Water Quality Management Practices

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Week-07

Lecture-32

Bacterial growth kinetics in ASP: Biomass mass balance and substrate mass balance

Hello everyone, welcome to this NPTEL online certification course on Water Quality Management Practices. My name is Gourav, Professor Gourav Bhowmick. I am from the Department of Agriculture and Food Engineering of Indian Institute of Technology, Kharagpur. Continuing with the module 7, Aerobic Waste Water Treatment System and in this particular lecture I will be discussing about the Bacterial Growth Kinetics in Activated Sludge Process, Biomass Mass Balance and the Substrate Mass Balance. This is the different concepts that we will be covering the bacterial growth kinetics, biomass mass balance assumptions in the mass balance equation and also the substrate mass balance to start with like you know. In case of aerobic bacterial treatment systems and all, so we know that our one of the main major component here is a living stuff.

It is a bacteria, it is a type of either bacteria, protozoa or fungi which are taking the lead role here in the in our aerobic treatment systems. So, what are what are their purposes, what they do? They their job is to is to consume the oxidized organic matter and convert it into either carbon dioxide, bacterial cell biomass or to release the ammonia. If you see this equation, equation number 1 it is represents this CHONS represents the a say like a organic matter.

$$\underset{\text{(Organic matter)}}{CHONS} + O_2 + \text{Nutrients} \xrightarrow{Bacteria} CO_2 + H_2O + NH_4^+ + \underset{\text{(New cells)}}{C_5H_7O_2N}$$

In presence of oxygen this organic matter acts like a electron donor and oxygen acts like a electron acceptor.

What it does? It in it converts into the carbon dioxide, ammonia, ammonium ion, water and some new bacterial cell. This C 5 H 7 O 2 N represents a new bacterial cell ok. So, now this oxidation and the synthesis reaction is responsible for the removal of biodegradable organic matter from the wastewater in the activated sludge process. And when this organic matter present in the wastewater is depleted, the bacteria is also flow forced to go for endogenous respiration. So, we all already know that what is endogenous respiration, but in case of endogenous respiration the cell protoplasm it converts into the elemental it like started breaking down and convert into elemental constituents like the like the equation you can see in the equation number 2.

$C_5H_2O_2N(113g) + 5O_2(160g) \rightarrow CO_2 + H_2O + NH_3$

Where you see this $C_5H_2O_2N$ if say like it represents a cell cell biomass. In the presence of oxygen 5 number of oxygen molecule it what is happening there? It converts into carbon dioxide H 2 O and ammonia. So, if you see the oxygen requirement here for 113 gram of biodegradable cell mass like you know it 160 gram of oxygen is required for the for its stabilization. So, stoichiometrically speaking the relationship tells that the cellular sludge mass required at oxygen value of equal to 1.42 milligram of oxygen per milligram of volatile suspended solid.

So, just 160 by 1.113 you will get this value ok. So, this is how actually this representations is very important you need to remember this thing will be needing it later on. In general the rate of biomass increase during the log growth phase is directly proportional to the initial biomass concentration and it follows the strictly the first order equation which is represented in equation number 3 that

$$\frac{dX}{dt} = \mu . X \qquad (3)$$

where is this what is this dx by dt is the growth rate of biomass in gram per meter cube per day and μ is the specific growth rate in per day and x is the biomass concentration in gram per meter cube per milligram per liter ok. This biomass concentration this capital X how we can find out it is a MLVSS ok, good.

So, we already started understanding the concepts and all. So, taking into account the endogenous respiration because it is not that only it is a continuous system and maybe some amount of biomass is also taking its own protoplasm because of the severity of the say like substrate what it does it consumes some amount its own protoplasm and it goes through the endogenous decay process. In this endogenous decay process we it is because of that we need to introduce this in this equation 3 as well and it convert it modified to equation number 4 you say

$$\frac{dX}{dt} = \mu X - K_d X \quad (4)$$

whereas, this Kd is the endogenous decay coefficient and X is the biomass concentration. And in case of in general the wastewater treatment by biological means depends on the versatility of the bacterial metabolism and the ability of microorganisms to carry out the metabolic processes at very low concentration of substrate in an aqueous environment. So, and we already know what is stoichiometry.

So, this is how the bacterial growth kinetics in case of log growth phase works. To discuss to understand further as the maximum bacterial growth takes place during the exponential growth phase or the log growth phase this μ is equal to μ max when the ample amount of substrate is available. The specific growth rate in operational activated sludge process will never be equal to the μ x μ max, but it because it depends upon the substrate concentration, but theoretically its maximum in case of exponential growth phase. In this growth condition we know the substrate is in the substrate limited growth conditions under which they will not be able to exhibit the maximum growth rate. Monod equation actually monod scientist monod's equation actually gives us the most perfectly pictured expression for which represents this specific growth rate of bacteria as a function of the substrate utilization which can be represented by equation number 5 you see the

$$\mu = \mu_m \frac{S}{K_s + S}$$

.

