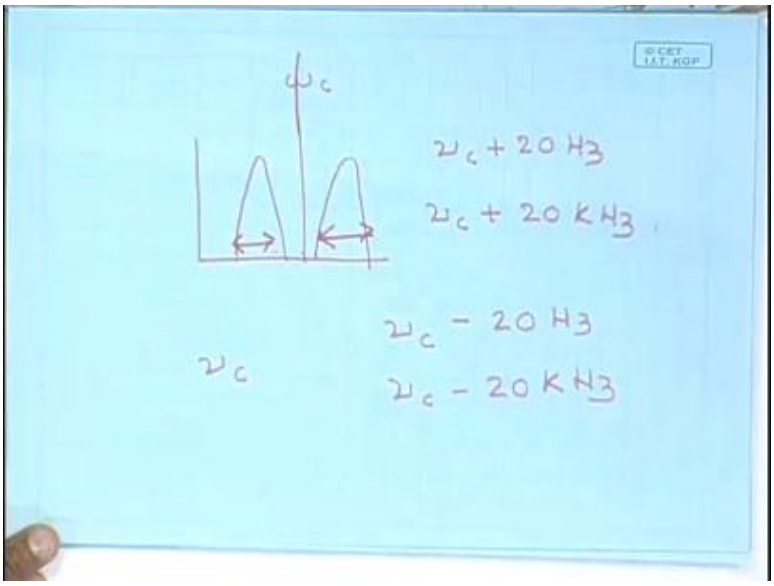

Instead of producing side bands, which at signal frequency, you are not going to produce side bands, which we look like this on both sides, you will have 2 bands and the band. So, this is the omega carrier and this is going to span or if I work it in terms of nu carrier, the carrier frequency is nu c the side bands are going to be from, nu c minus 20 hertz's to nu c minus 20 kilohertz's on one side. And this band over here is going to span from nu carrier frequency minus 20 hertz's to nu the carrier minus 20 kilohertz's, my voice span this entire frequency range.

So, the side band is also going to have the same band width as my voice. We also go to produce another side band over here, which is going to span from the carrier frequency plus 20 hertz's to the carrier frequency plus 20 kilohertz's. So, you are going to produce every radio transmitter is going to operate, if you are going to modulate the signal, you are going to modulate. The amplitude of the carrier wave with a signal every radio transmitter is going to actually, send out signals not at the carrier frequency, but in a band of frequency is around the carrier frequency typically.

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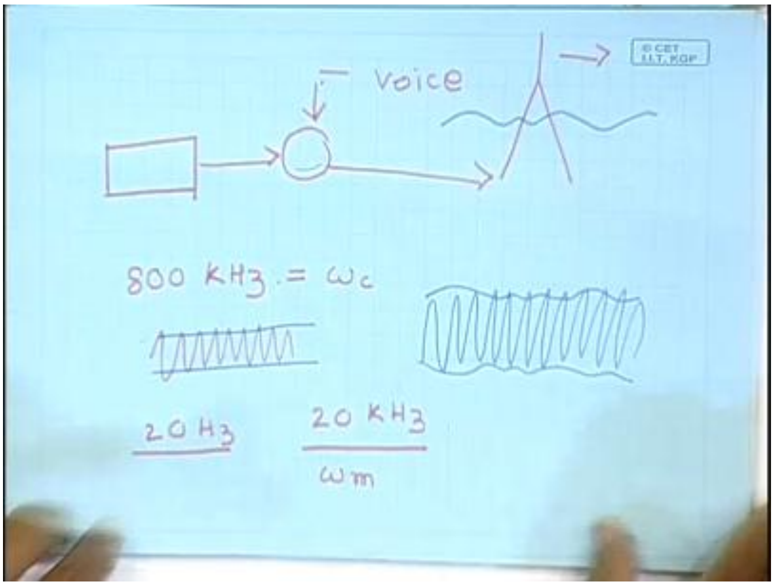

Typically, the high frequency parts are not recorded, they are not very crucial unless you really, want to have high feda fidelity transmission. And typically the band width the voice is recorded over is 10 kilohertz's not 20.

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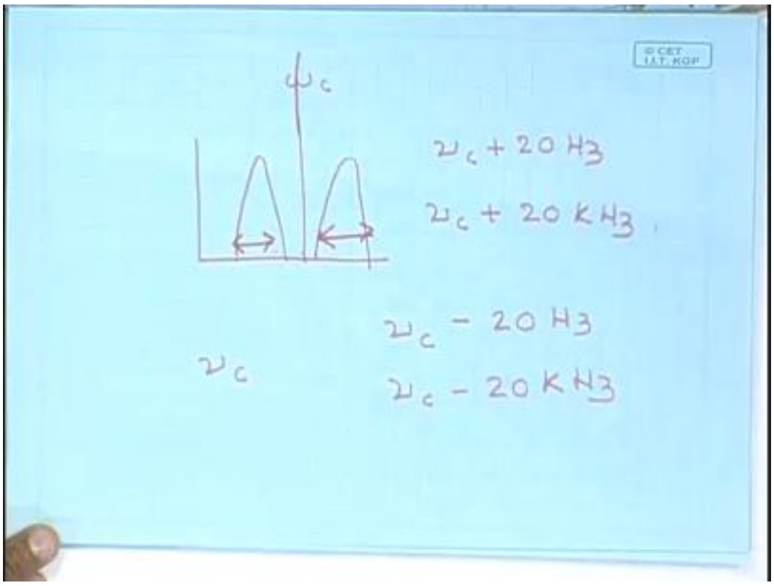

So, typically each transmitter will span nu c the carrier frequency plus, minus 10 kilohertz's. So, let us consider few implication of this.

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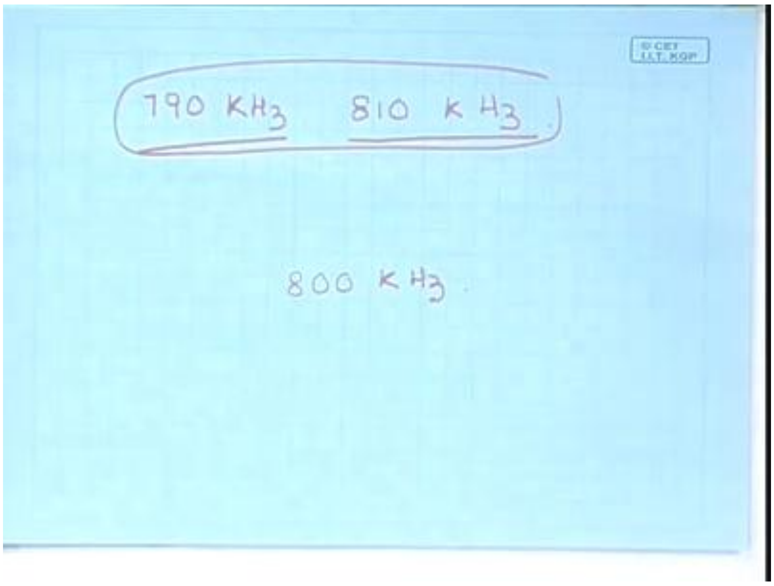

So, in this particular case the entire voice signal will be distributed from 790 kilohertz's 2 8 100 10 kilohertz. Now, consider a receiver, so, this transmission now, consider the receiver, which is tuned at 8. 100 kilohertz's, but which tuned to very narrow band width, let me just tell you how the receiver tuned receiver are tuned using LCR circuit. So, the antenna will be coupled to a LCR circuit the LCR circuit as a resonance decided by the values of a L C and R, so, by changing the capacitors the usually. The value of the capacitors is connected to a knob, the capacitor be connected to the knob.