So, here if you now use the equation number 4 represent like you know now put this value equation number 5 and equation number 4 you will get this equation number 6

$$\frac{dX}{dt} = \mu_m X\left(\frac{S}{K_s + S}\right) - K_d.X$$

Here the k S is what is the concentration of the substrate S when μ is equal to μ × 2 ok. It is represented by the unit of milligram per liter and if you see in this figure the specific growth rate versus limiting substrate concentration you see the at whenever the μ m by μ m is equal to like the in the graph when the in the specific growth rate value μ value is μ_m × 2 that time whatever the substrate concentration it is represented by the k S ok. So, please remember that. And for if all the substrate like in organic matter present in the system converted to biomass then the substrate utilization rate is obviously, equal to the biomass growth rate whatever the substrate is utilized it converted into biomass.

So, that should be this rate should be same the substrate utilization rate is equal to biomass growth rate, but it does not happen it never happened. Why because all the substrate cannot be converted into biomass because energy generation from the oxidation of organic matter is must for supporting the anabolic reaction the biomass synthesis that is happening in case of in case of biomass growth rate in that in that part of in that part of it. So, because of the this biochemical conversion you cannot have a exact value of substrate left for converting into the biomass value ok. So, there will be certain disproportions. So, in general that is why we introduce the cell yield coefficients or normally this value is less than 1 because that means, the all the substrate even the 100 percent of the substrate is convert like you know utilized it does not give you the 100 percent conversion into a cell biomass ok.

I mean like you cannot you cannot do it. So, because of that this yield coefficient value is less than 1 and it this correlation can be defined by equation 7 the

$$-\frac{dS}{dt}Y = \frac{dX}{dt}$$

whereas, the minus dS by dt if you replace this dS by dt with the monod equation it will become equation number 8

$$-\frac{dS}{dt} = \frac{1}{Y}\frac{\mu_m SX}{K_S + S}$$

Where this y value normally varies from 0.4 to 0.8 milligram of VSS per milligram of BOD in aerobic biological wastewater treatment systems.

Now, the mean cell residence time which we all know what it is like you know we already had a discussion about it that total mass of VSS present in the reactor divided by the VSS that we are wasting from the system per daily basis ok. So, this capital X divided by dx by dt is it is represented by the value of theta C.

$$\theta_c = \frac{Total\ mass\ of\ VSS\ in\ reactor}{VSS\ wasted\ per\ day} = \frac{X}{dX/dt}$$

If you now substitute this value in equation 6 like and rearrange it the you will get the equation number 10 isn't it because it is like 1 by theta C is equal to dx by dt divided by x and dx by dt if you replace this dx by dt with this equation with the equation number 6 you will be divided by x you will get rid of the x component in the right hand side.

$$\frac{1}{\theta_c} = \mu_m\left(\frac{S}{K_s + S}\right) - K_d$$

The same thing you can see in the equation number 10 just the upside down ok. Now, solving this equation number 10 for S value you will get this equation number 11 and which actually represents the which by this equation you can easily calculate or estimate the effluent soluble BOD if you all the other parameters are known to you.

In some cases in certain cases this S value is significantly less than the Ks and in that case this Ks plus S value in equation number 10 in the denominator it is nearly Ks. So, hence the term can be substituted by Ks and it makes this equation you know 10 must simplify to equation 12 as in the special case you see 1 by theta C equal to µ max multiplied by S by Ks minus Kd

$$\frac{1}{\theta_c} = \mu_{max}\left(\frac{S}{Ks}\right) - K_d$$

. So, this is how like you know your simplified version of the equation 10 can be derived. What are the assumptions that we normally take in case of mass balance equation? The biomass concentration in the effluent is considered negligible that means, the influent that is coming from say like primary treatment unit that influent should not be having any biomass theoretically speaking ok. It should only be containing the organic matter which needs to be treated in the aeration time.

So, whenever we go we did this substrate mass balance or the biomass balance and I mean like the mass balance equations whenever we are doing it we have to make sure that it is like it we follow this particular assumptions. Substrate concentration in the influent wastewater is considered to be constant that means, the BOD value is not varying. The influent BOD value should be the same for the design purpose ok. The waste stabilization is considered to be occur in the aeration tank only. There is no further aerobic decomposition happening in the secondary sedimentation tank theoretically, but actually speaking there is good amount of conversion taking place in the secondary sedimentation tank also because why? Because the same aerobic bacteria it goes along with the wastewater to the secondary sedimentation tank.