If you rotate the capacitance could change and by changing the capacitance, you change the angular frequency, the frequency corresponding to the resonance. And so, it will be tuned to different frequency now, suppose the resistance is the width of the resonance that is decided by the value of the resistance. If the resistance is 0, which then you will have a very sharp resonance, basically of no width, but there is all always, going to be resonance resistance. If you have a finite resistance is going to be a finite width, but if the resistance is very small width is going to be very small suppose my resonance is so, narrow.

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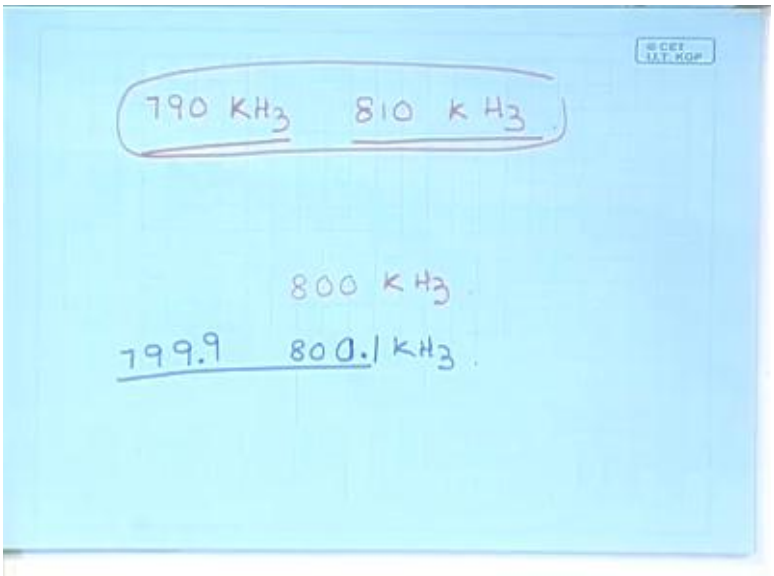

That my receiver is tuned from the frequency range 800 and 1 between the frequency range 799 to 800 and 1 kilohertz's only. Or let us see, even smaller, so, 8 and 99.9 to 800.1. So, suppose I tune my receiver is such that, it is tuned at eight 100 kilohertz's. But it can receive frequency it is going to be sensitive to frequency only in this very narrow range of frequency around this frequency. Question is can you reconstruct my voice from the radio signal? Only in this very narrow range of frequency known, because my voice spans the entire frequency ranges of ten kilohertz's. So, you have to your radio receiver, also as to be sensitive to the entire band, to the entire side band. The entire side bands to be able to receive, to be able to reconstruct, my voice from the signal that is being transmitted. So, the receiver, itself all has to also receive the entire side band 1 both the side bands contain my contain full voice signal.

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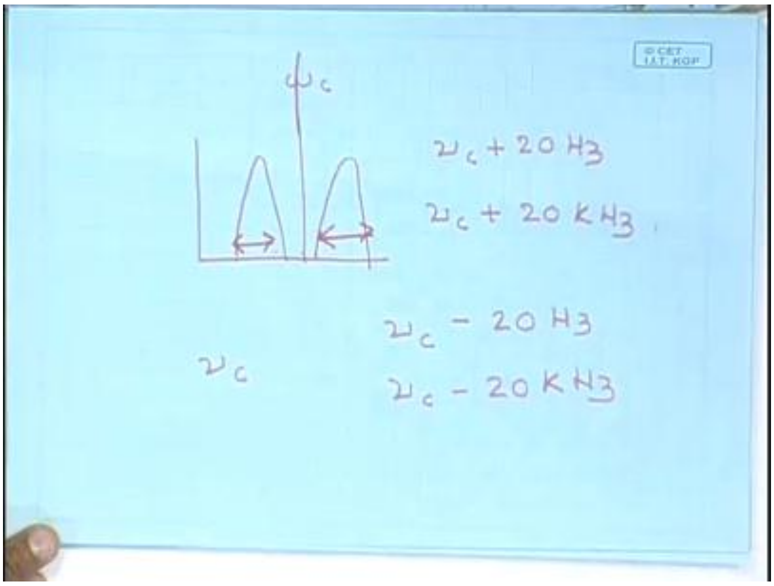

So, as long as radio receiver can receive 1 entire side band, it can reconstruct. So, you will able to hear, what is am saying. But if I cannot receive the full thing, it only receives part of, it you will not be able to make out what I am saying. May be the high frequency parts will be missing in the radio receiver in what you receive, what you hear in your radio receiver.

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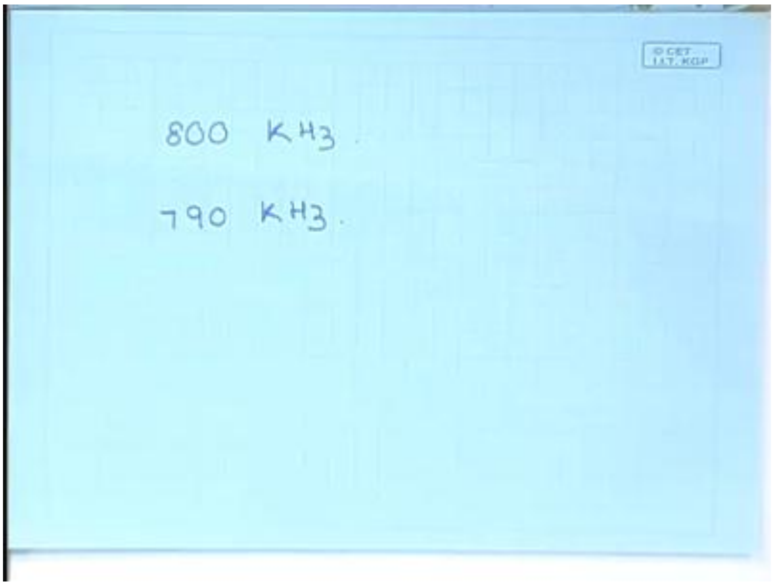

Consider another situation. where I have 1transmitter working at 800 kilohertz's another transmitter working at 790 kilohertz's now, the side bands of these two transmitters are going to overlap. And what you receive? What you are going to receive? Then is going to be some kind of a junk, so, it as to 1 as to ensure that the radio transmissions occur sufficiently, for a part in frequency. So, the side bands that are produce do not overlap and typically the whole frequency range from 500 kilohertz's.