So, there you are letting it stay for couple of minute hour and all so, that it will settle down. So, during this process of settling also it will still search for food and it will actually have some plenty of food around and it will start consuming and it will actually settle down. While settling down also it will start consuming some amount of organic matter in the system. So, theoretically speaking waste stabilization occurs also in the secondary sedimentation tank, but by for this assumption purpose we can only say that only in the aeration tank waste stabilization is taking place. There is no microbial degradation of organic matter happening and also the no biomass growth is happening inside the secondary clarifier.

Steady state condition is prevailing and the volume used for calculation for all mean cell residence time. It only includes the volume of the aeration tank. The aeration tank volume is only what we are considering for calculating the mean cell residence time and all. So, let us go for the biomass mass balance. A mass balance for the microorganisms in the completely mixed reactor in the figure 2 if you see can be written as given in the equation number 13.

The rate at which the biomass enters the system minus the rate at which the biomass leaves the system is obviously, the net gain the net rate of change in biomass within the system boundary that can be represented in the figure number 2. If you see in the figure number 2 the influent wastewater Q_0, S_0 and X_0. What is Q_0? Q_0 is the inflow rate. S_0 is what here the substrate concentration and the BOD say X_0 is the biomass concentration. Theoretically speaking as we understand from the assumptions this X_0 should be 0.

It should be it should not be having any value. So, I mean like the influent wastewater does not have any biomass. Then there come the aeration tank. In the aeration tank you have a volume V, S is the inlet biomass like you know their organic matter present in the system and X is the biomass concentration or the MLVSS of the aeration tank. This is a very important element this capital X.

Then there comes the $X_0 + X_R$. So, you can say where is this X R coming sorry Q R coming Q 0 plus Q R. Q 0 is the inlet inflow rate and Q R is the return sludge the return sludge flow rate return sludge which acts which will come into the picture just now only and then it will also be added into the Q 0. So, now, the amount of waste that is coming from treated waste coming from aeration tank to the secondary sedimentation tank is Q 0 plus Q into Q R and the obviously, the biomass concentration and the organic matter concentration will be the same as the combination as the aeration tank because in the aeration tank we are providing a proper mixing condition ok. Then it come then it comes to the secondary clarifier.

In the secondary clarifier theoretically what is happening there the clarification takes place the sedimentation takes place. After the sedimentation is done so, what will happen some amount of sludge is we are wasting the Q W amount of sludge we are wasting which has a organic matter concentration of S and the say like the biomass concentration is X X R.

This X R is quite huge obviously, and the same X R amount of the biomass concentration is there in case of return sludge also. So, in the return sludge the Q R is having the X R of the biomass concentration that also goes back to the aeration tank again and same amount of effluent we are releasing we are collecting or the as a from the supernatant position that is how much nothing, but the Q 0 minus Q W. So, this Q 0 minus Q W amount of in flow rate of outflow rate we are getting and from there the volume of sorry the amount of biomass should be X E and the substrate should be capital S that same S value only.

Now this X E according to our assumption this X E value should be also 0. They should not be any biomass they should not be leaving the system ok. Theoretically the outflow the effluent should have 0 biomass it should be clean of any biomass it should only be having some amount of organic matter still present in the system, but it should not be having any the cellular biomass. In general when we do the when you do the mass balance statement analysis we follow this equation 14 C inflow of biomass plus net growth of biomass minus outflow is equal to the accumulation. Now if we consider steady state condition there should not be any accumulation.

So, it will convert into equation 15. So, in influent biomass plus biomass production equal to effluent biomass plus the wasted biomass ok. So, this is the final equation that we will get. If you now convert it what is the influent biomass according to this here picture according to this picture in the figure 2. Biomass you understand the influent biomass we are talking about.

So, influent inlet the influent flow rate what is the flow rate here q 0 multiplied by x 0 is the x is the biomass concentration. If you multiplied the concentration with the biomass concentration with the flow rate you will get the total biomass. So, which is q 0 into x 0 what is the biomass production here? Biomass production is a dx by dt into V, V is the volume of the aeration tank. What is the effluent biomass here?.

$$Q_0X_0 + V\frac{dX}{dt} = (Q_0 - Q_W)X_e + Q_WX_R$$

Now, theoretically if you if you remember according to our assumption our influent biomass is 0 and the effluent biomass is also 0 because x e and x 0 is 0 this two value.

That means, we are left with only two factors.

$$V\frac{dX}{dt} = Q_W X_R$$

This is the actual final equation that we come up with ok. So, now this V dx by dt this dx by dt if you replace it with the equation number 17 it will become this as shown in the equation 18

$$V\left[\left(\frac{\mu_m S}{K_s + S}\right)X - K_d X\right] = Q_W X_R$$

and if you rearrange it a little bit it will become q it will become equation number 19

$$\left(\frac{\mu_m S}{K_s + S}\right) = \frac{Q_W X_R}{VX} + K_d$$

where also you have to understand that r dot g say which is say like named as net growth of microorganisms which definitely the net growth how we can say the net growth of microorganisms the waste by divided by the volume that is the amount of extra growth that is happening. So, why we are wasting it why we are wasting this excess microorganism right. So, excess microorganism we are wasting it.

So, that wasted total value divided by the volume you will get the net growth of microorganisms or we can write this q w into x r divided by V into x or R g dash by x that also possible. Now, can you remember this equation

$$Q_w \cdot X_R/V \cdot X = r_g{}'/X$$

does it remind you something this V into x divided by q w into x r is nothing, but the 1 by theta I mean like the SRT isn't it. So, from there also you can find out this equal to R g. So, it is like x by R g dash is equal to theta I mean like the SRT also you can get out anyway.

$$r_g{}' = -Y.r_{su} - K_d \cdot X$$

So, in another word this R g dash or the net growth of microorganisms can be also represented by minus y into R s u R s u it is the y I mean like you remember the I will coefficient and the R s u is a substrate utilization rate minus K d into x you remember this yeah you remember this equation from here we get equation number 7

$$-\frac{dS}{dt}Y = \frac{dX}{dt}$$

So, here also we use the same minus like minus y into R s u is the substrate ds by dt nothing, but ds by dt minus K d into x considering the endogenous decay value and from this equation 20 and 21 you can get the 22

$$Q_w \cdot X_R/V \cdot X = -(Y.r_{su}/X) - K_d$$

left hand side is nothing, but

$$1/\theta_c = -(Y \cdot r_{su}/X) - K_d$$

as mentioned in the equation number 23 and also R s u is nothing, but the substrate utilization rate right

So, that is this theta is what this theta is the hydraulic retention time or the HRT you understand and what is this theta c is the MCRT or the sludge retention time. So, this hydraulic retention time from there you can get the R s u value and now we can easily calculate this 1 by theta c from equation number 23. If you use the equation number 24

$$r_{su} = -Q(S_o - S)/V = (S_o - S)/\theta$$

in the 23 and now it convert into like if it will reshuffle and it may look like equation number 25.

$$1/\theta_c = [Y(S_o - S)/\theta_i X] - K_d$$

Now if you do some more solving for value V the volume of the reactor aeration tank you will get the equation number 26.

$$V = \frac{Q \cdot \theta_c \cdot Y(So - S)}{X(1 + K_d \cdot \theta_c)}$$

This is the most important derivation here derivation here like you know it is V equal to q into theta c into y into S 0 minus S divided by x into in parenthesis 1 plus K d into theta c. So, this equation actually gives us an estimation of the volume of aeration tank when the kinetic coefficients are known to us ok. This is very important. So, the F in the and the flow out F out and the H value and we know and all the other parameters are known to us then it is very easy for us to find out the volume of the aeration tank.

If you do the substrate mass balance as same way we did it for the mass balance just now here the inflow of a substrate the equal minus this consumption of substrate equal to the outflow substrate plus the wasted substrate. So, it can be converted into the equation

$$Q_0S_0 + V\left[\frac{1}{Y}\left(\frac{\mu_m SX}{k_S + S}\right)\right] = (Q_0 - Q_W)S + Q_W S$$

So, from there if you reshuffle it and use the equation number 8 the monoid equation here and after rearranging you will get the final the equation number 32 where you will get the exact the you will get the value of S ok.

The substrate concentration inside the aeration tank which can be easily you can easily get by this equation number 32

$$S = \frac{K_s(1 + K_d.\theta_c)}{\theta_c(YK - K_d) - 1}$$

So, with this equation 32 you can easily get an idea about the substrate concentration at any course of time inside your aeration tank with that we can easily find out the value volume of aeration tank that it requires for the activated sludge process.

In conclusion we understand the that in case of complexly mix activated sludge processes different kind of mass balance equations are needed and this mass balance equations are actually giving us exact idea the design information about what should be the criteria for microorganism or cryo microorganisms beneficial microorganisms growth what should be the volume what should be the perfect condition we can provide inside the aeration tank. So, that it will consume the maximum amount of organic matter from the wastewater possible we also understand the stoichiometric analysis of in case of biodegradable cell mass in a in a in a heterogeneous decay phase we understand about different biomass mass balance and the substrate mass balance. These are the references that you can follow and we will discuss more about this activated sludge process in the coming lecture as well and I think in lecture number 34 or we will be discussing about different type of numerical also I mean like different numerical sources so that your idea will be much more clearer I mean like you will be able to understand the whole concept in a much nicer way perfect. So, thank you so much thank you for the day we will see you in the next class.