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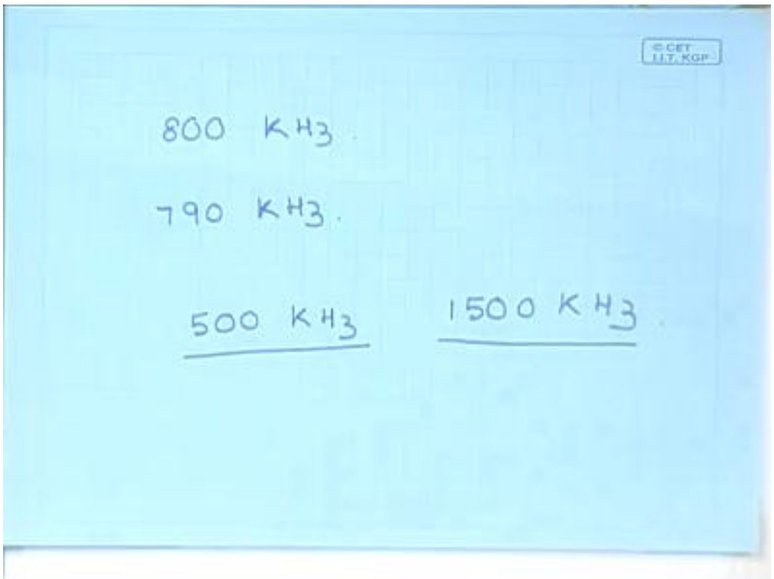

To 1500 kilohertz's is available for amplitude modulation am transmission and you can fit in a large number of radio stations in this entire range of frequencies. So, until now, we have been discussing the behavior of the super position of two different frequencies at a fixed position, only as a function of time. Let us now, discuss the full space time behavior of super position of two different frequencies. So, this is what is shown over here.

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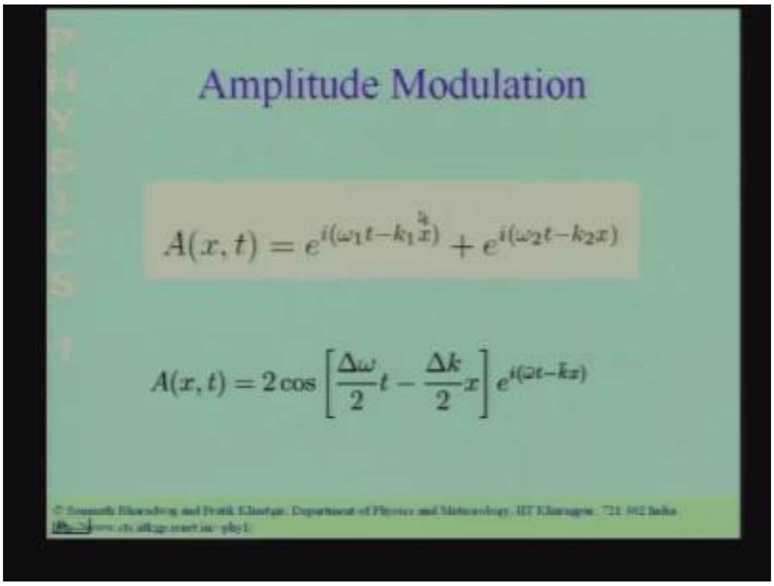

We have two waves of two different angular frequencies omega 1 and omega 2, we are also interested in the special evolution of these waves. So, the wave numbers are also different and we have written now, the full space time evolution now. So, we e to the power i omega 1 t minus k 1 x here, we have omega 2 k 2 instead of omega 1 and k 1. Now, we can repeat exactly the same exercises, which we had done, when we had looked at time evolution.

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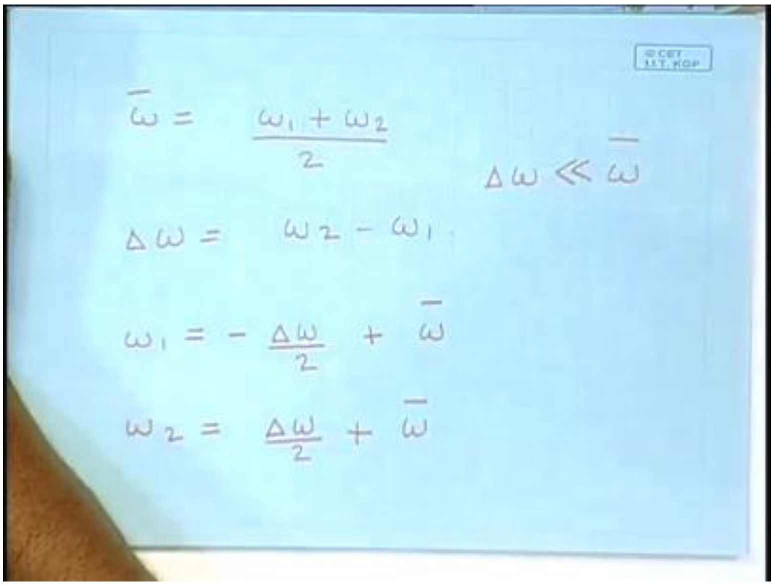

So, we could write omega 1 and omega 2 in terms of the mean angular frequency and the difference in angular frequencies, we can do exactly the same thing with the wave numbers also.

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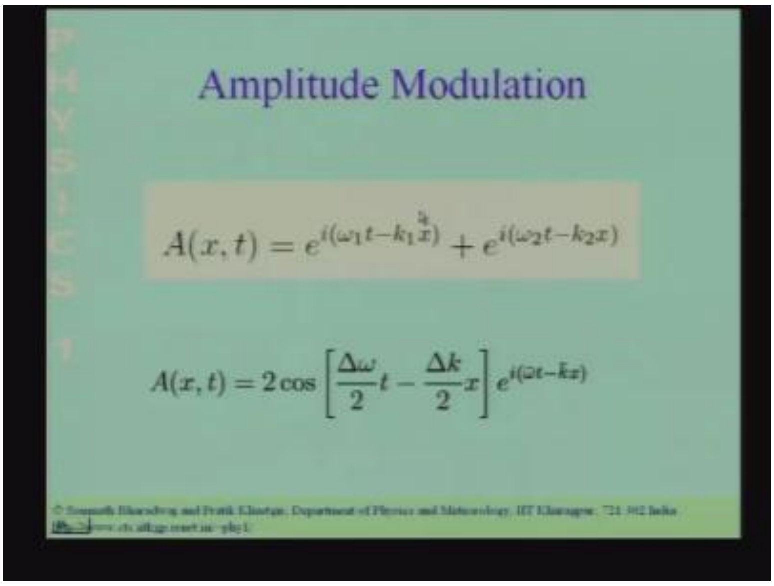

So, we could also introduce the average wave number and the different, difference in the wave numbers and the super position of this is a we have already, worked it out.

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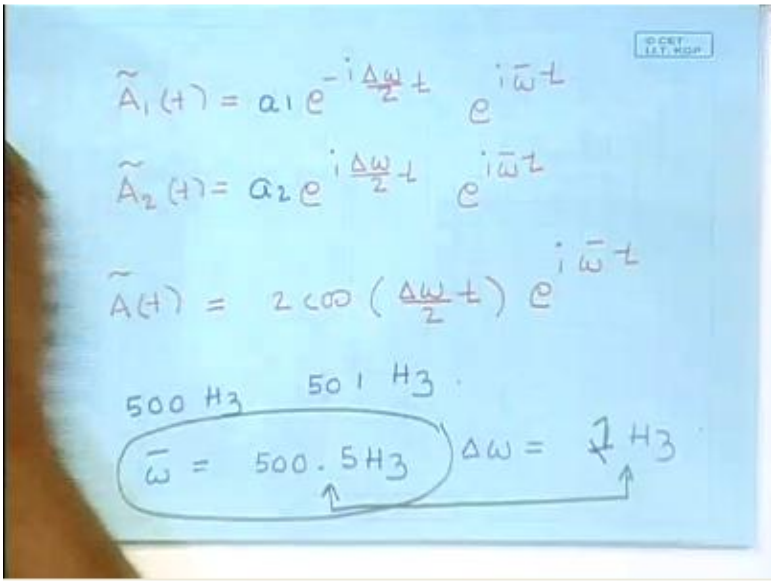

If you superpose these when we looked at it as a function of time alone, we had got this cosign into e to the power i omega of the average angular frequency.

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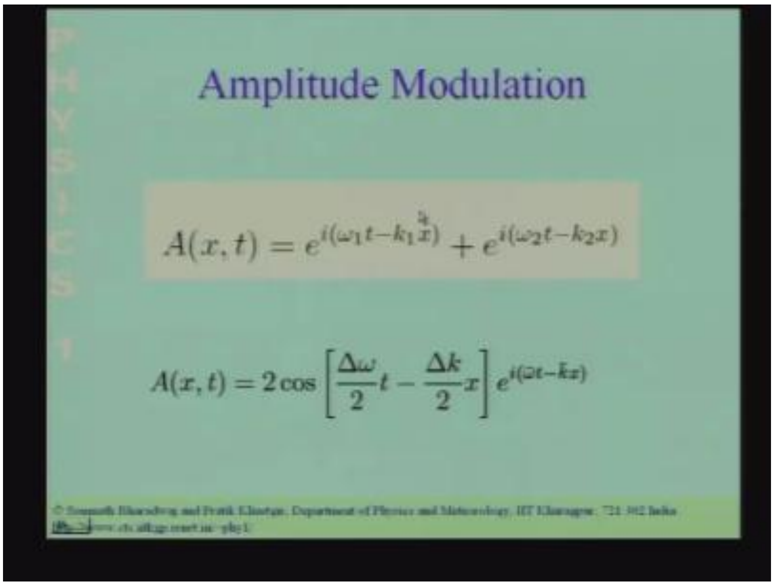

So, now, when I keep the special depends…What we get is again a cosign now, we have delta omega by 2 t, which we had earlier. But we also had have an extra term minus delta k by 2 x, which in co-operates the space in depends similar, we have minus I k bar x over here. So, it is a just a small generalization of the exercise, which we have already done in the being after this lecture, just that we have now, in co-operated the space independence also. So, now, you see that it is this the resultant is the super is a product of two different function of space and time. This a fast varying function of space time both of space and time. So, this is a faster varying function compare to this of space also, because k bar the average wave number is much larger than the difference in the wave numbers.

So, we assuming that delta omega and delta k are small compare to omega bar and k bar respectively. So, this function varies rapidly in space and rapidly in time the amplitude of this, wave vary slowly in space and slowly in time. Now, so, when we superpose two slightly different two waves of to slightly frequencies, what we get is a fast travelling wave again at the average frequency. But the amplitude of this wave gets modulated and it again itself, behaves like wave, which propagates, which has a small wave number and the small angle of frequency. So, this is this modulation of the amplitude is represented through, this. Now, let us ask the question what speak does the amplitude of the modulation of the wave propagate, the wave itself propagates?

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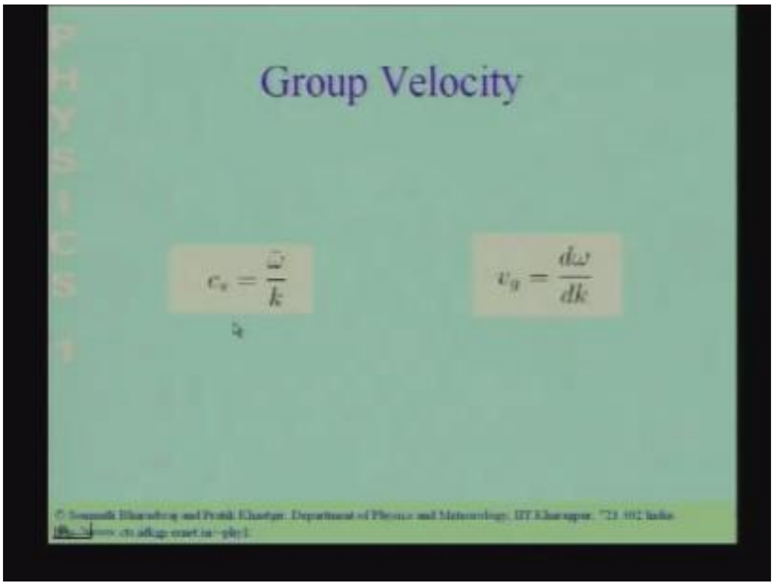

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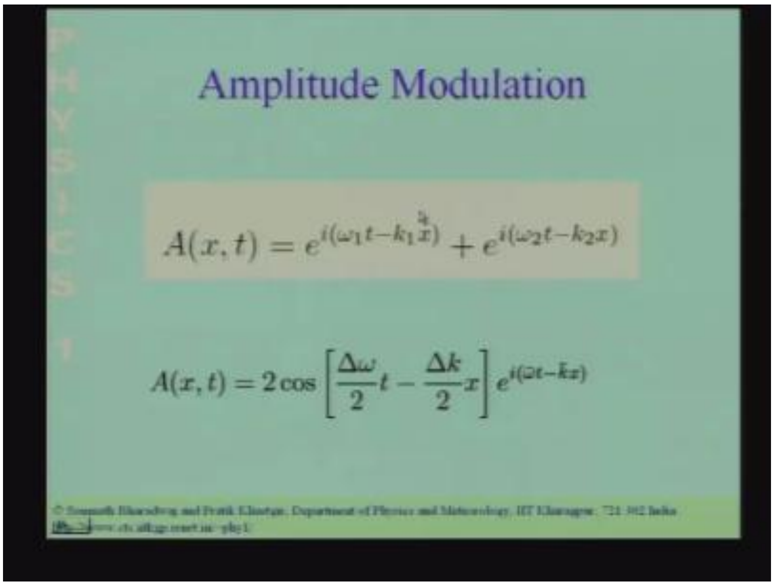

At the phase velocity which in this case… So, when we say that the wave itself propagates at the phase velocity the fast part propagates at the phase velocity which is omega bar by k bar.

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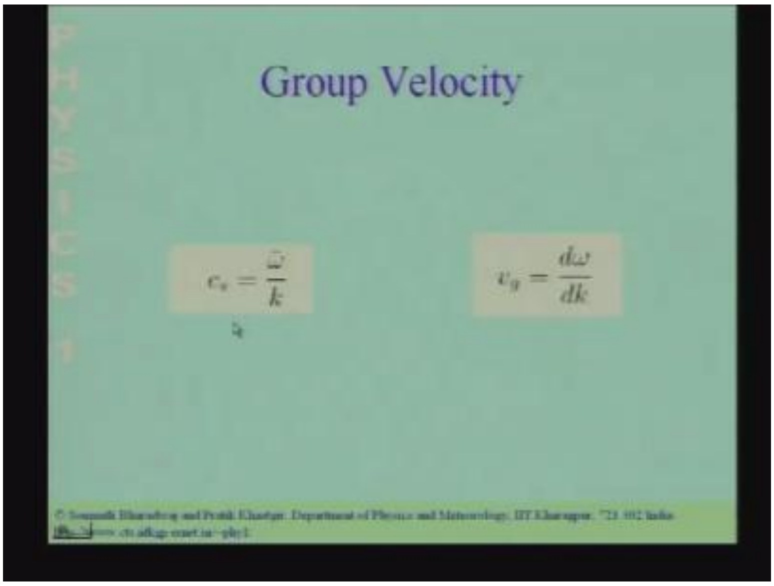

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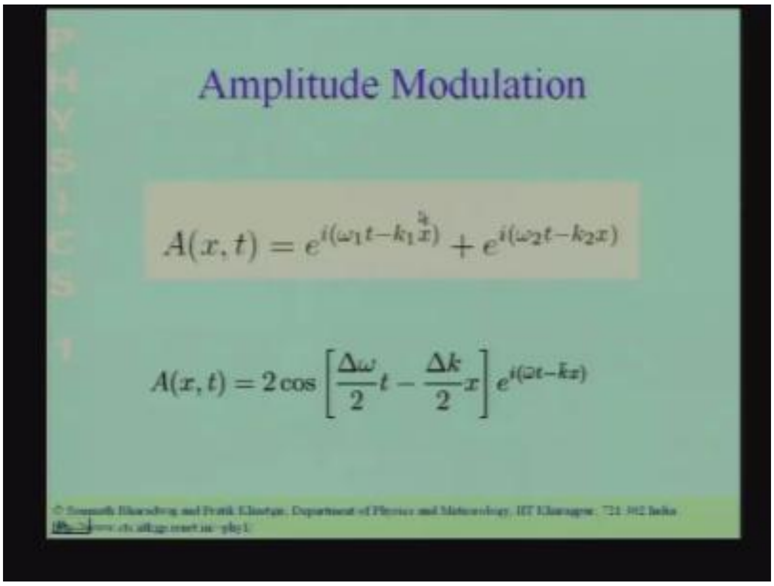

And the modulation of the amplitude itself propagate, you can determine the at, what speed the modulation propagates the modulation will propagates at the speed at delta omega by delta k. And if you take delta omega and delta k going in the limit, where the difference in the 2 frequencies is goes to 0.

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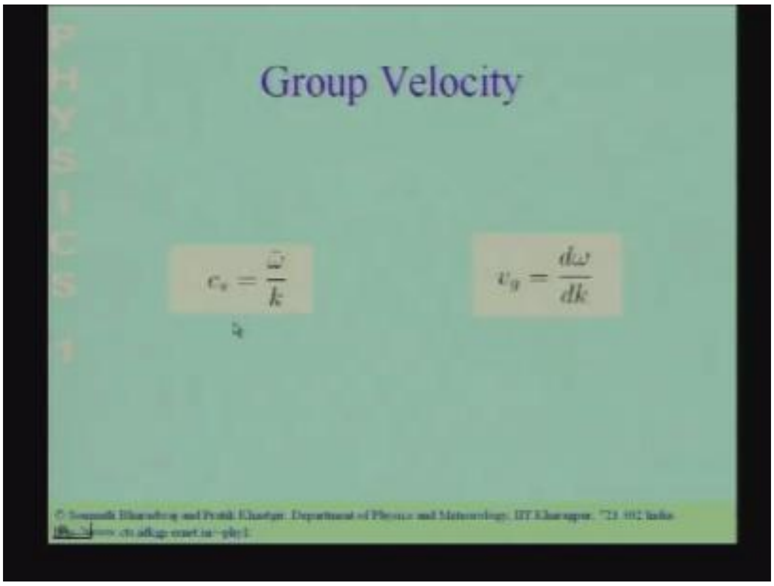

Then you get the speed at which the modulation propagates, that is call the group velocity which is d omega by d k.

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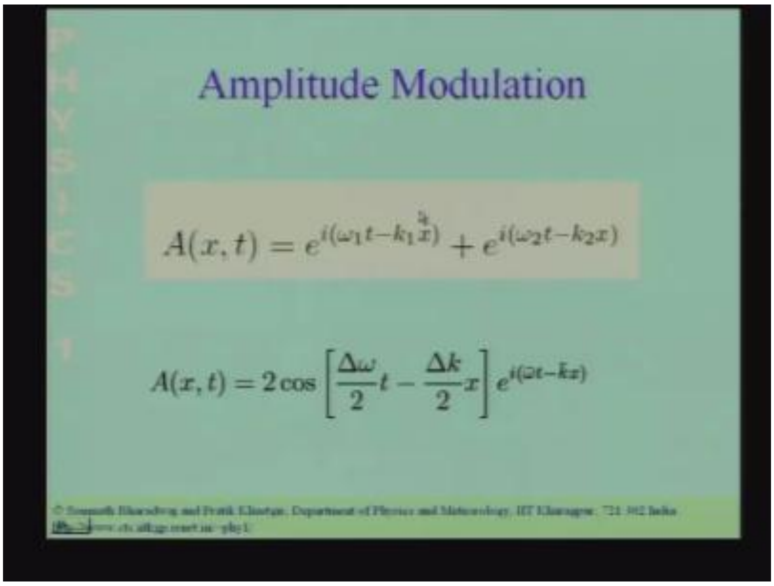

And we have already discussed that; the modulations can be used to send information.

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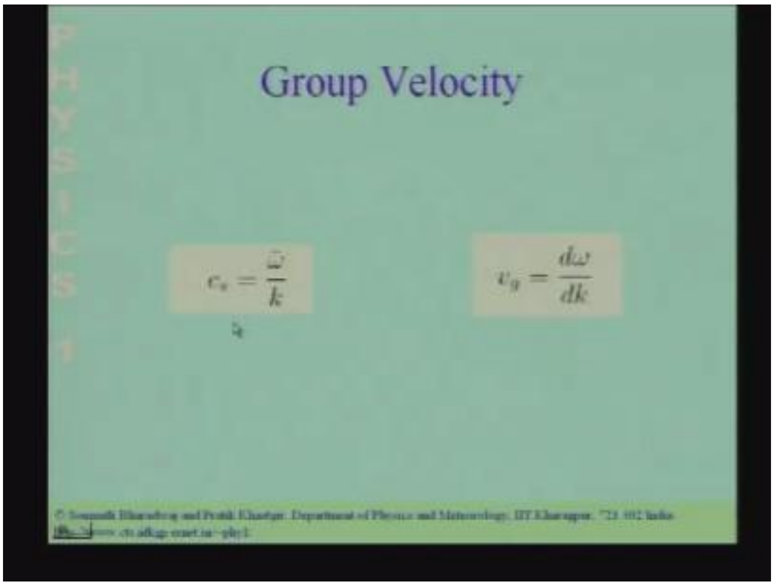

So, the group velocity usually tells us that the speed at which the information can be carry by the wave.

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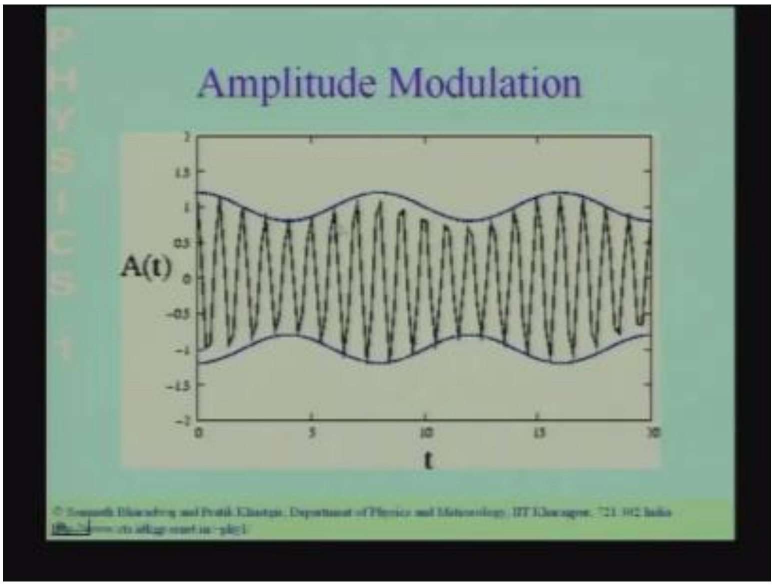

So, it the propagation of these modulations, that carries information that carry signal and the modulation in the amplitude itself propagates.

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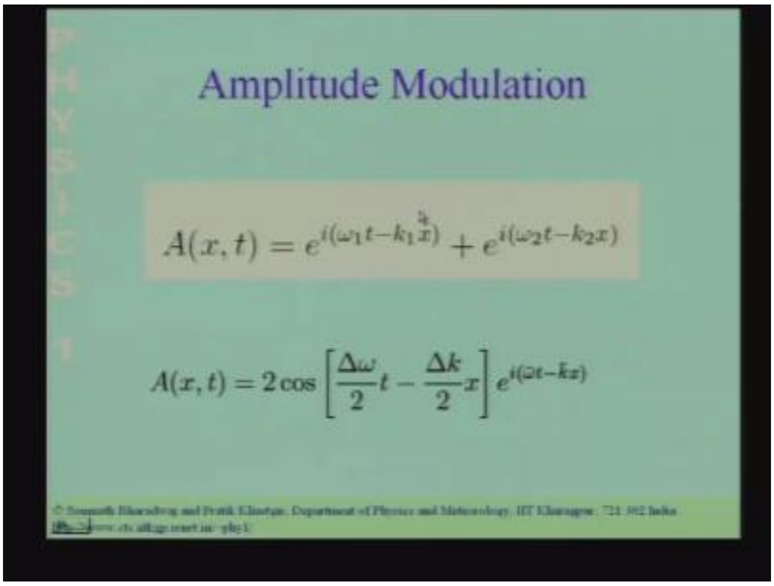

As you can see here at a speed, which is the ratio of this delta omega to this, delta k ratio of this co-efficient to this co-efficient.

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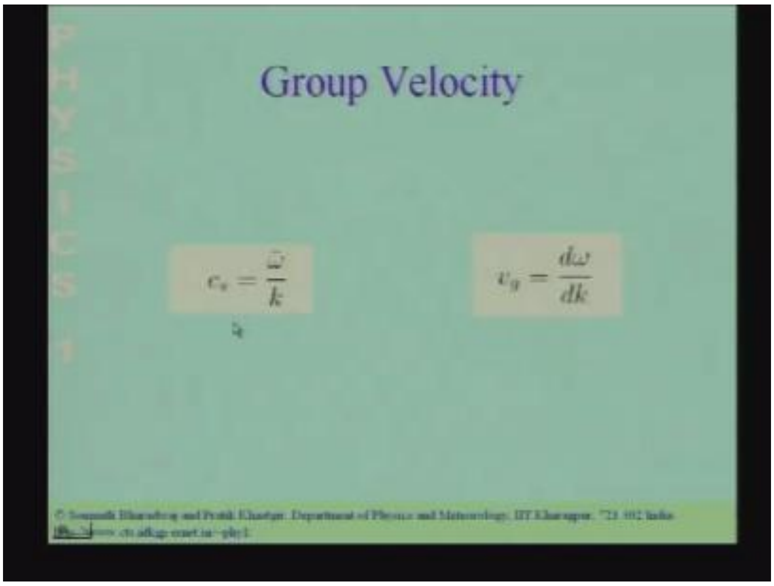

And this ratio is call the group velocity d omega d k and this tells us the speed at, which the modulation propagates. And this is usually, the speed at, which you can send the signals using the wave. So, this called the group velocity, so, let me summarize this, the phase velocity tells us the speed at, which the phase propagates. But this itself carries no information, no signal to send the signal, you have to modulate the amplitude and the modulation in the amplitude. They propagate at the group velocity, which is d omega d k not the ratio omega by k, but at the group velocity, which is d omega by d k. So, the modulations propagate at the group velocity and usually, the signals also propagate at the group velocity, D omega d k usually not always let me consider an example.

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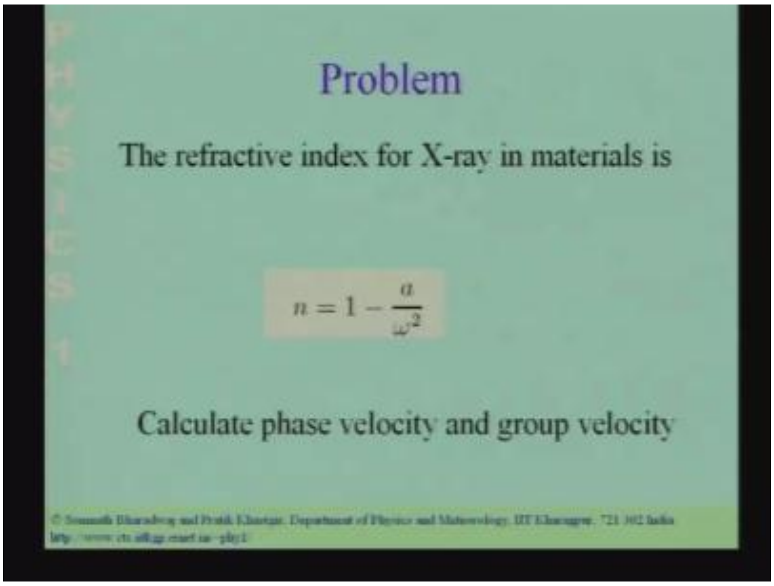

The example is as follows the refractive index for x-rays in material is known to be less than unity and it is given by the expression over here. So, refractive index for x-ray in materials is found to be of the form 1 minus a constant divided by omega square where the constant a is positive. So, refractive index for x-rays inside materials is less than 1 now, you must have heard that inside medium light. The refractive index, it is a speed of light changes inside medium and this the speed of light inside the medium, is given by refractive index. And usually the refractive index is more than 1 and the speed of light is c divided by the refractive index, which is less than 1 the less than speed of light in vacuum. But for x-rays, you have a different situation the refractive index is less than 1. So, the speed of light, inside medium the speed of x ray the phase velocity to emphasize for x-ray inside medium is also more than speed of light in vacuum?

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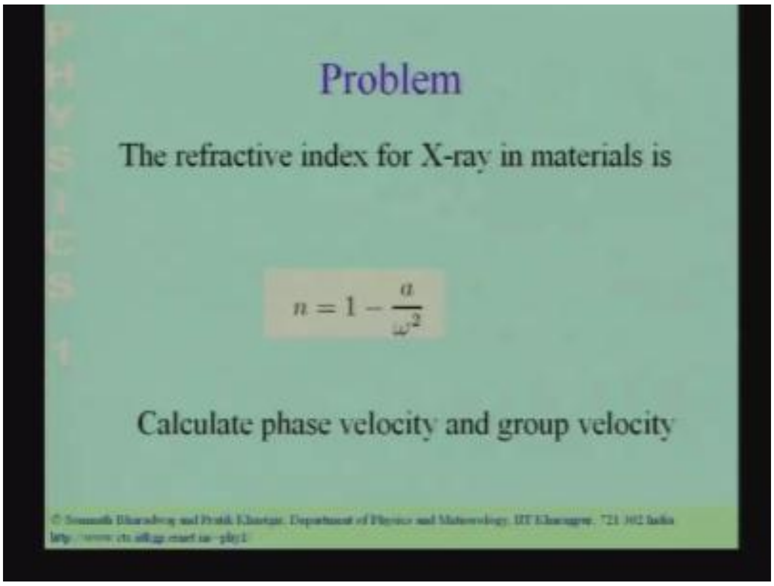

So, the problem, which we have giv over here is to calculate the phase velocity and the group velocity for x-rays inside the material.

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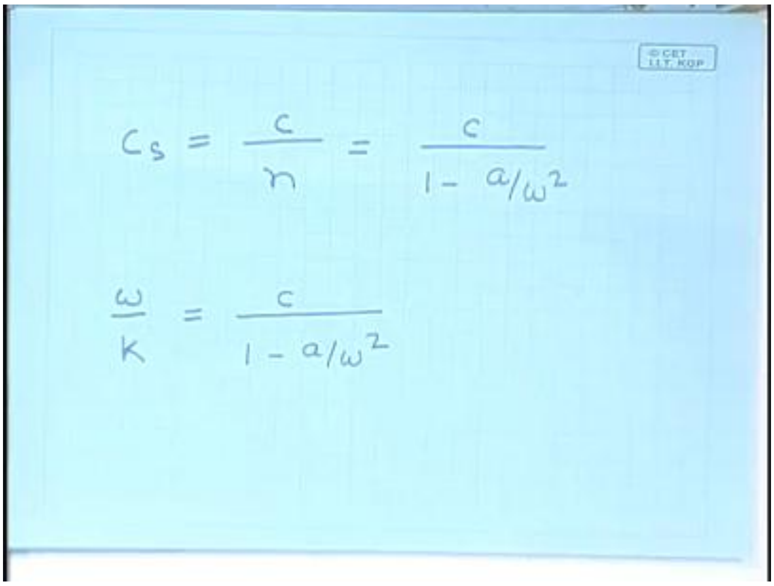

So, the phase velocity, you will use c s to denote the phase, you better use c s to denote the phase velocity the phase velocity is the speed of light in vacuum divided by the refractive index, which in this case is c divided by 1 minus a by omega square. So, that the phase velocity inside for x-ray inside the medium, which is more than the speed of light in vacuum. Now, the phase velocity, we know is the ratio of the angular frequency to the wave number. So, this is equal to c divided by 1 minus a by omega square. Now, we want to next calculate the group velocity to calculate the group velocity, we have to calculate the derivative of the angular frequency with respect to the wave number.

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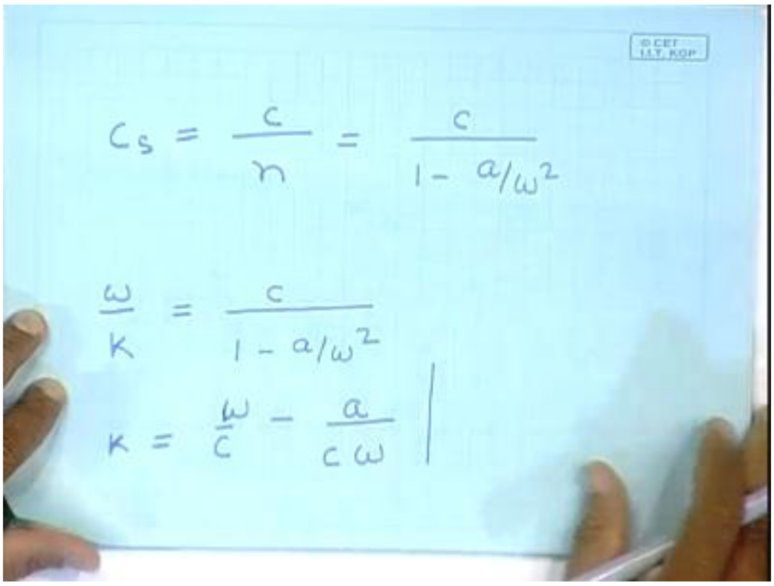

So, what we do is we obtain an expression for k in terms of omega, let us do that. So, k is going to be omega 1 by c 1 by c into omega times, this expression over here. So, we have 1 term, which is omega by c and then we have another term, which is going to be minus. So, the factor of a will come on top a by c omega right, that is the value of the wave number in terms of the angular frequency. And we can use this to calculate d k d omega, let us let us differentiate the expression for k with respect to omega.

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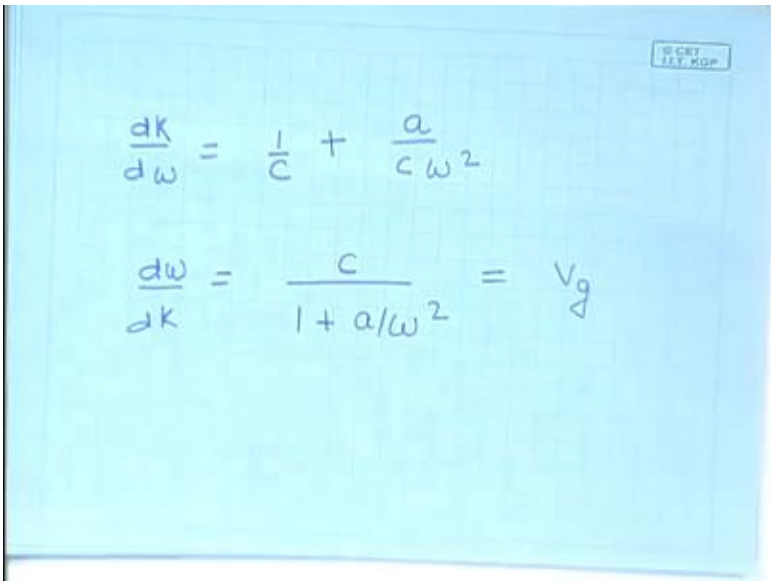

D k d omega is equal to 1 by c minus a by c now, if I differentiate this, with respect to omega I will get plus 1 by omega square, which gives us d omega by d k is equal to c divided by 1 plus a by omega square . So, the group velocity, we see is more than is less than the speed of light in vacuum. So, let me just explain, to you the significance of what we have done for x-rays in materials the refractive index is less than 1. It implies, that the phase velocity of x-ray in material in medium in materials is more than the speed of light in vacuum. But as we have already, discuss the phase velocity is on the speed at, which you can send signals using the light. So, if you use your x-ray to send a signal the modulations, you will have to modulate the wave and the modulations travel at the group velocity d omega by d k, which we have calculated.

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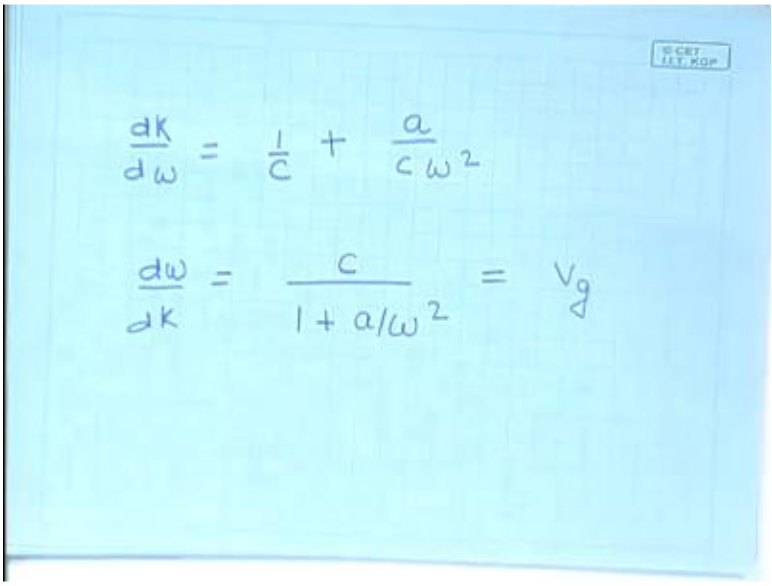

and the group velocity is c divided by 1 plus a by omega square. So, this factor is going to be more than 1, so, the group velocity is going to be less than the speed of light. So, the modulations do not propagate faster than, this speed of light in vacuum and you cannot send signals, at usually the group velocity tells, you the speed at, which you can use the wave to send the signal. So, it turns out, that you cannot send the signal at a speed faster than, the speed of light in vacuum using x-rays through by sending it through, medium cannot be done the group velocity is less than the speed light in vacuum. So, you cannot send signals faster than, the speed c using this x-ray in medium now, it is a fundamental postulate physics, which is a very important for Einstein Theorem. Specialty with fundamental postulate is that no signal can propagate at a speed faster than the speed of light in vacuum.

It is a fundamental postulate and till there as been no contradiction of this fundamental postulate. If there was a contradiction, then you would have to reverse the whole of physics as it is stands now. So, till there is no there no evidence, that you can actually send signal faster than, the speed of light in vacuum. And this is consistent with that postulate, because the phase velocity is more than c, but the group velocity, which is usually, corresponds to the speed at, which you can send signals. The group velocity usually tells us the speed at, which you can send signals and the group velocity is less than the speed of light in vacuum. So, let me end today's lecture over here and we going on to quite different topics from the next, we have finished interference diffraction and super position of waves. We are going to go on to quite different topics from the coming next lecture onwards.